Performance Analysis of the Nonhomogeneity Detector for STAP Applications

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Abstract — We consider the statistical analysis of a recently proposed non-homogeneity detector (NHD) for Gaussian interference statistics. We show that a formal goodness-of-fit test can be constructed by accounting for the statistics of the generalized inner product (GIP). Specifically, the Normalized-GIP follows a central-F distribution. This fact is used to derive the goodness-of-fit test in this paper. We also address the issue of space-time adaptive processing (STAP) algorithm performance using the NHD as a pre-processing step for training data selection. Performance results are reported using simulated as well as measured data.

I. INTRODUCTION

An important issue in space-time adaptive processing (STAP) for radar target detection is the formation and inversion of the covariance matrix underlying the clutter and interference. Typically, the unknown interference covariance matrix is estimated from a set of independent identically distributed (iid) target-free training data that is representative of the interference statistics in a cell under test. Frequently, the training data is subject to contamination by discrete scatterers or interfering targets. In either event, the training data becomes non-homogeneous. Consequently, it is not representative of the interference in the test cell. Estimates of the covariance matrix from non-homogeneous training data result in severely under-nulled clutter. Consequently, CFAR and detection performance suffer. Significant performance improvement can be achieved by employing pre-processing to select representative training data.

The problem of target detection using improved training strategies has been considered in [1-8]. The impact of non-homogeneity on STAP performance is considered in [9-11]. It was shown in [12] that the distribution information of a class of multivariate probability density functions (PDF) is succinctly determined through an equivalent univariate PDF of a quadratic form. An application of this result is the non-homogeneity detector (NHD) based on the generalized inner product (GIP) [1-4,8].

Non-homogeneity of the training data arises due to a number of factors such as contaminating targets, presence of strong discretes, and non-stationary reflectivity properties of the scattering surface. In these scenarios, the test cell disturbance covariance matrix, $\mathbf{R}_T$, differs significantly from the estimated covariance matrix, $\hat{\mathbf{R}}$ formed using target-free disturbance realizations from adjacent reference cells [13]. If a large number of test cell data realizations are available, the underlying non-homogeneity is characterized via the eigenvalues of $\hat{\mathbf{R}}^{-1}\mathbf{R}_T$ [14]. However, in radar applications, only a single realization of test cell data is usually available. Consequently, the resulting estimate of $\mathbf{R}_T$ is singular. Hence, [1-4,8] compared the empirically formed GIP with a theoretical mean corresponding to a 'known' covariance matrix. Large deviations of the GIP mean from the theoretical mean have been ascribed to non-homogeneity of the training data. Such an approach provides meaningful results in the limit of large training data size. In practice, the amount of training data available for a given application is limited by system considerations such as bandwidth and fast scanning arrays. Furthermore, the inherent temporal and spatial non-stationarity of the interference precludes the collection of large amounts of training data. Consequently, the approach of [1-4,8] can be misleading since it ignores finite data effects and the resulting variability in the covariance matrix estimate. In a recent paper [15], we derived significant results pertaining to the statistics of the GIP for Gaussian interference. Using (15) of [15] it follows that the empirical GIP mean using an estimated covariance matrix with finite data can be twice as large as the corresponding GIP mean for a known covariance matrix. Consequently, such a scenario can easily lead to incorrect classification of training data.

The main result of [15] is that the normalized GIP, $\bar{P}$, admits a remarkably simple stochastic representation as the ratio of two statistically independent Chi-Squared distributed random variables. As a result, the GIP follows a central-F distribution. This fact is exploited to construct a formal goodness-of-fit test for selecting homogeneous training data.

This paper presents performance analyses of the NHD using the goodness-of-fit test for the GIP. Performance results are presented using both simulated and measured data. We then employ the NHD as a pre-processing step for training data selection and assess performance of the adaptive matched filter (AMF) test [16,17,18]. Performance is reported in terms of the probability of detection versus output signal to noise ratio for simulated data. For measured data, a plot of the test statistic versus range is used as a performance metric.

