Modelling and Analysis of Piezoelectric Transformers

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ABSTRACT

Piezoelectric transformers are well known since the end of the 1950s. One of the most prominent applications today is in the backlighting of LCD displays. Key features of piezoelectric transformers, such as light-weight, flatness, high step-up at low volume and high efficiency, make them attractive for a broad range of applications. The present paper first explains the operating principle of piezoelectric transformers and then concentrates on the modeling and analysis of such devices. Recent developments and potential future applications will also be discussed.

I. INTRODUCTION

In many applications the magnetic components of power electronic circuits contribute significantly to the overall cost and weight of the system. Although remarkable reductions in size and weight of the magnetic components have been made possible by raising the switching frequencies there is still need to further miniaturize these components. This is particularly the case for transformers. Piezoelectric transformers can be an interesting alternative to magnetic transformers. Simple design models are needed in order to evaluate the potential benefits and the potential drawbacks of piezoelectric transformers with respect to magnetic transformers in the early design phase of a system. These design models need to be validated by experiments if quantitative judgments need to be made. Unfortunately there are only few published results concerning the quality of design models for piezoelectric transformers. In the following we will first explain the operating principle of piezoelectric transformers and then present a validated design model which can be used in early design phases.

Piezoelectric transformers use the piezoelectric effect in two steps: in a first step electric energy is transformed into mechanical vibrations and in a second step these vibrations are transformed back into electric energy. This principle can be embodied in many geometric configurations and many different vibration modes can be employed. A very simple design is shown in figure 1.

![Figure 1: Operation principle of piezoelectric transformers.](image-url)
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Piezoelectric transformers usually are driven at a resonance of the piezoelectric element. The transmission rate, i.e. the ratio of output voltage to input voltage, depends on the type of mode, the geometrical position of input and output electrodes, and of course the output load of the transformer, see [1] and [2].

First patents concerning piezoelectric transformers date back to 1931, when a “piezoelectric crystal converter-generator for polyphase currents and voltages” was described in [3]. Unfortunately, piezoelectric materials have not been as effective at that time as they are today, and no technical application was found in those early days.

The situation changed after the patent-applications of Rosen and his co-workers in 1954 and 1959. First technical use as band pass circuits and high voltage supplies, e.g. in x-ray devices and televisions, became possible due to the development of barium titanate ceramics, see [1], [4], [5], [6]. The breakthrough of piezoelectric transducers, however, was hindered by the costly and bulky power electronics needed for driving the piezoelectric transformers.

The development of piezoceramic multilayer technology, as well as new power electronic components and modern control strategies have finally contributed to the recent increase in applications of piezoelectric transformers, see [7], [8] and [9]. Today piezoelectric transformers are mainly used to generate high voltages for cold cathode fluorescent lamps in notebooks. Such transformers convert up to 10 W electrical energy at a high efficiency (larger than 95%) and at a high voltage step-up ratio of about 100, see [10].

The main advantages of piezoelectric transformers with respect to conventional electromagnetic devices are:

- higher efficiency (> 90 %) at comparable power-densities,
- large transformation ratio at low construction volume,
- two-dimensional flat structure,
- non-flammability and high insulation resistance,
- negligible electromagnetic emissions and insensitivity to electromagnetic fields,
- variable transmission rate and multiple output-configurations.

These properties make piezoelectric transformers attractive for applications wherein their characteristic advantages balance their costs. The limiting factor today seems to be the total converted electric power. Until now, only small-power applications (typically smaller than 40 W) have been reported. The desire for higher power rates leads to the demand for new mechanical designs and improved circuitry.

II. MODELING AND SIMULATION

In a general sense, models of technical devices can be regarded as a special form of encoded knowledge about their behavior. Modern design processes strongly rely on top-down and bottom-up analyses which require adequate mathematical models for the various components of the investigated system.

Piezoelectric transformers can be described in a straightforward manner as mechanical continuum structures with additional degrees of freedom for the electrical behavior, see [11]. Usually the electromechanical coupling is described by linear piezoelectric material laws and a system of linear partial differential equations with corresponding boundary conditions is obtained. The resulting equations can be solved for specific geometries and for some special cases like e.g. harmonic vibration. Using analytical and semi-analytical methods it is possible to reduce the equations in such a way that material data and geometrical parameters can easily be varied in parameter studies to elucidate their effects on the electromechanical behavior. These elementary models also help to understand the mechanisms of vibration generation and transfer. Thus they are well suitable for the first steps of development, like e.g. feasibility analyses. The main problem of this approach is that many simplifications are needed to find a closed analytical solution. It is generally not possible to take the anisotropic nonlinear material behavior into account or to compute the three-dimensional vibration patterns of a piezoelectric transformer fully analytically.
As an alternative to the analytical approach based on continuum mechanical modelling, the finite element method can be employed. Present day state-of-the-art programs allow modelling a variety of system features, including complex geometries. These models, however, often are very complex so that it becomes very difficult to employ them in the early design phases. They are much better suited for investigations which are typically performed at the end of the development process, when the geometric shape and size of the system is already fixed within certain well-defined limits.

