**4. TITLE AND SUBTITLE**
An Experimental Study of the Receptivity of a Compressible Laminar Boundary Layer and the Effects on Stability and Receptivity of 2-D and 3-D Pressure Gradients

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**14. ABSTRACT**
Techniques have been developed to provide, for the first time, pure tone acoustic forcing with a wave packet having a particular wave vector and at instability wave frequencies for a supersonic boundary layer. Receptivity to this pure tone forcing has been studied, initially with continuous forcing and now with well defined wave packets. In the latter case precise measurements can now be made of the detailed response to the free-stream disturbance. The wave velocity and wave vector in the free-stream have been determined consistently. A preliminary finding from the wave packet experiments is that the boundary layer response is broadly in two parts; one which is a local forced response in phase with the free-stream forcing and the other which, at downstream locations, has larger amplitude, is growing with downstream distance and has a significant time delay compared with the first forced response. The explanation for this second wave packet and its relation to the first is not yet clear and is the present subject of detailed investigation. Once the data and an understanding are obtained with the present U + a wave forcing attention will be focused on U - a waves. It was found in [1], that the stream-wise wavelengths of the instability waves and the acoustic U - a free-stream fluctuations were closely matched and it was speculated that this played an important role in receptivity. The

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An Experimental Study of the Receptivity of a Compressible Laminar boundary Layer and the Effects On Stability and Receptivity of 2-D and 3-D Pressure Gradients

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1. Motivation For the Research
The least well understood aspect of the onset of transition in a compressible boundary layer is the receptivity of the boundary layer to disturbances. Very well defined experiments are required to advance fundamental understanding and to validate CFD simulations. Advances in CFD, validated with high quality experiments, will enable new understanding and the development of new tools for the prediction of compressible transition. It remains true that in numerous compressible flows the prediction of transition and its (unknown) sensitivity to various disturbances has a major impact on system performance.

2. Comments on the Original Proposal Objectives
In the original proposal it was hoped to determine the receptivity to other sources of disturbance beyond acoustic disturbances (i.e. entropy and vorticity disturbances) and to examine the effects on stability of two-dimensional and three-dimensional pressure gradients. These further plans proved overly optimistic but would have been pursued if the acoustic receptivity work had not proved so successful. The value of the acoustic wave packet forcing is very considerable since there is so much power in the precise duplication of the disturbance and the precise time matching and ensemble-averaging which enables the signal in the boundary layer to be extracted from the random background noise (instability waves) in the frequency band of interest.

3. Concept for the Research
In the first stage of our work, emphasis was placed on characterizing the free stream disturbances in the wind-tunnel in the “naturally occurring” case, measuring the development of linear instability and measuring some principal features of transition. These measurements have been analyzed, written up and published [1]. The experiments showed that with laminar tunnel wall boundary layers the remaining corner disturbances were the primary source of free-stream fluctuations at low tunnel stagnation pressures, and that these fluctuations were acoustic. The non-dimensional propagation speed of these free-stream disturbances (with respect to free-stream velocity) was found to be \( \sim 0.64 \), a value which yields stream-wise wavelengths which are remarkably close to the wavelengths of the corresponding first mode instability waves. With assumptions for the receptivity, Fedorov, (Appendix in [1]) has made predictions from stability theory of the amplitude and growth rate which agree well with the measurements at frequencies within the unstable range, in this “naturally occurring” case. By contrast, at lower frequencies for which stability theory predicts a decaying disturbance, the measurements show a growing disturbance in the boundary layer (and also in the free stream). The receptivity to the free-stream mass flux fluctuation (density-velocity product) was found experimentally to give a relatively large ratio of 10 for the amplitude of the peak boundary layer fluctuation to the free-stream fluctuation at \( R \sim 700 \), and at approximately the most unstable frequency [1].
This work provided a foundation for the more recent work which has focused on the receptivity of the boundary layer and in particular the receptivity to an imposed single frequency acoustic free-stream disturbance. This allows detailed measurements to be made of the initial forced response in the boundary layer and of the evolution of this response at frequencies of unstable eigen modes as well as at lower frequencies where eigen modes are damped. Receptivity experiments for incompressible flows have focused on the isolation of the T-S wave from the background disturbances. Examples can be found in a recent review by Saric et al (2002). The major difficulty in the study of supersonic receptivity comes from the “naturally occurring” free-stream disturbances which are responsible for the “naturally occurring” linear instability waves in the boundary layer (Graziosi & Brown 2002). When an external acoustic forcing is applied, the forced boundary layer response is contaminated by this background T-S wave at the same frequency. Also, in the compressible case if the forcing is continuous, it is difficult to isolate the forced instability wave from a direct response of the boundary layer (Stokes wave) since it is expected that the principal receptivity is not confined to the leading edge region.

