

# Fuzzy Spatial Querying with Inexact Inference

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**Keywords:** uncertainty, inexact inference, fuzzy inference, spatial query, GIS, binary relationship, FuzzyClips implementation

## Abstract:

The issue of spatial database accuracy has been viewed as critical to the successful implementation and long-term viability of the geographic information system (GIS) technology. In order to improve the spatial querying accuracy and quality, the problems associated with the areas of fuzziness and uncertainty are of great common in the spatial databases. In this paper, we are dedicated to develop an approach that can perform fuzzy spatial querying under uncertainty. An inferring strategy under uncertainty is investigated. The study shows that the fuzzy set and the certainty factor can work together to deal with spatial querying. Querying examples implemented by FuzzyClips are also provided.

## 1. Introduction

Spatial queries in GIS involve human interpretation and knowledge, which are invariably imprecise, incomplete, or not totally reliable. To explore the realm of the uncertainty in spatial querying is always a major issue. There has been a strong demand to provide approaches that deal with inaccuracy and uncertainty in GIS [1].

Since the spatial querying deals with some concepts expressed by verbal language, the fuzziness and uncertainties are frequently involved. Hence, the

ability to query a spatial data under the fuzziness and uncertainty is one of the most important characteristics of any spatial database. In earlier works [2-4], a binary model for defining and representing topological and directional relationship between 2D objects was presented, which provided a basis for fuzzy querying capabilities. For implementation purpose, [5] introduced a modified data structure by using a unique geometry mapping, which preserves all of the binary relationships between two objects. A Clips-based implementation shows that this model can distinguish various cases of the same relationships, and perform a flexible querying.

However, in the implementation, the representation of the fuzzy variables is based on classical set theory where the membership can be clearly set to a set. Although classical sets are suitable for various applications and have proven to be an important tool for mathematics and computer science, they do not reflect the nature of human concepts and thoughts, which tend to be abstract and imprecise. The flaw comes from the sharp transition between inclusion and exclusion in a set. In this paper we are devoting to use the fuzzy set for dealing with the vague meaning of linguistic terms, in which the smooth transition is characterized by membership function.

Since queries expressed by verbal language often involve a mixture of uncertainties in the outcomes that are governed by the meaning of linguistic terms, there is an availability-related need for skilled inexact inferring approach to handle the uncertain feature. In this research, we intend to investigate an approach to

# REPORT DOCUMENTATION PAGE

Form Approved OMB No.  
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1. REPORT DATE (DD-MM-YYYY) 24-04-2002	2. REPORT TYPE	3. DATES COVERED (FROM - TO) xx-xx-2002 to xx-xx-2002
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4. TITLE AND SUBTITLE Fuzzy Spatial Querying With Inexact Inference Unclassified	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S) Yang, Huiqing ; Cobb, Maria ; Ali, Dia ; Rahimi, Shahram ; Petry, Frederick E. ;	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Computer Science & Statistics University of Southern Mississippi Hattiesburg, MS39406-5601	8. PERFORMING ORGANIZATION REPORT NUMBER
---	--

9. SPONSORING/MONITORING AGENCY NAME AND ADDRESS Office of Naval Research 800 North Quincy Street Alexandria, VA22217-5660	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT  
APUBLIC RELEASE

13. SUPPLEMENTARY NOTES

14. ABSTRACT  
See report.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:	17. LIMITATION OF ABSTRACT Public Release	18. NUMBER OF PAGES 9	19. NAME OF RESPONSIBLE PERSON email from ONR, (blank) lfenster@dtic.mil
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a. REPORT Unclassified	b. ABSTRACT Unclassified	c. THIS PAGE Unclassified	19b. TELEPHONE NUMBER International Area Code Area Code Telephone Number 703767-9007 DSN 427-9007
---------------------------	-----------------------------	------------------------------	--

**REPORT DOCUMENTATION PAGE**

Form Approved  
OMB No. 0704-0188

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.  
**PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.**

