**Title and Subtitle**

Experiment-Based Development and Validation of Mistuning Models for Bladed Disks

**Performing Organization**

Department of Mechanical Engineering
The University of Michigan
2350 Hayward Street
Ann Arbor MI 48109-2125

**Sponsoring/Monitoring Agency**

USAF/AFRL
AFOSR
801 N. Randolph Street
Arlington VA 22203

**Distributor/Availability Statement**

Distribution Statement A. Approved for public release; distribution is unlimited.

**Abstract**

The primary objective of this project was to perform experimental work to further the understanding of the fundamental physics of mistuned bladed disks. The experiments that were carried out during the course of this project have served to corroborate analytical and numerical findings, to validate the computational methods that have been developed to date, and to guide the development of improved models and methods that incorporate the relevant physics. In addition, a pioneering mistuning identification technique was developed, and alternative designs featuring intentional mistuning were examined in a series of simulations and experiments.
Final Report to

Air Force Office of Scientific Research
Aerospace and Materials Science
Structural Mechanics

EXPERIMENT-BASED DEVELOPMENT AND VALIDATION OF MISTUNING MODELS FOR BLADED DISKS

AFOSR GRANT F49620-01-1-0130

Christophe Pierre
Professor
Phone: (734) 764-8221  Fax: (734) 763-2447  Email: pierre@umich.edu

Steven L. Ceccio  Matthew P. Castanier  Sang-Ho Lim and Jia Li
Professor  Associate Research Scientist  Graduate Students

Department of Mechanical Engineering
The University of Michigan
2350 Hayward Street
Ann Arbor, MI 48109-2125

GRANT MONITOR
Capt. Clark Allred

OVERVIEW

The primary objective of this project was to perform experimental work to further the understanding of the fundamental physics of mistuned bladed disks. The experiments that were carried out during the course of this project have served to corroborate analytical and numerical findings, to validate the computational methods that have been developed to date, and to guide the development of improved models and methods that incorporate the relevant physics. In addition, a pioneering mistuning identification technique was developed, and alternative designs featuring intentional mistuning were examined in a series of simulations and experiments.
EXPERIMENTAL SETUP

During the course of this research grant and a previous research grant, an advanced experimental facility was established at the University of Michigan, called the Turbomachinery Vibration Laboratory. The purpose of this experimental facility is to conduct experiments on the vibration of bladed disks in a controlled environment. The major equipment in the laboratory is shown in Fig. 1. In order to check the experimental setup, the Air Force Research Laboratory provided the blisk (single-piece bladed disk) seen in Fig. 1, which will be referred to as the “Air Force blisk.” The Air Force blisk is shown mounted on a fixture that is attached to the laboratory’s vibration-isolation table.

![Image](a)

![Image](b)

Fig. 1: The Turbomachinery Vibration Laboratory at the University of Michigan. (a) An overview of the experimental facilities. (b) Full-field image capture of the vibration of an Air Force blisk. A 2-nodal-diameter response is seen on the monitor.

The laboratory features two types of laser devices for taking non-intrusive measurements of rotor vibration. One is an Electronic Speckle Pattern Interferometry (ESPI) system. This device is capable of observing full-field vibration, as depicted in Fig. 1b, so that mode shapes and forced response patterns of the system may be captured. In addition, the laboratory has a laser vibrometer for taking highly accurate measurements at specific points on the rotor. This vibrometer is mounted on a pair of linear actuators, as seen in Fig. 1, which allow it to be aimed at any point in the “x-y” plane. This actuation is computer-controlled, so that a point or a set of points can be programmed into the control routine, and the vibration is measured automatically at the same locations on each blade. It should be noted that the investigators have recently received DURIP funding from the AFOSR for the purchase of a scanning laser vibrometry system, which will greatly enhance the vibration measurement capabilities of the laboratory.
ACCOMPLISHMENTS AND NEW FINDINGS

Traveling Wave Excitation System

To date, all experiments have been stationary bench tests. However, a novel excitation system has been developed [1,4,8] that provides the same engine order excitation that the rotor would experience if it were rotating. This non-intrusive excitation system consists of an array of speakers as shown in Fig. 2.

