NMR 3D Diffractive Imaging technique
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Abstract — A new approach to MR angiography, the NMR diffractive imaging technique, has been investigated. The expression for NMR signals obtained in the NMR diffractive imaging technique is similar to the equation for Fresnel diffraction in light waves or sound waves. Therefore, it is possible to reconstruct three-dimensional images by converting the NMR signal to a hologram and using an optical image processing system from the data scanned two-dimensionally. Moreover, an image focusing on any plane in the depth direction can be reconstructed numerically from the data by changing an distance parameter in the reconstruction step. Experiments were performed using an ultra-low-field MRI scanner to acquire two-dimensional data in the proposed technique. Even though blurred images outside the focal plane are superimposed on the image in the focal plane, the three-dimensional distribution of the object can be recognized. This technique is expected to be useful in MR fast angiography.

Keywords — MRI, angiography, three-dimensional imaging, holography.

I. INTRODUCTION

Lately, a brain scanning examination, obtaining the image of angiography in the brain to discover any existence of abnormalities, has been utilized more frequently. Imaging angiography using an MRI scanner, however, requires considerable amount of time to collect three-dimensional data. Therefore, shortening the time for imaging is believed to relieve the burden of a study subject, followed by improving throughput because of increases in the number of subjects to be checked per hour. With potential possibilities, shortening the time for imaging seems to produce great effectiveness in the future.

In aiming to shorten the examination time, we studied utilization of the NMR Fresnel diffractive imaging technique, which enables us to obtain the images containing depth information by using the collected two-dimensional data. Although NMR phenomena is not a wave motion itself, it can take the similar form to the description of the wavefront in the Fresnel diffractive region by using non-linear field gradients. We have been studying the NMR imaging technique in which NMR signal has the form of Fresnel integral equation of two-dimensional object by scanning a nonlinear quadratic filed gradient over the imaging region[1], [2].

By developing the Fresnel transform imaging technique to three-dimensional one, it will be possible to obtain the NMR images containing depth information of the object by the signal scanned two-dimensionally as the hologram reproduces the three-dimensional images in the optical holography.

Diffractive tomography is a technique for obtaining subjective two-dimensional and three-dimensional distributions based on the measurement of field waves. This tomography has been applied to diffractive fields such as optical microscopes[3], [4] and techniques with supersonic waves[5], as suggested by Wolf concerning lighting problems[6].

In our study, aiming principally to image vascular tracts, we designed a coil system, which can obtain the same signal equations as those of three-dimensional Fresnel diffractive waves, and then experimented on its use.

According to the technique used, we can obtain the three-dimensional distribution of the subject in the time of two-dimensional acquisition, and we might be able to judge any existence of abnormalities rapidly.

II. NMR SIGNAL IN THE FRESNEL DIFRACTIVE IMAGING TECHNIQUE

Holography is a technique proposed by D.Gabor to record an object light wave scattered from the object on photosensitive materials by superimposing a reference light on the object light wave. Although wavefronts in two-dimensional distribution of the object are recorded, illuminating light wave on the hologram can reproduce the scattered wavefront from the object which can reconstruct natural three-dimensional images.

A. Wavefront equation in the Fresnel diffraction region

The diffractive wavefront $u(x_1, y_1)|_{z=z_1}$ scattered from the object $g(x_0, y_0, z_0)$ on the screen located at $z = z_1$ in the Fresnel region is written as Eq. (1) [8]

$$u(x_1, y_1)|_{z=z_1} = \frac{1}{j \lambda (z_1 - z_0)} \exp \{ jk(z_1 - z_0) \}$$

$$\cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x_0, y_0, z_0) \exp \left\{ jk \frac{(x-x_0)^2 + (y-y_0)^2}{2(z_1-z_0)} \right\} dx_0 dy_0 dk$$

where $\lambda$ is the wavelength of light source, $k$ is an wave number and $z_1$ represents the distance from the center of coordinates $(x_0, y_0, z_0)$ to the screen. Equation (1) is rewritten as follows by convolution equation:

$$u(x_1, y_1)|_{z=z_1} = g(x_1, y_1, z_1) \ast f(x_1, y_1, z_1), \quad (2)$$

where $f(x_1, y_1, z_1)$ is written as follows:

$$f(x_1, y_1, z_1) = \frac{1}{j \lambda z_1} \exp \left\{ jk \left[ z_1 + (x_1^2 + y_1^2)/2z_1 \right] \right\}. \quad (3)$$

