Influence of Wavelength on the Parameterization of Drag Coefficient and Surface Roughness

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Surface waves are the roughness element of the ocean surface, the air-sea interaction processes are influenced by the wave conditions. The dynamic influence of surface waves decays exponentially with distance from the air-water interface. The relevant length scale characterizing the decay rate is the wavelength. The parameterization of drag coefficient and surface roughness can be significantly improved by using wavelength as the reference length scale of atmospheric measurements. The wavelength scaling of drag coefficient and dynamic roughness also receives support from theoretical studies of wind and wave coupling.

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Surface waves are the roughness element of the ocean surface, the air-sea interaction processes are influenced by the wave conditions. The dynamic influence of surface waves decays exponentially with distance from the air-water interface. The relevant length scale characterizing the decay rate is the wavelength. The parameterization of drag coefficient and surface roughness can be significantly improved by using wavelength as the reference length scale of atmospheric measurements. The wavelength scaling of drag coefficient and dynamic roughness also receives support from theoretical studies of wind and wave coupling.

1. Introduction

Drag coefficient ($C_D$) and surface roughness ($z_0$) are important parameters for quantifying air-sea momentum and energy exchanges. For a long time the drag coefficient of the ocean surface has been considered a constant (e.g., Kraus, 1972). As the quality and quantity of measurements have improved, it has become evident that drag coefficient tends to increase with increasing wind speed (e.g., Garratt, 1977; Wu, 1980). Later, it was found that in addition to wind speed, stability stratification, sea state and wave conditions are also important in modifying the drag coefficient (e.g., Donelan, 1982, 1990; Toba et al., 1990; Jones and Toba, 2001). The study of drag coefficient can also be approached by investigating the surface roughness parameter, frequently expressed in the dimensionless form $Z_D = g z_0 / u^2$, where $g$ is the gravitational acceleration and $u$ is the wind friction velocity. The dimensionless roughness $Z_D$ is generally referred to as the Charnock parameter and was originally considered a constant (e.g., Charnock, 1955; Wu, 1980). Later on, more extensive measurements have convincingly illustrated that $Z_D$ does not remain constant and there seems to be good correlation between $Z_D$ and surface wave properties such as wave height and wave slope (e.g., Donelan, 1990; Donelan et al., 1993; Anctil and Donelan, 1996; Taylor and Yelland, 2001) or wave period (e.g., Toba et al., 1990).

Over many decades of intensive research, our knowledge of the stability correction has advanced significantly (e.g., Garratt, 1977; Donelan, 1990; Geernaert, 1999; Toba et al., 2001). In contrast, a consistent parameterization of $C_D$ with wave parameters remains elusive. Some earlier works have introduced wave height or wavelength into the parameterization of drag coefficient and surface roughness (e.g., Kitaigorodskii, 1973; Stewart, 1974; Donelan, 1990). Recently, Makin and Kudryavtsev (1999, 2002) and Makin (2003) developed wind-over-wave coupling (WOWC) theory, focusing on the dynamics of microscale air-sea interactions. The surface wind stress has been shown to be significantly influenced by the surface wave dynamics through the form drag, which is the correlation of wave-induced pressure field and the surface wave slope. The form drag is further divided into a non-separated sheltering component and the stress due to flow separation from breaking of both small-scale and dominant-scale waves. Numerical computations have been carried out to demonstrate the relative importance of the different surface stress components and their dependence on wind speed, wave age, finite water depth and other atmospheric and wave parameters. In the WOWC analysis, the wavelength at the spectral peak is a clear scaling length of air-sea interaction processes.

