Thermodynamic Limitations of Energy Conversion in Laser Propulsion

C.W. Larson, F.B. Mead, Jr., S.D. Knecht (AFRL/PRSP)

Air Force Research Laboratory (AFMC)
AFRL/PRS
5 Pollux Drive
Edwards AFB CA 93524-7048

Air Force Research Laboratory (AFMC)
AFRL/PRS
5 Pollux Drive
Edwards AFB CA 93524-7048

Approved for public release; distribution unlimited.

For presentation at the 10th International Workshop of Combustion & Propulsion – In-Space in La Spezia, Italy, taking place 21-25 September 2003.

16. SECURITY CLASSIFICATION OF:
   a. REPORT Unclassified
   b. ABSTRACT Unclassified
   c. THIS PAGE Unclassified

17. LIMITATION OF ABSTRACT
   A

18. NUMBER OF PAGES 22

19a. NAME OF RESPONSIBLE PERSON
    Leilani Richardson

19b. TELEPHONE NUMBER (include area code)
    (661) 275-5015
Thermodynamic Limitations on Energy Conversion in Laser Propulsion

C. William Larson, Franklin B. Mead, Jr., and Sean D. Knecht
Propulsion Directorate
Air Force Research Laboratory
Edwards AFB, CA 93524-7680

10th International Workshop on Combustion and Propulsion
IN-SPACE PROPULSION
21-25 September 2003
Lerici, La Spezia, Italy

Approved for public release; distribution unlimited.

Outline

Tour of White Sands Laboratory (HELSTF/PLVTS) and Video of Flight Testing.

Comparison of Constant Momentum Mission and Constant Specific Impulse Mission. \( \Delta v, v_{\text{jet}}, f, m_0, v_0, P_{\text{jet}}, m/E_{\text{jet}} \)

Efficiency of conversion of laser energy to propellant kinetic energy, \( \alpha \beta \).

Upper limit to conversion of laser energy to jet kinetic energy from energy conservation and definitions: \( C_{v_{\text{jet}}} = \alpha \beta \Phi < 1 \).

Comparing momentum quantities to energy quantities. The “Phi Factor” \( \Phi = \frac{\langle v^2 \rangle}{\langle v^2 \rangle} \) and velocity distributions in propellant jet. \( \Phi \) values for delta function, Maxwellian, Gaussian, Chunks and gas, supersonic expansion, etc.

Upper limits to performance based on chemical thermodynamics. Blowdown from defined equilibrium state \((u, \rho)\) of known volume.

Conclusions

Approved for public release; distribution unlimited.
Pulsed Laser Vulnerability Test System (PLVTS)

- Original Performance
  - 800 joules/pulse
  - 10 Hz
  - 30 μsec pulses
- Modified Performance
  - 1998
    - 400 joules/pulse
    - 28 Hz
    - 18 μsec pulses
  - 1999
    - 150 joules/pulse
    - 30 Hz
    - 5 μsec pulses

Approved for public release; distribution unlimited.

Field Test Telescope (FTT)
- 50 cm
- Cassegrainian
- Dynamic Focusing
- Minimum Acquisition Distance is 200 m

Laser Beam Handoff to This Telescope Should Allow Altitudes of ~300 m (1,000 ft)

Approved for public release; distribution unlimited.
Optical Bench Set Up At 500-Ft Mark

Optical Power vs Time:
  a) 2.5 \mu s; b) 5 \mu s; c) 18 \mu s; d) 35 \mu s

Approved for public release; distribution unlimited.
Laboratory Telescope Burn Patterns

Near Field At ~10 Ft

5 cm Ref. 500 Ft

Approved for public release; distribution unlimited.

FTT Beam Burn Patterns

500 Ft 1,000 Ft

11 cm Ref. 1,500 Ft

Approved for public release; distribution unlimited.
Figure 2. Cross-sectional view of Myrabo Laser Lightcraft, Model 200-3/4. The maximum diameter of the test article at the shroud is ~10 cm. The indicated ring of Delrin weighs ~10 g and has a volume of ~7 cm³ and a surface area ~25 cm². The idealized maximum plug nozzle exit area is ~350 cm².

