Design of a Compact Optically-Guided, Pinched, Megawatt Class FEL

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### Design of a Compact Optically-Guided, Pinched, Megawatt Class FEL

A conceptual design for a compact megawatt class FEL operating at 1 μm is presented. The proposed FEL consists of an optically guided, pinched amplifier configuration driven by an RF linac. The gain length, efficiency, electron pulse slippage and the distance between the wiggler and first relay mirror are determined for a megawatt class design. Of particular concern in the design is the overall length of the optical system, i.e., wiggler length and distance to the first relay mirror. In the present design the wiggler length is ~1 meter, and the distance between the first relay mirror and the wiggler is determined by the average intensity damage threshold on the mirror. By focusing the electron beam, the optical beam can be pinched upon exiting the wiggler. The pinched optical beam has a reduced Rayleigh length, which permits the first relay mirror to be relatively close to the wiggler. By pinching the optical beam and employing grazing incidence, the first relay mirror can be located within ~3 meters of the wiggler.
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I. Introduction

The free electron laser (FEL) is potentially capable of producing high average power at high efficiency without the conventional thermal management and waste issues associated with other laser systems [1]. In addition, the operating wavelength of the FEL can be chosen for optimized propagation in, for example, a maritime environment [2]. The FEL can be designed to operate over a wide range of wavelengths and is capable of generating high average power at high efficiency [3]. These unique features make the FEL a leading candidate for naval directed energy applications [4].

II. Optically Guided - High Gain, Pinched FEL

We present and discuss the key elements and issues associated with an optically guided-high gain FEL having a pinched optical beam. There are a number important characteristics and advantages of the proposed configuration. Optical guiding of the radiation beam is possible provided the electron beam current is sufficiently high. In this design the filling factor, i.e., the ratio of the electron beam to the optical beam cross-sectional area, remains relatively constant throughout the wiggler region, and the radiation increases exponentially until saturation. Upon exiting the wiggler the optical beam is pinched by focusing the electron beam with external focusing fields. The Rayleigh range of the pinched optical beam is short and as a result the first relay mirror can be placed close to the wiggler without exceeding the intensity damage threshold. The first relay mirror employs a grazing incidence reflector as indicated in Fig.1. In the high-gain regime electron slippage (lethargy) is shown to be significantly less than in the low gain regime and is not a concern for this design. In a uniform wiggler, enhanced efficiency can be obtained by frequency detuning the input signal. Megawatt FEL designs have been proposed which are based on short Rayleigh range oscillators [4] and amplifiers [5].

Using the Source Dependent Expansion (SDE) formulation [6], expressions for the optical growth rate, wavenumber shift, spot size and optical curvature of the optical beam in an FEL are obtained in the Appendix. The results obtained in the Appendix are used to design a compact MW FEL in the optically guided – high gain regime, i.e.,
having a matched optical spot size. In addition, focusing the electron beam at the wiggler exit is shown to result in a pinched optical beam with a shortened Rayleigh range. The following discussions are based on the design parameters listed in Tables I - IV.

a. Operating Wavelength

Atmospheric water vapor transmission bands exist near 1\( \mu \)m, 1.3\( \mu \)m, 1.6\( \mu \)m, and 2.2\( \mu \)m. In the present design the operating wavelength of a linearly polarized optical beam is \( \lambda = \lambda_w (1 + K^2 / 2) / \gamma_o^2 = 1 \mu \)m (see Tables I and II).

b. Gain Length

The power gain length in the optically guided regime is found from the Appendix to be given by

\[
L_e = 1/2(2 + 3f) \left( \frac{\nu}{\gamma_o} \right)^{1/2} \frac{K}{\sqrt{1 + K^2 / 2}} = 9 \text{ cm},
\]

where \( f = R_e^2 / R_i^2 = 1 \) is the filling factor, \( \nu = I_b[kA]/17 \) is Budker’s parameter, \( I_b \) is the electron beam current in kA, \( K = q B_w \lambda_w^2 / 2 \pi m c^2 = 1.6 \) is the wiggler strength parameter, \( B_w = 8.6 \text{kG} \) is the wiggler field and \( \lambda_w = 2 \text{ cm} \) is the wiggler period. The transverse profiles of the optical and electron beam are Gaussian with radii \( R_e \) and \( R_b \), respectively [see Eqs.(A2) and (A4)].

c. Efficiency

In the optically guided, high gain regime the efficiency of conversion from electron beam power to optical power is given by [7]

\[
\eta = \frac{\lambda_w \Delta k}{2\pi} = f^{1/2} \left( \frac{2 + 3f}{1 + 2f} \right)^{1/2} \left( \frac{\nu}{\gamma_o} \right)^{1/2} \frac{K}{\sqrt{1 + K^2 / 2}} = 0.8\%.
\]

The efficiency can be increased by frequency detuning and/or tapering the wiggler [5,7,8].

