

Multi-Sensor Single Target Bearing-Only Tracking in Clutter

Mahendra Mallick^a, T. Kirubarajan^b

^aALPHATECH, Inc.
50 Mall Rd, Burlington, MA 01803
Phone: (781) 273-3388x245, Fax: (781)-273-9345
mallick@alphatech.com

^bElectrical & Computer Engineering Department
McMaster University
1280 West Main Street, Hamilton, Ontario, Canada L8S 4K1
Phone: (905) 525 9140 x 24819 Fax: (905) 523 4407
kiruba@mcmaster.ca

ABSTRACT

Multiple unattended ground sensors are deployed for surveillance, monitoring the movement of troops, military vehicles, and targeting. Usually, the probability of detection (P_D) of an UGS is low and the false alarm density (FAD) is high. The particle filter (PF) and range-parametrized extended Kalman filter (RPEKF) have been used previously to produce improved results for the single sensor single target bearing-only tracking problem in the absence of clutter and with unity probability of detection. Although a great deal of work has been done for the single target single sensor bearing-only tracking problem, not much is known for the single target multi-sensor bearing-only problem with low P_D and high FAD. We present algorithms and numerical results using particle filter (PF) and probabilistic data association (PDA) for the single target multi-sensor bearing-only tracking problem with high false alarms and low probability of detection. Our numerical results show that the algorithm estimates the target state in a robust manner in realistic harsh conditions when the probability of detection is low and the false alarm density is high.

Keywords: Multi-sensor Single-target Tracking, Unattended Ground Sensors (UGS), Acoustic Sensors, Particle Filter (PF), Probabilistic Data Association (PDA)

1. INTRODUCTION

Unattended ground sensors (UGSs) [8],[9],[36] are deployed on the ground and are widely used in military and industrial applications to collect signals from remote sources. In the military domain, UGSs are used to detect, track, and classify ground vehicles and aircrafts. In the industrial domain, UGSs are used in mining operation that involves deep blasting. Common types of UGSs are acoustic, seismic, seismic/acoustic, magnetic, seismic/magnetic, seismic string, passive infrared (PIR), active infrared (AIR), and breakwire [36]. Current UGSs are usually low-cost, lightweight, and small size sensors compared to the previous UGSs.

Aerial surveillance and human intelligence were primarily used to detect and track the movement of ground forces until the mid 1960s. During the Vietnam War, it was realized that these approaches are ineffective to monitor the force movement in thick forests. Moreover, long range standoff airborne radar sensors are not capable of providing continuous surveillance data due to occlusion of the radar line-of-sight by terrain in certain scenarios. The UGSs can complement the airborne and spaceborne sensors to provide continuous surveillance data of the battlespace.

Report Documentation Page

Report Date 00 Oct 2001	Report Type N/A	Dates Covered (from... to) -
Title and Subtitle Multi-Sensor Single Target Bearing-Only Tracking in Clutter	Contract Number	
	Grant Number	
	Program Element Number	
Author(s)	Project Number	
	Task Number	
	Work Unit Number	
Performing Organization Name(s) and Address(es) ALPHATECH, Inc 50 Mall Rd Burlington, MA 01803	Performing Organization Report Number	
Sponsoring/Monitoring Agency Name(s) and Address(es) Department of the Army, CECOM RDEC Night Vision & Electronic Sensors Directorate AMSEL-RD-NV-D 10221 Burbeck Road Ft. Belvoir, VA 22060-5806	Sponsor/Monitor's Acronym(s)	
	Sponsor/Monitor's Report Number(s)	
Distribution/Availability Statement Approved for public release, distribution unlimited		
Supplementary Notes See also ADM201471, Papers from the Meeting of the MSS Specialty Group on Battlefield Acoustic and Seismic Sensing, Magnetic and Electric Field Sensors (2001) Held in Applied Physics Lab, Johns Hopkins Univ, Laurel, MD on 24-26 Oct 2001. Volume 2 (Also includes 1999 and 2000 Meetings), The original document contains color images.		
Abstract		
Subject Terms		
Report Classification unclassified	Classification of this page unclassified	
Classification of Abstract unclassified	Limitation of Abstract UU	
Number of Pages 17		

In this paper, we address the problem of single target tracking in clutter using multiple acoustic UGSs. An UGS determines the bearing of a target by processing the incoming acoustic signal. In addition to bearing, other derived measurements such as harmonic frequencies are also available. We restrict our analysis to single target tracking using bearing-only measurements from multiple acoustic UGSs. Usually, the probability of detection (P_D) for an UGS is low and the false alarm density (FAD) is high. Although a great deal of work has been done for the single sensor bearing-only tracking problem, not much is known for the single target multi-sensor bearing-only problem with low P_D and high FAD. The particle filter (PF) [2],[5],[10],[11],[14]-[17],[30],[32] and range-parametrized extended Kalman filter (RPEKF) in [2], [25] have been shown to produce improved results for the single sensor single target bearing-only tracking problem with unity P_D and no clutter. The difficulty in bearing-only tracking is the low information content of the measurement, in addition to the nonlinearity in the measurement model. The state estimates are also very sensitive to the relative geometry between the sensor and target. Often, the estimates are unreliable, i.e., they have very high variance [1],[13],[21]-[24],[33]-[35]. We present algorithms and numerical results using the PF for the single target multi-sensor bearing-only problem in clutter that represents realistic harsh conditions.

In Sections 2 and 3, we present the target kinematic model in two dimensions and acoustic measurement model, respectively. We describe the Bayesian approach that provides a rigorous framework for state estimation with general probability density functions for the state and measurement in Section 4. Analytic solutions using the Bayesian approach are not possible for cases that involve nonlinear dynamics for the state, nonlinear measurement models, and non-Gaussian distributions. The well-known Kalman filter estimator is a Bayesian estimator for linear dynamic model with additive Gaussian noise, linear measurement model with additive Gaussian noise, and prior Gaussian distribution for the state. The bearing measurement model is a nonlinear function of the target state. The commonly used extended Kalman filter (EKF) [3],[4],[5],[6],[19] is not an optimal estimator due to the nonlinear models for the measurement and state dynamics. When the degree of nonlinearity is small and the errors in the measurement and state dynamics are small, the EKF is a reasonable approximate estimator. The EKF has been used within other estimator configurations like the Interacting Multiple Model (IMM) estimator and the IMM probabilistic data association (PDA) estimator to solve different tracking problems [3],[4],[6],[7],[27],[28],[31]. When false alarms are present and the P_D is less than unity, we can have zero or multiple measurements from a single sensor in a scan for a single target. Commonly used approaches for this problem are the multiple hypothesis tracking (MHT) [6],[26] and probabilistic data association (PDA). The PF algorithm using the PDA [4],[11],[18] is described in Section 5. We present numerical simulation and results in Section 6 and conclusions in Section 7.