![Fig. 2: The non-contacting, acoustic excitation shown with and without the Air Force blisk mounted. This system consists of an array of speakers, one near each blade. The input signal to each adjacent speaker has a phase lag equal to the interblade phase angle. This novel system provides the equivalent of a rotating, engine order excitation in a controlled, stationary bench test environment.](image)

Each speaker is mounted near a blade, and a set of phase-synchronized function generators (see Fig. 1a) provides a sinusoidal input signal to each speaker. The signals are given a phase lag equal to the interblade phase angle of the desired engine order of excitation. Thus, traveling wave excitation is achieved. This engine order excitation system was a new contribution that enhances significantly the evaluation capabilities of bench tests for turbomachinery rotors. It allows for a realistic traveling wave excitation, while avoiding complexities associated with taking measurements in a rotating environment. This technology has been transferred from the University of Michigan (UM) to the Air Force Research Laboratory (AFRL) at Wright-Patterson Air Force Base. AFRL researchers have continued to make improvements to the system, and a Disclosure and Record of Invention (AF Form 1279) has been filed jointly by AFRL and UM researchers.

Once this excitation system was devised, there still remained the task of ensuring that a uniform forcing amplitude was generated by each speaker across the frequency range of
interest. To achieve this, the frequency transfer function of a speaker was measured in an anechoic chamber. Then, the amplitude of the input signal to the speaker was calibrated according to this frequency transfer function so that the speaker output would be approximately constant. This provided an even excitation, across the speaker array and across the frequency range. The acoustic pressure generated by the speakers has been found to provide sufficient blade forcing so that accurate measurements may be taken. Furthermore, this acoustic excitation is non-contacting, so that it does not change the mistuning pattern or other properties of the rotor. The combination of this non-contacting engine order excitation system and the non-contacting laser measurement devices combine to make the Turbomachinery Vibration Laboratory a state-of-the-art facility for observing mistuning-induced vibration phenomena and for validating analytical and numerical predictions.

Observation of Mistuning Phenomena

The Air Force blisk was used as a test specimen for checking that all of the hardware and attendant software in the Turbomachinery Vibration Laboratory was working properly. In addition, the vibration characteristics of this blisk were investigated experimentally.

Because no finite element model was available for the Air Force blisk, the natural frequencies and mode shapes had to be located and identified by experimental exploration—a true test of the experimental facility. First, a sinusoidal force input was provided to a single blade, and a frequency sweep was performed. By measuring blade response with the laser vibrometer, vibration resonances were found at various frequencies, each corresponding to vibration dominated by a single mode. Then, using this natural frequency information, the ESPI system was used to visualize the vibration patterns at these frequencies and thus identify the mode shapes.

![Fig. 3: Mode shape images captured by the ESPI system for the Air Force blisk. Vibration is indicated by dark fringes, with each fringe corresponding to a certain quantum level of vibration. More fringes in a blade indicate higher vibration amplitudes.](image)

Several representative mode shapes were captured digitally, and four of these are shown in Fig. 3. Some of the modes of the Air Force blisk featured clear nodal diameter shapes. This is because they were disk-dominated modes, which tend to be less sensitive to
mistuning than blade-dominated modes due to stronger coupling between the blades. Figure 3a shows a pair of 2-nodal-diameter modes. It can be seen that these modes are rotated versions of the same shape, and that they are orthogonal to each other. Furthermore, their natural frequencies are quite close, 905.2 Hz and 905.5 Hz. Note that in the ideal (tuned) case, the natural frequencies of such “double modes” would be identical. The existence of nodal diameter mode shapes, combined with the slightly separated natural frequencies, indicates a fairly low sensitivity to mistuning at these frequencies. That is, the rotor behaves almost as if the blades were identical for these modes, despite the presence of some unknown amount of blade mistuning.