1. REPORT DATE (DD-MM-YYYY) 24-APR-2002		2. REPORT TYPE Conference Proceedings, (refereed)		3. DATES COVERED (From - To)	
4. TITLE AND SUBTITLE Fuzzy Spatial Querying With Inexact Inference				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER 63782N	
6. AUTHOR(S) HUIQING YANG    MARIA COBB    DIA ALI    SHARHRAM RAHIMI    FREDERICK PETRY (Dr.) KEVIN B SHAW				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER 74-7960-D1	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Research Laboratory Marine Geoscience Division Stennis Space Center, MS 39529-5004				8. REPORTING ORGANIZATION REPORT NUMBER NRL/PP/7440--02-1012	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Office of Naval Research 800 North Quincy Street Arlington, VA 22217-5660				10. SPONSOR/MONITOR'S ACRONYM(S) ONR	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release, distribution is unlimited					
13. SUPPLEMENTARY NOTES					
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15. SUBJECT TERMS spatial database, geographic information system (GIS) technology					
15. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Kevin Shaw
unclassified	unclassified	unclassified	Unlimited	8	19b. TELEPHONE NUMBER (include area code) 228-688-4197

handle uncertainties. The aim is to improve spatial querying accuracy and quality.

The work is organized as follows. In section 2, previous works are briefly overviewed, it shows some basic technique and strategies to deal with fuzzy multiple relations in spatial querying. The main part is section 3 which will describe an approach to perform fuzzy querying under the uncertainties. As an implementation tool, the basic syntax that will be used to construct fuzzy variables and access the fuzzy components are given in section 4. Based on FuzzyCLIPS, section 5 provides some improved fuzzy querying results and analysis. The final conclusions are presented in last section.

## 2. Overview Previous Works

Based on a spatial binary model by Dr. Maria Cobb [2-4], the previous works [5] presented a modified binary data structure that can support the fuzzy queries in two dimensions, and investigated an implementation by means of the CLIPS tool (C Language Integrated Production System). The query technique and strategies can be briefly overviewed as follows.

### 2.1 Basic Spatial Querying

We use the minimum bounding rectangles (MBR) as the basis for object representation. Figure 1 shows two objects in 2-dimension. Based on the MBR representation, we take into account two major type of spatial relationships: topological and directional relationship.

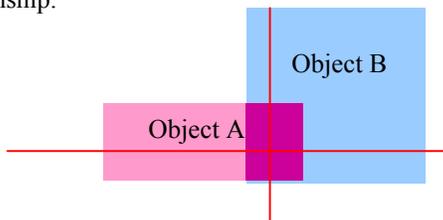


Figure 1. Two objects in 2-D

The topological relationships express the concepts of inclusion and neighborhood. A large body of related work has focused on the intersection mode that describes relations using intersections of object's interiors and boundaries. Based on this idea, we classified the topological relations as ten groups by means of geometrical similarity. Then we defined the qualitative topological relationship as a set:

$T = \{\text{disjoint, tangent, surrounded-by, partially-surrounded-by, partially-surrounds, overlaps,}$

$\text{overlapped-by, x-subspace, y-subspace, y-subspaced-by}\}$

The directional relationships are commonly concerned in everyday life. Most common directions are cardinal direction and their refinement. We defined the directional relations as a following set,

$D = \{\text{North, East, South, West, Northeast, Southeast, Southwest, Northwest}\}$ .

Each element in set T and D can be derived from 85 relationships obtained by extending 13 Allen's relations. [5] gives more details on this. Such relationships provided a significant resource for the basic binary spatial queries. The examples of such queries might look like these:

Object A **overlaps** Object B.  
Object A is **south** Object B.

### 2.2 Fuzzy Querying

Given two objects shown in Figure 1, object A overlaps object B in horizontal direction, and starts object B in vertical direction. It is obvious that this pair of objects belongs to multiple relationships, that is, 'object A is south, west, and southwest of object B'. Although the basic spatial queries can provide such multiple relations, these kinds of information do not associated with any degrees. This means it cannot distinguish the various cases of the same relationships. For example, given A overlaps B, does all of A overlap some of B, or does little of A overlaps most of B? In order to improve the spatial querying accuracy, we develop a technique deal with the multiple relations. Some basic ideas will describe as follows.