2. TARGET KINEMATIC MODEL

The nearly constant velocity model (NCVM) in two dimensions is a widely used kinematic model [3],[4],[6] for ground moving targets. The continuous-time target state for the NCVM is defined by

$$(2.1) \quad \mathbf{x}(t) := \begin{bmatrix} x(t) & y(t) & \dot{x}(t) & \dot{y}(t) \end{bmatrix},$$

where $(x(t), y(t))$ and $(\dot{x}(t), \dot{y}(t))$ are the Cartesian position and velocity coordinates, of the target at time t , respectively. The continuous-time NCVM is described by the linear stochastic differential equation

$$(2.2) \quad \dot{\mathbf{x}}(t) = \mathbf{F}(t)\mathbf{x}(t) + \mathbf{w}(t),$$

where

$$(2.3) \quad \mathbf{F}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$(2.4) \quad \mathbf{w}(t) = \begin{bmatrix} 0 & 0 & w_x(t) & w_y(t) \end{bmatrix}.$$

\mathbf{F} is known as the system matrix and $w_x(\cdot)$ and $w_y(\cdot)$ are mutually independent, zero-mean white Gaussian acceleration processes:

$$(2.5) \quad E\{w_x(t)\} = 0, \quad E\{w_x(t)w_x(\mathbf{t})\} = q_x \mathbf{d}(t-\mathbf{t}),$$

$$(2.6) \quad E\{w_y(t)\} = 0, \quad E\{w_y(t)w_y(\mathbf{t})\} = q_y \mathbf{d}(t-\mathbf{t}),$$

$$(2.7) \quad E\{w_x(t)w_y(\mathbf{t})\} = 0, \quad \text{for all } t, \mathbf{t}.$$

q_x and q_y represent the power spectral density (PSD) of $w_x(\cdot)$ and $w_y(\cdot)$, respectively [19].

We assume the initial state of the target to have the prior distribution

$$(2.8) \quad \mathbf{x}(0) \sim N(0, \mathbf{P}_0),$$

where \mathbf{P}_0 is the prior covariance of the state.

Sensor observations are available at discrete times $t_k, k = 0, 1, \dots$. In the current work, we do not address the out-of-sequence measurement (OOSM) that can arise in a multi-sensor tracking scenario. Since the measurements are available at discrete times, it is necessary to discretize the continuous-time model in (2.2) corresponding to the measurement times. Discretization of the continuous-time dynamics based on this sequence of times yields the following:

$$(2.8) \quad \mathbf{x}_{k+1} = \mathbf{F}_k \mathbf{x}_k + \mathbf{w}_k,$$

where

$$(2.9) \quad \mathbf{x}_k := \begin{bmatrix} x_k \\ y_k \\ \dot{x}_k \\ \dot{y}_k \end{bmatrix} = \begin{bmatrix} x(t_k) \\ y(t_k) \\ \dot{x}(t_k) \\ \dot{y}(t_k) \end{bmatrix},$$

$$(2.10) \quad \mathbf{F}_k := \mathbf{F}(t_{k+1}, t_k) = \begin{bmatrix} 1 & 0 & \mathbf{D}t_k & 0 \\ 0 & 1 & 0 & \mathbf{D}t_k \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \Delta t_k = t_{k+1} - t_k,$$

$$(2.11) \quad \mathbf{w}_k := \int_{t_k}^{t_{k+1}} \mathbf{F}(t_{k+1}, t) \mathbf{w}(t) dt, \quad k = 0, 1, \dots$$

We can prove that $\{\mathbf{w}_k, k \geq 0\}$ is a zero-mean white Gaussian process:

$$(2.12) \quad E\{\mathbf{w}_k\} = 0, \quad E[\mathbf{w}_k \mathbf{w}_l'] = \mathbf{d}_{kl} \mathbf{Q}_k, \quad k, l = 0, 1, \dots$$

$$(2.13) \quad \mathbf{w}_k \sim N(0, \mathbf{Q}_k), \quad k=0,1,\dots$$

$$(2.14) \quad \mathbf{Q}_k = \begin{bmatrix} \frac{1}{3}q_x(\mathbf{D}t_k)^3 & 0 & \frac{1}{2}q_x(\mathbf{D}t_k)^2 & 0 \\ 0 & \frac{1}{3}q_y(\mathbf{D}t_k)^3 & 0 & \frac{1}{2}q_y(\mathbf{D}t_k)^2 \\ \frac{1}{2}q_x(\mathbf{D}t_k)^2 & 0 & q_x\mathbf{D}t_k & 0 \\ 0 & \frac{1}{2}q_y(\mathbf{D}t_k)^2 & 0 & q_y\mathbf{D}t_k \end{bmatrix}.$$

\mathbf{Q}_k is known as the process noise covariance matrix.

3. ACOUSTIC UGS MEASUREMENT AND CLUTTER MODEL

The number of measurements in a scan can be zero or greater than zero due to $P_D < 1$ and the presence of false measurements or clutter. Let m_k denote the number of measurements in the k^{th} scan. The set of measurements \mathbf{z}_k in the k^{th} scan is defined by

$$(3.1) \quad \mathbf{z}_k := \{\mathbf{z}_{ki}\}_{i=1}^{m_k},$$

where \mathbf{z}_{ki} is the i^{th} measurement in the k^{th} scan. For the acoustic sensor \mathbf{z}_{ki} is a scalar. For the sake of generality, we consider \mathbf{z}_{ki} as a vector. All the measurements up to and including the k^{th} scan are defined by

$$(3.2) \quad \mathbf{Z}^k := \{\mathbf{z}_j\}_{j=1}^k.$$

We describe the acoustic sensor measurement model, false measurement model, and likelihood functions for both the target originated measurement and false alarm in Sections 3.1, 3.2, and 3.3, respectively.

3.1 Acoustic Sensor Measurement Model

Acoustic sensors are omni-directional sensors, which determine bearing by processing the incoming acoustic signal. The bearing is a nonlinear function h of the target state \mathbf{x}_k and the sensor state \mathbf{s}_k at time t_k . We assume that the acoustic sensor can detect a target with probability P_D when the distance of the target from the sensor is less than a maximum range, r_{\max} . We assume that the measurement noise v_k at time t_k is an additive independent Gaussian noise process. Thus, the nonlinear measurement model is described by

$$(3.3) \quad z_k = h(\mathbf{x}_k, \mathbf{s}_k) + v_k, \quad [(x_k - s_{xk})^2 + (y_k - s_{yk})^2]^{1/2} < r_{\max},$$

$$(3.4) \quad \mathbf{s}_k := \mathbf{s}(t_k) = [s_x(t_k) \quad s_y(t_k) \quad \dot{s}_x(t_k) \quad \dot{s}_y(t_k)]',$$

$$(3.5) \quad [s_{xk} \quad s_{yk} \quad \dot{s}_{xk} \quad \dot{s}_{yk}] := [s_x(t_k) \quad s_y(t_k) \quad \dot{s}_x(t_k) \quad \dot{s}_y(t_k)]',$$

