COMPARISON OF WAVELET PACKETS WITH COSINE-MODULATED PSEUDO-QMF BANK FOR ECG COMPRESSION

M. Blanco-Velasco¹, F. Cruz-Roldán¹, F. López-Ferreras², P. Martín-Martín²
¹Deparment of Ingeniería de Circuitos y Sistemas, Universidad Politécnica de Madrid, Madrid (Spain). E-mail: mblanco@ics.upm.es
²Department of Teoría de la Señal y Comunicaciones, Universidad de Alcalá, Alcalá de Henares-Madrid (Spain). E-mail: fernando.cruz@uah.es

Abstract—An electrocardiogram (ECG) compression technique based on subband coding has been developed in this paper to compare with compression techniques based on Wavelet Packets (WP). The filter bank designed is a cosine-modulated pseudo-QMF bank using the Kaiser Window Approach (KWA) method that does not have the perfect reconstruction (PR) property, as opposed to WP that are implemented by means of a PR filter bank. In both schemes, the coding process is the same to be able to compare the results. This is carried out using a very easy algorithm based on a thresholding technique, which provides good compression rate. Its main application is to encode long-term registers of digitized electrocardiogram signal, in order to reduce the bit rate. To preserve the reconstructed signal accuracy, the percentage root-mean-square difference (PRD) is used as an objective measurement parameter, which is selected before compression. The tests have been done for the twelve principal cardiac leads, and the compression degree measurement is evaluated by means of the mean number of bits per sample (MBPS) and the compression ratio (CR).

Key-Words - electrocardiogram (ECG), wavelet packets (WP), QMF banks, PRD.

I. INTRODUCTION

In the last few years, many data compression techniques have been developed to codify ECG signals. Most of them are transform methods in which a data transformation is applied as first stage to represent the input signal in the best way possible. Thus, the correlation among resulting coefficients after transforming original data is less. Therefore, the coefficients can be quantized with different precision in a second stage. In this context, the transforms based on the Discrete Wavelet Transform (DWT) have given good results because of their easy implementation and the quality of the recovered signal when it is compared with direct compression methods. These process directly digitized data taking advantage of redundancy among samples. In this paper, two compression techniques are compared. One of them is a transform method using WP. The second one is based on subband coding by using conventional cosine-modulated pseudo-QMF banks. By means of a filter bank, the signal is split in the frequency domain obtaining several subband signals, which contain the original information but non-uniform distributed. The block diagram of subband coding is similar to the previous one, but now, the first stage has to be changed. It should be an M-channel cosine-modulated filter bank and the response the set of subband signals. To be able to compare the two methods, both of them must apply the same compression scheme as well as the same procedure to process the signal.

As well known, the purpose of the ECG signal compression is to reduce the long-term register bit rate to solve storage and transmission needs. The algorithm done in this work has been developed to process the signal continuously without heartbeat segmentation. As there is information loss, the quality of the reconstructed signal has to be assured. The percentage root-mean-square difference (PRD) is used as an accepted objective measurement [1] to preserve the original waveform with a degree of acceptable quality:

\[
PRD = \frac{\sqrt{\sum(x[n]-\hat{x}[n])^2}}{\sqrt{\sum(x[n])^2}} \times 100
\]  

II WAVELET PACKETS

Multiresolution analysis is a very important point of view to understand and apply wavelet analysis [2]. Using this overall theory, a function \(f(t) \in L^2(R)\) can be represented as a succession of approximation in several scales. Defining two basic functions, a vector space is generated by scaling and translating them ((2), (3)) to represent the signal: wavelet function \((\mathcal{W}(t))\), which has the accurate details of the signal \(f(t)\), is one, and scaling function \((\varphi(t))\), which offers a non accurately approximation, is the other one:

\[
\varphi_{j,k}(t) = 2^{j/2} \varphi(2^j t - k) \tag{2}
\]

\[
\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k) \tag{3}
\]