#### Reduce the topological relationship set

The topological relations have been found useful for increasing the speed of spatial query [5]. By analyzing the geometric characteristics of topological relationships, it is easy to find that all relations, except the disjoint relation, have a similar geometry, i.e. the common area is part of both objects involved. Thus, the original topological relation set can be reduce or reclassified to a binary topological set:

$T = \{\text{disjoint, connect}\}$ .

#### Geometry Mapping

For implementation purpose, the reduced topological relationship is mainly considered. Taking the

connected topological relation as an example, we design geometry mapping illustrated in Figure 2. The basic idea is to partition each object into its sub-group according the reference area shown in Figure 2, and then map each sub-group to a node. In this way, each object is represented by 9 nodes in Figure 3. Each node has two weights – area weight and node weight that are used to determine the special degree. From mathematical viewpoints, the geometry mapping presented here is one-to-one which guarantees a unique representation .

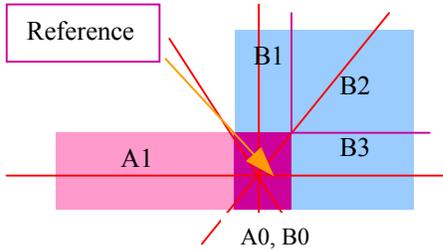


Figure 2. Partitioning two objects in 2D

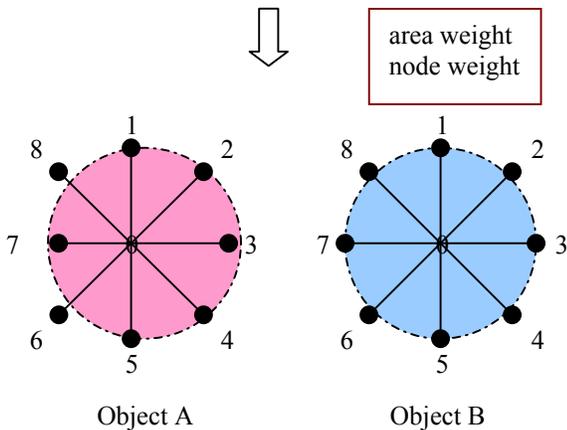


Figure 3. Geometry Mapping

Simply, area weight can be calculated by:

$$AW = (\text{area of an object sub-group}) / (\text{area of the entire object}).$$

Node weight can be obtained by

$$NW = AW \cdot (\text{Axis length}) / (\text{longest axis length}).$$

Fuzzy queries rely on the use of fuzzy qualifiers. Dr. Lotfi A. Zadeh, the founder of fuzzy logic theory, defined two kinds of qualifiers, absolute and relative [9]. We are interested in relative qualifiers. For performing qualitative queries, the resulting quantitative figures (AW, NW) are mapped to a range that corresponds to a term known as linguistic qualifiers.

Area Weight AW  $\rightarrow$  {all (0.96–1.0),  
most (0.6–0.95), some (0.3–0.59),  
little(0.06–0.29),none(0–0.05) }

Node Weight NW  $\rightarrow$  {directly(0.96–1.0),  
mostly(0.6–0.95), somewhat(0.3–0.59),  
slightly(0.06–0.29), not(0–0.05)}

These qualifiers allow users to distinguish the various cases of the same relationships. As shown in Figure 1, the based-Clips fuzzy query can provide the following information:

- Most of Object A overlaps Object B
- Object A overlaps some of Object B
- 
- $\Rightarrow$  Most of Object A overlaps some of Object B
- 
- Most of Object A is west of Object B
- Object A is mostly west of Object B
- 
- $\Rightarrow$  Most of Object A is mostly west of Object B

In this mapping, quantitative features of each object are stored in its associated nodes. By investigating querying information, we might say this structure will preserve all of the topological and directional relations that exist between two 2D objects.

