The Relationship Between Detection Algorithms for Hyperspectral and Radar Applications

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The Relationship Between Detection Algorithms for Hyperspectral and Radar Applications

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See Briefing Charts.

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Objective

- Overview of hyperspectral sensing

- Demonstrate how and why detection algorithms for hyperspectral imagery are related to detection algorithms for MTI radar
  - Similar physical assumptions
  - Common signal model

- Illustrate detection in hyperspectral imagery with real data and familiar detectors
Outline

- Introduction to hyperspectral sensing
- Signal models
- Detection models
- Hyperspectral detection results
- Conclusion
Hyperspectral Imaging (HSI) Concept

Each pixel contains a continuous spectrum that is used to identify the materials present in the pixel.

- **Signal**
- **Wavelength**

Pushbroom Class of Hyperspectral sensor

- **Spectral dimension**
- **Swath width**

Along track dimension built up by the motion of the spacecraft

Scene
Hyperspectral Sensing

- Hyperspectral imaging (HSI) is a form of passive imaging
  - Extension of multispectral sensing (e.g., Landsat)
  - Hundreds of contiguous, real-valued spectral bands
  - Spatial resolution is a function of Instantaneous Field of View (IFOV) and altitude
Outline

• Introduction to hyperspectral sensing
• Signal models
  – Hyperspectral sensing
  – MTI radar
• Detection models
• Hyperspectral detection results
• Conclusion
Modeling of Spatially Unresolved (Mixed) Pixels

**Physical Space**

- Spatially unresolved targets
- Measured spectrum

**Data Space**

- Band 1
- Band 2
- Band 3
- Pixel spectra

**Signal Processing**

- **Unmixing**
  - Find endmembers
  - Compute abundances
- **Classification**
- **Detection**
Linear Mixing Model (LMM)
Target and Background Modeling

Test pixel \( \mathbf{x} = \sum_{k=1}^{P_T} a_k s_k + \sum_{k=P_T+1}^{P_T+P_B} a_k s_k + n \)

\( P_T \) \( P_B \) \n
\( \text{abundance} \)

\( \text{end member} \)

\( \text{Target subspace} \) \( \text{Background subspace} \) \( \text{Noise hyper-sphere} \)

\( N(\mathbf{0}, \sigma^2 I) \)
MTI Radar

Two-dimensional filtering required to cancel interference

Space-Time Adaptive Processing (STAP)
Pulsed Radar Datacube

Measurement | Physical Quantity
---|---
Pulse | Doppler (velocity)
Element | Angle
Fast-time | Range
STAP Radar Signal Model

- Space-time snapshot for single target
  \[ x = t + c + n \quad \text{and} \quad t = \alpha v(\phi, f) \]

- \( v(\phi, f) \) is called the space-time steering vector

- Space-time interference (clutter, noise) covariance is
  \[ R = E \{(c + n)(c + n)^H\} = R_c + R_n \]
### Hyperspectral Imaging and MTI Radar

#### Summary of Properties

<table>
<thead>
<tr>
<th>Hyperspectral Imaging</th>
<th>MTI Radar</th>
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<tbody>
<tr>
<td><strong>System</strong></td>
<td><strong>Active, coherent sensing</strong></td>
</tr>
<tr>
<td><strong>Resolution</strong></td>
<td><strong>Resolution is a function of signal bandwidth and aperture length</strong></td>
</tr>
<tr>
<td>Passive, incoherent sensing</td>
<td>Components add linearly to yield received signal</td>
</tr>
<tr>
<td>Resolution is a function of detector IFOV and altitude</td>
<td>Complex array measurements are sum of steering vectors weighted by RCS values</td>
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<tr>
<th><strong>Signal Model</strong></th>
<th><strong>System</strong></th>
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<tbody>
<tr>
<td>LMM assumes distinct spectra mix linearly</td>
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</tr>
<tr>
<td>Real spectra are sum of endmembers weighted by abundances</td>
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<tr>
<td>$x = a s + b + n$</td>
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<tr>
<th><strong>Data Cube</strong></th>
<th><strong>Signal Model</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M bands</strong></td>
<td><strong>Components add linearly to yield received signal</strong></td>
</tr>
<tr>
<td><strong>Y</strong></td>
<td><strong>Complex array measurements are sum of steering vectors weighted by RCS values</strong></td>
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</table>