II. GIP STATISTICS

The generalized inner product is defined by $P = \mathbf{x}^H\hat{\mathbf{R}}^{-1}\mathbf{x}$ where $\mathbf{x}$ denotes a realization of test cell data and

$\hat{\mathbf{R}} = \frac{1}{K}\sum_{i}^{K} \mathbf{x}_i \mathbf{x}_i^H$ with $\mathbf{x}_i$, i = 1,2,…,K being independent identically distributed 'target free' training data vectors having zero mean and covariance matrix $\mathbf{R}$. Additionally, the test and training data are assumed to be statistically independent. For complex-Gaussian distributed $\mathbf{x}$ and $\mathbf{x}_i$, sharing the same covariance matrix, $\mathbf{R}$, it was shown in [15] that the probability density function (PDF) of the normalized GIP, P' = P/K , follows a central F distribution given by

$$f_{p'}(p') = \frac{(p')^{M-1}}{\beta(M, L)(1 + p')^{L+M}} \tag{2.1}$$

where M is the dimensionality of $\mathbf{x}$ and K = L – M + 1. Furthermore, it was shown in [14] that P' admits a stochastic representation as a ratio of two statistically independent Chi-squared distributed random variables. Consequently, the mean and variance of the GIP are readily expressed as [15]

$$E(P) = \frac{M}{[1 - M/K]}$$

$$\mathrm{Var}(P) = \sigma_p^2 = \frac{M}{\left[1 - \frac{M}{K}\right]^2 \left[1 - \frac{(M+1)}{K}\right]} \tag{2.2}$$

These facts are used to construct a mechanism for selecting homogeneous training data in the next section.

III. NON-HOMOGENEITY DETECTOR

We now present two methods for selecting homogeneous data from a set of training data. The first method exploits the central-F distribution of P' given by (2.1) to construct a formal goodness-of-fit test, while the second method relies upon a comparison of empirically formed P' with the theoretical mean predicted by (2.2) and discarding those realizations for which P' deviates significantly from the theoretical mean. The cumulative distribution function of P' is given by

$$\Pr(P' \le r) = 1 - \mathrm{betainc}\left(\frac{1}{r+1}, M, L\right) \tag{3.1a}$$

where

$$\mathrm{betainc}(x, m, n) = \frac{1}{\beta(m, n)} \int_0^x w^{m-1} (1-w)^{n-1} dw \tag{3.1b}$$

The goodness-of fit test consists of determining whether realizations of P' formed from a given set of training data are statistically consistent with the PDF of (2.1). For this purpose a suitable type-I error, α, is chosen. More precisely, α is simply the probability of incorrectly rejecting the hypothesis that a given realization of P' is statistically consistent with the PDF of (2.1). Specifically, we seek a threshold, λ, such that

$$\alpha = \Pr(P' > \lambda) = 1 - \Pr(P' \le \lambda) = betainc\left(\frac{1}{\lambda + 1}, M, L\right). \tag{3.2}$$

λ is determined from an inversion of (3.2). The goodness of fit test consists of forming realizations of P' from a set of training data and rejecting those training data vectors for which P' exceeds λ. The second method is based on comparing the realizations of P with the theoretically predicted mean of P given by (2.2) and retaining those realizations exhibiting least deviation from the theoretically predicted mean of (2.2). Examples that illustrate the two approaches are presented. For a given training data set, a moving window approach is used to form realizations of P'. This approach is sub-optimal because it does not guarantee statistical independence of the realizations of P'. However, we adopt this approach due to the limited training data support. For the examples presented here, data from the MCARM program [19] corresponding to 16 pulses and 8 channels from acquisition '220' on Flight 5, cycle 'e' is used. For this example, α is set to 0.1. The plot in Fig. 1 shows P' and λ as a function of range. A moving window approach is used to obtain P' for each range cell considered. Non-homogeneity of the training data is seen in those range cells for which P' exceeds λ. Fig. 2 plots the normalized GIP as a function of range. The normalized GIP theoretical mean is obtained from (2.2) with a simple normalization. Values of the normalized GIP, which exceed the theoretical mean correspond to non-homogeneous training data realizations. Observe that the second method is more sensitive to the presence of discrete scatterers in the training data.