The continuous work will explore an approach that can be used to improve querying accuracy.

### 3. Improved Fuzzy Queries

Because the spatial relationships depend on human interpretation, spatial querying should be related by fuzzy concepts. Some researchers have shown that the directional relationships are fuzzy concepts[6-7]. Actually, topological relationships are also fuzzy concepts. To support queries of the nature, previous works provided fuzzy queries without uncertainty that can handle the fuzziness by defining fuzzy qualifiers. However, in these kinds of fuzzy queries, the particular grades of membership have been defined as classical sets, where the membership can be clearly set to a set. There exist a gap between two members. To improve the fuzzy querying, the fuzzy set theory is concerned in our continuous research.

Uncertainty is an inevitable problem in GIS. Unfortunately, there are clear gaps in our understanding of how to incorporate uncertain reasoning into spatial querying purpose. Recently, models of uncertainty have been proposed for spatial information that incorporate ideas from natural

language processing, the value of information concept, non-monotonic logic and fuzzy set and evidential and probability theory. Each model is appropriate for a different type of inexactness in spatial data. In this research, we devote ourselves to explore an approach that can perform the fuzzy querying under uncertainties. The study exemplifies whether the fuzzy set and certainty factor can incorporate in spatial querying.

Moreover, above queries, whether basic queries or fuzzy queries are kind of qualitative queries, which provide subjective information. For spatial data analysis, we will also explore a technique that can provide objective information in spatial queries. This quantitative spatial querying might provide some supplementary information.

### 3.1 Fuzzy Querying under Uncertainty

Fuzziness occurs when the boundary of a piece of information is not clear-cut. Hence, fuzzy querying expands query capabilities by allowing for ambiguity and partial membership. The definition of the grades of membership is subjective and depends on the human interpretation. A way to eliminate subjectivity is another interested research field. Here simple membership functions will be considered.

Let us consider the area weight, node weight as fuzzy variables. Each variable has an associated fuzzy term set called “primary terms”, which is the set of values that the fuzzy variable may take.

For topological queries, the fuzzy variable (area weight) may have the primary term set {all, most, some, little, none}. We use topological qualifiers TQ to express it, i.e.

$$TQ = \{TQ_1, TQ_2, TQ_3, TQ_4, TQ_5\} \\ = \{all, most, some, little, none\}.$$

For directional queries, the node weight is mapped to the directional qualifiers, which can be represented as:

$$DQ = \{DQ_1, DQ_2, DQ_3, DQ_4, DQ_5\} \\ = \{directly, mostly, somewhat, slightly, not\}.$$

#### Define Membership Function Based on Classical Set

As mentioned in [9], relative qualifiers (fuzzy terms) can be represented as fuzzy subsets of the unit interval and use linguistic word. Based on the classical set, the membership function of qualifiers can be defined as a binary set, that is, complete membership has a value of 1, and no membership has a value of 0. The following tables give the quantifying description.

Table 1. Topological Qualifiers

Topological Qualifiers (TQ)	Area Weight (AW)
all	0.96 to 1.00
most	0.60 to 0.95
some	0.30 to 0.59
little	0.06 to 0.29
none	0.00 to 0.05

Table 2. Directional Qualifiers

Directional Qualifiers (DQ)	Node Weight (NW)
directly	0.96 to 1.00
mostly	0.60 to 0.95
somewhat	0.30 to 0.59
slightly	0.06 to 0.29
not	0.00 to 0.05

It is apparent this definition has shortcomings. Firstly, the definition is not exact. There is a big gap between two terms such as ‘all’ and ‘most’. Because a jump occurs, no qualifier is defined in some area such as interval [0.95, 0.96]. Secondly, the definition does not reflect the nature of human concepts and thoughts, which tend to be abstract and imprecise. As an alternative, a fuzzy set is mainly considered.