$$(3.6) \quad E\{v_k\} = 0, \quad E\{v_k v_l\} = \mathbf{d}_{kl} R_k, \quad v_k \sim N(0, R_k),$$

$$(3.7) \quad R_k = \mathbf{s}_{ka}^2,$$

where \mathbf{s}_{ka}^2 is the variance of the bearing measurement noise in the k^{th} scan. In practice, \mathbf{s}_{ka}^2 can vary with scan depending on the distance of the target from the sensor. For simplicity, we assume that the variance of the bearing measurement noise is a constant for all scans:

$$(3.8) \quad R_k = \mathbf{s}_a^2.$$

We also assume that the measurement noise v_k and the process noise \mathbf{w}_l are uncorrelated at all times:

$$(3.9) \quad E\{v_k \mathbf{w}_l'\} = 0, \quad \text{for all } k, l.$$

The bearing angle is measured from the Y-axis (usually chosen along the local North direction) in the clockwise direction and the measurement function is described by

$$(3.10) \quad h(\mathbf{x}_k, \mathbf{s}_k) = \tan^{-1}(x_k - s_{xk}, y_k - s_{yk}), \quad h(\mathbf{x}_k, \mathbf{s}_k) \in [0, 2\mathbf{p}).$$

The sensors are stationary with respect to the ground in our application. Therefore, $\dot{s}_{xk} = \dot{s}_{yk} = 0$, for all k .

3.2 Clutter Model

In addition to actual measurement originating from a true target, there may be false measurements observed by a sensor in a scan. We assume that the number of false measurements m_k in the k^{th} scan obeys the Poisson distribution. Thus the probability of observing m_k false measurements in the k^{th} scan is

$$(3.11) \quad P_{FA}(m_k) = \frac{e^{-\mathbf{I}A} (\mathbf{I}V)^{m_k}}{m_k!},$$

where \mathbf{I} is the average number of false measurements per unit area of measurement space per scan and V is the volume of the measurement space. For the omni-directional acoustic sensor

$$(3.12) \quad V = 2\mathbf{p}.$$

We assume that the false measurements are uniformly distributed in the measurement space.

3.3 Likelihood Function

The likelihood functions for target originated measurement and false measurement are

$$(3.13) \quad p(z_k | \mathbf{x}_k) = \begin{cases} N[z_k; h_k(\mathbf{x}_k, \mathbf{s}_k), R_k] = N[z_k(k) - h_k(\mathbf{x}_k, \mathbf{s}_k); 0, R_k], & \text{if } z_k \text{ is target originated,} \\ 1/V, & \text{if } z_k \text{ is not target originated.} \end{cases}$$

4. BAYESIAN ESTIMATION

A general discrete time dynamics for the state $\mathbf{x}_k \in \mathfrak{R}^n$ of a system is described by [11],[14],[15],[20],[30]

$$(4.1) \quad \mathbf{x}_k = \mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \mathbf{w}_{k-1}),$$

where $\mathbf{f}_{k-1} : \mathfrak{R}^n \times \mathfrak{R}^p \rightarrow \mathfrak{R}^n$ is the system evolution function and $\mathbf{w}_{k-1} \in \mathfrak{R}^p$ is a white noise sequence (known as the process noise) independent of the past and current states. We assume that the probability density function (pdf) of \mathbf{w}_{k-1} is known. The function \mathbf{f}_{k-1} may be a linear or nonlinear function of \mathbf{x}_{k-1} and \mathbf{w}_{k-1} , and the pdf of \mathbf{w}_{k-1} may be arbitrary.

We assume that measurements $\{\mathbf{z}_k \in \mathfrak{R}^m\}$ are available at discrete times and a functional relationship between the measurement \mathbf{z}_k and the state \mathbf{x}_k is known [11],[14],[15],[20],[30]:

$$(4.2) \quad \mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \mathbf{v}_k).$$

where $\mathbf{h}_k : \mathfrak{R}^n \times \mathfrak{R}^r \rightarrow \mathfrak{R}^m$ is the measurement model function and $\mathbf{v}_k \in \mathfrak{R}^r$ is a white noise sequence (known as the measurement noise) with known pdf. We assume that \mathbf{v}_k is independent of the past and current states and process noise. The measurement function \mathbf{h}_k may be a linear or nonlinear function of \mathbf{x}_k and \mathbf{v}_k , and the pdf of \mathbf{v}_k may be arbitrary. Let \mathbf{Z}^k denote the set of measurements $\{\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_k\}$.

Our objective is to compute the conditional density $p(\mathbf{x}_k | \mathbf{Z}^k)$ of the state \mathbf{x}_k given all the measurements \mathbf{Z}^k at time k . Suppose $p(\mathbf{x}_{k-1} | \mathbf{Z}^{k-1})$ is known. Then $p(\mathbf{x}_k | \mathbf{Z}^k)$ can be computed from $p(\mathbf{x}_{k-1} | \mathbf{Z}^{k-1})$ using the prediction and the measurement update steps. Using the prediction step, the prior pdf of the state at time t_k is given by [11],[14],[15],[20],[30]

$$(4.3) \quad \begin{aligned} p(\mathbf{x}_k | \mathbf{Z}^k) &= \int p(\mathbf{x}_k | \mathbf{x}_{k-1}, \mathbf{Z}^{k-1}) p(\mathbf{x}_{k-1} | \mathbf{Z}^{k-1}) d\mathbf{x}_{k-1} \\ &= \int p(\mathbf{x}_k | \mathbf{x}_{k-1}) p(\mathbf{x}_{k-1} | \mathbf{Z}^{k-1}) d\mathbf{x}_{k-1} \end{aligned},$$

where $p(\mathbf{x}_k | \mathbf{x}_{k-1})$ is known as the state transition density and is determined by the system dynamics model (4.1). The second step on the RHS of (4.3) is obtained from the first using the system dynamics model (4.1). Then the prior pdf of the state $p(\mathbf{x}_k | \mathbf{Z}^k)$ can be updated at time k by Bayes' rule using the measurement \mathbf{z}_k [11],[14],[15],[20],[30]:

$$(4.4) \quad p(\mathbf{x}_k | \mathbf{Z}^k) = \frac{p(\mathbf{z}_k | \mathbf{x}_k, \mathbf{Z}^{k-1}) p(\mathbf{x}_k | \mathbf{Z}^{k-1})}{p(\mathbf{z}_k | \mathbf{Z}^{k-1})} = \frac{p(\mathbf{z}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathbf{Z}^{k-1})}{p(\mathbf{z}_k | \mathbf{Z}^{k-1})},$$

$$(4.5) \quad p(\mathbf{z}_k | \mathbf{Z}^{k-1}) = \int p(\mathbf{z}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathbf{Z}^{k-1}) d\mathbf{x}_k.$$

$p(\mathbf{z}_k | \mathbf{x}_k)$ is known as the likelihood function and is determined by the measurement model (4.2). Equations (4.3) and (4.4) represent a general recursive solution for the state estimation problem in the Bayesian framework. These equations are valid for any pdf of the state, process noise, and measurement noise with general system dynamics and measurement models. However, closed form solutions are not always possible. When \mathbf{f}_k and \mathbf{h}_k are linear, and \mathbf{w}_k and \mathbf{v}_k are additive Gaussian noises with known pdf, (4.3) and (4.4) give rise to the well-known Kalman filter (KF) algorithm [3],[4],[6],[19]. Since \mathbf{h}_k for the acoustic measurement model is nonlinear, the EKF is not an optimal estimator for the problem.