However, most modes were blade-dominated modes, and these modes featured localization, indicating sensitivity to the presence of mistuning in the Air Force blisk. In Fig. 3b, two examples of localized modes are shown. These modes are extremely close in frequency, yet they do not resemble each other at all. The vibration is localized to mostly a few blades in these mode shapes. This highlights the dramatic effects of mistuning on the system response.

In addition to developing an engine order excitation system and examining the resonant modes, a fundamental excitation-mode interaction mechanism was identified and explained. In particular, it was observed that traveling wave excitation could lead to standing wave response. This was explained by the fact that a traveling wave may be considered to be a combination of two standing wave modes with the same spatial harmonic. In the ideal, tuned case, the modes in a mode pair have exactly the same frequency. However, even a small amount of mistuning causes the natural frequencies of a mode pair to separate. This is known as “peak splitting,” because if the damping in the system is low enough, the resonant peaks of the two modes will be distinct.

Therefore, for a system with mistuning and light damping, the bladed disk will vibrate primarily in one standing wave mode near one resonant frequency, and primarily in the other standing wave mode near the other resonant frequency. In between the two peaks, traveling wave response may be seen, although the response amplitudes will not be as large due to the off-resonance condition. This traveling/standing wave phenomenon is illustrated in Fig. 4.
Fig. 4: Blade amplitude and phase data for the forced response of the Air Force blisk at 908 Hz (left) and 911 Hz (right). The peak-splitting phenomenon is seen in the plot at center: two orthogonal, standing wave modes have slightly different resonant frequencies due to mistuning. Because of this separation, traveling wave excitation yields a standing wave response near one of the resonant frequencies, as seen on the left. In between the two peaks, the combination of standing wave modes yields traveling wave response, as seen on the right.

Validation of Mistuning Theory

Although the experimental study of the Air Force blisk’s vibration provided an excellent validation of the experimental facility, it did not serve to validate quantitatively any mistuning theories or modeling codes. In fact, most industrial rotors are ill suited to validating mistuning theory, since the actual mistuning cannot be determined nor controlled in a precise manner.

With this in mind, a blisk was designed and manufactured specifically for use in experimental validation of mistuning theory. The finite element mesh for this blisk, which is referred to as the “validation blisk,” is shown in Fig. 5a.

The design of the validation blisk was performed with significant input and feedback from AFRL engineers at the Turbine Engine Fatigue Facility (TEFF) at Wright-Patterson
Air Force Base. Several "iterations" were performed in the design process. The UM researchers began with a basic geometry that satisfied two initial criteria: (1) the key characteristics were representative of an industrial bladed disk, and (2) it was compatible with the hardware in the TEFF spin rig. At each design iteration, the REDUCE code was used to predict the vibration characteristics for that design. Based on that analysis, changes were made in the finite element model for the next design iteration.

![Diagram](image1)

(a)

![Diagram](image2)

(b)

**Fig. 5:** The validation blisk. (a) Finite element mesh of the final design. (b) Two views of the manufactured blisk.

The final design of the validation blisk, as shown in Fig. 5, has the following features that make it ideal for experimental validation of mistuning theory:

- **Thick blades:** the thickness decreases the sensitivity to mistuning due to manufacturing tolerances
- **24 sectors:** this number was selected so that every engine order of excitation for the system that corresponds to a nodal diameter mode shape (0–12) can be produced by the assembly of phase-synchronized function generators in the laboratory
- **A raised hub:** keeping the mounting surface away from the main disk lessens the impact of the boundary conditions, thus increasing the repeatability of experiments and reducing unmodeled effects
- **Flat blades:** specific mistuning patterns may be added intentionally by attaching masses to the blades

Once the design was set, the blisk was manufactured from a single piece of annealed steel. Computer-numerically-controlled machining was used to hold the manufacturing tolerances to less than $5 \times 10^{-4}$ inches.
Some of the mode shapes of the validation blisk are shown in Fig. 6. Note that these modes are all nodal diameter modes, despite the fact that some of them are blade-dominated modes that would typically be sensitive to mistuning. In addition, the natural frequencies are plotted as a function of the number of nodal diameters in Fig. 6d. Note that there is excellent agreement between the predictions from finite element analysis of the nominal (tuned) design and the measurements for the actual validation blisk. The combination of targeted design and precision manufacturing has successfully produced a test specimen that features “tuned” behavior.