#### Define Membership Function Based on Fuzzy Set

A fuzzy set is a set without a crisp boundary. The smooth transition is characterized by membership functions that give fuzzy sets flexibility in modeling commonly used linguistic expressions. More formally a fuzzy set in a universe is characterized by a membership function  $\mu: U \rightarrow [0,1]$ . Figure 4 illustrates the primary term of fuzzy variable area weight. Each term represents a specific fuzzy set.

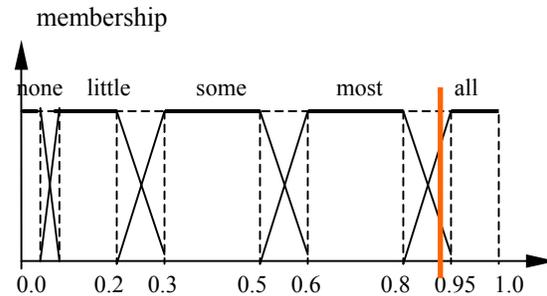


Figure 4. Topological qualifiers of variable AW

The fuzzy set functions for topological qualifiers can be described as:

$$\mu_{\text{all}}(AW) = \begin{cases} 1.0 & \text{if } 0.95 \leq AW \leq 1.0 \\ 20(AW - 0.80) / 3 & \text{if } 0.8 \leq AW \leq 0.95 \end{cases}$$

$$\mu_{\text{most}}(AW) = \begin{cases} 20(0.95 - AW) / 3 & \text{if } 0.8 \leq AW \leq 0.95 \\ 1.0 & \text{if } 0.6 \leq AW \leq 0.80 \\ 10(AW - 0.5) & \text{if } 0.5 \leq AW \leq 0.6 \end{cases}$$

$$\mu_{\text{some}}(AW) = \begin{cases} 10(0.6 - AW) & \text{if } 0.5 \leq AW \leq 0.6 \\ 1.0 & \text{if } 0.3 \leq AW \leq 0.5 \\ 10(AW - 0.2) & \text{if } 0.2 \leq AW \leq 0.3 \end{cases}$$

$$\mu_{\text{little}}(AW) = \begin{cases} 10(0.3 - AW) & \text{if } 0.2 \leq AW \leq 0.3 \\ 1.0 & \text{if } 0.02 \leq AW \leq 0.2 \\ 100(AW - 0.01) & \text{if } 0.01 \leq AW \leq 0.02 \end{cases}$$

$$\mu_{\text{none}}(AW) = \begin{cases} 100(0.02 - AW) & \text{if } 0.01 \leq AW \leq 0.02 \\ 1.0 & \text{if } 0.0 \leq AW \leq 0.01 \end{cases}$$

In the same way, the fuzzy set functions for directional querying can be described as:

$$\mu_{\text{directly}}(NW) = \begin{cases} 1.0 & \text{if } 0.95 \leq NW \leq 1.0 \\ 20(NW - 0.80) / 3 & \text{if } 0.8 \leq NW \leq 0.95 \end{cases}$$

$$\mu_{\text{mostly}}(NW) = \begin{cases} 20(0.95 - NW) / 3 & \text{if } 0.8 \leq NW \leq 0.95 \\ 1.0 & \text{if } 0.6 \leq NW \leq 0.80 \\ 10(NW - 0.5) & \text{if } 0.5 \leq NW \leq 0.6 \end{cases}$$

$$\mu_{\text{somewhat}}(NW) = \begin{cases} 10(0.6 - NW) & \text{if } 0.5 \leq NW \leq 0.6 \\ 1.0 & \text{if } 0.3 \leq NW \leq 0.5 \\ 10(NW - 0.2) & \text{if } 0.2 \leq NW \leq 0.3 \end{cases}$$

$$\mu_{\text{slightly}}(NW) = \begin{cases} 10(0.3 - NW) & \text{if } 0.2 \leq NW \leq 0.3 \\ 1.0 & \text{if } 0.02 \leq NW \leq 0.2 \\ 100(NW - 0.01) & \text{if } 0.01 \leq NW \leq 0.02 \end{cases}$$

$$\mu_{\text{not}}(NW) = \begin{cases} 100(0.02 - NW) & \text{if } 0.01 \leq NW \leq 0.02 \\ 1.0 & \text{if } 0.0 \leq NW \leq 0.01 \end{cases}$$

Unlike classical set theory that can describe membership to a set clearly, in fuzzy set theory membership of a term to a set is partial, i.e., a term belongs to a set with a certain grade of membership. Although it solves the gap problem in classical set expression, a new problem is coming, that is overlapping problem. For example, there are two different terms (all, most) associated with node weight  $AW=0.90$ .