Given \mathbf{Z}^k , the conditional mean estimator [11],[14],[15]

$$(4.6) \quad \hat{\mathbf{x}}_{k|k} := E\{\mathbf{x}_k | \mathbf{Z}^k\} = \int \mathbf{x}_k p(\mathbf{x}_k | \mathbf{Z}^k) d\mathbf{x}_k$$

represents the minimum variance or minimum mean-square error (MMSE) estimator [3],[4]. The covariance of the conditional mean estimate is given by [11],[14],[15]

$$(4.7) \quad \mathbf{P}_{k|k} := E\{(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k})(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k})' | \mathbf{Z}^k\} = \int (\mathbf{x}_k - \hat{\mathbf{x}}_{k|k})(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k})' p(\mathbf{x}_k | \mathbf{Z}^k) d\mathbf{x}_k.$$

We emphasize that if the posterior distribution is multi-modal, then the conditional mean and associated covariance are not sufficient statistics.

5. SINGLE TARGET TRACKING IN CLUTTER USING PARTICLE FILTER

The computation of the likelihood function in the absence of clutter is straightforward. We use the PDA approach [4],[11],[18] to compute the likelihood function required by the PF. First we present the algorithm to compute the likelihood function and then we present the steps of the PF algorithm.

5.1 Likelihood Function

When there are m_k measurements in the k^{th} scan, we have the following mutually exclusive and exhaustive hypotheses [4]:

$$(5.1) \quad \mathbf{q}_{kj} = \begin{cases} \{\mathbf{z}_{kj} \text{ is the target originated measurement}\}, & j = 1, 2, \dots, m_k, \\ \{\text{none of the measurements is target originated}\}, & j = 0. \end{cases}$$

Using the total probability theorem [3]

$$(5.2) \quad p(\mathbf{z}_k | \mathbf{x}_k) = \sum_{j=0}^{m_k} p(\mathbf{z}_k | \mathbf{q}_{kj}, \mathbf{x}_k) P(\mathbf{q}_{kj} | \mathbf{x}_k),$$

where $P(\mathbf{q}_{kj} | \mathbf{x}_k)$ and $p(\mathbf{z}_k | \mathbf{q}_{kj}, \mathbf{x}_k)$ are the probability and the likelihood, respectively, of the hypothesis \mathbf{q}_{kj} . Since the measurements are conditionally independent,

$$(5.3) \quad p(\mathbf{z}_k | \mathbf{q}_{kj}, \mathbf{x}_k) = \prod_{r=0}^{m_k} p(\mathbf{z}_{kr} | \mathbf{q}_{kj}, \mathbf{x}_k).$$

Using the measurement model (3.3) and likelihood in (3.13)

$$(5.4) \quad p(\mathbf{z}_{kj} | \mathbf{q}_{kj}, \mathbf{x}_k) = \begin{cases} N[\mathbf{z}_{kj}; \mathbf{h}_k(\mathbf{x}_k), \mathbf{R}_k] = N[\mathbf{z}_{kj}(k) - \mathbf{h}_k(\mathbf{x}_k); 0, \mathbf{R}_k], & \text{if } \mathbf{z}_{kj} \text{ is target originated,} \\ 1/V, & \text{if } \mathbf{z}_{kj} \text{ is not target originated.} \end{cases}$$

where V is the volume of the measurement space. For generality, we treat \mathbf{z}_{kj} as a vector measurement. Using (5.4) in (5.3), we get

$$(5.5) \quad p(\mathbf{z}_k | \mathbf{q}_j(k), \mathbf{x}_k) = \begin{cases} V^{-m_k+1} N(\mathbf{?}_{kj}; 0, \mathbf{R}_k), & j = 1, 2, \dots, m_k, \\ V^{-m_k}, & j = 0, \end{cases}$$

$$(5.6) \quad \mathbf{?}_{kj} := \mathbf{z}_{kj} - \mathbf{h}_k(\mathbf{x}_k),$$

$$(5.7) \quad N(\mathbf{?}_{kj}; 0, \mathbf{R}_k) = \frac{\mathbf{e}_{kj}}{\mathbf{h}_k},$$

$$(5.8) \quad e_j(k) := \exp\left[-\frac{1}{2} \mathbf{?}'_{kj}(k) \mathbf{R}_k^{-1} \mathbf{?}_{kj}\right],$$

$$(5.9) \quad \mathbf{h}_k := [(2p)^m |\mathbf{R}_k|]^{1/2}.$$

For simplicity in notation, define

$$(5.10) \quad \mathbf{g}_j := P(\mathbf{q}_j(k) | \mathbf{x}_k).$$

Then we get [4]

$$(5.11) \quad \mathbf{g}_j = \begin{cases} \frac{1}{m_k} P_D \left[P_D + (1 - P_D) \frac{P_{FA}(m_k)}{P_{FA}(m_k - 1)} \right]^{-1}, & j = 1, 2, \dots, m_k \\ (1 - P_D) \frac{P_{FA}(m_k)}{P_{FA}(m_k - 1)} \left[P_D + (1 - P_D) \frac{P_{FA}(m_k)}{P_{FA}(m_k - 1)} \right]^{-1}, & j = 0. \end{cases}$$

Using the Poisson model (3.11) for clutter, we get

$$(5.12) \quad \frac{P_{FA}(m_k)}{P_{FA}(m_k - 1)} = \frac{(m_k - 1)! (\mathbf{I}V)^{m_k}}{m_k! (\mathbf{I}V)^{m_k - 1}} = \frac{\mathbf{I}V}{m_k}.$$

Substituting (5.12) in (5.11), we get

$$(5.13) \quad \mathbf{g}_j = \begin{cases} P_D [m_k P_D + (1 - P_D) \mathbf{I}V]^{-1}, & j = 1, 2, \dots, m_k \\ (1 - P_D) \mathbf{I}V [m_k P_D + (1 - P_D) \mathbf{I}V]^{-1}, & j = 0. \end{cases}$$

Define

$$(5.14) \quad \mathbf{b}_{kj} := p(\mathbf{z}_k | \mathbf{q}_{kj}, \mathbf{x}_k) P(\mathbf{q}_{kj} | \mathbf{x}_k), \quad j = 0, 1, \dots, m_k.$$

