Abstract

As feasibility study has been conducted to investigate whether compact adaptive optical (AO) systems based on the use of transparent wavefront modulators are viable. The results presented here consider the generic properties of suitable wavefront sensor technologies and establish sufficient conditions for their exploitation. Schemes by means of which these properties might be exploited in a multi-conjugate AO system are then considered. It is shown that the prospects for such systems are good, but further work is required to establish necessary conditions and to optimise the system.

1. Background

This programme is a brief feasibility study intended to establish whether a compact adaptive optical system based on transparent wavefront modulators (e.g. liquid crystal SLMs) and phase-diversity wavefront sensing, is realistic.

The rationale for the approach adopted was that a compact Adaptive Optical (AO) system, based on the use of diffractive optical elements (DOEs), phase-diverse wavefront sensing and transparent wavefront modulators, offered the potential for construction of AO systems with benefits such as:
1. Minimisation of non common-path errors by combining the wavefront sensor data and the corrected image in a single focal plane with essentially no separation of the ‘science’ and wavefront sensing optical trains
2. The ability of place wavefront modulators conjugate to multiple planes in the object space whilst preserving a compact and robust optical train that would be particularly well-suited to multi-conjugate AO (MCAO)
3. Avoidance of the requirement for conventional optics that increase the size and weight of the equipment, such as beam splitters and conventional bulk optical components for re-imaging conjugate planes.

The programme commenced in August 2001 and this report provides a summary of results achieved during the programme and progress during the final 3-month period from May 2002 to July 2002.
This report results from a contract tasking Heriot-Watt University as follows: The contractor will conduct a brief feasibility study to establish whether compact adaptive optical systems, based on phase-diverse wavefront sensing and transparent wavefront modulators, are viable. Such systems, if developed, would offer several significant advantages. The development of compact (and lightweight) adaptive optical systems is important in military applications where size and weight are always at a premium. Furthermore, the development of adaptive systems free from the need for a co-operative wavefront sensing beacon and able to operate with extended, low-contrast scenes would greatly extend the applicability of adaptive optics in military scenarios.
The programme consists of two distinct work packages and the results achieved are reported against each of these below.
2. Data processing scheme – work package 1

Data collection

Measurements were made using a set of 4 DOEs manufactured with quadratic distortion\(^1\) such that multiple image planes are recorded suitable for implementation of phase-diverse wavefront sensing in the system entrance pupil\(^2\). This implementation is very similar to ‘wavefront curvature sensing’\(^3\) although the term phase-diversity is retained here since other implementations will be discussed that represent a generalisation of this specific implementation.

The DOEs discussed above were used to test the robustness of the data collection system to misalignment of the optical system. Work conducted within a programme\(^4\) funded by the UK Engineering and Physical Sciences Research Council (EPSRC) has indicated that data collection systems based on these DOEs can provide high-accuracy measurement of wavefront shape\(^5\). In addition the system can be designed to maintain accurate control over system magnification (thus eliminating the need for data processing to compensate for magnification differences). Such simplification of the data processing is considered highly desirable in order to realise compact AO systems capable of high-temporal frequency correction.

Despite problems in the manufacture of the DOEs, experimental work was conducted to ascertain the dependence of image magnification on the positioning of the DOEs in the optical system. As reported in Progress Report 3, this work indicated that the positioning requirements on the DOE were not too stringent, with a 5% magnification error resulting from a 5% misalignment. Under typical operation this leads to a tolerance on DOE positioning of ~50\(\mu\)m, which is considered practical under most circumstances.

The basic principles behind this use DOEs manufactured in the form of quadratically-distorted diffraction gratings was developed in Prof Greenaway’s team while he worked at DERA (now QinetiQ). Through reciprocal agreements with the UK MoD, who funded that programme, it is understood that the US Government has access to the IP from that work.

Algorithm

\(^4\) GR/R321185/01, Greenaway AH & Taghizadeh MR, *Three-dimensional Imaging in True Colour*
The algorithmic work to be conducted under this programme was intended to re-design the data processing in order to ensure that the algorithm was consistent with the use of discontinuous wavefront modulators (e.g. pixellated liquid crystal devices). However, during the later phases of the programme it became progressively more obvious that the data processing, even when extremely simple, was likely to be difficult to accomplish in the multi-conjugate (anisoplanatic) conditions that were the prime motivation for the work. For that reason increasing emphasis was placed on the use of null sensing rather than wavefront reconstruction. In this mode of operation the DOE is used as before to produce images of the wavefront on either side of the entrance pupil of the optical system.

Figure 1, below, indicates the general scheme for phase diverse wavefront sensing using DOEs.

Figure 1: Generic AO scheme using a DOE. The DOEs based on the use of quadratically-distorted diffraction grating have been trade marked by QinetiQ under the name IMP®. In application it is possible to arrange that the image field 2 is an image of the scene under examination and image fields 1 and 3 have equal magnification and represent the data required for phase-diverse (wavefront curvature) wavefront sensing (see reference 2).