Almost all the earlier analyses of $C_D$ and $z_0$ use the reference wind speed at 10 m elevation ($U_{10}$). Although the employment of $U_{10}$ in the analyses provides a consistent reference level of wind measurements (as compared to earlier reports using "mast height" or "anemometer height"), the dynamical significance of the 10-m elevation in the marine boundary layer is not clear. From a heuristic point of view, surface waves are the ocean surface roughness, the air-sea interaction processes are influenced by the wave conditions. Because the influence...
of surface waves decays exponentially with the wavelength serving as the vertical length scale, the dynamically meaningful reference elevation should be the characteristic wavelength of the peak component of surface wave spectrum, \( \lambda \). As described in the last paragraph, this common sense vertical length scale also receives theoretical support from the study of wind over waves coupling. This paper presents an analysis with \( U_{\lambda/2} \) serving as the reference wind speed, where \( U_{\lambda/2} \) is the wind speed at the elevation equal to one-half of the surface wavelength. The general conclusions of this paper are not changed by using different fractions of the wavelength as the vertical length scale of wind speed reference; more discussions on this point will be presented later. The functional expressions of the drag coefficient referenced to \( U_{\lambda/2} \) (expressed as \( C_{\lambda/2} \) to distinguish it from \( C_{10} \), the drag coefficient referenced to \( U_{10} \) and surface roughness and their dependence on the wind and wave parameters are described. The dimensionless roughness in the form of \( k_p z_0 \) falls out naturally when wavelength is used as the reference length scale, where \( k_p = 2\pi/\lambda \) is the wavenumber at the spectral peak. The relation between \( k_p z_0 \) and \( z_0^* \) can be easily derived.

The detail of the analysis with \( U_{\lambda/2} \) as the reference wind speed and the parameterization of surface roughness in terms of \( k_p z_0 \) and \( z_0^* \) is given in Section 2. Field data with wind stress measurements under conditions dominated by wind seas are assembled to verify the results obtained from the analysis (Section 3). A comparison of \( C_{\lambda/2} \) and \( C_{10} \) illustrates the superiority of the former for grouping together datasets from different sources. Similarly, the dimensionless roughness represented by \( k_p z_0 \) consolidates data better than \( z_0^* \). Section 4 presents discussions on the issue of possible spurious correlation when scaling with \( u_\ast \) (Smith et al., 1992) in the parameterization function of the drag coefficient and surface roughness. The analysis indicates that spurious correlation does not pose a problem in the datasets used in the present study. The summary and conclusions are given in Section 5.

### 2. Analysis

In this paper, the discussion is limited to neutral stratification or to the situation that the stability correction has been performed. The vertical wind speed distribution is described by the logarithmic wind profile,

\[
U(z) = \frac{u_\ast}{\kappa} \ln \frac{z}{z_0},
\]

where \( U \) is the equivalent wind speed at neutral condition, \( \kappa \) is the von Kármán constant (0.4), and \( z \) is the vertical elevation measured from the mean water surface.

Taking the reference length scale for wind speed measurements at the level equal to one-half of the wavelength, the drag coefficient \( C_{\lambda/2} = u_\ast^2 / U_{\lambda/2}^2 \) can be written as

\[
C_{\lambda/2} = \kappa^2 \left( \frac{\ln \frac{\pi}{k_p z_0}}{k_p z_0} \right)^2.
\]

The approach of using \( k_p \) for the characteristic wave component is expected to work well in wind-sea dominated wave conditions. The influence of swell on the drag coefficient and surface roughness requires further investigation.

Equation (2) suggests that a natural expression of the dimensionless surface roughness is \( k_p z_0 \), and from (2)

\[
k_p z_0 = \pi \exp(-\kappa C_{\lambda/2}^{0.5}).
\]

Using the dispersion relation of surface waves,

\[
\omega_p^2 = g k_p \tanh k_p h,
\]

\( k_p z_0 \) can appear in several different forms

\[
k_p z_0 = z_0^* \left( \frac{u_\ast}{C_p} \right)^2 \tanh k_p h = z_0^* \left( \frac{U_{\lambda/2}}{C_p} \right)^2 C_{\lambda/2} \tanh k_p h = \frac{z_0 \omega_p^2}{\tanh k_p h} = \frac{z_0^* \omega_p^2 C_{\lambda/2}}{\tanh k_p h},
\]

where \( \omega_p = \omega_\ast / g \) and \( \omega_\ast = U_{\lambda/2} \omega_p / g \), both can be interpreted as the dimensionless frequency of the air-sea coupled system; \( \omega_p \) and \( C_p \) are the angular frequency and phase speed, respectively, of the peak spectral component. For deep water, the dimensionless frequency is identical to the inverse wave age \( u_\ast / C_p \) or \( U_{\lambda/2} / C_p \).