![Mode Shapes and Natural Frequencies](image)

**(a) 2 nodal diameters**  
**(b) 4 nodal diameters**  
**(c) 6 nodal diameters**

**(d) Predicted and measured natural frequencies versus nodal diameters**

*Fig. 6: Modes and natural frequencies of the validation blisk.*

As mentioned above, mistuning can be added by attaching masses to the blades. (The effects of implementing mass mistuning in this manner have been investigated [10].) Thus, the mistuning in the blisk can be controlled, and the blisk can be used to validate modeling predictions for both tuned and mistuned systems. The validation blisk has been used to verify the mistuning identification technique and the intentional mistuning design strategy that will be described below.
**Mistuning Identification**

An important practical consideration for mistuning research is how to identify the mistuning that is actually present in a manufactured bladed disk. Mistuning identification is important because it can be used for the following critical tasks:

- To determine mistuning parameters that are used in vibration models
- To assess the quality of the manufacturing process
- To validate the integrity of a manufactured rotor before placing it in service
- To perform in-service structural health monitoring of jet engine rotors (e.g., to identify a cracked blade)

The pioneering mistuning identification technique developed in this research program has provided a major step forward in this area.

For rotor stages with inserted blades, the blade natural frequencies can be measured individually. However, for a blisk—a one-piece bladed disk—the blades cannot be removed from the assembly. Therefore, a mistuning identification technique [2-4,7] was developed by the investigators to determine the mistuning pattern for each blade mode family of interest.

In order to identify individual blade mistuning from the dynamics of an entire bladed disk, two sources of information are used. The first is a theoretical model of the bladed disk, containing enough information to predict its response accurately if all structural parameters were known. The second is a set of measurements of the response of the actual blisk, which can be used together with the information in the model to determine the mistuning parameters responsible for such behavior. A reduced-order modeling technique previously developed by the investigators makes use of component mode synthesis (CMS) to reduce the size of the finite element model, and then follows this with a secondary modal analysis, further reducing the model size using a set of modes of the CMS model. In the mistuning identification process, the same CMS method is used, but it is followed by a secondary modal analysis that condenses only the disk and disk-blade interface portions of the CMS model. The result is an extremely reduced model that retains blade modal stiffnesses explicitly. Then a small set of experimental measurements of the system response can be used to determine the mistuning for the isolated blade modal stiffnesses. Either mode shape measurements or forced response measurements can be used [7].

The accuracy of the mistuning identification technique was checked using the validation blisk in a sequence of test cases [4]. One of these test cases involved identifying the small mistuning inherently present in the manufactured validation blisk, and then trying to improve the tuning by adding to the blades a set of lead weights with slightly different mass values designed to cancel out the mistuning pattern identified for the blisk. First, the mistuning in the validation blisk was identified from a set of system mode shape measurements, and it was found that the modal stiffness mistuning had a standard deviation of 0.35%. Then, a pattern of lead weights was chosen and attached to the blade tips (see Fig. 7a) in an attempt to give every blade the same modal stiffness in the first
flexural mode. After applying the pattern of weights, measurements of the free vibration mode shapes were made, and the mistuning identification algorithm was again used to determine the mistuning that was actually present in this nearly tuned case.

Fig. 7: (a) A lead weight attached to a blade tip of the validation blisk. Weights of various sizes were attached to the blade tips to control the blade mistuning. (b) Improved tuning of the validation blisk based on identifying the mistuning and then attaching a set of weights to try to cancel out the identified mistuning pattern. The standard deviation of mistuning was reduced from 0.35% to 0.20%.