IV. PERFORMANCE ANALYSIS OF THE AMF TEST

In this section, we consider the performance analysis of the AMF test [15,18,19] in non-homogeneous training data. The AMF test is given by

$$\Lambda = \frac{\left|\mathbf{s}^H \hat{\mathbf{R}}^{-1} \mathbf{x}\right|^2}{\left|\mathbf{s}^H \hat{\mathbf{R}}^{-1} \mathbf{s}\right|} \underset{<}{>} \lambda_{AMF} \tag{4.1}$$

where $\mathbf{s}$ is the spatio-temporal steering vector, $\mathbf{x}$ is the received data vector, $\hat{\mathbf{R}}$ is the sample covariance matrix given by $\hat{\mathbf{R}} = \frac{1}{K}\sum_{i}^{K} \mathbf{x}_i \mathbf{x}_i^H$ with $\mathbf{x}_i$ denoting independent identically distributed training data and λ_{AMF} is a threshold selected to obtain a desired probability of false alarm.

For the case of homogeneous training data, analytical expressions for the probability of false alarm and probability

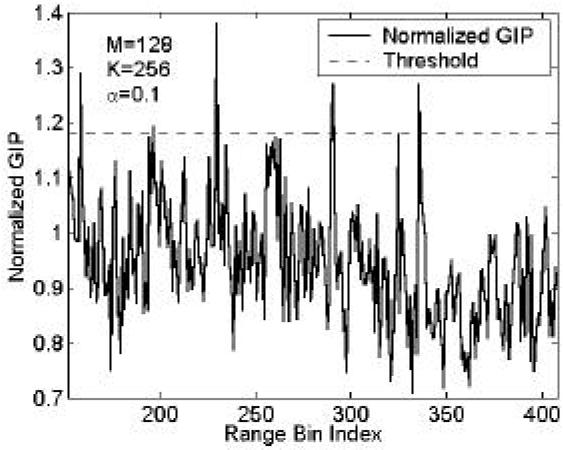

Fig. 1. Normalized GIP versus Range

of detection are given by [17]

$$P_{fa} = \int_0^1 \frac{f_\rho(\rho)d\rho}{(1+\lambda_{AMF}\rho)^L} \tag{4.2}$$

$$P_d = 1 - \int_0^1 \sum_{k=1}^{L} \binom{L}{k} \rho^k \lambda_{AMF}^k G_k\left[\frac{\rho b}{(1+\lambda_{AMF}\rho)}\right] \frac{f_\rho(\rho)d\rho}{(1+\lambda_{AMF}\rho)^L} \tag{4.3}$$

where

$$f_\rho(\rho) = \frac{(1-\rho)^{M-2}\rho^L}{\beta(M-1, L+1)} \qquad L = K\text{-}M\text{+}1 \tag{4.4}$$

$$G_k(x) = \exp(-x)\sum_{n=0}^{k-1}\frac{x^n}{n!} \tag{4.5}$$

and ‘b’ is related to the output signal to noise ratio (SNR). For $K \to \infty$, the sample covariance matrix tends to the true clutter covariance matrix, **R**. Consequently, the AMF test converges to the matched filter (optimal receiver in Gaussian

