Analysis of Bearing Errors from Acoustic Arrays

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Abstract

This report presents a limited analysis of bearing estimates based on acoustic sensors. Significant biases are found in the bearing estimates of all the acoustic sensor arrays involved in the experiment. The explanation of the biases in terms of other factors, such as wind, range and SNR was beyond the scope of this study. However, the biases do explain much of the track bias observed in recent tracker evaluations.

1 Introduction

This analysis was motivated by observations of substantial biases in tracks based on bearing data from acoustical arrays. The particular data analyzed here was provided by the ARMY Center of Excellence in Acoustics, TACOM/ARDEC, AMSTA-AR-FSF-R, Picatinny Arsenal. The geometric configuration of the four sensor arrays is presented in Figure 1 along with the oval roadway that was traversed by the test vehicle.

Figure 1: Array sensor positions and centerline of test track.
The tracking exercise consisted of 4 counter-clockwise loops and 4 clockwise loops made by an acoustic target. This study is limited to the first counter clockwise loop, denoted here as CCW#1. The position estimates provided with the data were developed by a sequential localizer which gathered data into groups and performed a position estimate using something like a least squares method. (The exact method is unknown to us.) The positions developed by this method are plotted in Figure 2 along with a dashed line to a point marked with "x" where the target was observed. Figure 3 presents

Figure 2: Sequential localizer position estimates (black) along with time aligned truth track (red "x") for segment CCW#1.

the positional errors relative to the mean X error $m_X = -10.52$ meters and the mean Y error $m_Y = -32.7$ meters. The standard deviation of the first principal component (along the line of slope 138.8°) is 26.3 meters and along the 2nd principal component it is 15.1 meters. So the bias $m_Y$ is very large (negatively, or to the south) compared to the variation in the first principal component. The histogram of miss distances is presented in Figure 4 which also shows the mean miss distance to be 43.6 meters.

A batch tracker is one in which all relevant measurement data in a time window are used to estimate the parameters of target motion. In this case target motion was assumed to be along a straight line. In statistical language, the data are regressed on the function

$$\tilde{P}(t) = [x(t_0) + v_x(t - t_0), y(t_0) + v_y(t - t_0), v_x, v_y]$$

where $x(t_0), y(t_0)$ are the coordinates at some reference time $t_0$ and $v_x, v_y$ are the velocity components. Further details of the batch tracking algorithm will be documented separately. Figure 5 shows the positional errors of the batch tracker relative to the mean X error $m_X = -3.49$ meters and the mean Y error $m_Y = -16.7$ meters. The standard deviation

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Figure 3: Scatter plot of position errors of sequential localizer from segment CCW#1.
of the first principal component (along the line of slope 201.5°) is 18.3 meters and along
the 2nd principal component it is 12.8 meters. So the mean errors of the tracker positions
are about half of the sequential localizer and the standard deviations along the principal
components are reduced as well. However, the southerly bias \( m_Y = -16.7 \) is still substantial.
The histogram of miss distances is presented in Figure 6 which also shows the mean miss
distance to be 22.3 meters.

In order to see the effect of biases in the bearings, the experimental bearings were
replaced with simulated normally distributed bearings (taken modulo 360deg) all having
zero mean and \( \sigma = 3 \) degrees. Figure 7 shows the positional errors of the batch tracker
relative to the mean \( X \) error \( m_X = .14 \) meters and the mean \( Y \) error \( m_Y = -.97 \) meters.
The standard deviation of the first principal component (along the line of slope 93.2°) is
10.8 meters and along the 2nd principal component it is 5.7 meters. So the mean errors
of the tracker positions based on simulated bearings are less than 1/10 th of the mean
errors of those produced by operational bearing estimates. The standard deviations along
the principal components are also about half those based on the operational data. The
histogram of miss distances presented in Figure 8 shows the mean miss distance to be 9.76
meters, less than half that of the operational data shown in Figure 6. The various miss
distances and principal component standard deviations are summarized for the three cases
in Table 1.
Figure 5: Scatter plot of position errors of batch tracker from segment CCW#1.
Figure 6: Histogram of miss distances of batch tracker from segment CCW#1. Mean miss distance is 22.3 meters.