Driving a free stream acoustic field in the test section of a supersonic tunnel using an acoustic source in the settling chamber has been attempted by a number of investigators in the past but abandoned because a measurable disturbance, at frequencies of interest, was not obtained. For the conditions of the present experiment, however, the laminar boundary layer is relatively thick so that the frequencies of interest (4kHz – 15kHz) are relatively low and well within the range of a loudspeaker. In addition to the fact that in the “naturally occurring” case the free-stream disturbances were acoustic, there are many compelling experimental advantages in using a pure tone disturbance and we have therefore made a further major attempt to obtain acoustic forcing with a pure tone and a well defined wave vector.

Figure 1 shows the schematic of the Low Turbulence Variable Geometry Mach 3 wind tunnel. The tunnel is operated in these experiments at a low stagnation pressure of 4.0 psia. At this pressure the tunnel wall boundary layers are laminar (with the exception of residual disturbances measured in the nozzle corner) and the free stream turbulence level is < 0.11%.

This work began with the recognition that high frequency oblique acoustic waves should be readily transmitted through the throat and that the acoustic wave vector would be refracted by the increase in velocity. Initially we placed a loud speaker just outside the wall of the upstream settling chamber and used a 2 in. diameter tube through the wall as a wave guide to bring the acoustic wave into the settling chamber (figure 2). Early experiments employed a wave duct with a 30-degree turning angle. Figure 3 shows the auto power spectra measured across the boundary layer with a 1.96 kHz driving frequency. As the frequency was increased, the signal level dropped substantially. It was speculated that at a higher frequency (above about 4 kHz) only a small fraction of the acoustic power was transmitted directly into the test section. Results at a frequency of 2.35kHz with a finite wave train of 50-cycles showed that the free-stream signal in the
test section consisted of two parts: the first had a time delay of approximately 7 ms and the second 32 ms when compared with the driving voltage. These time delays correspond with the propagation time of the directly transmitted wave and of a reflected wave which has propagated upstream from the contraction before being again reflected and returned to the test section. To reduce this reflected power, and better account for the refraction of the wave packet, a 66° wave duct aimed more directly towards the throat, was used (Figure 2). Free-stream forcing at 8.9 kHz (Figure 4) was then readily measured.

4. Experiments and Data Processing

4.1 Continuous Forcing

Initially continuous wave forcing at a specific frequency was used to investigate the boundary layer mass-flux oscillations and their phase. Two hotwire probes were placed \( \frac{1}{4} \) inch apart vertically, one remaining in the free-stream and the other moving across the boundary layer. Cross-correlation functions between the hotwire outputs and the reference signal (speaker driving voltage) were calculated and the amplitude of the coherent component was determined from the maximum correlation and the phase difference by the time delay corresponding to this maximum value. Figure 5 shows the resulting measurements of disturbance amplitude across the boundary layer scaled by the free-stream amplitude and the corresponding ratio for the naturally occurring case (using a narrow band filter at the same frequency) at \( R \sim 700 \). A peak ratio of approximately 4 was found at a forcing frequency of 8.9 kHz, while the corresponding ratio is approximately 10 for the naturally occurring disturbance at this frequency. It was speculated that the difference between the receptivity of the boundary layer to the naturally occurring disturbances and to the continuous acoustic forcing arises from the difference in the wave modes for the two cases. From the measurements in the naturally occurring case the acoustic disturbance is an upstream propagating \( U \) - a wave (arising from the residual corner disturbances) while in the forced case it seemed likely to be a \( U + a \) wave. With this much faster speed of propagation the disturbance would couple less well with boundary layer waves.