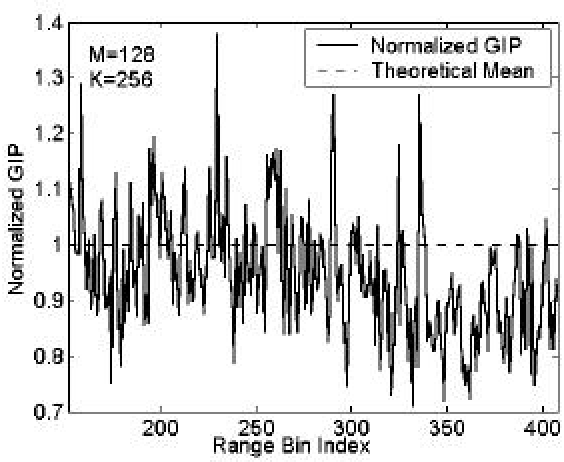

Fig. 2. Normalized GIP versus Range

clutter) for large K. The expressions for the matched filter P_{fa} and P_d are given by [17]

$$P_{fa} = \exp(-\lambda_{MF}) \tag{4.6}$$

$$P_d = \exp(-A)\sum_{k=0}^{\infty}\frac{A^k}{k!}[1 - G_k(?_{MF})] \tag{4.7}$$

where A is related to the output SNR and l_{MF} is the threshold.

Fig. 3 presents P_d versus output signal-to-interference plus noise ratio (SINR). Relevant test parameters are reported in the plot. The matched filter (MF) curve obtained from (4.7) corresponds to the optimal performance in Gaussian clutter. The P_d curve for the AMF operating in homogeneous Gaussian clutter follows from (4.3) and exhibits performance to within 3 dB of the MF. The AMF performance operating in non-homogeneous training data with and without NHD pre-processing is carried out by Monte Carlo simulation at AFRL. For this example, the training data contained thirty high-amplitude, mainbeam discrete targets located at various range cells and Doppler frequencies. Initial sample support for NHD pre-processing is 6M. A sliding window approach is used to select a subset consisting of 4M training data realizations. Each GIP value obtained at a specific range cell is computed using $\hat{\mathbf{R}}$ formed from 2M adjacent training data vectors. Previously, we noted the sub-optimality of this scheme. In practice, its use is dictated by training data size limitations. In this manner 4M GIP values are obtained. The NHD pre-processing used in this example is based on a comparison of the empirical GIP with its theoretical mean value given by (2.2). The training data used in forming $\hat{\mathbf{R}}$ after NHD processing is obtained by sorting the GIP values and retaining K=2M realizations corresponding to the smallest GIP deviation from the theoretical mean of (2.2). Observe that the AMF performance in non-homogenous clutter degrades severely. Also note that NHD pre-processing restores the AMF performance to its analytical value.