The analysis of the logarithmic wind profile leads to one equation (Eq. (2) or (3)) connecting two dimensionless unknowns (\( C_{\lambda/2} \) and \( k_p z_0 \)). To make further progress, either another wind-wave coupling function or assumption needs to be introduced or an empirical relation of \( C_{\lambda/2} \) or \( k_p z_0 \) needs to be established. The next section presents an analysis of results from several field experiments (Donelan, 1979; Merzi and Graf, 1985; Antcill and Donelan, 1996; Janssen, 1997). The selected datasets are characterized by wind-sea dominated wave conditions and the tabulated data list sufficient information on wave properties and water depth to derive the wavelength. The investigation shows that among the dependence of \( C_{\lambda/2} \), \( k_p z_0 \) and \( z_0^* \) on \( u_\ast / C_p \), \( U_{\lambda/2} / C_p \), \( \omega_\ast \) or \( \omega_\ast \), the
Fig. 1. (a) Drag coefficient $C_{1/2}(\omega_\ast)$ measured in wind-sea dominated wave conditions. The numbers within the square brackets in the legends are the maximum and minimum wavelengths in each individual dataset; the curve represents the parameterization equation (6). (b) Dimensionless roughness in terms of $k_p z_0(\omega_\ast)$, the curve represents the parameterization equation (7). (c) Dimensionless roughness in terms of $z_0(\omega_\ast)$, the curve represents the parameterization equation (8). (c) also shows the curve from least square power-law fitting of $z_0(C/\omega_\ast)$. The functions given by Maat et al. (1991) and Smith et al. (1992) are also illustrated (Section 4) for comparison.

The parameterization of $C_{1/2}(\omega_\ast)$ yields a simple power-law function,

$$C_{1/2} = A_c \omega_\ast^{1.2},$$

where $A_c = 1.22 \times 10^{-2}$ and $\alpha_c = 0.704$. More details of the analysis of the field data are presented in the next section. The dimensionless roughness becomes

$$k_p z_0 = \pi \exp \left[-\kappa(A_c \omega_\ast^{1.2})^{-0.5}\right].$$

The dimensionless roughness expressed as the Charnock coefficient can be obtained from substitution of (5) in (7),

$$z_0 = \pi \tanh kh \omega_\ast^2 \exp \left[-\kappa(A_c \omega_\ast^{1.2})^{-0.5}\right].$$

Other expressions of the dimensionless roughness in terms of $u_c/C_p$ or $U_{1/2}/C_p$ can also be constructed and will not be further elaborated here.

3. Comparison with Field Measurements

In order to verify the analysis presented in the last section, tabulated field data reported by Donelan (1979), Merzi and Graf (1985), Anctil and Donelan (1996) and Janssen (1997) have been assembled. All these datasets share several common attributes, including data quality control screening for steady wind events, fetch-limited wave conditions, direct wind stress measurements using either the profile method or the eddy correlation technique, and they contain sufficient information to derive the peak wavelength. The listed wind velocity is corrected for the stability influences. Moreover, most of the data have been collected in finite-depth waters; therefore, a proper accounting of the depth effect as described in (5) or (8) should be applied when the Charnock parameter is used for the dimensionless surface roughness. However, the data scatter is rather large, the depth effect is masked and the scatter plots of $z_0(\omega_\ast)$ and $z_0 \tanh(kh)/(\omega_\ast)$ are not very different (not shown). A brief description of these experiments is given in Appendix A.