The input light distribution, $g(x_0, y_0, z_0)$, has the equation convoluted by quadratic phase term $f(x_1, y_1, z_1)$ which is the function of $z_1$. The function of $f(x_1, y_1, z_1)$ is called “Point Spread Function”.
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B. NMR signal equation

In this section, we examine possibilities of an NMR signal equation to obtain the similar equations as those of the three-dimensional diffracted wavefront of Eq. (1). Since producing the same form signals as those of Eq. (1) makes magnetic field designing quite difficult, we planned to find ways to make magnetic fields produced as easy as possible and approximate the function $k/(2(z_0 - z))$ of quadratic phase term in Eq. (1) into a first order equation as $z$ coordinate. Based on the condition above, we use a field gradient, whose field intensity changes in a quadratic form on $x - y$ plane, and whose coefficient of the quadratic field gradient varies in the $z$ direction by $\alpha$, as shown in Eq. (4).

$$\Delta B = b(1 + \alpha z) \left\{ (x' - x)^2 + (y' - y)^2 \right\},$$  \hspace{1cm} (4)

where $b$ is a coefficient of a quadratic field gradient at $z = 0$ and coordinates $(x', y')$ are the center of this quadratic field gradient which can be set to optional places by the field given from outside. Therefore, Eq. (5) is the target equation of NMR signals, which can be obtained by scanning this quadratic field gradient on $xy$ plane

$$v(x', y') = P \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x, y, z) \cdot \exp \left\{ -j \gamma b \tau (1 + \alpha z) \left[ (x' - x)^2 + (y' - y)^2 \right] \right\} dx dy dz,$$  \hspace{1cm} (5)

where $\rho(x, y)$ represents the spin density distribution in the subject, $\gamma$ is the magnetogyric ratio, $\tau$ represents its impressed time, and $P$ is a constant.

In order to obtain the signal like the form of Eq. (5), the coefficients of the field must be varied in the $z$ direction not only the scanning field but also the field applied in the time reading direction. Therefore, magnetic field equations should all be varied into Eqs. (6) to (9),

(a) quadratic field: $\Delta B(x, y, z) = b(1 + \alpha z) \left\{ (x^2 + y^2) \right\};$  \hspace{1cm} (6)

(b) scanning field: $\Delta b_{xy}(z) = \sqrt{1 + \alpha z} b_{xy};$  \hspace{1cm} (7)

(c) sweeping field: $\Delta b_{xy}(z) = (1 + \alpha z) b_{xy};$  \hspace{1cm} (8)

(d) field gradient: $\Delta G_{yz}(y, z) = (1 + \alpha z) G_{yz}$  \hspace{1cm} (9)

Figure 1 shows the pulse sequence for the NMR diffractive imaging technique. After excitation of spins in the subject, a quadratic field gradient($a$) and scanning field($b$) are applied in the phase encoding direction only for $\tau$ of time. Scanning field is used to scan the center of a quadratic field, so when the amount of scanning is set to be $x'$, the quadratic field after scanning is $\Delta B = b(1 + \alpha z) \left\{ (x' - x)^2 + y^2 \right\}$. Next, the sweeping field($c$) and the field gradient($d$) are applied at the same time. The echo signal obtained by the method above can be written as Eq. (10) by setting the origin of the time at the center of the echo signal

$$v(t, y') = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x, y, z) \exp \left\{ -j \gamma b \tau (1 + \alpha z) \left[ (x' - x)^2 + y^2 \right] \right\} \cdot \exp \left\{ -j \gamma b t \left( \frac{G_{yz}^2}{4br^2} t - \frac{b_{xy}^2}{2br^2} \right) \right\} \cdot \exp \left\{ j \frac{G_{yz}^2}{4br^2} \right\} \cdot \exp \left\{ -j \gamma b t \left( \frac{G_{yz}^2}{4br^2} - \frac{b_{xy}^2}{2br^2} \right) \right\} dx dy dz,$$  \hspace{1cm} (10)