### Certainty Factor (CF)

Uncertainty occurs when one is not absolutely certain about a piece of information. Uncertainty is referred to the lack of adequate and correct information to make decision. Given  $AW=0.90$ , the fuzzy querying can give the following querying phrase: 'All of Object A overlaps Object B and Most of Object A overlaps Object B.' How do we make the decision according to the information? Which querying information is reliable?

This reveals important deficiencies in areas such as the reliability of queries and the ability to detect inconsistencies in the knowledge. Because we cannot be completely certain that some qualifiers are true or others are false, we construct a certainty factor (CF) to evaluate the degree of certainty.

The degree of certainty is usually represented by a crisp numerical value one a scale from 0 to 1. A certainty factor of 1 indicates that it is very certain that a fact is true, and a certainty factor of 0 indicates that it is very uncertain that a fact is true. Some key ideas relevant to the determination the CF are discussed as following.

**Case 1.** Considered a single qualifier in each querying result.

- If the given weight only belongs to single fuzzy set, there is only one qualifier is involved in querying information such as:

All of Object A overlaps Object B  
(fuzzy qualifier is 'all', and  $AW_{ai}=0.99$ )

Object A is directly west of Object B.  
(fuzzy qualifier is 'directly' and  $NW_{ai}=0.99$ )

In this case, the grade of membership  $\mu( )$  can be used as a CF that represents the degree of belief. The results will look like:

All of Object A overlaps Object B  
with  $CF=\mu(\text{all}, AW_{ai}=0.99)=1.0$

Object A is directly west of Object B  
with  $CF=\mu(\text{directly}, NW_{ai}=0.99)=1.0$

Where  $AW_{ai}$  is the area weight of a sub-group associated with object A;  $NW_{ai}$  is the node weight of a subgroup associated with object A; and  $i, j \in I[0, 8]$ ,  $I$  represents an integer set.

- If the given weight is in the overlapping area, two qualifiers will be related. For example, **All** of Object A overlaps Object B (fuzzy qualifier is 'all', and  $AW_{ai}=0.90$ )

**Most** of Object A overlaps Object B (fuzzy qualifier is 'most' and  $AW_{ai}=0.90$ )

Object A is **directly** west of Object B. (fuzzy qualifier is 'directly', and  $NW_{ai}=0.90$ )

Object A is **mostly** west of Object B. (fuzzy qualifier is 'mostly', and  $NW_{ai}=0.90$ )

It is acceptable if we take the qualifier that has a larger grade of membership. The certainty factor can be determined by the maximum value, that is,

$$CF = \max \{ \mu_{all}(AW_{ai}=0.90), \mu_{most}(AW_{ai}=0.90) \} \\ = \mu_{all}(AW_{ai}=0.90).$$

$$CF = \max \{ \mu_{directly}(NW_{ai}=0.90), \mu_{mostly}(NW_{ai}=0.90) \} \\ = \mu_{directly}(NW_{ai}=0.90).$$

The final querying results should be **All** of Object A overlaps Object B with  $CF = \mu_{all}(AW_{ai}=0.90)$ .

Object A is **directly** west of Object B with  $CF = \mu_{directly}(NW_{ai}=0.90)$ .