Fig. 7b shows the mistuning pattern of the blisk before and after the masses were added. The y-axis shows the percentage of the original design modal stiffness on the left, and the percentage of the new intended baseline modal stiffness on the right (note that adding mass lowers the blade eigenvalues, which are also the blade modal stiffnesses assuming mass normalization of the modes). It is clear that progress has been made in tuning the blisk: the standard deviation of the new nearly tuned pattern is about 0.20%, versus 0.35% previously, and the difference between the maximum and minimum values has been significantly reduced. It is believed that the remaining mistuning cannot be eliminated, due to the level of uncertainty for the mass values of the lead weights and the glue used for attachment.

Recently, this mistuning identification technique was improved by changing the formulation to take advantage of recent advances in mistuning modeling. In particular, the component mode mistuning (CMM) method [9], which was recently introduced by the investigators, forms the basis of the enhanced mistuning identification technique [11]. The latest results have shown a significant improvement in the accuracy of the identified mistuning patterns for a test case.
Intentional Mistuning

Another important aspect of this research program has been the exploration of novel designs that mitigate the damaging effects of mistuning. In particular, the use of “intentional mistuning” has been investigated [5,6,12]. This is a novel, alternative bladed disk design strategy that was proposed and examined in this research program.

Intentional mistuning is the deliberate implementation of blade-to-blade differences in the nominal design. Instead of trying to make all blades exactly the same, the blades are made different by using two or more blade designs featuring different natural frequencies. The reason for doing this is simple: forced response does not increase monotonically with increasing mistuning. There is often a critical region of mistuning strength that leads to the largest blade amplitudes. If the mistuning is below that critical zone, then the system modes are close to the nodal diameter modes of the tuned case. In this case, the vibration energy flows freely from blade to blade, and thus the energy is distributed throughout the system. If the mistuning is above the critical zone, then the modes become highly localized, and the forcing at one blade has little effect on other blades. In this case, the energy is distributed among the blades because it cannot be transferred well between blades. In the critical region, there is sufficient communication between blades to allow energy transfer, but the modes are sufficiently localized that the vibration energy is trapped in small regions of the system, leading to a few blades with large amplitudes. Therefore, intentional mistuning in the design may provide a mechanism for keeping the mistuning above the critical region for any level of “random mistuning” (mistuning in the traditional sense—e.g., parameter differences due to manufacturing tolerances and in-operation wear).

This “peak phenomenon” of the vibration amplitude as a function of mistuning strength is illustrated in Fig. 8 for an industrial blisk. Figure 8 shows the 99th percentile of the amplitude magnification, which is the ratio of the maximum mistuned forced response to the maximum tuned forced response. Thus, an amplitude magnification of 1.5 would indicate a 50% increase in the maximum vibration amplitude due to mistuning. It can be seen in Fig. 8 that the original design suffers a roughly 90% increase in vibration amplitude at a small level of mistuning. At higher levels of random mistuning, this drops to 50% higher than the tuned vibration level. The other four lines in this plot are for nominal designs that include intentional mistuning in various harmonic patterns. These intentional mistuning patterns have 10% amplitude in stiffness (i.e., the blade stiffness varies from 1.1 to 0.9 times the original blade stiffness), and the stiffness of each blade varies in a sinusoidal manner (i.e., spatially harmonic patterns of intentional mistuning, harmonics 5–8). The four harmonic intentional mistuning patterns shown all lead to a significant reduction in the amplitude magnification. In fact, the peak amplitude magnification is cut in half, from around 90% above the tuned level for the original design, to at most 45% above the tuned level for the designs with intentional mistuning. Furthermore, it can be seen that the designs with intentional mistuning are relatively unaffected by the level of random mistuning. This implies that intentional mistuning makes the design more robust with respect to random mistuning.
Fig. 8: An example of how the use of intentional mistuning can reduce forced response amplitudes and stresses. The 99th percentile of amplitude magnification (maximum mistuned response divided by maximum tuned response) is shown as a function of random mistuning strength for various designs of an industrial blisk. The original design (no intentional mistuning) shows a maximum amplitude increase of around 90% due to random mistuning. Four different designs with sinusoidal patterns of intentional mistuning (harmonics 5-8) yield a significant reduction in the amplitude magnification.