Figure 7: Scatter plot of position errors of batch tracker from segment CCW#1 using simulated bearings with $\sigma = 3$. 

Figure 8: Histogram of miss distances of batch tracker from segment CCW#1 using simulated bearings with $\sigma = 3$. Mean miss distance is 9.76 meters.
<table>
<thead>
<tr>
<th>Case</th>
<th>$m_X$</th>
<th>$m_Y$</th>
<th>$m_d$</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seq. Loc. CCW#1, Run No. 33</td>
<td>-10.52</td>
<td>-32.7</td>
<td>43.6</td>
<td>26.3</td>
<td>15.1</td>
</tr>
<tr>
<td>Bat. Tkr. CCW#1, Run No. 33</td>
<td>-3.49</td>
<td>-16.7</td>
<td>22.3</td>
<td>18.3</td>
<td>12.8</td>
</tr>
<tr>
<td>Bat. Tkr. simulated data</td>
<td>.14</td>
<td>-.97</td>
<td>9.76</td>
<td>10.8</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Table 1: Summary of positional errors. $m_d$ is mean miss discance, $\sigma_1$ is for largest principal component, corresponding to the semi-major axis of the ellipse derived from the sample covariance; $\sigma_2$ is for second largest principal component, corresponding to the semi-minor axis.

Figure 9: Red: Node 1 bearings. Black: true bearings.

2 Analysis of bearing errors

Bearings from each of the four nodes are presented in Figures 9 through 12. The measured bearings are in red and the true bearings in black. The blue dashed lines are at spacing of 45 degrees. In all these plots, the bearings that produce large errors are possibly due to interfering signals that capture the bearing estimation process for short times. We exclude these in this analysis as tracking programs often have methods of removing outliers. But in a multiple target environment, the problem needs to be given further thought. Note that the target bearings cross 0 degrees during the target trajectory for nodes 2 and 3, while this does not occur for nodes 1 and 4. For all nodes, the long stretch of constant bearing correspond to the target remaining fixed, or nearly fixed at the top of the roadway between the transits. Even at the scale used for these figures we can see evidence of "localized" biases.
Figure 10: Red: Node 2 bearings. Black: true bearings.

Figure 11: Red: Node 3 bearings. Black: true bearings.
**Figure 9, Node 1.** The stretches where the bearing is nearly constant at the top of the track produce biases that are negative (true bearing is larger than that reported); note this corresponds to a northerly bias. On the other hand, when the target is at the bottom of the track, the bearing bias is positive which would cause a southerly bias. The bias switches from negative to positive during the transit. Figure 13, in which the bearings around and during transit CCW#1 are presented, shows the bias sign change more clearly.

**Figure 10, Node 2.** In the northern part of the track, when the true bearings are around 85 degrees, the bias is negative, producing southerly and easterly biases in tracks. For bearings corresponding to the southern part of a transit, the biases are small or slightly negative which again produces southerly track bias.

**Figure 11, Node 3.** In the northern part of the track, when the true bearings are around 85 degrees, the bias is positive, producing northerly and westerly track bias. For bearings corresponding to the southern part of a transit, the bearing bias switches from positive to negative (in all of the transits) when the true bearing is approximately 280 degrees. Note that the minimum bearing in this region does not correspond to the most southerly point on the transit, which occurs later, roughly at the time when the bias switches. This would suggest, for this node, that the estimated bearing is consistently lagging the target in this region. This effect does not seem to be prevalent, but still it should be studied further.
Figure 12, Node 4. Bearing biases appear always to be positive. In both the northern and southern parts of a transit this will produce a southerly track bias.

Figure 13: Red: Node 1 bearings during CCW#1. Black: true bearings.

Since localized biases can be easily perceived in plots 9 through 12, histograms of bearing error were produced in which there was conditioning (as in conditional probability) on sectors of true bearing. The sectors were 30 deg in width and were chosen to cover the range of bearings in as few sectors as possible. The resulting histograms appear in Figures 15 through 18. The sample means and standard deviations are presented in Table 2. The N/S (north/south) effect on track bias entries are determined by consideration of the true target positions relative to the sensors at the time interval during which the measurements were made.