As a result, the CF in case 1 can be obtained by

$$CF = \max \{ \mu_{TQ_k}(AW_i = \text{const}), k \in I[1,5], i \in I[0,8] \} \\ CF = \max \{ \mu_{DQ_k}(NW_j = \text{const}), k \in I[1,5], j \in I[0,8] \}$$

Where  $TQ_k$  is a topological qualifier;  
 $DQ_k$  is a directional qualifier;  
 $AW_i$  is an area weight associated object i-node  
 $NW_j$  is a node weight associated object i-node

**Case 2.** Considered multiple qualifiers in each querying result.

In the querying results, many pieces of fuzzy terms are conjoined (i.e. they are joined by AND), or disjointed (i.e. joined by OR). The examples of this type of queries are as follows:

**Most** of Object A overlaps **some** of Object B .  
**Some** of Object A is **slightly** south of Object B

Hence, to perform these kinds of queries, we have to handle multiple fuzzy qualifiers.

It is easy to understand that the relationship between different object qualifiers is conjunction, and the relationship between the same object qualifiers is disjointed. According to the fuzzy set theory, the conjunction and disjunction of fuzzy term can be respectively defined as the minimum and maximum of the involved facts. Therefore, the certainty factor contained multiple qualifiers can be determined by the following formulas:

Consider topological relationships

$$CF = \min \{ \max \{ \mu(TQ_k, AW_{ai} = \alpha) \}, \\ \max \{ \mu(TQ_k, AW_{bj} = \alpha) \}, \\ \text{where } k \in I[1,5] \ \& \ i, j \in I[0,8] \}$$

Consider topological/directional relationships

$$CF = \min \{ \max \{ \mu(TQ_k, AW_{ai} = \beta) \}, \\ \max \{ \mu(DQ_k, NW_{aj} = \beta) \}, \\ \text{where } k \in I[1,5] \ \& \ i, j \in I[0,8] \}$$

As seen above, an approach in which the fuzzy set and uncertainty can incorporate to perform the fuzzy queries is developed.

#### 4. FuzzyCLIPS Implementation

FuzzyCLIPS is an enhanced version of CLIPS developed at the National Research Council of Canada to allow the implementation of fuzzy expert systems. The modifications made to CLIPS contain the capability of handling fuzzy concepts and reasoning. It allows any mix of fuzzy and normal terms, numeric-comparison logic controls, and uncertainties in the rule and facts. Fuzzy sets and relations deal with fuzziness in approximate reasoning, while certainty factors for rules and facts manipulate the uncertainty.

In the process of our implementation, all fuzzy variables must be predefined with the *deftemplate* statement. This is an extension of the standard *deftemplate* construct in CLIPS. The extended syntax of this construct is as follows:

```
(deftemplate <name> [“<comments>”]
  <from> <to> [<unit>];
  ( t1
    :           ; list of primary terms
    tn
  )
)
```

where is used to identify the fuzzy variable. The description of the universe of discourse consists of three elements. The <from> and <to> should be floating point numbers. They represent the begin and end of the interval that describes the domain of the

fuzzy variable. The <unit> is a word that represents the unit of measurement (optional). The primary terms  $t_i$  are specifications for the fuzzy terms used to describe the fuzzy variable.

A primary term has the form:

(<name> <description of fuzzy set>),

where <name> represents the name of a primary term used to describe a fuzzy concept, and <description of fuzzy set> defines a membership function of the given primary term.

In our situation, fuzzy variables can be declared in *deftemplate* constructs as following:

```
(deftemplate TFVariable
  0 1; define the fuzzy variable area-weight
  ((all (0.8 0.0)(0.95 1.0)(1.0 1.0))
   (most (0.5 0.0)(0.6 1.0)(0.8 1.0) (0.95 0.0))
   (some (0.2 0.0)(0.3 1.0)(0.5 1.0) (0.6 0.0))
   (little(0.01 0.0)(0.02 1.0)(0.2 1.0) (0.3 0.0))
   (none (0.0 1.0)(0.01 1.0)(0.02 0.0)) ) )

(deftemplate DFVariable
  0 1; define the fuzzy variable node-weight
  ((directly (0.8 0.0)(0.95 1.0)(1.00 1.0))
   (mostly (0.5 0.0)(0.60 1.0)(0.80 1.0)(0.95 0.0))
   (somewhat (0.2 0.0)(0.30 1.0)(0.50 1.0)(0.60 0.0))
   (slightly (0.01 0.0)(0.02 1.0)(0.20 1.0)(0.30 0.0))
   (not (0.0 1.0)(0.01 1.0)(0.02 0.0)) ) )
```