When the parameters are set to Eq. (11), and variables are also transformed, Eq. (10) can be rewritten as the following equation (12), and the target equation of Eq. (5) is obtained

$$\frac{G_{yz}^2}{4br^2} = \frac{b_{xy}^2}{2br^2}, \quad y' = -\frac{G_{yz} t}{2br^2};$$  \hspace{1cm} (11)

$$v(x', y') = P \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x, y, z) \cdot \exp \left\{ -j \gamma b \tau (1 + \alpha z) \left[ (x' - x)^2 + (y' - y)^2 \right] \right\} dx dy dz.$$  \hspace{1cm} (12)

Where $P$ is set as $P = \exp \left\{ -j \gamma b t_0 / 2t_0^2 \right\}$ and $\gamma b (1 + \alpha z)$, equivalent to a distance parameter, became the same form as written in Eq. (5). There are two methods for reconstructing images from Fresnel integral equation. One is a technique using the inverse Fourier transformation once after multiplying the quadratic phase term. The other method solve the convolution integral by the inverse filtering[1]. In this paper, we will discuss the method reconstructing images by the inverse filtering technique, in which the pixel width is not changed by the imaging parameters depending on the focal plane.

Let $F_{xy}$ be the Fourier transformation with respect to $x$, $y$, and it is applied to Eq. (12), then Eq. (13) can be obtained as follows:

$$F_{xy} \left\{ v(x', y') \right\} = P \int_{-\infty}^{\infty} \exp \left\{ -j \frac{\pi}{\gamma b \tau (1 + \alpha z)} \right\} \cdot R(k_x, k_y, z) \exp \left\{ j \frac{k_x^2 + k_y^2}{4\gamma b \tau (1 + \alpha z)} \right\} dz.$$  \hspace{1cm} (13)

![Fig. 1. Pulse sequence for NMR Fresnel diffractive imaging technique.](image-url)
where $R(k_x, k_y, z)$ denotes the Fourier transform of $p(x, y, z)$ with respect to $x$ and $y$ coordinates. By arranging Eq.(13), Eq.(14) is obtained

$$F_{xy} \{v(x', y')\} = P \exp (-j \frac{\pi}{2}) \gamma br \int _{-\infty} ^{\infty} \frac{1}{1 + \alpha z}$$

$$\cdot R(k_x, k_y, z) \exp \left( \frac{\alpha z}{4 \gamma br (1 + \alpha z')} \right) dz.$$  \hspace{1cm} (14)

Image focused on $z'$ plane is obtained by multiplying the inverse function of modulation transfer function on $z'$ plane to Eq.(14) using the distance parameter of $\gamma br(1+\alpha z')$, and taking the inverse Fourier transform of it. Let $\rho(x', y', z')$ denote the image focused on $z'$ plane which can be obtained by the following equation:

$$\rho(x', y', z') = \frac{1}{P} \exp \left( j \frac{\pi}{2} \right) F^{-1}_{xy} \left[ F_{xy} \{v(x', y')\} \right]$$

$$\cdot \exp \left\{ -j \frac{k_x^2 + k_y^2}{4 \gamma br (1 + \alpha z')} \right\} \hspace{1cm} (15)$$

$$= \frac{1}{P} \exp \left( j \frac{\pi}{2} \right) F^{-1}_{xy} \left[ \int _{-\infty} ^{\infty} \frac{1}{1 + \alpha z} \cdot R(k_x, k_y, z) \right.$$  

$$\cdot \exp \left\{ \frac{\alpha z - z'}{4 \gamma br (1 + \alpha z')} \right\} \} dz.$$ \hspace{1cm} (16)

From Eq.(16), we can obtain the image focused on optional plane by giving the focal plane coordinate $z'$ in the distance parameter of $\gamma br(1+\alpha z)$ in the reconstruction procedure.

III. EXPERIMENTS

To support this imaging technique, a coil system which generate the fields written by Eqs.(6) to (9) was designed and fabricated. Fig.2 show the outlook of the coil system. Experiments were performed using an ultra-low-field MRI scanner which generates the static magnetic field of $B_0 = 0.0183$ T by the solenoid coil (resonant frequency $f_0 = 779$ kHz). The parameters of experiments are as follows: $\gamma br = 1.49 \text{ rad/cm}^2$, the repetition time for the pulse sequence TR =300ms, $\Delta x' = \Delta y' = 0.2 \text{ cm}$, and the data matrix of the NMR signal is set to $64 \times 64$.