Figure 1(a) displays the dependence of $C_{1/2}$ on $\omega_\ast$ processed from the assembled data. Although the data scatter is large, the result is a substantial improvement.
over the expressions of $C_{10}$. Figure A1 in Appendix A
shows several presentations of $C_{10}$, including (a) $C_{10}(U_{10})$, (b) $C_{10}(\omega_{*})$, and (c) $C_{10}(U_{10}/C_{p})$. The $C_{10}$ representations
result in sets of curves and clearly require a secondary parameter.
Comparing the results presented in Figs. 1 and A1, it is obvious that $C_{10}(\omega_{*})$ does indeed provide a better parameterization than $C_{10}$ regarding the surface wave effects on the drag coefficient. Polynomial fitting of a power law function to the combined data presented in
Fig. 1(a) yields the empirical function of $C_{10}(\omega_{*})$ given in
(6). (Different parameterizations of $C_{10}$ with $u_{*}/C_{p}$ and
other combinations of wind and wave parameters have also been investigated. For example, the regression constant and exponent for $u_{*}/C_{p}$ parameterization are $1.09 \times 10^{-2}$ and 0.696, respectively. The root mean square (rms) difference between the regression curve and the measurements is $2.78 \times 10^{-2}$, which is about 20 percent larger than $C_{10}(\omega_{*})$ parameterization with an rms difference of $2.26 \times 10^{-4}$. The issue of the fraction of wavelength as the vertical reference level is also studied. Examples of using $U_{3/2}$, $U_{1/2}$, $U_{5/3}$ as reference wind speeds are given in Appendix B, showing that the wavelength scaling of the drag coefficient is not influenced by the choice of the proportionality constant.

The dimensionless roughness $k_{z}/\omega_{*}(\omega_{*})$ is shown in
Fig. 1(b). The measured data again display a smooth variation with $\omega_{*}$ and measurements from different sources collapse very well following the proposed parameterization function (7), given as the solid curve in the figure. In terms of the Charnock parameter, $z_{0}$, (Fig. 1(c)), the dimensionless roughness increases with $\omega_{*}$ in the parameter range of the available data. The parameterization function (8) predicts that the dependence of $z_{0}(\omega_{*})$ reverses to a decreasing trend at $\omega_{*} > 0.25$ and $z_{0}$ approaches $\omega_{*}^{-2}$ asymptotically as $\omega_{*} \to \infty$. Field data at very high $\omega_{*}$ are difficult to obtain so such non-monotonic behavior cannot be positively confirmed from the field measurements assembled here. The inverse relationship of $z_{0}(\omega_{*})$ at high $\omega_{*}$ range has been observed in laboratory measurements (e.g., Toba et al., 1990).

4. Discussions

From the point of view of dimensional analysis, (6) can be interpreted as choosing peak wavelength and phase velocity of surface waves as the length and velocity scales for organizing the observations. As discussed earlier, these scaling factors are logical choices because the roughness of the ocean surface is primarily composed of surface waves. Moreover, the study of wind over wave coupling supports the suggestion of peak wavelength as the vertical scaling length for parameterizations (Makin and Kudryavtsev, 1999, 2002; Makin, 2003).

Smith et al. (1992) caution about the possible haz-

ards of self-correlation in scaling with $u_{*}$, because the exponential dependence of $z_{0}$ on $u_{*}$ (and that wind speeds at different elevations—e.g., from instrument height to $z = 10 \text{ m}$ or $\lambda/2$—are subsequently computed from $z_{0}$ and $u_{*}$ using the function describing the wind speed profile). They provide two test criteria for determining whether the spurious correlation is negligible in a dataset (p. 134, ibid.) with $z_{0} = (C_{p}/u_{*})^a$.

$$\text{var}(\ln Z) \ll \text{var}(\ln Y), \text{ and } \text{var}(\ln W) \ll \text{var}(\ln X),$$

where $Z = u_{*}^a, Y = (g z/u_{*}^2) \exp(-\kappa U_{10}/u_{*}), W = u_{*}, X = C_{p}$
and the function var is the variance of the argument in parentheses. They further comment that the second condition is usually not met in most field data (in the example cited by them, the four variances are $\text{var}(\ln Z) = 0.146$, $\text{var}(\ln Y) = 0.486$, $\text{var}(\ln W) = 0.146$, and $\text{var}(\ln X) = 0.0057$), thus “the variability of $\ln u_{*}$ is not smaller than that of $\ln C_{p}$ variation of $\ln(C_{p}/u_{*})$ is due more to $u_{*}$ than to $C_{p}$, and self-correlation has influenced the fit to obtain the parameterization equation.” They further comment that “This does not disprove the parameterized equation ...” and that “As far as we know, other similar data sets would lead to the same impasse if tested for spuriousness.”

