Monolithic Compiler Experiments Using C++ Expression Templates*

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** Dr. Mullin participated in this work while on sabbatical leave from the Dept. of Computer Science, University of Albany, State University of New York, Albany, NY.
**Report Documentation Page**

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Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18
Outline

• Overview
  – Motivation
  – The Psi Calculus
  – Expression Templates
• Implementing the Psi Calculus with Expression Templates
• Experiments
• Future Work and Conclusions
Motivation: The Mapping Problem

Mathematics of Arrays
- Math and indexing operations in same expression
- Framework for design space search
  - Rigorous and provably correct
  - Extensible to complex architectures

Example: “raising” array dimensionality

x: < 0 1 2 ... 35 >

Map:

```
< 0 1 2 >
< 3 4 5 >
< 6 7 8 >
< 9 10 11 >
< 12 13 14 >
< 15 16 17 >
< 18 19 20 >
< 21 22 23 >
< 24 25 26 >
< 27 28 29 >
< 30 31 32 >
< 33 34 35 >
```
Basic Idea

Combining Expression Templates and Psi Calculus yields an optimal implementation of array operations.

- **Expression Templates**
  - Efficient high-level container operations
  - C++

- **Psi Calculus**
  - Array operations that compose efficiently
  - Minimum number of memory reads/writes

**Benefits**
- Theory based
- High level API
- Efficient

**Implementation**

**Theory**

**PETE Style Array Operations**
Psi Calculus$^1$ Key Concepts

Denotational Normal Form (DNF):
- Minimum number of memory reads/writes for a given array expression
- Independent of data storage order

Operational Normal Form (ONF):
- Like DNF, but takes data storage into account
- For 1-d expressions, consists of one or more loops of the form:
  \[ x[i] = y[\text{stride} \cdot i + \text{offset}], \quad l \leq i < u \]
- Easily translated into an efficient implementation

Gamma function:
- Specifies data storage order

Psi Calculus rules are applied mechanically to produce the DNF, which is optimal in terms of memory accesses.
- The Gamma function is applied to the DNF to produce the ONF, which is easily translated to an efficient implementation.

---

### Some Psi Calculus Operations

<table>
<thead>
<tr>
<th>Operations</th>
<th>Arguments</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>take</td>
<td>Vector A, int N</td>
<td>Forms a Vector of the first N elements of A</td>
</tr>
<tr>
<td>drop</td>
<td>Vector A, int N</td>
<td>Forms a Vector of the last (A.size-N) elements of A</td>
</tr>
<tr>
<td>rotate</td>
<td>Vector A, int N</td>
<td>Forms a Vector of the last N elements of A concatenated to the other elements of A</td>
</tr>
<tr>
<td>cat</td>
<td>Vector A, Vector B</td>
<td>Forms a Vector that is the concatenation of A and B</td>
</tr>
<tr>
<td>unaryOmega</td>
<td>Operation Op, dimension D, Array A</td>
<td>Applies unary operator Op to D-dimensional components of A (like a for all loop)</td>
</tr>
<tr>
<td>binaryOmega</td>
<td>Operation Op, Dimension Adim. Array A, Dimension Bdim, Array B</td>
<td>Applies binary operator Op to Adim-dimensional components of A and Bdim-dimensional components of B (like a for all loop)</td>
</tr>
<tr>
<td>reshape</td>
<td>Vector A, Vector B</td>
<td>Reshapes B into an array having A.size dimensions, where the length in each dimension is given by the corresponding element of A</td>
</tr>
<tr>
<td>iota</td>
<td>int N</td>
<td>Forms a vector of size N, containing values 0 . . N-1</td>
</tr>
</tbody>
</table>
# Convolution: Psi Calculus Decomposition

## Definition of $y = \text{conv}(h,x)$

$$y[n] = \sum_{k=0}^{M-1} h[k] x'[n-k]$$

where $x$ has $N$ elements, $h$ has $M$ elements, $0 \leq n < N+M-1$, and $x'$ is $x$ padded by $M-1$ zeros on either end.