IV. IMAGE RECONSTRUCTION

A. Adjusting focal plane experiments

We conducted the experiment in order to examine whether images at optional focal planes can be obtained from two-dimensional scanned signals by parameters used in the numerical reconstruction procedure. Fig.3(a) shows the results of experiments using the phantom, having four water poles placed in the same intervals in the $x$ direction and the $z$ direction. As the distance parameters used in numerical reconstruction are followed by $\gamma br (1 + \alpha z)$, computerized images was reconstructed by being varied from 1.44 to 1.56 rad/cm$^2$ in 0.44 steps, as shown in Fig.3. Fig.3 shows the images focusing on a water pole located in the place, $z = 9 \text{ mm}$, and (b) to (d) show the reconstructed images focusing on each water pole after focal plane moves by $-0.06 \text{ cm}$. The results show that the water pole on focal plane is imaged most clearly. On the other hand image of water pole become obscure as a focal plane is moving gradually farther from the plane where water pole is located. The results of this experiment indicates that, even in collecting two-dimensional scanned signal, NMR signal in the NMR diffractive imaging technique can reconstruct images focusing on optional plane according to the choice of distance parameters depending on $z$ coordinate.

B. Imaging experiments with 3D tube

We performed the experiment using a vascular tract model. Phantom shown in Fig.4 was used for this experiment after being devised with a 4 mm diameter tube including water. Numerically reconstructed images obtained by adjusting the focal plane $z'$ are shown in Fig.4(a)–(d) whose focal plane is shifted every $-8 \text{ mm}$ from a point $z = 12 \text{ mm}$. Fig.4(a) and (b) show the images each focusing on the tubes on the right and left sides of a small-sized loop. Fig.4(c) and (d) show the images each focusing on the tubes on the right and left sides of a large-sized loop. Although these images include unnecessary image components out of focus, the three-dimensional distributions of phantom are understood by three-dimensional information.

Using a proposed diffractive imaging technique, there are some cases where images are emphasized by the superimposition of blurred image components outside the focal plane on the focal plane one. The area indicated as A in Fig.4(d) is the example of that obscure image. In this case, it may not be possible to distinguish normal parts from diseased parts. In order to reduce this interference problem, we had
performed the experiments for imaging from different angles to change the way of interference. Fig.5(a)-(d) show the results of an imaging experience by revolved phantom. Since the way of interference is changed, it is the judgement whether parts with high intensity is generated by interference or by other reasons.

V. Discussion

In the experiment, we could obtain images focusing on an optional depth by the signals scanned two-dimensionally. We can recognize that an object is located around focal plane by the fact that the image is clearly reconstructed. However, there are some cases where we have found it difficult to judge this clearly. In this case, we can recognize three-dimensional distributions of an object more easily when we can observe images as animation by continuously displaying the images focused on each z coordinate. This is considered to be because we can spatially recognize the obscure amount of images located in the front and rear parts of focuses by observing continuous images in the z direction.

However, given the considerable shortage of information about the subject on the use of this technique, we will have to confront problems of judging whether an observed place is abnormal because the image intensity on the plane will become greater when vascular tracts are crossing in the depth direction. In this case, although it doubles the amount of imaging time, we might solve the problem by observing the subject from different angles. There are two ways to move a visual point: one is to move an object and the other is to create an equivalent effect as moving a visual point by adding magnetic fields in the imaging experiment. We consider that the latter technique - moving visual points by fields - is the more practical.

Since the description of the NMR signal has the similar form to the equation of the wavefront in the Fresnel region, a hologram which can produce image in the coherent optical system is produced easily by transforming the signal distribution into the gray-scale pattern [9]. In principle, it is possible to reconstruct natural three-dimensional images using the holographic technique. We will investigate the holographic reconstruction in the next step.

VI. Conclusion

A new approach to MR angiography, the NMR Fresnel diffractive imaging technique, which can obtain the signal similar to the equation of the Fresnel diffractive images, is proposed. To support this imaging technique, a coil system composed of six coils was designed and imaging experiments using the water tube model were performed. The results shows that images focusing on optional plane in the depth direction can be reconstructed from data scanned two-dimensionally. Even though blurred images outside the focal plane are superimposed on the image in the focal plane, the three-dimensional distribution of the object can be recognized by moving the focal plane in the depth direction. To attain supplemental information for the object, acquiring images from different angles is helpful for recognizing the spatial distribution of the object more precisely. The proposed imaging technique is expected to be useful in MR fast angiography.

References