3 Conclusions

Target positions estimated using bearing measurements from the bearing data show definite a southerly bias. The southerly biases were $m_Y = -32.7$ the sequential localizer and $m_Y = -16.7$ for the batch tracker. These biases suggested the possibility of biases in the bearing measurements. Histograms of bearing errors conditioned on 30 degree true bearing sectors show the presence of significant bearing bias that appears consistent with the reported positional bias. Although bearing sector provides a method of analysis that is informative, the causes of bearing bias need further investigation. Possible causes for bearing bias include (a) sound speed errors including wind effects, (b) array configuration
Figure 14: (Top in all figures) Histograms of bearing errors from indicated node. (Bottom in all figures) Histograms of bearing errors in the range $[-20, 20]$ degrees.
Figure 15: Histograms of bearing errors by sector. Node 1 Cluster 1.

Figure 16: Histograms of bearing errors by sector. Node 2 Cluster 1.
Figure 17: Histograms of bearing errors by sector. Node 3 Cluster 1.

Figure 18: Histograms of bearing errors by sector. Node 4 Cluster 1.
Table 2: Summary of bearing analysis by sectors.

<table>
<thead>
<tr>
<th>Node</th>
<th>Sector endpoints</th>
<th>N</th>
<th>$m_e$</th>
<th>$\sigma_e$</th>
<th>Track bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(110,140)</td>
<td>721</td>
<td>-2.97</td>
<td>3.37</td>
<td>N</td>
</tr>
<tr>
<td>1</td>
<td>(140,170)</td>
<td>94</td>
<td>.21</td>
<td>2.90</td>
<td>S</td>
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<tr>
<td>1</td>
<td>(170,200)</td>
<td>154</td>
<td>1.39</td>
<td>1.37</td>
<td>S</td>
</tr>
<tr>
<td>1</td>
<td>(200,230)</td>
<td>123</td>
<td>2.23</td>
<td>1.47</td>
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<tr>
<td>2</td>
<td>(0, 30)</td>
<td>44</td>
<td>-6.50</td>
<td>2.80</td>
<td>S</td>
</tr>
<tr>
<td>2</td>
<td>(30,60)</td>
<td>72</td>
<td>-3.51</td>
<td>2.59</td>
<td>S</td>
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<tr>
<td>2</td>
<td>(60,90)</td>
<td>830</td>
<td>-2.47</td>
<td>2.20</td>
<td>S</td>
</tr>
<tr>
<td>2</td>
<td>(270,300)</td>
<td>99</td>
<td>-3.06</td>
<td>2.39</td>
<td>S</td>
</tr>
<tr>
<td>2</td>
<td>(300,330)</td>
<td>63</td>
<td>-5.0</td>
<td>3.73</td>
<td>S</td>
</tr>
<tr>
<td>2</td>
<td>(330,360)</td>
<td>31</td>
<td>-6.35</td>
<td>2.05</td>
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</tr>
<tr>
<td>3</td>
<td>(0, 30)</td>
<td>35</td>
<td>-3.56</td>
<td>2.91</td>
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<tr>
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<td>(30,60)</td>
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<td>1.56</td>
<td>2.21</td>
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<tr>
<td>3</td>
<td>(60,90)</td>
<td>623</td>
<td>.81</td>
<td>2.95</td>
<td>N</td>
</tr>
<tr>
<td>3</td>
<td>(270,300)</td>
<td>400</td>
<td>.09</td>
<td>3.08</td>
<td>N</td>
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<tr>
<td>3</td>
<td>(300,330)</td>
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<td>-7.25</td>
<td>2.84</td>
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<tr>
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<td>(330,360)</td>
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<td>S</td>
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<tr>
<td>4</td>
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<td>4</td>
<td>(180,210)</td>
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<tr>
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<td>99</td>
<td>3.88</td>
<td>3.64</td>
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</tbody>
</table>

Efforts should be undertaken to better understand these effects.