The four datasets assembled in this paper cover a considerably wider range of wind sea conditions. The least square fitting of the combined data yields $z_{0} = 1.50(C_{p}/u_{*})^{-136}$. The fitted power-law function is shown in Fig. 1(c) in terms of $z_{0}(\omega_{*})$, where $C_{p}/u_{*} = \omega_{*}^{-1}$ for deep water condition. As noted earlier, the large data scatter has masked the depth effect in these datasets. For comparison, the functions presented by Maat et al. (1991) and Smith et al. (1992), $z_{0} = 0.8(C_{p}/u_{*})^{-1}$ and $z_{0} = 0.48(C_{p}/u_{*})^{-1}$, respectively, are also illustrated. The computed variances are $\text{var}(\ln Z) = 0.278, \text{var}(\ln Y) = 0.687, \text{var}(\ln W) = 0.150$, and $\text{var}(\ln X) = 0.202$. The problem of spurious self-correlation is not expected to be serious in the present analysis.

5. Summary and Conclusions

The dynamic influence of surface waves decays exponentially with distance from the air-water interface and the decay rate is inversely proportional to the characteristic wavelength. These dynamic properties of surface waves suggest that the reference wind speed for atmospheric measurements on air-sea coupling parameters should be the wind speed at an elevation proportional to the wavelength. The wavelength scaling also receives strong support from wind over wave coupling theory (e.g., Makin and Kudryavtsev, 1999, 2002; Makin, 2003). The analytical expressions of the drag coefficient and surface roughness with $U_{3/2}$ as reference wind speed are established from the logarithmic wind profile (Section 2) for
stratification-corrected conditions and wind-sea dominated wave fields. The analysis suggests that a natural expression of the dimensionless surface roughness is \( k_p \varepsilon_0 \). Other expressions of the nondimensional surface roughness, such as the Charnock parameter, can be obtained by the substitution of \( k_p \) with \( \omega_p \) using the dispersion relation of surface gravity waves.

Several field experiments with wind-sea dominated wave fields and tabulated data that provide sufficient information to derive wavelengths have been analyzed. The results show that the measurements of the drag coefficient from different experiments can be consolidated in a more organized manner when it is presented in terms of \( \lambda_{2/3} \) than in terms of \( C_{10} \) (compare Figs. 1(a) and A1). With the consideration that wavelength and wave phase speed are the proper length and velocity scales of the ocean surface roughness, the parameterization of \( C_{1/2} \) is given by an empirical power-law function (6). The dimensionless roughness can then be represented by (7) for \( k_p \varepsilon_0 (\omega_p) \) or by (8) for \( \varepsilon_0 (\omega_p) \). Surface roughness in terms of \( k_p \varepsilon_0 \) increases monotonically with \( \omega_p \). When expressed in terms of \( \varepsilon_0 \), the roughness increases with \( \omega_p \) for \( \omega_p < -0.25 \) and decreases with \( \omega_p \) for \( \omega_p > -0.25 \). The range of \( \omega_p \) in the field data is usually much less than 0.25 but laboratory data extend to a much higher \( \omega_p \) range (e.g., Toba et al., 1990).

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**Appendix A. Descriptions of the Assembled Datasets**

Donelan (1979) acquires wind and wave measurements from a fixed tower at the western side of Lake Ontario. The tower is located 1100 m from the coast, and the local water depth is 12 m. An extensive suite of instruments are mounted on the tower. Data relevant to this study include wind velocity, direct wind stress measurement using the eddy correlation method, wave variance and peak frequency of the wave spectrum. The wind conditions selected for his analysis are steady offshore blowing events with directions of both winds and waves staying within 25 degrees to the beach normal, and that the peak frequency of a companion waverider buoy deployed further offshore to be less than 3.14 rad/s. All together, six cases are reported. The maximum and minimum wavelengths in this dataset are 5.7 and 3.3 m.