## Algorithm and Psi Calculus Decomposition

<table>
<thead>
<tr>
<th>Step</th>
<th>Algorithm step</th>
<th>Psi Calculus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial step</td>
<td>$x= &lt;1\ 2\ 3\ 4&gt;$ $h= &lt;5\ 6\ 7&gt;$</td>
<td>$x= &lt;1\ 2\ 3\ 4&gt;$ $h= &lt;5\ 6\ 7&gt;$</td>
</tr>
<tr>
<td>Form $x'$</td>
<td>$x'=\text{cat}(\text{reshape(&lt;k-1&gt;,&lt;0&gt;), cat(x, reshape(&lt;k-1&gt;,&lt;0&gt;)))}$</td>
<td>$x'= &lt;0\ 0\ 0\ \ldots\ 4\ 0\ 0&gt;$</td>
</tr>
<tr>
<td>Rotate $x'$ ($N+M-1$) times</td>
<td>$x'_\text{rot}=\text{binaryOmega}(\text{rotate,0,iota(N+M-1),1,x'})$</td>
<td>$x'_\text{rot}= &lt;0\ 0\ 1\ \ldots\ &gt;$</td>
</tr>
<tr>
<td>Take the “interesting” part of $x'_\text{rot}$</td>
<td>$x'<em>\text{final}=\text{binaryOmega}(\text{take,0,reshape(&lt;N+M-1&gt;,&lt;M&gt;),1,x'</em>\text{rot}})$</td>
<td>$x'_\text{final}= &lt;0\ 0\ 1\ &gt;$</td>
</tr>
<tr>
<td>Multiply</td>
<td>Prod=\text{binaryOmega}(*,1,h,1,x'_\text{final})</td>
<td>Prod$= &lt;0\ 0\ 7\ &gt;$</td>
</tr>
<tr>
<td>Sum</td>
<td>$Y=\text{unaryOmega}($sum,1,Prod$)$</td>
<td>$Y= &lt;7\ 20\ 38\ \ldots\ &gt;$</td>
</tr>
</tbody>
</table>

Psi Calculus reduces this to DNF with minimum memory accesses.
Main
1. Pass B and C references to operator +
2. Create temporary result vector
3. Calculate results, store in temporary
4. Return copy of temporary
5. Pass results reference to operator =
6. Perform assignment

Operator +

2 temporary vectors created

Operator =

Example: A = B + C vector add

Additional Memory Use
- Static memory
- Dynamic memory (also affects execution time)

Additional Execution Time
- Cache misses/page faults
- Time to create a new vector
- Time to create a copy of a vector
- Time to destruct both temporaries
C++ Expression Templates and PETE

- **PETE, the Portable Expression Template Engine, is available from the Advanced Computing Laboratory at Los Alamos National Laboratory**
  - PETE provides:
    - Expression template capability
    - Facilities to help navigate and evaluating parse trees

**Expression**

\[ A = B + C \]

**Expression Type**

```
BinaryNode<OpAdd, Reference<Vector>, Reference<Vector> >
```

**Main**

1. Pass B and C references to operator +
2. Create expression parse tree
3. Return expression parse tree
4. Pass expression tree reference to operator
5. Calculate result and perform assignment

Parse trees, not vectors, created

- **Reduced Memory Use**
  - Parse tree only contains references

- **Reduced Execution Time**
  - Better cache use
  - Loop fusion style optimization
  - Compile-time expression tree manipulation

**PETE:** http://www.acl.lanl.gov/pete
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• Implementing the Psi Calculus with Expression Templates
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Implementing Psi Calculus with Expression Templates

Example:
A = take(4, drop(3, rev(B)))
B = <1 2 3 4 5 6 7 8 9 10>
A = <7 6 5 4>

1. Form expression tree
   - take
     - 4
   - drop
     - 3
   - rev
     - B
Example:
A=take(4,drop(3,rev(B)))
B=<1 2 3 4 5 6 7 8 9 10>
A=<7 6 5 4>
Example:
A = \text{take}(4, \text{drop}(3, \text{rev}(B)))

B = \langle 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \rangle 
A = \langle 7 \ 6 \ 5 \ 4 \rangle
Example:
A = take(4, drop(3, rev(B)))

B = [1 2 3 4 5 6 7 8 9 10]
A = [7 6 5 4]
Implementing Psi Calculus with Expression Templates

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Example:
A = \text{take}(4, \text{drop}(3, \text{rev}(B)))

B = <1 2 3 4 5 6 7 8 9 10>
A = <7 6 5 4>

1. Form expression tree
2. Add size information
3. Apply Psi Reduction rules

\text{take} 4
\text{drop} 3
\text{rev} B

size = 4
size = 7
size = 10

size = 10
A[i] = B[i]
Implementing Psi Calculus with Expression Templates

Example:
A = take(4, drop(3, rev(B)))

B = [1 2 3 4 5 6 7 8 9 10]
A = [7 6 5 4]