Measurements of the phase speed in the free-stream, however, and phase changes across the boundary layer did not give simple results and suggested that reflections contributed to a number of different waves and wave vector directions in the free-stream and possible wave interactions across the boundary layer. It was clear that these issues could be best resolved by forcing with a single wave packet and by achieving a much higher signal to noise ratio in the measurements.

4.2 Wave Packet Forcing

With wave packet forcing the overall signal level is much less than in the continuous case and a particular effort is required to extract the wave packet from the naturally occurring disturbances. With increasing understanding numerous refinements have been made to the experimental details for the creation of the wave packet and to the signal processing
to extract the wave packet in the free stream and the boundary layer. The key to a very much higher signal to noise ratio is to exploit the precise wave propagation time from the loud speaker to the hotwire in the free stream. The corresponding noise rejection is then due to the random phase of the noise with respect to this time. The speaker driving voltage was used as the reference signal and used as the trigger for the data acquisition to ensure that all signal records were aligned with respect to the origin of the wave packet. The records were first filtered with 8 to 12.5 kHz band filters and then ensemble averaged to extract the wave packet and reject the noise. Figure 6 shows the driving voltage to the loud speaker for a packet of 30 waves and the corresponding, ensemble-averaged, delayed response (due to their distance from the speaker) measured by two hotwires both in the free-stream. These wires are separated in x and y and Figure 7 shows an expanded view of the time delay between the two signals. When the same hotwires are separated only in y, Figure 8 shows that the time delay between these signals is zero. As expected, there is therefore a negligible y component of the wave vector. (Note in Figure 8 the very close matching of the two hotwire, ensemble-averaged, signals) From the time delay evident in Figure 7 the phase velocity and wave vector of the acoustic disturbance can be determined. It was found that the wave vector has been refracted from an angle of 24 degrees at the wave duct in the settling chamber to approximately 68 degrees in the test section. This test section wave angle is comparable with that of the most amplified eigenmode but in this case the stream-wise velocity of the wave in the free-stream is 1160 m/sec and is very much larger than free-stream velocity. This confirms that the disturbance is propagating as a U + a wave.

Figure 9 shows the two signals when one hotwire remains in the free-stream and the other is located in the boundary layer at a value of the similarity coordinate where the amplitude for an eigenfunction disturbance is near a maximum and at a distance of 4.25 inches from the leading edge. Figure 10 gives an expanded view of the free-stream disturbance and the boundary layer response, and shows a zero time delay between the leading waves of the two signals.

It was noticed that the wave packets in the free-stream were spread over a longer time than the driving voltage and the boundary layer response over a still longer time. This led to further refinements. The number of waves in the packet was reduced and the length of duct to the loud speaker outside the settling chamber was increased to ensure that the wave packet transmitted down the duct is separated from the reflection of the waves from the angle and open end of the duct.

Figure 11 shows the similar response for a packet of 8 waves in which the ensemble average has been obtained from 2500 wave packets and Figure 12 shows the corresponding expanded view of the free-stream disturbance and the boundary layer response at R=724. Figures 13 and 14 show the response upstream at R=614. These boundary layer responses clearly show the development of a second wave packet which arrives after the initial packet and is much larger than the initial forced response. Note the amplification of this second packet between R=614 and R=724. Similar measurements are being obtained with the same wave packet at other locations. Figure 15 shows measurements from two different runs in the boundary layer for Δx = 0.5". The
second packet is shifted 30\(\mu\)s from \(x = 9.05''\) to \(x = 9.55''\), and the corresponding stream-wise velocity is 418 m/sec. To determine any possible signal drift between these two different runs, the reference signal at the same location in the free stream was recorded and this measurement showed a zero time shift. It is too early to speculate on an explanation for the second packet but it is clear that its amplitude increases with downstream distance and it is traveling at a lower wave speed than the initial wave packet.