Of course, it is important to understand how intentional mistuning works. On this topic, the investigators have identified a key physical mechanism [5]. A mode shape of a system with only random mistuning may be fairly localized, but there still tends to be a dominant nodal diameter component. That is, the mode is a perturbation of a nodal diameter mode. With intentional mistuning, each mode tends to be more localized; but it is comprised of many nodal diameter modes, with no single nodal diameter mode being dominant. This means that the mode of a system with intentional mistuning is less likely to be excited by an engine order excitation. Thus, intentional mistuning leads to an increase in mode localization, but it leads to a decrease in the forced response by effectively reducing the modal forces.

Furthermore, the effectiveness of intentional mistuning has been verified experimentally [6]. In conducting the experiment, four different sets of lead weights were glued to the tips of the blades in order to modify the effective stiffnesses of the blades. Each of the four sets of weights corresponded to a mistuning pattern chosen to demonstrate the effects of random and intentional mistuning on the dynamic behavior of the blisk:

- **Nearly Tuned**: In the first set, nearly equal weights were used in order to provide a tuned baseline, from which the other three mistuning patterns diverge (see Fig. 7).
• **Random Mistuning**: The second set was a "random" pattern, consisting of weights selected to give the blisk a mistuning pattern demonstrating the high amplitude magnification effects of random mistuning. This pattern was chosen based on Monte Carlo simulations of the rotor's mistuned forced response to engine order 7 excitation, with 2% standard deviation of mistuning.

• **Intentional Mistuning**: The third set was a square-wave intentional mistuning pattern, chosen from simulations to be a more effective design that is less susceptible to the harmful impact of random mistuning. The square-wave pattern consisted of two periods with 3% amplitude: the first six blades each had modal stiffness 3% above the original design, the next six blades each had modal stiffness 3% below the original design, and then this pattern was repeated for the other 12 blades.

• **Combined Mistuning**: The fourth set was a combination of the intentional and random mistuning patterns, which was obtained by adding the mistuning values of patterns 2 and 3.

![Graph](image)

**Fig. 9**: Comparison of maximum blade amplitudes for the four mistuning cases as determined from a reduced order model (ROM), a finite element model (FEM), and experimental measurements (Expt).

Figure 9 shows the relative maximum amplitudes for the four cases studied, with three values shown for each case: numerical predictions from a 95-DOF reduced order model (ROM), numerical predictions from a 141,840-DOF finite element model (FEM), and experimental measurements (Expt.). All values are normalized by the corresponding
nearly tuned maximum amplitude, and the relative magnitude predicted for a perfectly tuned case is indicated by a dashed line.

Two significant effects of intentional mistuning can be seen. First, the maximum amplitude of the blisk with intentional mistuning alone is somewhat lower than the maximum amplitude of the nearly tuned case; in fact, it is similar in magnitude to what would be found if a perfectly tuned blisk could be created. Second, and more significantly, the presence of intentional mistuning has prevented significant amplitude magnification effects due to the addition of random mistuning. The case with combined "random" and intentional mistuning has maximum amplitudes no higher than those of the nearly tuned case, despite the large amount of mistuning present. These results clearly demonstrate the potential for significant amplitude increases due to random mistuning, as well as the beneficial effects of intentional mistuning.

Finally, a key aspect of the intentional mistuning design strategy is the selection of the intentional mistuning pattern. In recent work, three design guidelines were found for selecting the pattern of intentional mistuning: (1) assign an equal or nearly equal number of blades to each blade type; (2) distribute the blades of each type so that they are "well balanced" about the disk; and (3) assign an equal or nearly equal number of blades to each group of consecutive blades of the same type. Intentional mistuning configurations that satisfy these guidelines include square-wave, sawtooth, and staircase patterns. Numerical simulations indicate that these guidelines can be used to reduce the design space dramatically, yet the reduced design space includes optimal or near-optimal intentional mistuning patterns [12].