d. Optical Guiding

The optical beam can undergo refractive guiding within the wiggler. An envelope equation for the optical beam spot size can be obtained by using the results in the Appendix, and the condition for a matched optical beam can be obtained. This condition
states that when the power gain length is nearly equal to the free space Rayleigh range optical guiding takes place. The precise condition is

$$L_e = \frac{(2 + 3 f)^{1/2}}{2 f^{1/2} (1 + 2 f)} Z_R,$$

(3)

where $Z_R = \pi R_i^2 / \lambda = 22 \text{ cm}$ is the Rayleigh range. The optical guiding condition in Eq. (3) can be rewritten in terms of the beam current,

$$I_b [kA] = 1.1 \times 10^{-2} \frac{f^3 (1 + 2 f)^6 \left( \frac{\lambda}{R_b} \right)^4 \left( 1 + K^2 / 2 \right)^3}{(2 + 3 f)^3} = 2 \text{ kA}.$$  

4. Saturated Peak and Average Optical Power

The peak optical power at saturation is $P = \eta I_b V_b = 1.2 \text{ GW}$, where $V_b = 76 \text{ MV}$ is the electron beam voltage. The average power at saturation is

$$\langle P \rangle = \eta D I_b V_b = 1.2 \text{ MW},$$

(5)

where $D = 10^{-3}$ is the duty factor.

5. Electron Pulse Slippage (lethargy)

In a high gain FEL the electron pulse slippage is found to be significantly reduced compared to that in the low gain regime. The slippage length, i.e., the separation between the optical and electron pulse, is $S \equiv (v_g - v_z) L_w / c$, where $v_g = \partial \omega / \partial k$ is the optical pulse group velocity and $v_z \equiv c (1 - 1 / 2 \gamma_z^2)$ is the axial electron beam velocity. Making use of the high gain FEL dispersion relation [7], the group velocity is

$$v_g \equiv c \left( 1 - \frac{1}{3 \gamma_z^2} - \frac{1}{2} \left( \frac{\lambda}{\pi R_L} \right)^2 \right),$$

and the slippage length is

$$S = \lambda N_w \left( \frac{1}{3} - \left( \frac{\lambda \gamma_z}{\pi R_L} \right)^2 \right) \leq 17 \mu m,$$

(6)

where $N_w = L_w / \lambda_w$ is the number of wiggler periods. The well known slippage length in the low gain regime is $\lambda N_w$. Slippage in the high gain regime is reduced by more
than a factor of three. Slippage can be completely eliminated if the laser spot size is
\[ R_L = \sqrt{3} \frac{\lambda \gamma c}{\pi}. \]

g. Pinched Optical Beam

The intensity on the first relay mirror must be below a damage threshold level. To further reduce the intensity on the mirror the optical beam can be pinched at the wiggler exit in order to shorten the Rayleigh range. By employing external focusing fields near the wiggler exit the electron beam and in turn the optical beam can be pinched. The output of the FEL is thus an optical beam with a shortened Rayleigh range. Making use of a grazing incidence geometry the intensity on the mirror can be further reduced. As a result of these measures, the intensity on the mirror can be kept well below the damage threshold level in a relatively compact configuration.

The spot size of the optical beam \( R_L(z) \) and the radius of the electron beam \( R_{e}(z) \) are plotted as functions of the propagation distance \( z \) in Fig. 2. The entrance to the wiggler is at \( z = 0 \) and extends to \( z = 1 \text{ m} \). In Fig. 2(a) the radius of the electron beam is constant and the optical beam is guided through the wiggler with a constant spot size. Upon exiting the wiggler, the optical beam diffracts. In the example of Fig. 2(b) the electron beam is focused by external fields applied near the wiggler exit. This pinches the optical beam, thereby shortening the Rayleigh range. As a result, the optical beam diffracts more rapidly than in the unpinched example.

h. Distance Between First Relay Mirror and Wiggler

To avoid damage, the distance between the wiggler and relay mirror must exceed
\[ L_{\text{relay}} = Z_{R_{\text{pinch}}} \left( \frac{\langle I_{\text{pinch}} \rangle}{\langle I_{\text{damage}} \rangle} \right)^{1/2} \sin^{1/2} \Theta = 3 \text{ m}, \tag{7} \]
where \( \Theta = 5^\circ \) is the tilt angle (Fig. 1), \( Z_{R_{\text{pinch}}} = \pi R_{\text{pinch}}^2 / \lambda = 3.6 \text{ cm} \) is the Rayleigh range beyond the pinch point, \( \langle I_{\text{pinch}} \rangle = 2 \langle P \rangle / \pi R_{\text{pinch}}^2 = 6.7 \text{ GW/cm}^2 \) is the average intensity at the pinch point and \( \langle I_{\text{damage}} \rangle = 100 \text{ kW/cm}^2 \) is the average intensity damage threshold of the mirror.
III. Discussion and Conclusions

A point design for a compact MW class FEL amplifier operating in the optically guided, high gain regime is given. Optical guiding allows the interaction to be sufficiently long to achieve substantial growth of the input signal. By pinching the optical beam at the wiggler exit the Rayleigh range is shortened, thus reducing the intensity on the first relay mirror. The optical beam is pinched by focusing down the electron beam using external magnetic fields. A grazing incidence geometry permits a further reduction in intensity to avoid damage to the mirror.