1. Form expression tree
2. Add size information
3. Apply Psi Reduction rules

Size info
Reduction
Example:
A = take(4, drop(3, rev(B)))

B = <1 2 3 4 5 6 7 8 9 10>
A = <7 6 5 4>

1. Form expression tree

2. Add size information

3. Apply Psi Reduction rules

size = 7
A[i] = B[-(i+3)+9]
   = B[-i+6]

size = 10
A[i] = B[-i+B.size-1]
   = B[-i+9]

size = 10
A[i] = B[i]
Implementing Psi Calculus with Expression Templates

Example:
A = take(4, drop(3, rev(B)))

B = <1 2 3 4 5 6 7 8 9 10>
A = <7 6 5 4>

Recall:
Psi Reduction for 1-d arrays always yields one or more expressions of the form:

\[ x[i] = y[\text{stride} \cdot i + \text{offset}] \]

\( i \leq u \)

1. Form expression tree
2. Add size information
3. Apply Psi Reduction rules
Implementing Psi Calculus with Expression Templates

Example:
A = take(4, drop(3, rev(B)))
B = <1 2 3 4 5 6 7 8 9 10>
A = <7 6 5 4>

1. Form expression tree
2. Add size information
3. Apply Psi Reduction rules
4. Rewrite as sub-expressions with iterators at the leaves, and loop bounds information at the root

Recall:
Psi Reduction for 1-d arrays always yields one or more expressions of the form:
\[ x[i] = y[\text{stride} \times i + \text{offset}] \]
\[ l \leq i < u \]

- Iterators used for efficiency, rather than recalculating indices for each \( i \)
- One “for” loop to evaluate each sub-expression
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Experiments

Results

• Loop implementation achieves good performance, but is problem specific and low level.

• Traditional C++ operator implementation is general and high level, but performs poorly when composing many operations.

• PETE/Psi array operators perform almost as well as the loop implementation, compose well, are general, and are high level.

Execution Time Normalized to Loop Implementation (vector size = 1024)

Test ability to compose operations
Experimental Platform and Method

Hardware
- **DY4 CHAMP-AV Board**
  - Contains 4 MPC7400’s and 1 MPC 8420
- **MPC7400 (G4)**
  - 450 MHz
  - 32 KB L1 data cache
  - 2 MB L2 cache
  - 64 MB memory/processor

Software
- **VxWorks 5.2**
  - Real-time OS
- **GCC 2.95.4 (non-official release)**
  - GCC 2.95.3 with patches for VxWorks
  - Optimization flags: -O3 -funroll-loops -fstrict-aliasing

Method
- Run many iterations, report average, minimum, maximum time
  - From 10,000,000 iterations for small data sizes, to 1000 for large data sizes
- All approaches run on same data
- Only average times shown here
- Only one G4 processor used

• Use of the VxWorks OS resulted in very low variability in timing
• High degree of confidence in results
Experiment 1:
A=rev(B)

- PETE/Psi implementation performs nearly as well as hand coded loop, and much better than regular C++ implementation
- Some overhead associated with expression tree manipulation
Experiment 2:
\[ a = \text{rev} (\text{take}(N, \text{drop}(M, \text{rev}(b)))) \]

- Larger gap between regular C++ performance and performance of other implementations \( \rightarrow \) regular C++ operators do not compose efficiently
- Larger overhead associated with expression-tree manipulation due to more complex expression
Experiment 3:
\( a = \text{cat}(b+c, d+e) \)

- Still larger overhead associated with tree manipulation due to \text{cat}()
- Overhead can be mitigated by “setup” step prior to assignment
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Future Work

• **Multiple Dimensions:** Extend this work to N-dimensional arrays (N is any non-negative integer)
• **Parallelism:** Explore dimension lifting to exploit multiple processors
• **Memory Hierarchy:** Explore dimension lifting to exploit levels of memory
• **Mechanize Index Decomposition:** Currently a time consuming process done by hand
• **Program Block Optimizations:** PETE-style optimizations across statements to eliminate unnecessary temporaries
Conclusions

• Psi calculus provides rules to reduce array expressions to the minimum of number of reads and writes

• Expression templates provide the ability to perform compiler preprocessor-style optimizations (expression tree manipulation)

• Combining Psi calculus with expression templates results in array operators that
  – Compose efficiently
  – Are high performance
  – Are high level

• The C++ template mechanism can be applied to a wide variety of problems (e.g. tree traversal ala PETE, graph traversal, list traversal) to gain run-time speedup at the expense of compile time/space