Merzi and Graf (1985) conduct wind stress measurements in Lake Geneva. The measurement station is on the northern part of the lake. The local water depth is 3 m. The records selected for analysis are based on steady
southwesterly events and well-behaved wind profiles (60 cases). The friction velocity is derived from the profile method. Thermal stratification is corrected. The friction velocity used in this analysis is calculated from their tabulated neutral drag coefficient and $U_{10}$. The maximum and minimum wavelengths in this dataset are 19.0 and 8.6 m.

Anctil and Donelan (1996) report momentum flux observations from four towers at different depths along a shore-normal line at the west end of Lake Ontario, Canada. The nominal depths of the towers are 2, 4, 8, and 12 m, the wind stress is measured by the eddy correlation method at anemometer heights of 6.2, 7.8, 7.4 and 12 m for the four towers. The drag coefficients and wind speeds at both anemometer heights and 10 m reference elevation are tabulated along with the stability parameter, instantaneous water depth, root mean square wave height, peak wave period, peak phase speed, wave angle and steepness. All together, 18 cases are reported. The maximum and minimum wavelengths in this dataset are 75.8 and 28.6 m.

Janssen (1997) carries out a statistical error analysis of the HEXMAX data to investigate the sea-state influence on wind-stress. The HEXMAX data are collected from a research platform at a nominal depth of 18 m. The wind stress is calculated by the eddy correlation method. The dataset (58 cases) has been carefully screened to remove unsteady events and swell-contaminated cases. The wind stress, wind speed at 10 m elevation, peak phase speed and significant wave height are tabulated. The characteristic wavelength needed for the present analysis is derived by the dispersion relation using the tabulated phase speed and the nominal water depth. The maximum and minimum wavelengths in this dataset are 101.6 and 53.4 m.

Figure A1 shows several presentations of $C_{10}$ including (a) $C_{10}(U_{10})$, (b) $C_{10} (\omega_*)$, and (c) $C_{10}(U_{10}/C_p)$. The dimensionally consistent expressions (Figs. A1(b) and (c)) reveal the stratification of $C_{10}$ with wavelength. The wavelength ranges of the four datasets are (101.6 m, 53.4 m) in Janssen (1997), (75.8 m, 28.6 m) in Anctil and Donelan (1996), (19.0 m, 8.6 m) in Merzi and Graf (1985), and (5.7 m, 3.3 m) in Donelan (1979). For dimensionally inconsistent expression such as $C_{10}(U_{10})$ shown in Fig. A1(a), such a wave effect is still discernable. The results shown in Fig. A1 suggest that the drag coefficient in terms of $C_{10}$ cannot be expressed as a single-parameter function. Two-parameter functions, such as $C_{10}(U_{10}, \omega/C_p)$ and $C_{10}(U_{10}, U_{10}/C_p)$ have been reported.

Appendix B. A Comparison of $C_{2/3}$ and $C_{5/3}$

The drag coefficient, being the square of the ratio between the wind friction velocity and a reference wind speed, is quantitatively affected by the choice of the reference wind speed. It is demonstrated in Figs. 1 and A1 that the same group of data can be better organized when presented in terms of $C_{2/3}$ in comparison with $C_{10}$. If an elevation other than one-half of the wavelength is used for the vertical length scale, the equation of the drag coefficient can be written as
\[ C_{α/κ} = \frac{1}{k_p^2 \left( \ln \frac{α}{k_p^2} \right)^2}, \]  
\[ (B1) \]

where \( k_p = 2π/λ \). The drag coefficient \( C_{\alpha/κ}(α, \omega) \), \( C_{\lambda/κ}(α, \omega) \), \( C_{\lambda/κ}(α, \lambda) \), and \( C_{\lambda/κ}(α, \omega) \) of the assembled data are displayed in Fig. B1. As expected, the collapsing of measurements is not influenced by the choice of the proportionality constant for wavelength scaling.

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