In this design the length of the wiggler is ~1 m and the entire optical system, including the first relay mirror, is ~ 4 m in length. For a 2 kA, 76 MV electron beam and a wiggler field of 8.6 kG, the power gain length is 9 cm, the intrinsic efficiency is ~ 0.8%, the peak optical power is 1.2 GW and the average output power is 1.2 MW, assuming a duty factor of $10^{-3}$ for the RF linac. By focusing the electron beam radius down by a factor ~ 3 at the wiggler exit, a similar reduction in the optical beam radius is obtained and the Rayleigh range is reduced by a factor of ~ 6. With a relay mirror tilted by 5° relative the propagation direction and at a few meters from the wiggler, the intensity on the mirror can be kept well below the damage threshold.

The conceptual design presented here assumes that the FEL is driven by a high quality RF linac generated electron beam. Simulations of the FEL, taking into account emittance and energy spread, are required to fully evaluate the design. While estimates indicate that a modest external magnetic field is sufficient to focus the beam, more refined analysis is required to design the pinch region of the wiggler. Another issue is the non-uniformity of the optical guiding due to variation of the electron current within the pulse.

Acknowledgments

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REFERENCES

Figure 1. Schematic of high gain FEL amplifier with a grazing relay mirror. The input signal can be obtained from a solid state laser or FEL oscillator. The radiation beam is optically guided in the wiggler and optically pinched at the exit. The pinched optical beam has a shortened Rayleigh range and undergoes rapid diffraction upon exiting the wiggler. Employing a grazing incidence configuration the resultant footprint on the relay mirror can be made sufficiently large to avoid damage.
Figure 2 Plots of optical beam radius \( R_L \) and electron beam radius \( R_b \) versus propagation distance \( z \). Wiggler entrance is at \( z = 0 \) and the exit is at \( z = L_w \sim 1 \text{ m} \). The FEL operates in the high-gain regime, where the optical beam maintains a constant spot size by optical guiding. Outside the wiggler the optical beam diffracts and spreads transversely, as indicated in (a). By focusing the electron beam at the wiggler exit the optical beam can be pinched down to \( R_{L,pinch} = 0.011 \text{ cm} \) as indicated in (b). The pinched optical beam diffracts more rapidly compared to the unpinched example, i.e., has a shorter Rayleigh range, \( Z_{R,pinch} = \pi R_{L,pinch}^2 / \lambda = 3.6 \text{ cm} \).
### Table I. Wiggler Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wiggler Field, $B_w$</td>
<td>8.6 kG</td>
</tr>
<tr>
<td>Wiggler Parameter, $K$</td>
<td>1.6</td>
</tr>
<tr>
<td>Period, $\lambda_w$</td>
<td>2 cm</td>
</tr>
<tr>
<td>Length, $L_w$</td>
<td>1 m</td>
</tr>
</tbody>
</table>

### Table II. Electron Beam Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy, $E_b$</td>
<td>76 MeV</td>
</tr>
<tr>
<td>Current, $I_b$</td>
<td>2 kA</td>
</tr>
<tr>
<td>Radius, $R_b$</td>
<td>0.025 cm</td>
</tr>
<tr>
<td>Normalized Emittance, $\epsilon_n$</td>
<td>15 mm-mrad</td>
</tr>
<tr>
<td>Duty Factor, $D$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>RF Linac Frequency</td>
<td>750 MHz</td>
</tr>
<tr>
<td>Micro Pulse Duration</td>
<td>1.3 psec</td>
</tr>
</tbody>
</table>
Table III. Optical Beam Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength, ( \lambda )</td>
<td>1(\mu)m</td>
</tr>
<tr>
<td>Spot Size in Wiggler, ( R_L )</td>
<td>0.026 cm</td>
</tr>
<tr>
<td>Filling Factor, ( f )</td>
<td>0.9</td>
</tr>
<tr>
<td>Rayleigh Range (inside wiggler), ( Z_R )</td>
<td>22 cm</td>
</tr>
<tr>
<td>Gain Length (power), ( L_e )</td>
<td>9 cm</td>
</tr>
<tr>
<td>Efficiency (uniform wiggler), ( \eta )</td>
<td>0.8 %</td>
</tr>
<tr>
<td>Average Intensity in Wiggler, ( \langle I \rangle )</td>
<td>1.1 GW/cm(^2)</td>
</tr>
<tr>
<td>Pinched Spot Size, ( R_{L, \text{pinch}} )</td>
<td>0.011 cm</td>
</tr>
<tr>
<td>Rayleigh Range (outside wiggler), ( Z_{R, \text{pinch}} )</td>
<td>3.6 cm</td>
</tr>
<tr>
<td>Average Pinched Intensity, ( \langle I_{\text{pinch}} \rangle )</td>
<td>6.7 GW/cm(^2)</td>
</tr>
<tr>
<td>Peak Saturated Power, ( P )</td>
<td>1.2 GW</td>
</tr>
<tr>
<td>Average Output Power, ( \langle P \rangle )</td>
<td>1.2 MW</td>
</tr>
<tr>
<td>Number of Power E-Folds, ( L_w / L_e )</td>
<td>11</td>
</tr>
<tr>
<td>Average Input Power, ( \langle P_{\text{input}} \rangle )</td>
<td>18 W</td>
</tr>
</tbody>
</table>