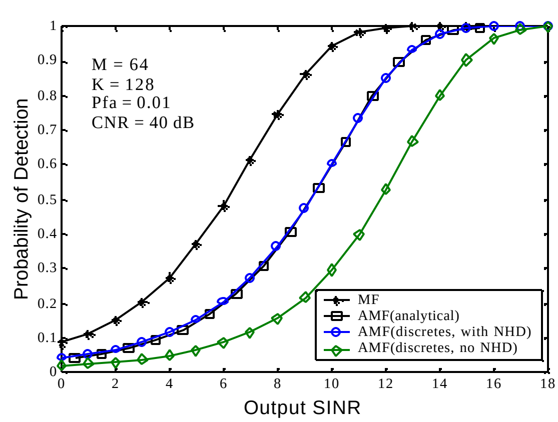

Fig. 3. Performance of the AMF with and without NHD

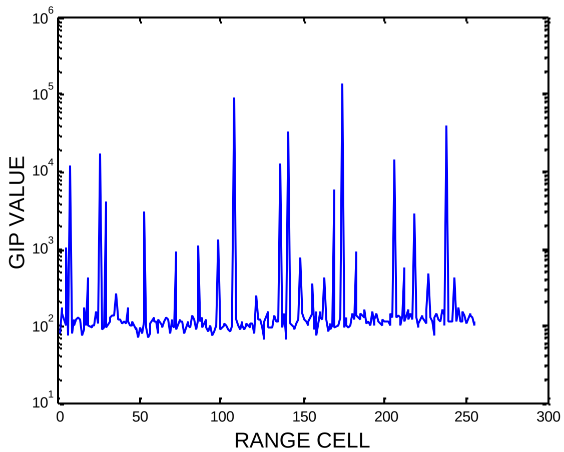

Fig. 4. GIP versus Range

Fig. 4 shows a plot of the GIP versus range prior to NHD pre-processing for the simulated data used in carrying out the performance analysis of Fig. 3. Fig. 5 shows a plot of the sorted absolute value of the difference between the GIP and its theoretical mean versus range after NHD pre-processing for the example in Fig. 3. Observe the absence of discretes in the first K=2M range cells.

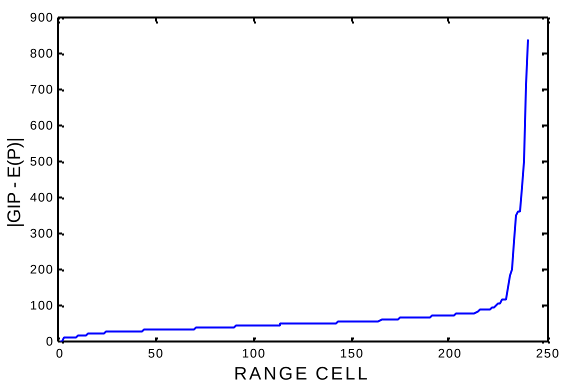

Fig. 5. Absolute value of difference between GIP and Theoretical GIP mean vs Range

Fig. 6 depicts performance using measured data from the MCARM program [19]. For this case, it is not possible to present performance in terms of detection probability versus SINR. This is due to the fact that only one realization of target present data is available. Hence, we present a plot of the detection test statistic versus range.

Since the AMF test statistic is an ad-hoc estimate of the output SINR, and since the probability of detection is a monotonically increasing function of the output SINR, this is an acceptable performance metric. Performance of the AMF without NHD processing degrades significantly in non-homogeneous clutter. Performance improvement is noted when the AMF is employed in non-homogeneous data with NHD pre-processing. Consequently, the use of NHD affords moderate performance improvement of the AMF test in non-homogeneous clutter. The performance with measured data is characterized by the ratio of the test statistic at the test cell to the mean of the test statistics formed from adjacent cells, ψ_1, and the ratio of the test statistic at the test cell to the highest test statistic formed from adjacent cells, ψ_2, respectively. Table 1 shows these values for the AMF test with and without NHD pre-processing.

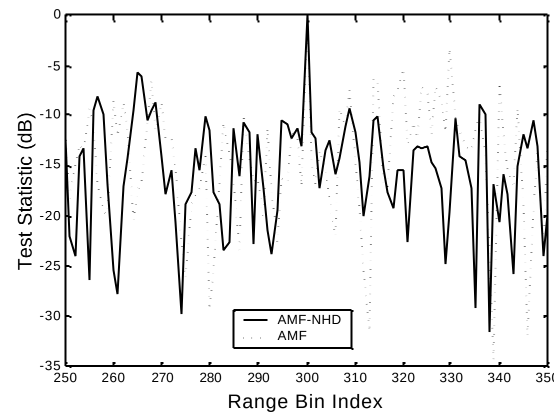

Fig. 6. Test Statistic vs Range

Table 1: AMF performance with measured data

Algorithm	ψ_1(dB)	ψ_2(dB)
AMF with NHD	13.25	5.68
AMF	11.83	3.38

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V. CONCLUSIONS

We presented a performance analysis of the AMF test in non-homogeneous clutter scenarios. We showed that the performance of the AMF test degrades severely in non-homogeneous clutter. A new implementation of the NHD based on finite sample support for covariance estimation was presented. Examples of the NHD performance with simulated and measured data were shown. Our results demonstrate that the use of NHD pre-processing affords considerable performance improvement for the AMF test.

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