Combining both approximations, the function \(f(t)\) is exactly obtained:

\[
f(t) = \sum_k c_{0,k}(t) \varphi_{0,k}(t) + \sum_{j=0}^{\infty} \sum_k d_{j,k}(t) \psi_{j,k}(t) \tag{4}
\]

In expression (4), two kind of coefficients, called Discrete Wavelet Transform (DWT), are used as projections in the vector space: the scaling coefficients \(c_{0,k}(t)\) which are coarse details, and the wavelet coefficients \(d_{j,k}(t)\) which are finer details. The advantage of multiresolution analysis is that the implementation algorithm may be achieved by means of a two-order filter bank that has PR property and whose impulsive responses \(h_0[n]\) and \(h_1[n]\) are low-pass and high-pass FIR causal filters with cutoff frequency at \(\pi/2\). This filter bank is applied successively at the low-pass filter output, which represents the coarse details. In this work, Daubechies filters are used, which defined an orthogonal base.

The DWT can be generalized decomposing the high-pass filter output too, that is, the finer details. In this way, we get a binary tree filter bank with a level number dependent on desired scale resolution (Fig. 1).

The binary tree can be considered as a library of bases called the Wavelet Packets (WP) [3]. The purpose is to select the best base to represent the signal in the best way pruning the tree conveniently. This is done using some criterion to measure the information cost of each node. In this paper, Shannon entropy has been used [3]. Information cost must be compared between the root or parent node and the sum of information...
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**Author(s)**

**Performing Organization Name(s) and Address(es)**  
Department of Ingenieria de Circuitos y Sistemas Universidad Politecnica de Madrid Madrid Spain

**Sponsoring/Monitoring Agency Name(s) and Address(es)**  
US Army Research, Development & Standardization Group (UK) PSC 802 Box 15 FPO AE 09499-1500

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**Abstract**

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cost of the following generations or children nodes in the binary tree. The remaining branches are those with higher value (Fig. 1).

On the other hand, the input signal is processed taking blocks of power of two consecutive samples. Each segment has its best base, so each one is processed with different filter bank structure. In order to recover the signal without information loss, each segment has to be processed independently. This is achieved taking the periodic extension of every segment and calculating its coefficients by means of periodic convolution. This is equivalent to considering each segment as a period of a periodic signal. In this way, the same periodic signal, and therefore the same segment, must be recovered applying the corresponding synthesis filter bank.

III. M-CHELLE Channel Maximally Decimated Filter Banks

The M-channel maximally decimated filter banks with a parallel structure have received widespread attention (see, e. g., [1] or [4] for a list of references). Pseudo-QMF banks can be an alternative to perfect reconstruction systems, to avoid the highly nonlinear optimization necessary to obtain the filter coefficients. In cosine-modulated filter banks, analysis and synthesis filters are cosine-modulated versions of a low-pass prototype filter. The design of the whole filter bank thus comes down to the design of the prototype filter.

Several methods have been proposed to facilitate the design of the pseudo-QMF banks’ prototype filter and to improve the characteristics of the resultant system [4, 5, 6]. One of these methods is the Kaiser Window Approach (KWA) to the design of prototype filters of cosine-modulated filter banks [8]. The design process of the prototype filter is the following. Let \( p[n] \) be a filter designed through Kaiser window technique. We define \( G(e^{j\omega}) = |p(e^{j\omega})|^2 \). In the KWA technique, the design process of the prototype filter is reduced to the optimization of the ideal filter cutoff frequency \( \omega_c \) in order to minimize the objective function given by

\[
\phi_{new} = \max_{n,\omega \neq 0} |g(2Mn)|. 
\]

