COMBUSTION INSTABILITY IN LAMINAR BOUNDARY LAYERS

by

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SUMMARY

This paper presents the results of two studies, aimed at understanding the basic mechanisms causing self-sustained flame oscillations. One study concerns the oscillations of a premixed flame in a forced-convective boundary layer of a combustible mixture flowing over a heated surface. Another study deals with the oscillations of a diffusion flame around a simulated fuel droplet in a natural-convective boundary layer. In both cases, the results obtained indicate strongly that the onset of the self-sustained flame oscillations is due to the instability of travelling Tollmien-Schlichting waves --- the same mechanism which leads to the onset of transition from laminar to turbulent flows.

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INTRODUCTION

The problem of combustion instability has intrigued researchers for many years. Although much progress has been achieved during the past decade through intensive effort in research and development, several fundamental questions still remain unresolved. One of these questions relates to the triggering mechanisms which lead to the onset of acoustic oscillations in a combustor. Another of equal importance is concerned with possible interactions of the acoustic oscillations (once triggered) with the triggering mechanisms. With an objective of answering such and related questions, a research program has been started at the Massachusetts Institute of Technology. This paper presents some of the results obtained in this program.

The first phase of this program has been devoted to two studies, both aimed at understanding the basic mechanisms causing self-sustained flame oscillations. One study concerns the oscillations of a premixed flame in a forced-convective flow of a combustible mixture over a heated surface. Another study deals with the oscillations of a diffusion flame around a simulated fuel droplet in a natural-convective flow. In both cases, the results obtained indicate strongly that the onset of the self-sustained flame oscillations is due to the instability of travelling Tollmien-
Schlichting waves --- the same mechanism which leads to the transition from laminar to turbulent flows.

Current study on diffusion flames burning at different positions inside of a vertical glass duct demonstrates vividly the strong interactions between acoustic and flame oscillations. These results suggest the relevancy of the self-sustained flame oscillations as one important triggering mechanism of acoustic oscillations. The details of this study, however, will be presented in another publication.
SELF-SUSTAINED OSCILLATIONS OF A PREMIXED FLAME IN
A FORCED-CONVECTIVE FLOW

In conjunction with the study of the mechanism of flame holding in a forced-convective boundary layer adjacent to an isothermal flat plate, the premixed flame has been found (by the use of high-speed motion photography) to oscillate at a well-defined frequency under most of the experimental conditions. Furthermore, the speeds at which the flame advances upstream and retreats downstream, and the amplitude and frequency of the oscillation depend on the flow conditions, including the free-stream velocity of the combustible mixture, the surface temperature of the flat plate and the relative confinement of the flow as controlled by the length of another plate opposite to the isothermal surface. The details of this study have been reported recently by Wu and Toong¹. For reasons of space, therefore, only the important findings will be presented here.

The test section of the combustion tunnel is one inch square and four inches long. A flat plate, which serves as one of the walls of the test section, is heated externally to a uniform temperature by 32 electrical heating elements. The temperature distribution of the plate is measured by nine thermocouples embedded in it. A suction slot is provided in the plate just downstream of the entrance to the test section, and the plate is cooled upstream of this slot.
Thus, both the velocity and thermal boundary layers begin to grow at the slot. The two walls of the test section adjacent to the heated plate are transparent so that photographic techniques can be employed to study the flame oscillations. These two walls are also provided with air films to minimize three-dimensional effects. The fourth wall opposite to the isothermal plate can be varied in length to control the confinement of the flow and the flame.

A stoichiometric mixture of ethanol and air was used throughout the experiments and all tests were conducted at atmospheric pressure. High-speed motion pictures (taken under various conditions at approximately 1000 frames per second) provide insight into the oscillating phenomenon. Fig. 1 shows consecutive frames of a typical oscillation cycle and Fig. 2 shows the corresponding plot of the position of the upstream-most tip of the oscillating flame versus time, immediately after ignition by means of a spark plug located at the trailing edge of the heated plate. As the mixture was ignited, the flame propagated upstream to the foremost position of the oscillating mode at a relatively high speed of 16.8 ft/sec. This speed far exceeds the laminar-propagation speed of a stoichiometric ethanol-air flame near a heated plate, the surface temperature of which is

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**Footnote:** Measured values of flame-propagation speed of this mixture in a heated laminar boundary layer have been reported by Toong, et al.2

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only 623° F, (thus suggesting the possible presence of turbulence in the neighborhood of the propagating flame to augment the propagation speed). After the flame reached its foremost position, the oscillation pattern was immediately established and repeated itself exactly at each cycle, unlike the ordinary second-order oscillation system which usually overshoots when it is underdamped. The flame was clearly seen from the motion pictures to be propagating through the boundary layer. Under the conditions specified in Figs. 1 and 2, the oscillation frequency was 11.5 cps, the amplitude was 0.90 inch, the maximum flame advance speed was 3.96 ft/sec and the maximum flame retreat speed was 2.50 ft/sec.