Approved for public release; distribution unlimited.
Overall Energy Conversion in Laser Propulsion Mission

\[ E_r = \frac{1}{2} m_r v_r^2 = \eta \alpha \beta \gamma \delta E_{\text{wall}} \]

\( \eta = \) propulsion efficiency (jet kinetic energy to vehicle kinetic energy)

\( \alpha = \) expansion efficiency (internal propellant energy to jet kinetic energy)

\( \beta = \) absorption efficiency (laser energy at vehicle to internal propellant energy)

\( \gamma = \) transmission efficiency (laser energy at ground to laser energy at vehicle)

\( \delta = \) laser efficiency (electric energy to laser energy at ground)

***** Issue: separability of \( \eta \alpha \beta \gamma \) *****

"$500 worth of electricity to put 1 kg into LEO."

At $0.10/KWH, $500 buys 18,000 MJ (1 KWH = 3.6 MJ); 1 kg at 10 km/s \( \rightarrow E_r = 50 \text{ MJ} \), so \( \eta \alpha \beta \gamma \delta \geq 0.0028 \)

Phipps, Reilly, Campbell, Laser & Particle Beams 18 (2001) 661-695

Approved for public release; distribution unlimited.

INSTANTANEOUS PROPULSION EFFICIENCY

\[ \eta_i = \frac{2 (v / v_{\text{jet}})}{1 + (v / v_{\text{jet}})^2} \]

CONSTANT MOMENTUM COMPARED TO CONSTANT SPECIFIC IMPULSE MISSION

The Constant Specific Impulse Mission

\[ \int_{m_0}^{m} \frac{dm}{m} = -\frac{1}{v_{\text{jet}}} \int_{v_0}^{v} dv \]

\[ f = \frac{m}{m_0} = \exp \left( \frac{v - v_0}{v_{\text{jet}}} \right) = \exp \left( \frac{-\Delta v}{v_{\text{jet}}} \right) \]

The Constant Momentum Mission

\[ \int_{m_0}^{m} \frac{dm}{m} = -\frac{1}{f'} \int_{v}^{v_0} dv \]

\[ f' = \frac{m}{m_0} = \frac{v_0}{v} = 1 - \frac{\Delta v}{v} = \left( 1 + \frac{\Delta v}{v_0} \right)^{-1} \]

Approved for public release; distribution unlimited.
Figures of Merit for Laser Propulsion: $m/E_{jet}$

The Constant Specific Impulse Mission

$$E_{jet} = -\frac{1}{2} \int v_{jet}^2 \, dm = \frac{1}{2} (m_0 - m) v_{jet}^2$$

$$B = \frac{m}{\frac{1}{2} (m_0 - m) v_{jet}^2} = \frac{2x^2}{(e^x - 1)[(\Delta v)^2]} = \frac{2f(ln f)^2}{(1 - f)[(\Delta v)^2]}$$

The Constant Momentum Mission

$$E_{jet} = -\frac{1}{2} \int v^2 \, dm = -\frac{1}{2} (m v_0)^2 \int \frac{dm}{m_0} = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 = \frac{1}{2} m v \Delta v$$

$$B' = \frac{m}{\frac{1}{2} m v \Delta v} = \frac{2(1 - f')}{[\Delta v]^2}$$
MISSION TIME FOR CONSTANT SPECIFIC IMPULSE AND CONSTANT MOMENTUM MISSIONS

**Constant Specific Impulse**

\[ P_{\text{jet}} = \frac{1}{2} v_{\text{jet}}^2 \frac{dm}{dt} = \frac{1}{2} F v_{\text{jet}} \]

\[ f = \frac{m}{m_0} = 1 - \frac{2P_{\text{jet}}}{m_0 v_{\text{jet}}^2} \]

\[ t = \frac{m_0}{2P_{\text{jet}}} \frac{(\Delta v)^2 (1-f)}{(\ln f)^2} \]

\[ \Delta v = -v_{\text{jet}} \ln \left(1 - \frac{2P_{\text{jet}}}{m_0 v_{\text{jet}}^2} \right) \]

\[ t' = \frac{m}{m_0} = \left[1 + \frac{2P_{\text{jet}}}{m_0 v_{\text{jet}}^2} t' \right]^{-1} \]

\[ t' = \frac{m_0}{2P_{\text{jet}}} (\Delta v)^2 \frac{f}{(1-f)} \]