A number of commands supplied in FuzzyCLIPS are very helpful for user to access fuzzy components that they need. In our application, when the weights (fuzzy variables) are calculated, the only interested information is the value of the fuzzy set at the specified weight value. The command *get-fs-value* provides us a tool to access the value. The syntax of the command is:

(get-fs-value ?<fact-variable> <number> )or  
 (get-fs-value <integer> <number> )or  
 (get-fs-value <fuzzy-value> <number> ),

where <number> is a value that must lie between the lower and upper bound of the universe of discourse for the fuzzy set. A simple example just look like:

```
(assert (TFVariable most) )
(defrule Get-CF
  ?f<- (TFVariable ?cf)
=>
  (printout t "CF for “ ?cf “ is = "
             (get-fs-value ?f AW) crlf)
  (retract ?f)
```

## 5. Querying Examples

Given two objects A(1, 1) (7, 2) and B(2, 1)(9, 4). The previous works based on CLIPS will provide the following query information.

```
=====
      Query results of binary
      spatial relationships
=====
2D physical relationship:  A |os| B.
topological relationship:
      Object A {overlaps } Object B.

Directional relationship:
      Object A {South } Object B
      Object A {South-West } Object B
      Object A {west }      Object B

      Little of Object A is West of Object B
      Object A is slightly West of Object B
-----
=> Little of A is slightly West of B.
```

Based FuzzyCLIPS, the querying results would be look like:

```
=====
Fuzzy Query results with certainty factor
=====
Topological information:
      83% of A overlaps 23.8% of B
      Most of A overlaps some of B with CF=0.778

Directional information:
      Little of A is West of B with CF = 1.0
      A is slightly West of B with CF = 1.0
-----
⇒ Little of A is slightly West of B
⇒ with CF= 1.0
```

More details for analysis are provided as following. Table 3 shows part of quantitative information stored in nodes associated with object.

Table 3. quantitative information stored in object nodes

Object Name	Node	AW	NW
Object A	center	0.8333	
	west	0.1667	0.1667
Object B	center	0.2380	
	north	0.4762	0.2655
	:	:	:

From these data, we know

- $AW_{a0} = 0.8333$ , TQ  $\rightarrow$  { all, most};
- $AW_{a7} = 0.1667$ , TQ  $\rightarrow$  { little};
- $AW_{b0} = 0.4762$ , TQ  $\rightarrow$  { some};
- $NW_{a7} = 0.1667$ , DQ  $\rightarrow$  { slightly}.

$$\begin{array}{l} \mu_{\text{all}}(AW_{a0}=0.8333) = 0.222 \\ \mu_{\text{most}}(AW_{a0}=0.8333) = 0.778 \\ \mu_{\text{some}}(AW_{b0}=0.4762) = 1.0 \end{array} \left. \begin{array}{l} \text{max=} \\ 0.778 \\ \text{min=} \\ 0.778 \end{array} \right\}$$

$$\begin{array}{l} \mu_{\text{little}}(AW_{a0}=0.1667) = 1.0 \\ \mu_{\text{slightly}}(NW_{b0}=0.1667) = 1.0 \end{array} \left. \right\} \text{min} = 1.0$$

## 6. Conclusion

Fuzziness and uncertainty can occur simultaneously. To improve spatial querying accuracy, our research investigates an approach that can perform fuzzy querying under uncertainty. The reliability of querying information is judged by a certainty factor (CF). The improved fuzzy querying is very flexible, and it can return spatial information in a wider variety of forms.

## 7. Acknowledgement

The authors would like to thank the ONR, PE 63782N and Dr. Dick Root, Program Manager, for funding this effort under the GECE Program.

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