Equivalent electric circuit or mechanical models have found widespread use in the early design steps, see [11]. Figure 2 shows such an equivalent circuit model for a piezoelectric transformer. The parameters of such a simplified model can be derived from an analytical continuum mechanic model or from a finite element analysis. In many cases it is also possible to describe the effect of parameter variations in the complex overall model on the parameters of the simplified model. An additional advantage of the equivalent electric circuit models is that their parameters can easily be measured if a prototype is available and that the parameters of the equivalent electric circuit model can be used in quality control.

![Figure 2: Equivalent electric circuit model for a piezoelectric transformer](image)

The simple model of figure 2 describes the electromechanical behavior of a piezoelectric transformer in the vicinity of a selected mode shape. The modal parameters used in it can be derived by comparison of the underlying mathematical equations with the solutions of analytical models or by experimental identification.

The electromechanical analogy of mechanical and electrical components is helpful for engineers of different disciplines to understand each other and to find a common base for modeling. Additionally, the structure of these models allows to include them into superior models e.g. for resonance control. This is especially helpful in the case of piezoelectric transformers, because the transmission ratio and efficiency can then be calculated as a function of the output load and operational frequency in advance.

The equations of motion of the equivalent electric circuit model of figure 2 are given by

\[
\begin{align*}
    u_{in} &= \frac{1}{C_1} \int i_1 dt + R_1 i_1 \\
    i_{in} &= i_1 + A_1 \ddot{x} \\
    u_{out} &= \frac{1}{C_2} \int i_2 dt + R_2 i_2 \\
    i_{out} &= -i_2 + A_2 \ddot{x}
\end{align*}
\]

and for the specific case of harmonic vibrations, the resulting admittance equations can be obtained as
The ratio of output voltage to input voltage, sometimes also called the „step-up-ratio“, is then given by

\[
\frac{\hat{u}_{\text{out}}}{\hat{u}_{\text{in}}} \bigg| _{\hat{u}_{\text{in}}=0} = \left( \frac{j\Omega}{1 + R_1 j\Omega} + \frac{A_1^2 j\Omega}{-m\Omega^2 + df\Omega + c} \right)
\]

\[
\frac{\hat{i}_{\text{in}}}{\hat{u}_{\text{out}}} \bigg| _{\hat{u}_{\text{in}}=0} = -\frac{A_1 A_2 j\Omega}{-m\Omega^2 + df\Omega + c}
\]

\[
\frac{\hat{i}_{\text{out}}}{\hat{u}_{\text{out}}} \bigg| _{\hat{u}_{\text{in}}=0} = \left( \frac{j\Omega}{1 + R_2 j\Omega} + \frac{A_2^2 j\Omega}{-m\Omega^2 + df\Omega + c} \right)
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\[
\frac{\hat{i}_{\text{out}}}{\hat{i}_{\text{in}}} \bigg| _{\hat{u}_{\text{in}}=0} = \frac{A_1 A_2 j\Omega}{-m\Omega^2 + df\Omega + c}
\]

It should be mentioned that the step-up-ratio above is defined for the unloaded output port of the system (i_{out} = 0).

**III. Experimental Validation**

In order to check the quality of the model of figure 2 a simple experiment was carried out. The piezoelectric transformer shown in figure 3 consists of a single rectangular plate (l x w x t = 30 x 8 x 2.5 mm³) made of piezoelectric ceramic (SONOX® P88). The polarization of the plate is homogenous and the electrodes are divided into two parts corresponding to the topology shown in figure 1. The characteristics of the transformer have been measured experimentally. They are compared to the calculated results in figure 4.

Experimental and calculated results do agree quite well for the experiments performed in the vicinity of the investigated mode shape. It should be mentioned that it is possible to model more than one mode shape in the equivalent electrical circuit. This can be done by inserting additional elements into the equivalent electrical circuit. For each additional mode an additional LC-parallel circuit has to be included.
Figure 3: Test specimen according figure 1.

Figure 4: Transmission ratio of the piezoelectric transformer of figure 3 as a function of
a) frequency and b) electrical output load.
The simple equivalent electrical circuit model presented in this paper can be used in the early phases of development for any kind of model-based analyses. Like e.g. feasibility studies or the like. A typical design scenario includes the following steps:

- identification of power electric requirements,
- choice of rough geometry and operating principle using simple analytical continuum models,
- detailed analysis of the electromechanical behavior of the overall system on the basis of the equivalent electrical circuit models,
- detailed design of the transformer structure using finite element models.

Taking into account the recent progress made in the power density of piezoelectric transformers, see figure 5, it seems likely that new application fields for piezoelectric transformers will be identified in the near future. One such application is in the ignition of automobile gas discharge lamps (Xenon). As piezoelectric transformers can deliver very high voltages they might be used in ignition circuits, see [13]. Taking into account the specific load characteristics of gas discharge lamps, it seems to be possible to use them as an ignition and driving unit for gas discharge lamps all at once. This would help to save much weight and construction volume. For this and other applications, however, an increase of the total power rate to a minimum of about 50 W is necessary.

![Figure 5: Development of the power-density of piezoelectric transformers during the last years [18].](image)

Piezoelectric transformers can have considerable advantages compared to conventional electromagnetic devices. Today the amount of transmittable power is limited to some watts, thus the field of applications is strongly restricted. Due to the high transmission ratio at a small construction volume piezoelectric transformers are used in portable devices mainly. A very thorough discussion of further application perspectives can be found in [19].

### V. REFERENCES