**Constant Momentum**

\[ P_{\text{jet}} = \frac{1}{2} v^2 \frac{dm}{dt} = \frac{1}{2} F v \]

\[ f' = \frac{m}{m_0} = \left[1 + \frac{2P_{\text{jet}}}{m_0 v^2} t' \right]^{-1} \]

\[ t' = \frac{2P_{\text{jet}}}{m_0} \left( \frac{1-f}{f'} \right) \]

\[ t' = \frac{2}{B} - \frac{2P_{\text{jet}}}{m_0} t' \]

Approved for public release; distribution unlimited.

---

(a) Constant Specific Impulse Mission

(b) Constant Momentum Mission

---

Approved for public release; distribution unlimited.

*trajmod* Aug 25, 2003 9:00:16 AM
Definitions and Energy Conservation

Propellant Kinetic Energy: \[ E_p = \frac{1}{2} m_p \langle v_e^2 \rangle = \alpha \beta E_L \Rightarrow \langle v_e^2 \rangle = \frac{\int_0^r \int d(\rho \nu_e^2)}{\rho_f} \]

Propellant Impulse: \[ I = m_p \langle v_e \rangle \Rightarrow \langle v_e \rangle = \frac{\int_0^r \int d(\rho \nu_e)}{\rho_f} \]

Coupling Coefficient: \[ C = \frac{I}{E_L} \Rightarrow C = \frac{2\alpha \beta}{\langle v_e \rangle} \left(\frac{\langle v_e \rangle^2}{\langle v_e^2 \rangle} - 1\right) \]

Energy Conservation: \[ \alpha \beta \Phi = \frac{I^2}{2 m_p E_L} = \frac{C \langle v_e \rangle}{2} = \frac{1}{2E_L} \leq 1 \]

Propellant Internal Energy: \[ Q^* = u_e - u_0 = \frac{\beta E_L}{m_p} \Rightarrow C = \frac{\beta \langle v_e \rangle}{u_e - u_0} \]

Propellant with added chemical energy, \( \Delta u \): \( (\alpha \beta \Phi)^{\text{apparent}} = \alpha \Phi(\beta + m_p \Delta u/E_L) \)

Approved for public release; distribution unlimited.
Maxwell distribution of velocity in three dimensions

\[ f(v) \, dv = \frac{1}{(2\pi)^{3/2}m/kT} v^2 \exp(-mv^2/2kT) \, dv \]

\[ \langle v \rangle = \sqrt{8kT/\pi m} \]

\[ \langle v^2 \rangle^{1/2} = \sqrt{3kT/m} \]

\[ \Phi = \frac{\langle v^2 \rangle}{\langle v \rangle^2} = \frac{8}{3\pi} \approx 0.848826 \]
Approved for public release: distribution unlimited.

G(v) = \frac{11.2 \text{m}^2}{\text{y}^2} \exp \left( \frac{v^2}{v_0^2} \right)

\text{velocity (Km/s)}

Laser Sustained Plasma Jet as a Tool for Propulsion.
\( \Phi \) for Bimodal velocity distribution

Chunks of propellant, \( f_{\text{heavy}} \) mass fraction, \( v_{\text{slow}} \) velocity
Hot gases, \( f_{\text{light}} \) mass fraction, \( v_{\text{fast}} \) velocity

\[ <v^2> = (f_{\text{heavy}}v_{\text{slow}} + f_{\text{light}}v_{\text{fast}})^2 \]

\[ <v^2> = f_{\text{heavy}}v_{\text{slow}}^2 + f_{\text{light}}v_{\text{fast}}^2 \]

\( \Phi = <v^2>/<v^2> = (f_{\text{heavy}} + f_{\text{light}})r^2/(f_{\text{heavy}} + f_{\text{light}})(r^2) \) where \( r = v_{\text{fast}}/v_{\text{slow}} > 1 \)

Approved for public release; distribution unlimited.
Conclusions

When $P_{\text{Laser}}/m_o \sim 0.05 \text{ MW/kg}$ small payloads (2 to 4 kg) may be launched into low earth orbit, $\Delta v \sim 10,000 \text{ m/s}$.