Then

$$(5.15) \quad \mathbf{b}_{kj} = \begin{cases} V^{-m_k+1} N(\mathbf{?}_{kj}; 0, \mathbf{R}_k) \mathbf{g}_j, & j = 1, \dots, m_k, \\ V^{-m_k} \mathbf{g}_j, & j = 0. \end{cases}$$

Using (5.7) and (5.13) in (5.15), we get

$$(5.16) \quad \mathbf{b}_{kj} = \mathbf{x} \begin{cases} e_{kj}, & j = 1, \dots, m_k, \\ b_k, & j = 0, \end{cases}$$

$$(5.17) \quad b_k := [(1 - P_D) / P_D] (\mathbf{I} \mathbf{h}_k),$$

where \mathbf{x} is a constant. Substitution of (5.16) in (5.2) gives

$$(5.18) \quad p(\mathbf{z}_k | \mathbf{x}_k) = \sum_{j=0}^{m_k} p(\mathbf{z}_k | \mathbf{q}_{kj}, \mathbf{x}_k) P(\mathbf{q}_{kj} | \mathbf{x}_k) = \mathbf{x} [b_k + \sum_{j=1}^{m_k} e_{kj}].$$

5.2 Steps of PF Algorithm

1. Initialization

Set scan index $k = 0$.

Sample $\{\mathbf{x}_0^i \sim p_0(\mathbf{x})\}_{i=1}^N$.

2. Increment scan index k : $k = k + 1$.

3. Prediction

Generate N samples of the process noise:

$$(5.19) \quad \{\mathbf{w}^i(k, k-1) \sim N(0, \mathbf{Q}(k, k-1))\}_{i=1}^N.$$

Compute N predicted state vectors:

$$(5.20) \quad \{\mathbf{x}_k^i = \mathbf{F}(k, k-1) \mathbf{x}_{k-1}^i + \mathbf{w}^i(k, k-1)\}_{i=1}^N.$$

4. Compute likelihoods $\{p(\mathbf{z}_k | \mathbf{x}_k^i)\}_{i=1}^N$.

$$(5.21) \quad \mathbf{n}_{kj}^i := \mathbf{z}_{kj} - \mathbf{h}_k(\mathbf{x}_k^i).$$

We note that the predicted measurement is the same for all measurements $\{\mathbf{z}_{kj}\}_{j=1}^{m_k}$.

$$(5.22) \quad e_{kj}^i = \exp[-\frac{1}{2}(\mathbf{z}_{kj}^i)' \mathbf{R}_k^{-1} \mathbf{z}_{kj}^i] = \exp[-\frac{1}{2}[\mathbf{z}_{kj} - \mathbf{h}_k(\mathbf{x}_k^i)]' \mathbf{R}_k^{-1} [\mathbf{z}_{kj} - \mathbf{h}_k(\mathbf{x}_k^i)]].$$

$$(5.23) \quad l_k^i = [b_k + \sum_{j=1}^{m_k} e_{kj}^i], \quad i = 1, 2, \dots, N.$$

5. Update weights and normalize

$$(5.24) \quad w_k^i = \frac{w_{k-1}^i l_k^i}{\sum_{i=1}^N w_{k-1}^i l_k^i}, \quad i = 1, 2, \dots, N.$$

6. Compute the measurement updated state estimate:

$$(5.25) \quad \hat{\mathbf{x}}_{k|k} = \sum_{i=1}^N w_k^i \mathbf{x}_k^i$$

7. Compute effective sample size (N_{eff})

$$(5.26) \quad N_{\text{eff}} = 1 / \sum_{i=1}^N (w_k^i)^2.$$

If $N_{\text{eff}} > N_{\text{thres}}$, then

Go to step 2.

Else

8. Resample a new set $\{\mathbf{x}_k^{i*}\}_{i=1}^N$ by sampling with replacement N times from the discrete set

$\{\mathbf{x}_k^i\}_{i=1}^N$ where $\Pr(\mathbf{x}_k^{i*} = \mathbf{x}_k^j) = w_k^j$.

Set $\{w_k^i = 1/N\}_{i=1}^N$.

Go to step 3.

End

6. NUMERICAL SIMULATIONS AND RESULTS

The truth trajectory of the target and the positions of four acoustic UGSs are shown in Figure 6.1. We use the nearly constant velocity model to generate the truth trajectory. The initial position and speed of the target are (-220 m, 300m) and 40 km/hr, respectively. The initial azimuth angle of the target is 45 degrees. The PSD of each component the acceleration process noise (q) for the truth trajectory is $0.01 \text{ m}^2 \text{ s}^{-3}$. We use 5000 particles throughout our simulation. The detections and false measurements at various scans for the case when $P_D = 0.9$ and the average number of false measurements per scan is one, are shown in Figures 6.2. The truth and PF estimated position and velocity for a benign scenario where the $P_D = 0.9$ and average number of false measurements per scan is one, are shown in Figures 6.3-6.5. We observe from Figures 6.3-6.5. that the PF algorithm estimates the position and velocity of the target accurately for this benign scenario. The root mean square (RMS) position and velocity errors are presented in Table 6.1. We present results for other cases in Table 6.1 by varying the P_D and average number of false measurements per scan. We observe from Figures 6.3-6.5 and 6.7-6.9 that the large RMS errors are mainly due to transients in the early part of estimation process. After the transient phase, the PF estimated state is close to the true state.

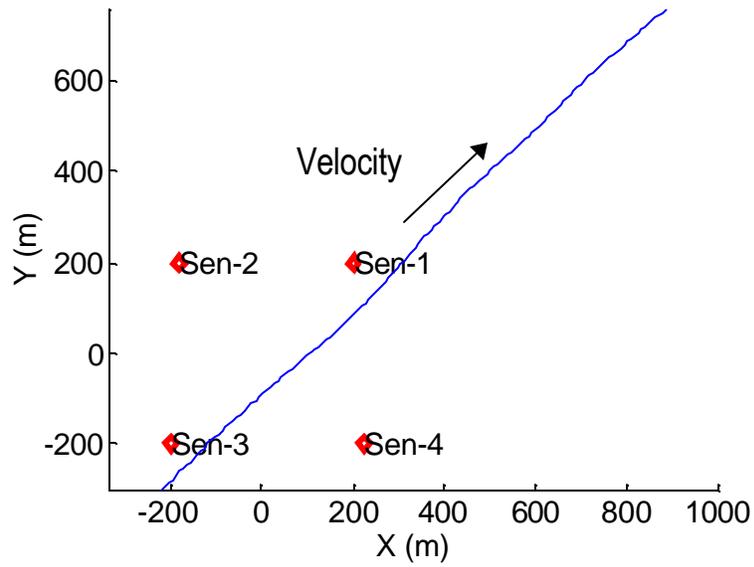


Figure 6.1. Sensor Locations and Truth Trajectory of the Target using the Nearly Constant Velocity Motion