Table IV. First Relay Mirror

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Intensity Damage Threshold, ( \langle I_{\text{damage}} \rangle )</td>
<td>100 kW/cm(^2)</td>
</tr>
<tr>
<td>Distance from Wiggler, ( L_{\text{relay}} )</td>
<td>3 m</td>
</tr>
<tr>
<td>Tilt Angle, ( \Theta )</td>
<td>5°</td>
</tr>
<tr>
<td>Area Enhancement Factor, ( 1/\sin\Theta )</td>
<td>11</td>
</tr>
<tr>
<td>Optical Foot Print on Relay Mirror</td>
<td>1cm by 11cm</td>
</tr>
</tbody>
</table>
The source dependent expansion (SDE) formulation [6] can be used to analyze the dynamics of the optical beam in FELs. A summary of the principal SDE equations is given in this appendix. For a planar wiggler the electric field of the radiation can be taken to be linearly polarized, and given by

\[ E(r,z) = \frac{1}{2} E_o(r,z) \exp(i\omega(z/c - t)) + c.c. \]  
(A1)

For the fundamental Gaussian mode the complex envelope \( E_o(r,z) \) is written as

\[ E_o(r,z) = A_o \exp(i\Psi(z)) \exp \left( -(1 - i\alpha(z))r^2 / R_L^2(z) \right), \]  
(A2)

where \( A_o \) is the amplitude on axis,

\[ \Psi(z) = \int_k \left( \Delta k(z') - i\Gamma(z') \right) dz', \]  
(A3)

\( \Delta k(z) \) is the wavenumber shift, \( \Gamma(z) \) is the spatial growth rate, \( \alpha(z) \) is related to the curvature of the optical wavefronts and \( R_L(z) \) is the spot size of the beam. To allow for an electron beam with a radius \( R_b(z) \) that varies along the propagation axis, the electron beam density is taken to be

\[ n_b(z,r) = n_b(0) \left( R_b^2(0) / R_b^2(z) \right) \exp \left( -r^2 / R_b^2(z) \right). \]  
(A4)

The SDE equations for the spot size and \( \alpha(z) \) are given by

\[ \frac{dR_b(z)}{dz} - \frac{2c\alpha(z)}{\omega} \frac{1}{R_L(z)} = -H_r(z)R_L(z), \]  
(A5)

and

\[ \frac{d\alpha(z)}{dz} \left( \frac{2c}{\omega} \right) \frac{1 + \alpha^2(z)}{R_L^2(z)} = 2H_r(z) - \alpha(z)H_i(z), \]  
(A6)

respectively. Here,

\[ H(z) = \frac{4v}{\gamma_o} \frac{c}{\omega} \left( \frac{K^2 k_w^2}{1 + K^2 / 2} \right) \frac{1}{R_b^2(z)} \frac{f(z)}{(1 + 2f(z))^2} \frac{1}{(\Delta k(z) - i\Gamma(z))^2}, \]  
(A7)

where \( v = I_b[kA]/17 \) is Budker’s parameter, \( k_w = 2\pi/\lambda_w \), \( f(z) = R_b^2(z) / R_L^2(z) \) is the filling factor and the subscripts R and I denote real and imaginary parts. The final SDE equation for the phase shift and growth rate is given by
\[ \Delta k(z) - i \Gamma(z) + \left( \frac{c}{\omega} \right) \left( 1 + \alpha^2(z) \right) - i \frac{(1-i \alpha(z))}{R_L(z)} \frac{dR_L(z)}{dz} + \frac{1}{2} \frac{d\alpha(z)}{dz} = -(1 + 2f(z))H(z) \]  

(A8)

Equations (A5), (A6) and (A8) describe the optical beam in the high gain FEL amplifier. The electron beam can be focused to produce a pinched optical beam.