At the same mass fraction, $f = 0.2$, $m/E_{\text{jet}}$ for constant momentum mission is 23% greater than for constant specific impulse mission.

For $\Delta v = 10,000 \text{ m/s}$, $m_o/P_{\text{jet}} = 20 \text{ kg/MW}$, $f = 0.2$, $v_o = 0$, the mission time for constant specific impulse propulsion is $\sim 315 \text{ sec}$.

For $\Delta v = 10,000 \text{ m/s}$, $m_o/P_{\text{jet}} = 20 \text{ kg/MW}$, $f = 0.2$, $v_o = 2000 \text{ m/s}$, the mission time for constant momentum propulsion is $\sim 155 \text{ sec}$.

At the same $m/E_{\text{jet}} = 0.013 \text{ kg/MJ}$ and $\Delta v$, $f(\text{constant momentum}) = 0.35$, and $f(\text{constant specific impulse}) = 0.20$.

Based on measured $I$, $E_L$, and ablated mass, overall energy conversion efficiencies (laser energy to jet kinetic energy) of $\alpha \beta \sim 50\%$ were obtained with Delrin propellant in the laser lightcraft.

Jet exit velocities of $\sim 2000 \text{ m/s}$ with Delrin (based on measured mass) and $\sim 3000 \text{ m/s}$ with air (based on estimated mass).

Approved for public release; distribution unlimited.

---

THE COUPLING COEFFICIENT AND THE SPECIFIC IMPULSE

$Q^* = \beta E_L/m$

$E_{\text{jet}} = \frac{1}{2} m <v^2> = \alpha m Q^* = \alpha \beta E_L$

$I = m <v>$

$C = \frac{I}{E_L}$

$\frac{1}{2} C <v> = \alpha \beta \Phi \leq 1$

$p_L = \omega E_L$

$F = \omega E_L \dot{C}$

$\frac{1}{2} F <v> = \alpha \beta \Phi p_L$

$p_{\text{jet}} = \frac{1}{2} \frac{F <v>}{\Phi} = \alpha \beta p_L$

$(\alpha \beta \Phi)_{\text{apparent}} = \alpha \Phi (\beta + m \Delta u_{\text{circ}}/E_L)$

Approved for public release; distribution unlimited.
Laser Lightcraft Flights with Air: \[ h = \frac{1}{2} \frac{1}{T} \left( \frac{C_E \cdot w}{M \cdot g} \right) = \frac{1}{2} g t^2 (T/W - 1) \]

Model 200-3/4, \( M = 0.04 \text{ kg, 10 kW at } w = 25 \text{ Hz, } E_L = 400 \text{ J/pulse} \)

Approved for public release; distribution unlimited.

---

Model 200-3/4 Lightcraft with Air and Delrin
- Delrin with tight focus
- Air with tight focus
- Air with loose focus