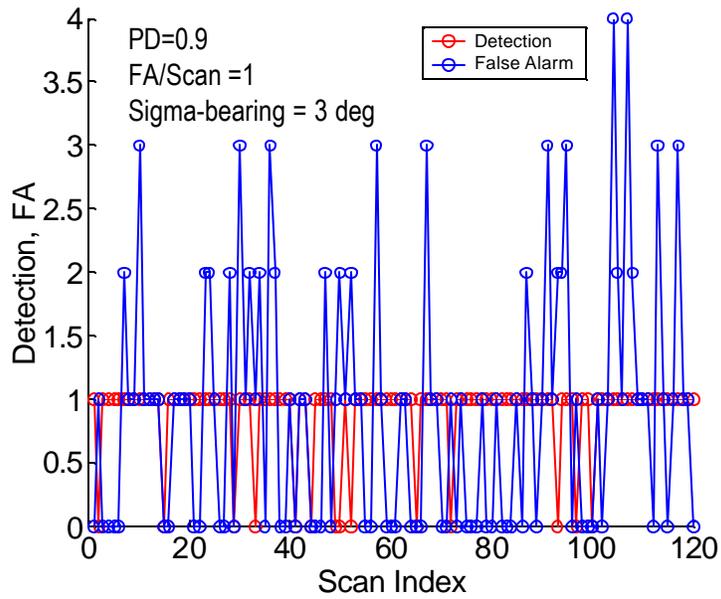


Figure 6.2. Detection and false alarms at various scans for high probability of detection and low clutter.

Table 6.1. PF-estimated position and velocity RMS errors using 120 scans.

Bearing Sigma (deg)	P_D	Average Number of False Alarms per Scan	RMS Position Error (m)	RMS Velocity Error (m/s)
3	0.9	1	38.824	3.279
3	0.7	1	67.682	3.315
3	0.7	2	94.076	3.654
3	0.7	3	132.235	4.118

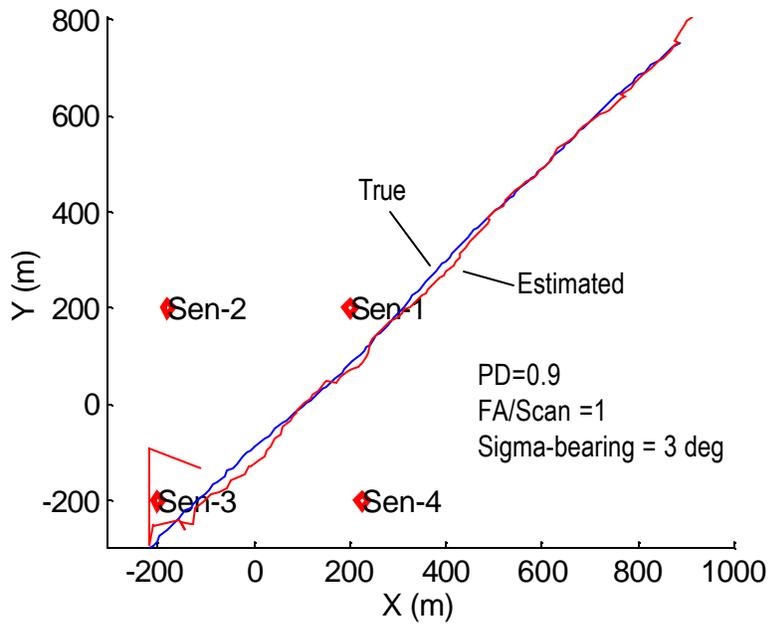


Figure 6.3. Truth and PF-estimated trajectories for high probability of detection and low clutter.

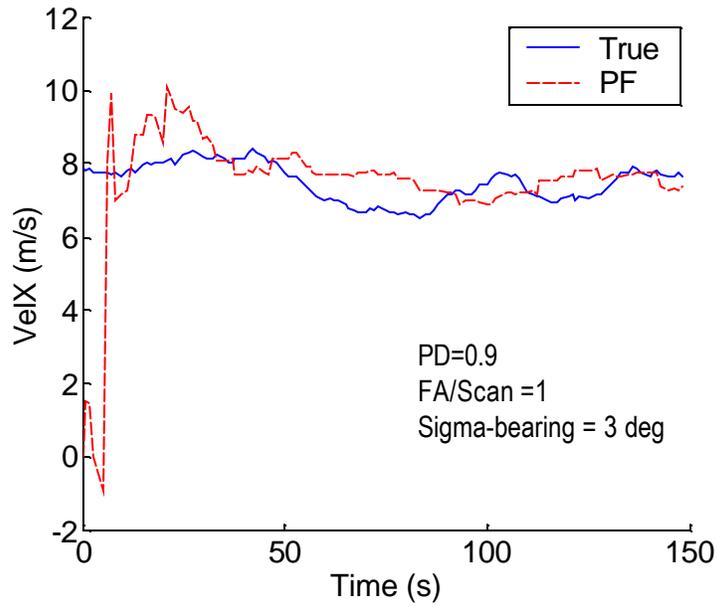


Figure 6.4. Truth and PF-estimated X-component of velocity for high probability of detection and low clutter.

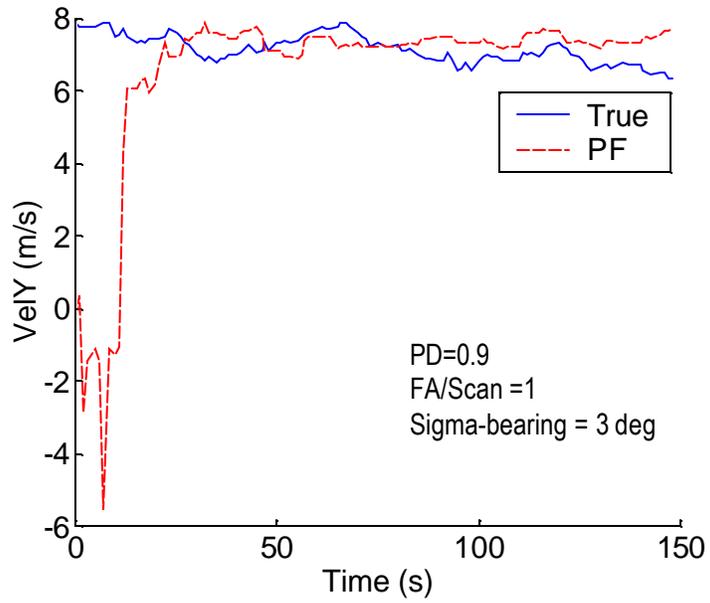


Figure 6.5. Truth and PF-estimated Y-component of velocity for high probability of detection and low clutter.