Specially relevant for this work are the conventional cosine-modulated pseudo-QMF banks [4, 7]. We have used these kinds of filter banks for the purpose of dividing the incoming signal into 16 separate subband signals. In conventional modulation, the real coefficients impulse response of the analysis \( h_k[n] \) and synthesis filters \( f_k[n] \), \( 0 \leq n \leq N - 1 \), \( 0 \leq k \leq M - 1 \), are given by

\[
h_k[n] = 2 \cdot p[n] \cos \left( \frac{(2k + 1)\pi}{M} \left( n - \frac{N - 1}{2} \right) + (-1)^k \frac{\pi}{4} \right) \]

\[
f_k[n] = 2 \cdot p[n] \cos \left( \frac{(2k + 1)\pi}{M} \left( n - \frac{N - 1}{2} \right) - (-1)^k \frac{\pi}{4} \right). 
\]

In order to evaluate the quality of the resulting filter banks, we can use several measures. We measure the peak difference (maximum amplitude distortion) on the magnitude response of the overall distortion transfer function \( T_0(z) \) (Fig. 2), where

\[
T_0(z) = \sum_{k=0}^{M-1} h_k(z) \cdot F_k(z). 
\]

Total aliasing error can be obtained using the aliasing function \( T_{al}(z) \) (Fig. 3) [4, 8], where

\[
T_{al}(z) = \sqrt{\sum_{z=0}^{M-1} \frac{1}{M} \sum_{k=0}^{M-1} F_k(z) \cdot h_k(z W_M^k \mod 1)^2} 
\]

This function provides an accumulated measure of the contributions from each one of the aliasing transfer functions. The maximum value of the aliasing function \( T_{al}(z) \) is useful in order to measure aliasing distortion because it is the worst peak aliasing distortion possible.
IV. COMPRESSION SCHEME

As we have said before, the same compression idea is applied to both coding techniques compared in this work. One of them is a transform method where the transform used is the WP using Daubechies filters. The output samples will be the coefficients of the transform. The other one is based on subband coding where the first stage is substituted by a pseudo-QMF cosine-modulated bank. The output samples will be the subband signals. The transform and QMF bank is followed by two common stages: a quantizer and an entropy coder.

To calculate WP, the signal must be segmented into consecutive blocks whose lengths are a power of two. The same segmentation is applied to the compression based on QMF bank.

On the other hand, the compression scheme is based on previous works [9] where an ECG signal coding was designed by using WP. The results were obtained varying four parameters: the order of filters, the number of levels of the decomposition tree, the length of signal segment and the PRD value. To conclusion, good results were obtained for a filter order 12 and no more than 4 layers of the decomposition tree. Therefore, in this work, the WP are designed using 12 order Daubechies filters and decomposition level up to 4.

The WP can be seen as a binary tree, which splits the spectral domain by 16 when the decomposition level is up to 4. Thus, the QMF bank selected to carry out the comparison with the WP of 4 layers must be 16-channels. To calculate the order of filters of 16-channel QMF bank, the nobles identities for multirate systems have to be used [5] resulting in 156. In this work, the order selected is not exactly the previous one because of the design procedure. The final order chosen is 160. The tests have been done with a cosine-modulated pseudo-QMF bank designed by the KWA method, which is called clkwa16161.

The quantizer designed for both methods is based on a thresholding technique. Coefficients with amplitude less than a certain value are discarded, that is, zero valued, maintaining only the largest and assuring the quality of the reconstructed signal with regard to the original. The reconstructed signal quality is selected before the compression as a predetermined PRD value. This is applied to each input segment.

For the entropy coder stage, a run-length coding is used as a means to join the null samples. The non-discarded samples of each segment processed are sent or stored without varying the original precision. Since the previous section is a thresholding technique, there will be unused codes in each set of samples processed called escape codes. In this case, the threshold can be used as an escape code to indicate the zero position. Two samples must be included as a header in every segment: the first will be a word indicating the beginning of the segment; the second one the escape code of the current segment. The next samples are the informative content of the segment. Non-discarded samples are encoded with the original precision (16 bits) until a zero stream appears, which is indicated by the escape code. Then, the number of consecutive zeros is encoded with different precision. As the first step, with 4 bits when the number of zeros are less than sixteen. The bit number '1111' marks an overflow (more than fifteen consecutive zeros). In this case, the zero stream is encoded with the previous 4 bits plus a number of bits enough to complete the length of the segment.