Approved for public release; distribution unlimited.
Table 1. Normalized absorption volume for air at 1.18 kg/m³ as a function of internal energy and laser energy.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42.3</td>
<td>48.9</td>
<td>57.1</td>
<td>69.4</td>
<td>83.7</td>
</tr>
<tr>
<td>2</td>
<td>21.1</td>
<td>28.3</td>
<td>38.4</td>
<td>51.7</td>
<td>67.4</td>
</tr>
<tr>
<td>3</td>
<td>14.1</td>
<td>28.2</td>
<td>34.5</td>
<td>41.5</td>
<td>51.7</td>
</tr>
<tr>
<td>4</td>
<td>10.1</td>
<td>21.5</td>
<td>34.7</td>
<td>41.3</td>
<td>51.5</td>
</tr>
<tr>
<td>5</td>
<td>8.67</td>
<td>16.9</td>
<td>19.4</td>
<td>24.5</td>
<td>30.3</td>
</tr>
<tr>
<td>6</td>
<td>7.63</td>
<td>14.0</td>
<td>17.6</td>
<td>23.6</td>
<td>29.1</td>
</tr>
<tr>
<td>7</td>
<td>6.87</td>
<td>12.5</td>
<td>16.1</td>
<td>22.4</td>
<td>27.0</td>
</tr>
<tr>
<td>8</td>
<td>5.93</td>
<td>10.5</td>
<td>14.8</td>
<td>20.1</td>
<td>23.7</td>
</tr>
<tr>
<td>9</td>
<td>4.71</td>
<td>9.22</td>
<td>13.1</td>
<td>18.2</td>
<td>21.3</td>
</tr>
<tr>
<td>10</td>
<td>4.24</td>
<td>8.47</td>
<td>12.7</td>
<td>16.9</td>
<td>20.0</td>
</tr>
<tr>
<td>15</td>
<td>2.82</td>
<td>5.65</td>
<td>8.47</td>
<td>11.3</td>
<td>16.5</td>
</tr>
<tr>
<td>20</td>
<td>2.12</td>
<td>4.24</td>
<td>6.96</td>
<td>10.7</td>
<td>15.2</td>
</tr>
<tr>
<td>30</td>
<td>1.41</td>
<td>2.82</td>
<td>4.24</td>
<td>6.58</td>
<td>9.73</td>
</tr>
<tr>
<td>40</td>
<td>1.06</td>
<td>2.12</td>
<td>3.18</td>
<td>4.24</td>
<td>6.36</td>
</tr>
<tr>
<td>50</td>
<td>0.85</td>
<td>1.69</td>
<td>2.54</td>
<td>3.59</td>
<td>5.09</td>
</tr>
<tr>
<td>60</td>
<td>0.71</td>
<td>1.41</td>
<td>2.12</td>
<td>2.82</td>
<td>4.04</td>
</tr>
<tr>
<td>70</td>
<td>0.61</td>
<td>1.21</td>
<td>1.82</td>
<td>2.42</td>
<td>3.63</td>
</tr>
<tr>
<td>80</td>
<td>0.53</td>
<td>1.06</td>
<td>1.59</td>
<td>2.12</td>
<td>3.18</td>
</tr>
<tr>
<td>90</td>
<td>0.47</td>
<td>0.94</td>
<td>1.41</td>
<td>1.88</td>
<td>2.82</td>
</tr>
<tr>
<td>100</td>
<td>0.42</td>
<td>0.83</td>
<td>1.37</td>
<td>1.69</td>
<td>2.54</td>
</tr>
<tr>
<td>110</td>
<td>0.39</td>
<td>0.77</td>
<td>1.46</td>
<td>1.54</td>
<td>2.31</td>
</tr>
</tbody>
</table>

Figure 1. Cross-sectional view of Nyrobo Laser Light® Model 200-3W. The maximum diameter of the test article at the shroud is ~10 cm. The indicated ring of Delrin weighs ~10 g and has a volume of ~7 cm³ and a surface area ~25 cm². The identified plume nozzle exit area is ~330 cm².

Approved for public release; distribution unlimited.

Approved for public release; distribution unlimited.
Approved for public release; distribution unlimited.
Figure 10. Comparison of Equilibrium expansion and frozen expansion of the Th-cold and Th-frozen expanded runs with the Th-0.1 expanded runs. The comparison shows that the differences in the normalized coupling coefficients, $C_{\text{norm}}$, are small, as are the changes in the free energy ($\Delta G$), temperature ($\Delta T$), and alpha ($\Delta \alpha$) for the Th-cold and Th-frozen expanded runs.
Figure 11. Comparison of equilibrium expansion from isentropic STP air (1.18 kgs/m^3) and Mach 5 air at stagnation density (5.9 kgs/m^3). In the STP air diagram on left, the circles and nearby crosses represent the blowdown quantities obtained from initial [u,e]^* states of 2E3, and 1E4 J/kg for the frozen expansion and 2E3, 6E3, 1E4, and 4E4 J/kg for the equilibrium expansion.

H_2, O_2, H_2O, OH, H, O Equilibrium ideal mixture of ideal gases (O/F)_{mass} = 6 

u = 0  

\[ h_{propellant} = 0.985655 \times 10^6 \text{ J/kg} \] 

\[ h_{ambient} = 0.983268 \times 10^6 \text{ J/kg} \] 

\[ u_{min} = 13.636 \times 10^6 \text{ J/kg} \] 

specific enthalpy (J/kg) 

specific entropy (J/kg-K) 

area ratio