$x = a s + b + n$

$x = \alpha v + c + n$
Outline

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• Signal models
• Detection models
  – Hyperspectral sensing
  – MTI radar
• Hyperspectral detection results
• Conclusion
Adaptive HSI Detection
Known and Unknown Targets

\[ \hat{R} = \frac{1}{N} \sum_{n=1}^{N} x(n)x^T(n) \]
Adaptive Detection in STAP Radar

Radar data cube

Estimate interference using this data (training region)

Estimate STAP Weights

\[ w = \hat{R}^{-1} v(\phi, f) \]

Hypothesis Testing:

\[ H_0: x = t + c + n \]
\[ H_1: x = c + n \]

Target present
No target
Replacement and Additive Target Models

- Hyperspectral detection has replacement targets
  \[ H_0 : \quad x = b + n \]
  \[ H_1 : \quad x = f t + (1 - f)b + n \]

- Interference statistics
  - Varies with \( f, 0 \leq f \leq 1 \)
  - Target displaces background

- Detection results
  - Insufficient target data for ROC curves
  - No theoretical models

- MTI radar detection has additive targets
  \[ H_0 : \quad x = c + n \]
  \[ H_1 : \quad x = t + c + n \]

- Interference statistics
  - Independent of target
  - Measure locally

- Detection results
  - ROC curves indicate \( P_D/P_{FA} \) values
  - Theoretical models for target
## Comparison of HSI and MTI Detection

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<thead>
<tr>
<th>Hyperspectral Imaging</th>
<th>MTI Radar</th>
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<tr>
<td><strong>Task</strong></td>
<td><strong>Strategy</strong></td>
</tr>
<tr>
<td><strong>Known target</strong></td>
<td><strong>Additive target model</strong></td>
</tr>
<tr>
<td>Detect target spectrum amid background</td>
<td>Moving target</td>
</tr>
<tr>
<td><strong>Unknown target</strong></td>
<td>Exploit coherency through beamforming and Doppler filtering</td>
</tr>
<tr>
<td>Detect pixels anomalous from background</td>
<td>RCS and velocity are key parameters for target visibility</td>
</tr>
<tr>
<td><strong>Covariance</strong></td>
<td><strong>Covariance</strong></td>
</tr>
<tr>
<td><strong>known target</strong></td>
<td><strong>Interference covariance estimated from local subset of pulse/element/range measurements</strong></td>
</tr>
<tr>
<td>Dimension equals number of bands (~ 100--200)</td>
<td>Better estimate</td>
</tr>
<tr>
<td>Can use subset of bands</td>
<td>Avoids non-stationarity</td>
</tr>
<tr>
<td><strong>Replacement target model</strong></td>
<td><strong>Interference covariance estimated from sample pixels</strong></td>
</tr>
<tr>
<td><strong>Known target</strong></td>
<td><strong>Interference covariance estimated from local subset of pulse/element/range measurements</strong></td>
</tr>
<tr>
<td>Measure spectral angle</td>
<td>Better estimate</td>
</tr>
<tr>
<td><strong>Unknown target</strong></td>
<td>Avoids non-stationarity</td>
</tr>
<tr>
<td>Measure magnitude</td>
<td></td>
</tr>
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</table>
Outline

• Introduction to hyperspectral sensing
• Signal models
• Detection models
• Hyperspectral detection results
  – Detection taxonomy
  – Sub-pixel target detection
• Conclusion
# Taxonomy of Hyperspectral Detectors