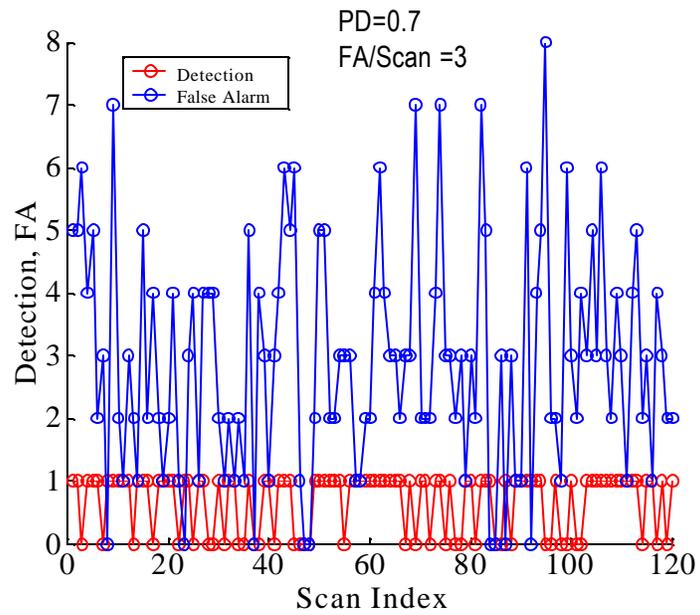


Figure 6.6. Detection and false alarms at various scans for low probability of detection and high clutter.

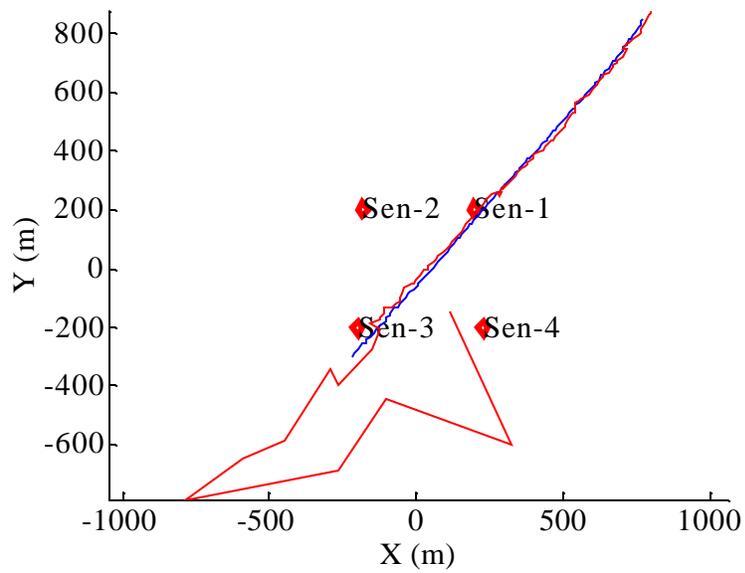


Figure 6.7. Truth and PF-estimated trajectories for low probability of detection and high clutter.

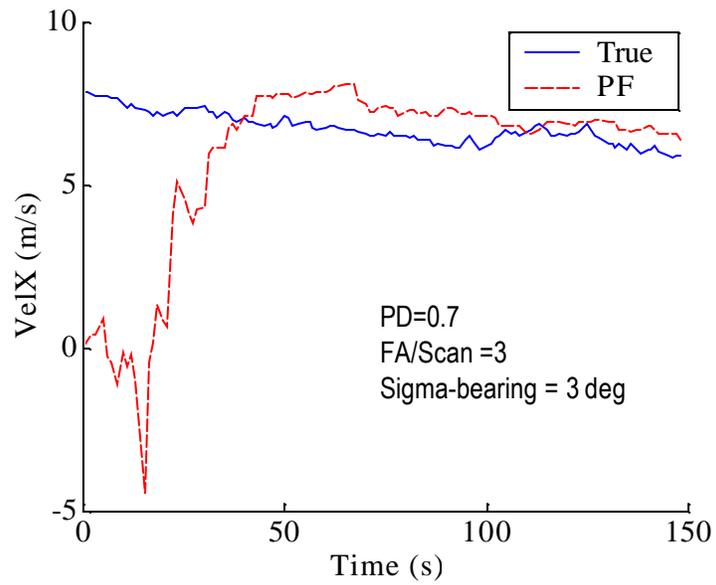


Figure 6.8. Truth and PF-estimated X-component of velocity for low probability of detection and high clutter.

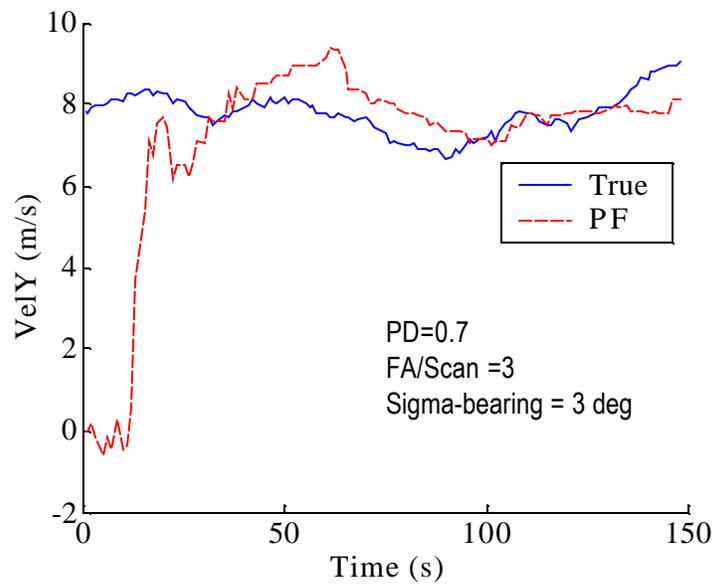


Figure 6.9. Truth and PF-estimated Y-component of velocity for low probability of detection and high clutter.

7. SUMMARY AND CONCLUSIONS

In this paper, we have addressed the single target multiple acoustic UGS tracking in clutter using the particle filter (PF) algorithm. We have used realistic values for the probability of detection and false alarm. We have demonstrated that the PF algorithm works in a robust manner when the probability of detection is low and the false alarm is high as is the case in realistic harsh scenarios. In our future work, we plan to compare the performance of the PF with the EKF using the PDA approach and analyze the estimation accuracy by varying the accuracy of the acoustic sensor measurement.

ACKNOWLEDGEMENT

This material is based on work supported by the Air Force Office of Scientific Research under Contract No. F49620-98-C-0010.

