Abstract - Various mechanical characteristics of stents were analyzed and mathematical models were developed in order to predict expansive pressure of stents. Given the geometry and material properties of a stent, one can utilize these models to predict its expansive pressure properties. Then, these models were verified with the test results derived from some prototype and commercially available stents. The models allow for the characterization of mechanical properties of stents and may be instrumental in developing clinically efficacious stents.

Keywords – Self-expandable metal stent, radial expansive pressure, analytic model

I. INTRODUCTION

Successful stenting is crucial for maintaining the patency of the organs with luminal obstruction. Three types of stents are commonly used: bare stent (wire only), coated stent (polymer coating on wire), and covered stent (polymer membrane on its peripheral surface). The clinical applicability of these stents largely depends on thorough understanding of their mechanical properties, one of which is radial expansive pressure. In the present study, we propose the mathematical models by which one can predict radial expansive pressures of coated and covered stents.

II. METHODOLOGY

A. Bare type

Jedwab and Clerc [1] simplified a bare type stent as combination of open-coiled helical springs. Each wire of bare stent was regarded as a spring and the equations for open-coiled helical spring by Wahl [2] was employed for wire model (Fig. 1). We use wire model of Jedwab and Clerc [1] for bare stents and wires for coated and covered stents. In this section, we rearranged the equations for open-coiled helical spring used in the study of Jedwab and Clerc [1].

A load is applied to elongate a stent in longitudinal direction. The load acting on the stent modeled with combination of \( n \) wires can be expressed as a function of pitch angle as in the following equation:

\[
F_{\text{wire}} = 2n \left[ \frac{G I_p \cos \beta}{K_3} \left( \frac{2 \sin \beta}{K_1} - K_1 \right) - \frac{E I \tan \beta}{K_2} \left( \frac{2 \cos \beta}{K_1} - K_2 \right) \right]
\]  

where \( K_1, K_2, K_3 \) are constants given by

\[
K_1 = \frac{\sin 2\beta_0}{D_0}, \quad K_2 = \frac{2 \cos^2 \beta_0}{D_0}, \quad K_3 = \frac{D_0}{\cos \beta_0}
\]

and \( \beta \) is pitch angle, \( I \) moment of inertia, \( I_p \) polar moment of inertia, \( E \) Young’s modulus, \( G \) shear modulus, and \( n \) number of wires of a stent.

Then, the radial expansive pressure of a stent is

\[
P_{\text{wire}} = \frac{2F_{\text{wire}} c}{D L \tan \beta}
\]  

B. Coated type

A coated stent exerts two types of forces against radial compression: the spring restoring force \( F_{\text{wire}} \) exerted by metal wires and polymer knot force \( F_{\text{coat}} \) derived from the moments by the knots of polymer coating. The helical spring model [1] is employed to calculate wire spring force in this type of stent. The polymer knots are considered as torsional springs and the spring moments are calculated.

Fig. 2 shows stent in initial state and compressed state in \( \theta-z \) plane (cylindrical coordination). When stent is compressed in radial direction as shown in Fig. 2(D), the moment from compressed torsional springs at each knots are exerted on wires of stent. The moment by one knot \( (M_{\text{knot}}) \) is

\[
M_{\text{knot}} = k_{\text{knot}} (\beta - \beta_0)
\]

where \( k_{\text{knot}} \) is a torsional spring constant. The moment exerted on one wire becomes

\[
M_{\text{wire}} = k_{\text{knot}} (\beta - \beta_0) \times \text{KNOTS}_{\text{wire}}
\]

where \( \text{KNOTS}_{\text{wire}} \) is the number of knots in the wire. Thus, polymer knot force of a stent \( (F_{\text{coat}}) \) can be expressed as
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Analytical Models for Predicting Mechanical Properties of Self-Expandable Metal Stents with Cover Membrane

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The method to determine the torsional spring constant of the polymer coating will be described in the following. When the radial expansive pressures for both bare type and coated type stents are measured, the difference in their expansive pressures is caused by the torsional springs at knots \( P_{coat} \) only. That is,

\[
P_{coat} = P_{coated stent} - P_{bare stent}
\]