<table>
<thead>
<tr>
<th>Noise model</th>
<th>Signal model</th>
<th>Available data</th>
<th>Test statistic $T(x)$</th>
<th>References</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbf{R} = \text{completely unknown interference (unstructured)}$</td>
<td>$s = as_i$ known direction</td>
<td>$x =$ test measurement ({x_n}_{n=1}^{N} = \text{“signal-free” training data})</td>
<td>$\frac{</td>
<td>s_i^T \mathbf{R}^{-1} x</td>
<td>}{(s_i^T \mathbf{R}^{-1} s_i)(1 + x^T \mathbf{R}^{-1} x)}$</td>
</tr>
<tr>
<td>$\mathbf{R} = \sum_{k=1}^{P} a_k s_k = \mathbf{S} a$</td>
<td>$x =$ test measurement ({x_n}_{n=1}^{N} = \text{“signal-free” training data})</td>
<td>$\mathbf{R} = \sum_{n=1}^{N} x_n x_n^T$, $\mathbf{R} = \frac{1}{N}$</td>
<td>$\frac{</td>
<td>s_i^T \mathbf{R}^{-1} x</td>
<td>}{(s_i^T \mathbf{R}^{-1} s_i)(1 + x^T \mathbf{R}^{-1} x)}$</td>
</tr>
<tr>
<td>$s = \sum_{k=1}^{P} a_k s_k = \mathbf{S} a$</td>
<td>$x =$ test measurement</td>
<td>$\hat{S} = [s_1, s_2, \ldots, s_P]$, $\mathbf{Z} = [z_1, z_2, \ldots, z_Q]$</td>
<td>Classical F-test for linear statistical models; Signal processing interpretations Matched Subspace Detector (MSD), Scharf-Friedlander (1994)</td>
<td>Orthogonal subspace projection (OSP): $T(x) = s_i^T \mathbf{P}_z^T x$</td>
<td>$T(x) = \frac{\mathbf{T}(x)}{P}$, $P = M \Rightarrow T(x) = x^T \mathbf{R}^{-1} x$, $\mathbf{P} = \mathbf{M} \Rightarrow$ Classical F-test for linear statistical models; Signal processing interpretations Matched Subspace Detector (MSD), Scharf-Friedlander (1994)</td>
</tr>
<tr>
<td>$\mathbf{R} = \sigma^2 \mathbf{I} + \sum_{k=1}^{Q} z_k z_k^T$ structured interference</td>
<td>$s = as_i$</td>
<td>$x =$ test measurement</td>
<td>$\hat{S} = [s_1, s_2, \ldots, s_P]$, $\hat{Z} = [z_1, z_2, \ldots, z_Q]$</td>
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\[ T'(x) = \frac{x^T \mathbf{P}_z \mathbf{P}_G \mathbf{P}_z^T x}{x^T \mathbf{P}_z \mathbf{P}_G \mathbf{P}_z^T x} \]

\[ P_G = \mathbf{G} (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T \]

\[ \mathbf{G} = \mathbf{P}_z \mathbf{S} \mathbf{P}_G = \mathbf{I} - \mathbf{P}_G \]
Hyperspectral Detection Results

- **HYDICE (HYperspectral Digital Imagery Collection Experiment)**
  - Airborne sensor
- **210 spectral bands**
  - 399-2501 nm
  - Channel widths ~ 3 – 11 nm
  - Spatial resolution, 1m x 1m
- **Look for sub-pixel targets**
Comparative Detector Performance
Sub-pixel Targets

- 8232 tree pixels
- 8232 synthetic mixed pixels
  - 25% / 75%
  - 50% / 50%
  - 75% / 25%
- Two detectors
  - SAM ("unwhitened")
    \[ T_{SAM}(x) = \frac{(s^T x)}{\sqrt{(s^T s) \sqrt{(x^T x)}}} \]
  - GLRT
    \[ T_{GLRT}(x) = \frac{(s^T \bar{R}_b^{-1} x)^2}{(s^T \bar{R}_b^{-1} s)(1 + x^T \bar{R}_b^{-1} x)} \]
- Measure range of test statistics

\[ T(x), \text{ Detector Statistic Value} \]

\[ \text{Target Percentage} \]

\[ \text{Skgd. range} \]

\[ \text{Target range} \]
Conclusions

• Under LMM, hyperspectral sensing shares a common signal model with MTI radar
  – Endmembers ↔ Steering vectors
  – Abundances ↔ RCS

• Hyperspectral processing has leveraged optimal detection algorithms from radar
  – Exploit spectral differences between targets and background

• Successful sub-pixel target detection depends upon
  – Target/background subspace relationship
  – Fraction of target present