5. Summary Of Accomplishments

Techniques have been developed to provide, for the first time, pure tone acoustic forcing with a wave packet having a particular wave vector and at instability wave frequencies for a supersonic boundary layer. Receptivity to this pure tone forcing has been studied, initially with continuous forcing and now with well defined wave packets. In the latter case precise measurements can now be made of the detailed response to the free-stream disturbance. The wave velocity and wave vector in the free-stream have been determined consistently. A preliminary finding from the wave packet experiments is that the boundary layer response is broadly in two parts; one which is a local forced response in phase with the free-stream forcing and the other which, at downstream locations, has larger amplitude, is growing with downstream distance and has a significant time delay compared with the first forced response. The explanation for this second wave packet and its relation to the first is not yet clear and is the present subject of detailed investigation. Once the data and an understanding are obtained with the present \(U + a\) wave forcing attention will be focused on \(U - a\) waves. It was found in [1], that the stream-wise wavelengths of the instability waves and the acoustic \(U - a\) free-stream fluctuations were closely matched and it was speculated that this played an important role in receptivity. The comparison (Figure 5) between the boundary layer response to continuous pure tone forcing and the response to the naturally occurring free-stream fluctuations in a narrow band at the same frequency is consistent with this suggestion.

6. Acknowledgement/Disclaimer

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7. Personnel Supported
Garry Brown, Professor, Princeton University Princeton, NJ
Xuejun Fan, Research Staff Member, Princeton University, Princeton, NJ
William Stokes, Senior Technician, Princeton University, Princeton, NJ

7. Publications
2. Fan, Xuejun and Brown, G.L., Development of Acoustic Forcing for the Studies of Supersonic Boundary Layer Receptivity In final preparation

8. Interactions/Transitions
This research is closely coupled with the research led by Dr. Hermann Fasel at the University of Arizona and with Dr Alexander Federov of Moscow Institute of Physics and Technology. The published experimental results with Dr Paolo Graziosi have provided a starting point for the numerical calculation of receptivity. Dr Xuesong Wu and his PhD student at Imperial College in the UK are also using these results and they are trying to reproduce theoretically the data in Figure 20 of Graziosi and Brown.

9. AFRL Point of Contact
Dr. Roger Kimmel Tel 937 255 6813. Contact has been by telephone and email and by a personal visit of Dr Kimmel to Princeton in June 2003.

10. References

Figure 1. Schematic of the LTVG wind tunnel

Figure 2. Speaker driver with 66° wave duct.
Figure 3. Power spectra with CW acoustic forcing: $M = 2.98$, $R = 705$, $30^\circ$ wave duct.

Figure 4. Averaged power spectrum in the free stream at $f = 8.9$ kHz: $M = 2.98$, $R = 705$, 4000 samples, $66^\circ$ wave duct.
Figure 5. Non-dimensional amplitude distribution in the boundary layer with continuous forcing compared with the “naturally occurring” case, f = 8.9 kHz, R = 700.

Figure 6. Driving voltage and wave packet in the free stream: f = 8.9khz, 30 waves.
Figure 7. An expanded view of Figure 6 which leads to the measurement of the phase speed $C_x$ and wave angle in the free stream.

Figure 8. Two hotwire, wave packet signals: $f = 8.9$ kHz. The phase difference leads to $k_f = 0$. 
Figure 9. Boundary layer response to the free stream acoustic forcing: $x = 4.25''$, $R = 483$, $f = 8.9$khz, 30 waves.

Figure 10. An expanded view of Figure 9 which shows the leading waves in the boundary layer and in the free stream
Figure 11. Boundary layer response to the free stream acoustic forcing: $x = 9.55''$, $R = 724$, $f = 8.9$khz, 8 waves, 2500 samples.

Figure 12. Expanded view: $x = 9.55''$, $R = 724$, $f = 8.9$khz, 8 waves, 2500 samples.
Figure 13. Boundary Layer and free-stream wave packets $x = 6.88''$, $R = 614$, $f = 8.9$kHz, 8 waves, 2500 samples.

Figure 14. Boundary Layer and free-stream wave packets $x = 6.88''$, $R = 614$, $f = 8.9$kHz, 8 waves, 2500 samples, (wave duct withdrawn 1.0'').
Figure 15. Measurements in the boundary layer over $\Delta x = 0.5''$. These show a 30 $\mu$s time delay for the second packet of waves, $y/(x/R) = 7.10$, $z = 0$, (wave duct withdrawn 1.0'').