PERSONNEL SUPPORTED
The following personnel were supported by and/or associated with this research program:

- Christophe Pierre, Professor
- Steven L. Ceccio, Professor
- Matthew P. Castanier, Associate Research Scientist
- John Judge, Graduate Student Research Assistant
- Jia Li, Graduate Student Research Assistant
- Sang-Ho Lim, Graduate Student Research Assistant
PUBLICATIONS


INTERACTIONS AND TRANSITIONS

Participation/Presentations at Meetings and Conferences
The results from this research program have been presented at the following conferences:
- AIAA/ASME/ASCE/AHS Structures, Structural Dynamics, and Materials Conference and Exhibit
- ASME Turbo Expo
- International Conference on Structural Dynamics Modelling
- International Forum on Aeroelasticity and Structural Dynamics
- National Turbine Engine High Cycle Fatigue Conference
A list of conference papers is included in the previous section of this report.

Consultative and Advisory Functions to Other Laboratories and Agencies
The investigators have worked closely with members of the Turbine Engine Fatigue Facility of the Air Force Research Laboratory (AFRL) at Wright-Patterson Air Force Base. In particular, the investigators collaborated with Dr. Charles Cross in designing the validation blisk and with Capt. Keith Jones in transitioning the experimental setup to AFRL (see the “Transitions” section below).

In addition, the investigators were selected by the NASA Glenn Research Center to perform vibration testing and mistuning identification on a prototype of a next-generation bladed disk design, which is shown in Fig. 10. NASA selected the investigators because the mistuning identification technique developed in this research program was the first practical method for identifying mistuning in integrally bladed rotors. First, the mistuning present in the manufactured rotor was identified. Then, free and forced vibration response predictions were calculated from numerical simulations using the identified mistuning parameters, and these predictions were compared to experimental measurements. Results are shown in Fig. 10 for a mode shape found at 734.1 Hz. It can see that the predictions agree well with experimental results.

Fig. 10: Left: NASA rotor being tested at the UM Turbomachinery Vibration Laboratory. Right: Comparison of a mode shape from experimental measurements to the results from a numerical prediction based on the mistuning ID technique.
**Transitions**

An important part of the Turbomachinery Vibrations Laboratory is the laser vibrometer and the computer-controlled positioning and traveling-wave excitation systems (see Figs. 1 and 2). The vibrometer-positioning system was developed as a means of taking accurate measurements at precise locations on each blade of a rotor. Furthermore, the acoustic excitation system with phase-synchronized and gain-adjusted speakers provides non-contacting, traveling-wave excitation. This effectively mimics engine order excitation in a bench test environment. The experimental setup has proven to be very successful for collecting quantitative data, and it has been transitioned to the Air Force: a similar system has been installed at the Turbine Engine Fatigue Facility at Wright-Patterson Air Force Base. In fact, AFRL and UM researchers have filed a Disclosure and Record of Invention (AF Form 1279) for the technology developed for the excitation system ("Programmable Multi-Channel Amplitude and Phase Shifting Circuit," K. W. Jones, C. Pierre, S. L. Ceccio, J. Judge, S. Fuchs, Air Force Research Laboratories, April 2002). Also, a patent for this invention is in the process of being filed.

The mistuning identification technique developed in this research is a pioneering method that has had a major impact on the turbine engine community. It has sparked the development of similar techniques by researchers at other universities, and experimental mistuning identification systems are now installed or planned at several government and industry laboratories.

The intentional mistuning research has also had a significant impact on the field. Researchers and engineers at the Air Force and turbine engine companies are currently pursuing this design strategy in order to develop more robust and reliable turbomachinery rotors.