REFERENCES

1. V. Aidala, Kalman Filter Behavior in Bearing-Only Tracking Applications, IEEE Transactions on Aerospace and Electronic Systems, Vol. 15(1), January 1979.
2. S. Arulampalam and B. Ristic, Comparison of the Particle Filter with Range-Parametrised and Modified Polar EKFs for Angle-Only Tracking, Signal and Data Processing of Small Targets: Proceedings of SPIE, vol. 4048, pp. 288-299, April 2000.
3. Y. Bar-Shalom, X. R. Li and T. Kirubarajan, Estimation with Applications to Tracking and Navigation, Wiley & Sons, 2001.
4. Y. Bar-Shalom and X. Rong Li, Multitarget-Multisensor Tracking: Principles and Techniques, Storrs, CT: YBS Publishing, 1995.
5. N. Bergman, Recursive Bayesian Estimation, Navigation and Tracking Applications, Ph.D. thesis, Linkoping University, Sweden, 1999.
6. S. Blackman and R. Popoli, Design and Analysis of Modern Tracking Systems, Chapter 9, Artech House, 1999.
7. H. A. P. Blom, and Y. Bar-Shalom, The Interacting Multiple Model Algorithm for Systems with Markovian Switching Coefficients, IEEE Transactions on Automatic Control, 22(3): 302-312, 1977.
8. E. M. Carapezza, and T. M. Hintz, Editors, Unattended Ground Sensor Technologies and Applications II: Proceedings of the SPIE, vol. 4040, 24 April, 2000, Orlando, Florida.
9. E. M. Carapezza, D. B. Law, and K. T. Stalker, Editors, Unattended Ground Sensor Technologies and Applications, Proceedings of the SPIE, vol. 3713, 8-9 April, 1999, Orlando, Florida.
10. J. Carpenter, P. Clifford, and P. Fernhead, An improved particle filter for nonlinear problems, IEE Proceedings-Radar, Sonar, Navig. , vol. 146, pp. 2-7, Feb. 1999.
11. S. Challa, and N. J. Gordon, Target Tracking Using Particle Filters, Proceedings of the Workshop on Estimation, Tracking, and Fusion: A Tribute to Yaakov Bar-Shalom, Monterey, CA, May 2001.
12. H. Chen, T. Kirubarajan, and Y. Bar-Shalom, Decentralized vs. Centralized Tracking for Air-to-air Scenarios, Signal and Data Processing of Small Targets: Proceedings of SPIE, vol. 4048, pp. 440-451, April 2000.
13. F. M. Dommermuth, Exploiting sound absorption for target motion analysis, Journal of the Acoustical Society of America, Vol. 91:3, (March 1992), 1545-1551.
14. A. Doucet, On sequential Monte Carlo sampling methods for Bayesian filtering, Technical Report, University of Cambridge, 1998.
15. A. Doucet, N. De Freitas, and N. Gordon, N. J., Editors, Sequential Monte Carlo Methods in Practice, Springer Verlag, 2001.
16. A. Doucet, S. Godsill, and C. Andrieu, On sequential Monte Carlo sampling methods for Bayesian filtering, Statistics and Computing, vol. 10, pp. 197-208, 2000.
17. P. Fernhead, Sequential Monte Carlo methods in filter theory, Ph.D. thesis, Merton College, University of Oxford, 1998.
18. T. E. Fortmann, Bar-Shalom, Y., Scheffe, M., and Gelfand, S., Detection threshold for tracking in clutter - A connection between estimation and signal processing, IEEE Transactions in Automatic Control, AC-30 (Mar. 1985), 221-229.
19. A. Gelb, Ed., Applied Optimal Estimation, The MIT Press, 1974.

20. N. J. Gordon, D. J. Salmond and A. F. M. Smith, Novel approach to nonlinear/nonGaussian Bayesian state estimation, IEE Proceedings-F, vol. 140, pp. 107-113, April 1993.
21. A. Holtsberg, A Statistical Analysis of Bearings-only Tracking, Ph.D. Thesis, Lund Institute of Technology, Sweden, 1992.
22. P. E. Howland, Passive Tracking of Airborne Targets Using Only Doppler and DOA Information, IEE Colloq. on Algorithms for Target Tracking Digest, (# 104), London, May 1995.
23. C. Jauffret, and Y. Bar-Shalom, Track formation with bearing and frequency measurements in clutter, IEEE Transactions on Aerospace and Electronic Systems, AES-26, (Nov. 1990), 999-1010.
24. T. Kirubarajan and Y. Bar-Shalom, Target Motion Analysis in Clutter for Passive Sonar Using Amplitude Information, IEEE Trans. Aerospace and Electronic Systems, AES-32(4):1367-1384, October 1996.
25. T. R. Kronhamn, Bearing-only target motion analysis on a multihypothesis Kalman filter and adaptive ownship motion control, IEE Proc. Radar Sonar Navig. vol. 145, pp. 247-252, Aug. 1998.
26. T. Kurien, Issues in the Design of Practical Multitarget Tracking Algorithms, in Multitarget-Multisensor Tracking, Y. Bar-Shalom (ed.), pp. 43-83, Artech House, 1990.
27. D. Lerro, and Y. Bar-Shalom, Interacting multiple model tracking with target amplitude feature, IEEE Transactions on Aerospace and Electronic Systems, AES-29, (Apr. 1993), 494-509.
28. X. Li and Y. Bar-Shalom, Multiple-model estimation with variable structure, IEEE Transactions on Automatic Control, 41(4), 1996.
29. J. S. Liu, and R. Chen, Sequential Monte Carlo methods for dynamic systems, Journal of the American Statistical Assoc., vol. 93, pp. 1032-1044, 1998.
30. M. Mallick, T. Kirubarajan, and S. Arulampalam, Comparison of Nonlinear Filtering Algorithms in Ground Moving Target Indicator (GMTI) Tracking, Proceedings of the 4th International Conference on Information Fusion, Montreal, Canada, August 2001.
31. A. Mazor, J. Averbuch, Y. Bar-Shalom, and S. Dayan, Interacting Multiple Model Methods in Target Tracking: A Survey, IEEE Transactions on Aerospace and Electronic Systems, 34(1), 1998.
32. S. Musick, J. Greenwald, C. Kreucher, and K. Kastella, Comparison of Particle Method and Finite Difference Nonlinear Filters for Low SNR Target Tracking, Proceedings of the 4th International Conference on Information Fusion, Montreal, Canada, August 2001.
33. S. C. Nardone, A. G. Lindgren, and K. F. Gong, Fundamental properties and performances of conventional bearing-only target motion analysis, IEEE Transactions on Automatic Control, AC-29 (Sept. 1984), 775-787.
34. R. O. Nielsen, Sonar Signal Processing, Boston, MA: Artech House, 1991.
35. H. M. Shertukde, and Y. Bar-Shalom, Detection and estimation for multiple targets with two omnidirectional sensors in the presence of false alarms, IEEE Transactions on Acoustics, Speech and Signal Processing, ASSP-38 (May 1990), 749-763.
36. L. B. Stotts, Unattended ground sensor related technologies: an Army Perspective, Unattended Ground Sensor Technologies and Applications II: Carapezza, E. M. and Hintz, T. M., Editors, Proceedings of the SPIE, vol. 4040, 24 April, 2000, Orlando, Florida.