The wavefront reconstruction used in the QinetiQ approach is based on inversion of the Intensity Transport Equation (ITE) using Green’s functions (reference 2, the theory is described in a paper by Woods and Greenaway and recently submitted to J Opt Soc Am A).

The ITE effectively solves the problem of finding a complex wavefunction that matches the measured intensity on both ends of a cylinder. As illustrated by the lower half of figure 2, this solution cannot be unique unless the wavefront curvature is relatively modest.
A wavefront that is concave converges toward a focus and thus leads to increasing intensity as it propagates from left to right (upper section of both diagrams). A wavefront that is convex diverges from focus and leads to a decreasing intensity as it propagates (lower section of both diagrams). If the focus is well outside the volume enclosed between the measurement surfaces the ITE solution works well (upper figure) but a focus within the volume produces the same data and an ambiguous situation (c.f. lower figure). Avoidance of the ITE approximation can produce more accurate solutions as the focus approaches the volume, but a third measurement is required to resolve the ambiguity when the focus is within the volume.

The Green’s function solution uses the difference between the intensity measured on the ends of the cylinder to reconstruct the wavefront shape (phase)

\[ -\frac{k}{I} \frac{\partial I}{\partial z} = \nabla^2 \phi = -\frac{k}{I} \frac{I_2 - I_1}{\Delta} \]

where \( \Delta \) is the length of the cylinder. The Green’s function is dependent on the measurement geometry, but for a fixed geometry the above wavefront reconstruction reduces to a simple matrix multiply. On a 450MHz PC the solution for each wavefront mode (e.g. defocus mode or wavefront phase at a defined pixel) takes about 50\( \mu \)s to compute under the QinetiQ implementation.

For control of an AO system, however, it is not strictly necessary to reconstruct the input wavefront. A sufficient condition for satisfactory operation of an AO system is the ability to drive a wavefront modulator using a null sensor, where a control signal derived from a wavefront sensor system indicates the size, location and direction of the wavefront error. Thus, if the wavefront modulator is providing full correction of the input wavefront error, the control signal will be zero and the wavefront modulator will not be driven from its present position.

The use of the IMP\textsuperscript{®} DOE indicated in figure 1 corresponds to an image of the input intensity on two planes located symmetrically about the plane on which the wavefront is to be reconstructed. If the input wavefront is plane, the intensity on the two measurement planes will be equal and the difference evaluated according to equation 1 will be zero. The quadratic curvature of the IMP\textsuperscript{®} grating provides the differential focus that defines which planes are imaged.
A natural question to ask at this point is ‘what, if anything, is unique about the use of a defocus?’

Suppose, for the present, that plane 2 in figure 1 represents the plane in which the wavefront is to be controlled. The presence of the DOE imposes aberrations on the images of plane 2 that are formed in the + and – diffraction orders. Suppose that: 

\( \psi(r) \) is the input wavefront on plane 2
\( r \) is the argument in plane 2
\( p_\pm(r) \) is the point spread function associated with the imaging system (lens + DOE) in the + or – diffraction order.

Let:

\( \Psi(\xi) \) and \( P_\pm(\xi) \) be the Fourier transforms of \( \psi \) and \( p_\pm \) respectively.

Then the images in the diffraction orders can be expressed

\[
I_\pm(r) = \left| \int d\xi \ e^{-i\xi \cdot r} \ \Psi(\xi) \ P_\pm(\xi) \right|^2
\]

Now let

\[
\Psi(\xi) = |\Psi(\xi)| e^{i\psi(r)}
\]

and

\[
P_\pm(\xi) = |P_\pm(\xi)| e^{i\beta(r)}
\]

then, since the DOE produces the complex conjugate phase aberration in the + and – diffraction orders, equation 2 becomes

\[
I_\pm(r) = \left| \int d\xi \ e^{-i\xi \cdot r} \ |\Psi(\xi)| \ e^{i\psi(r)} \ |P_\pm(\xi)| e^{\pm i\beta(r)} \right|^2
\]

Now when the input wavefunction is plane (i.e. when no AO corrections are to be made) \( \Psi(\xi) \) has Hermitian symmetry and \( I_+(r) = I_-(r) \forall \text{ even} \beta(r) \). Further, for any non-Hermitian \( \Psi(\xi) \) we find that \( I_+(r) \neq I_-(r) \), so an error signal is generated if the wavefront is not plane, as required. Thus, according to equation 3 any non-trivial even phase aberration is sufficient, but not necessary, for the difference between the two images in the + diffraction orders to be usable as a null sensor.

Such a procedure has the advantage that no inversion to evaluate the wavefunction is required for operation of an AO system. Although there has been insufficient time to consider the optimisation of \( \beta(r) \) for the ensemble of \( \psi(r) \) expected in atmospheric propagation problems, this is clearly an optimisation that could be very valuable in operational terms. The ability to choose a wide variety of functions \( \beta(r) \) for this task justifies retention of the generic description ‘phase diversity’ for this approach rather than using the terminology ‘wavefront curvature’.