With known \( P_{coat} \), \( D \), \( \beta \), and \( L \), we can obtain polymer knot force \( F_{coat} \) of a stent:

\[
F_{coat} = \frac{P_{coat} DL \tan \beta}{2n}
\]

Then, the total moment \( M_{wire} \) exerting on the knots of a wire is expressed as

\[
M_{wire} = \frac{L_{wire} F_{coat} \cos \beta}{n}
\]

Applying linear regression to the \( M_{wire} \) from (5) and (9), the torsional spring constant \( k_{knot} \) can be obtained. Fig. 3 shows a regression result for a commercial stent, where the \( k_{knot} \) was 0.00133 (Nm/rad).

Given the longitudinal forces from wire and knots \( (F_{wire}, F_{coat}) \), number of spring turns \( (c) \), diameter \( (D) \), length \( (L) \), and pitch angle \( (\beta) \) of a compressed stent, one can calculate the radial expansive pressure of coated stent by the following equation:

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P_{coated stent} = \frac{2(F_{wire} + F_{coat}) c}{DL \tan \beta}
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\]
Similarly, the strain between B and C is same as (15). Therefore, it is concluded that the strain ($\delta/L$) is uniform and independent of its location on the stent.

If the cover membrane is assumed to be uniform in material and thickness, the tensile force of the polymer cover is

$$F_{\text{cover}} = \frac{AE}{L} \delta$$

(16)

where $A$ is cross-sectional area and $E$ is Young’s modulus.

The expansive pressure $P_{\text{cover}}$ due to the tensile force of cover membrane is then calculated as

$$P_{\text{cover}} = \frac{2F_{\text{cover}}C}{DL \tan \beta}$$

(17)

Unlike metals, the stress-strain curve of polymers does not demonstrate any linear portion even on initial tension. Thus, constant Young’s modulus could not be used. In order to precisely determine the mechanical behavior of the polymer membrane, tensile test was performed and the resulting nonlinear stress-strain curve was employed to calculate the cover membrane tensile force. As shown in Fig. 5, the expansive pressure due to cover membrane demonstrates rapid increase during the initial phase of compression followed by insignificant change in pressure. Therefore, we can conclude that the cover membrane behaves as initial offset in total expansive pressure.

The actual amount of coating on wires of covered stent is not same as that of coated stent. In order to take into account this difference, the empirical coefficient $\alpha$ is employed, such that,

$$P_{\text{covered stent}} = P_{\text{wire}} + P_{\text{cover}} + \alpha P_{\text{coat}}$$

(18)

Thus, $P_{\text{covered stent}}$ is equal to

$$P_{\text{covered stent}} = \frac{2(F_{\text{wire}} + F_{\text{cover}} + \alpha F_{\text{coat}})C}{DL \tan \beta}$$

(19)

III. RESULTS

A. Bare type

Fig. 6 shows the plot of both simulation and experimental results for bare type stent. Circles represent actual measurements conducted by Moon et. al [3] and solid line is calculated with the model (3). We can see the model is well matched to the experimental data.

B. Coated type

Fig. 7 shows the plot of simulation and experimental results for coated type stent. The model is well matched to the experimental data. Also, $P_{\text{bare}}$ is about two to three times bigger than $P_{\text{coat}}$. That is, the force due to coating is dominant in coated type stent.

C. Covered type

Fig. 8 shows the plot of simulation and experimental results for covered type stent. The coefficient $\alpha$ was 0.5. The expansive pressure increases rapidly during the initial phase of compression region (0-10%) and increases slowly during the rest of compression (more than 10%). Similarly shown in Fig. 5, the nonlinear increase in expansive pressure is due to the expansion characteristics of polyurethane cover on the peripheral surface of stent.

D. Commercial stent

Similar characteristics of rapid increase in expansive pressure during the initial phase of compression were also observed in various commercially available covered type stents.
IV. DISCUSSION

Physical properties of metal wire, polymer knots, and cover membrane are all elements in determining expansive pressure characteristics. Given these properties, the models presented in the present study allow accurate prediction on expansive pressure. Further, physical variables can be conveniently altered to design and manufacture stents with desired expansion characteristics.

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