For the scheme based on WP, additional information must be considered: the base used to decompose each segment. In a binary tree library, the number of bases can be calculated recursively [3] and in the particular case of 4 layers, there are 677 different bases. Therefore, maintaining a table with the different kinds of decomposition tree, this information is included as a 16 bit word in the header of run-length coding of WP scheme.

V. RESULTS

The group of Electro-physiology Laboratory (Cardiology floor) of the Hospital Gregorio Marañón of Madrid supplies us the database used to carry out the test, which is composed by the twelve standard leads. An atrial fibrillation is the pathology contained in the database. Every lead is sampled at 360 Hz and each sample is encoded in PCM with 16 bits per sample. The results presented here have been obtained applying the compression system to frames of each lead, which last 5 minutes each. Apart from the order of filters for both schemes as well as the decomposition level for WP, there are still two parameters free: the segment length to split the input signal and the PRD value to select the quality of the recovered signal.

As we have studied on previous works [10] using a similar compression technique based on WP, the compression ratio were better by increasing the segment length from 128 to 2048 samples. This behavior has been the same with the run-length coding designed in this article for the transform method. Using the QMF bank, good results are obtained to segment length from 512 samples. Therefore, the tests have been done to both techniques taking block lengths of input signal from 512 samples to 2048.

Concerning the quality of the reconstructed signal, the PRD is selected before compression to assure a final determined accuracy. However, this performance measure is not enough. It must be validated by visual inspection by a clinical expert. After studying the bibliography regarding this subject ([1], [10]) we have decided to select only PRD values from 0.5 % to 5%.

On the other hand, it is interesting to not forget that the QMF bank used in this work does not have the property of PR as opposite to WP. The quality of several recovered signals without compression are shown in table 1. We note that because another point of view from the mathematical theory of wavelets is as filter bank theory where each output nodes may be seen as subband signals.

The results of both compression schemes for the lead ARV can be seen on Fig. 4 and 5 as a three-dimensional representation for WP and for the cosine-modulated pseudo-QMF bank clkwa16161 respectively. The MBPS is represented as a function of PRD and the segment length. The compression results are the mean compression value of all segments processed except the last, which is zero padded before compression to complete the segment length. The compression depends on the segment length, specially when the WP compression scheme is used, though the best results are for

| TABLE 1 |
| QUALITY OF RECOVERED SIGNALS WITHOUT COMPRESSION |
| Lead | clkwa16161 |
| I | 0.3562 |
| ARV | 0.3586 |
| V1 | 0.3626 |
2048 samples. Comparing both sheets, the corresponding to cosine-modulated pseudo-QMF bank (Fig. 5) is always lower than the corresponding to WP (Fig. 4). This means that compression by means of cosine-modulated pseudo-QMF bank provides better results with the compression scheme applied in this work. As a particular case, in Fig. 6, we can see both the original lead ARV on continuous line and the reconstructed version superimposed on dashed line for a PRD of 3 % and for a segment length of 2048. The compression was done with the clkwa16161 bank obtaining a CR of 11.50 (1.3913 MBPS) whereas with WP, the CR was 10.26 (1.5595 MBPS)

VI. CONCLUSION

A transform method of ECG compression based on WP has been compared to another one based on subband coding using a cosine-modulated pseudo-QMF bank. The same compression scheme was applied to both schemes. A lot of results have been obtained as a function of the quality of reconstructed signal and the segment length of input signal. In conclusion, the scheme based on subband coding always provides best compression degree. This one can be improved increasing the segment length. The tests were done for the twelve cardiac leads and the behavior of the system was the same for all of them, obtaining similar results on the same conditions of compression.

REFERENCES