In optimising the function \( \beta(r) \) cognisance will also need to be taken of the requirement for an adequate signal to noise ratio in the measurement. In this respect there is no difference from the ‘wavefront curvature’ approach, in which the defocus must be chosen
to generate a difference between the measured intensities that exceeds significantly the noise associated with the detection process.

Within the present programme there has been insufficient time available to give consideration to this optimisation procedure. It would however, be surprising if the use of a defocus phase diversity kernel \( P_\pm \) was not optimal for a process such as atmospheric transmission, in which the defocus term is likely to represent the most serious error term. However, the selection of other aberration terms and the possibility to programme the phase diversity kernel in real time with a combination of other terms, may offer the scope to select a kernel that provides good sensitivity to all aberration modes expected to have amplitudes above some pre-determined threshold.

3. Multi-conjugate measurement – work package 2

Completion of the programme requires consideration of the possibilities of multi-conjugate AO operation. In the event there has been insufficient time to simulate the results from the above null-sensor approach and, therefore, no time to simulate the results from multi-conjugate applications. However, the following considerations suggest that the null-sensor approach is a viable mechanism for application in multi-conjugate AO.

For simplicity the following description is given in terms of application to a star field rather than to the extended terrestrial targets of greatest military interest. However, if the stars are considered to be representative of selected areas from an extended scene the following description should remain valid.

![Figure 3: Cause of anisoplanatism. By similar triangles the intersection of the dotted lines is such that \( \frac{R_2}{R_1} = \frac{D}{d} \). To the right of this intersection the radiation from opposite sides of the object does not pass through the same atmospheric regions and the atmospherically induced aberrations are independent, leading to significant anisoplanatism unless the object imaged is significantly small than the imaging aperture diameter.](image)

Anisoplanatism results from aberrations induced in planes other than the entrance pupil of the instrument (or planes conjugate thereto). Aberrations located in the entrance pupil affect all image plane points equally because radiation from all object points passes equally through all points on the aberration function. Aberrations associated with other planes do not affect all object points equally, as illustrated in the figure above.
However, if the conditions are isoplanatic all points in the image scene will suffer the same aberrations. In consequence, the average of $I_\pm$ for each image point will be equal to the intensity for each individual image point (to within noise etc). If however, the imaging conditions are anisoplanatic the average of $I_\pm$ for many image points will preserve only those components of the intensity patterns that are common between the points imaged – i.e. the isoplanatic components are preserved and the anisoplanatic components average out. A wavefront modulator driven using the average of all $I_\pm$ should, therefore, be positioned conjugate to the entrance pupil. If the average $I_\pm$ data is now subtracted from the data associated with each image point the residual signal in each case is the anisoplanatic component.

Thus averaging the signal from various star images when the phase diversity sensor is used as a null sensor provides a mechanism for separation of isoplanatic and anisoplanatic components.

It may be supposed that residual correlations between the aberrations for each image point will be associated with aberrations due to turbulence layers to the left of the intersection of the dotted lines in figure 3. However, insufficient time was available for investigation of these possibilities by simulation or by experiment.
4. Summary

The results presented suggest that construction of a compact AO system based on phase-diverse measurement of wavefront aberrations is feasible and should be straightforward, at least for application under isoplanatic conditions.

It has been shown that defocus aberration is only one of a set of aberration functions that can be used to give phase-diverse wavefront sensors that can be operated as a null sensor.

Insufficient time was available to simulate the use of this approach using either computer simulations or laboratory simulations.

It has been shown that an average of phase-diverse wavefront sensor data over many image points can be used as a mechanism to separate isoplanatic and anisoplanatic aberration sensing. However no validation of this result has been possible and no attempt has been made to formalise a mechanism by means of which the anisoplanatic components might be corrected by other than trial and error approaches.

5. Future work

Future work in this area should firstly confirm the result from the first work package, initially by computer simulation and subsequently by laboratory experiment. In particular, for optimisation of the wavefront sensor it would be useful to establish necessary conditions on $P_\pm$.

Secondly, the assertions concerning anisoplanatic measurements should be confirmed by laboratory (or, as a last resort, computer) simulations. The interpretation of anisoplanatic residuals should then be considered.
6. Compliance

In accordance with the requirements of the Order under which this programme was granted, I certify that there were no subject inventions to declare as defined in FAR 52.227-13, during the performance of this programme.

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Heriot-Watt does not have any rights over the IP in IMP® gratings – the IP here is the property of QineiQ and no rights of access are implied by mention of that technology in this report.

The contractor, Heriot-Watt University hereby declares that, to the best of its knowledge and belief, the technical data delivered herewith under Contract Number F61775-01-WE063 is complete, accurate and complies with all requirements of the contract.

A H Greenaway
4 August 2002.