A Mechanism of Flame Oscillations

The oscillation phenomenon described earlier and represented by typical figures like Figs. 1 and 2 is quite reproducible. Furthermore, the amplitude and frequency of the flame oscillations and the flame advance and retreat speeds depend definitely on the flow conditions, including the free-stream velocity of the combustible mixture, the surface temperature of the flat plate and the relative confinement of the flow as controlled by the length of the plate opposite to the flat plate. (These results are summarized in Table 1.) Thus, it is conceivable that the observed oscillation is an in-
herent characteristic of the flame held in a boundary layer
(similar to that in a flow behind a cylinder with eddies
shodding alternately on each side).

The mechanism postulated to explain these self-sustained
oscillations is based on the theories of boundary-layer in-
stability, flame propagation and flame holding. According to
the theory of boundary-layer instability\(^3\), the onset of the
transition of a laminar boundary layer to turbulence is
caused by the amplification of travelling (Tollmien-Schlich-
ting) disturbance waves when the Reynolds number exceeds a
critical value. Furthermore, it is found that this critical
Reynolds number decreases in the presence of an adverse
pressure gradient or a heated wall. In other words, the lam-
inar boundary layer becomes more unstable under these con-
ditions.

According to the equilibrium theory of flame holding in
a boundary layer adjacent to a smooth surface, a stationary
flame is obtained at a position where the tendency for the
flame to propagate at the local burning velocity is in
stable equilibrium with the fluid motion\(^1\). Thus, the flame
front must assume a blunt-nose shape in the boundary layer.
Due to the adverse pressure gradient upstream of the flame
associated with this shape, the unburned mixture approaching
the nose is retarded, deflected to the sides and finally
passed through the flame and burned. This adverse pressure gradient also makes a laminar flow highly unstable and leads to the growth of disturbance waves if the Reynolds number corresponding to the flame position is larger than the critical value. According to the theory of flame propagation, the growth of such disturbances would give rise to an increase in the flame propagation speed due to augmentation of rates of transport processes involved. The resulting unbalance between the flame propagation and the fluid motion causes the flame to advance upstream to a region of lower Reynolds number --- a region where the amplitudes of the disturbance waves are smaller (or negligible) and the corresponding increase in the flame propagation speed is also smaller (or negligible). However, this is also a region where the velocity gradient of the unburned mixture at the wall is larger. Thus, the unbalance between the fluid motion and the flame propagation may reverse itself. As a result, the flame advance is slowed down, stops and eventually occurs in the opposite direction (or the flame retreats). As the flame retreats downstream to a region of higher Reynolds number, the disturbance waves grow. The accompanying increase in the flame propagation speed coupled with a decrease in the boundary velocity gradient at the flame position would slow down the flame retreat, bring it
to a stop, and eventually make the flame advance upstream. In this manner, the cycle repeats itself.

Supporting Evidences

A direct and critical test of the postulated mechanism described above would be the presence of the disturbance waves upstream of the oscillating flame. However, its detection is rather difficult. Hot-wire anemometry is not practical in the present situation because the flame would burn up the fine wire and the probe would not be able to follow physically the oscillating flame. Schlieren photography also proved to be not applicable, because the region where the small disturbances were likely to be present was overshadowed by the strong density gradient near the flame front and the heated plate. Nevertheless, experiments have been performed to provide supporting evidences to the postulated mechanism. As the alternate advance and retreat of the flame are due to the alternate growth and damping of the disturbance waves in the boundary layer, such oscillations can be eliminated should the disturbances not be allowed either to grow or to be damped. Experiments were thus designed accordingly to break up the links involved in the oscillation mechanism.

It has been noted earlier in conjunction with the theory of boundary-layer instability that the disturbances would not grow if the Reynolds number corresponding to the equilibrium
flame position is less than the critical value. Under such conditions, according to the postulated mechanism, the flame should be stationary. As the critical Reynolds number increases when the surface temperature decreases with respect to the freestream temperature, it is possible to obtain a stationary flame adjacent to a cooled wall. This has indeed been observed when the free-stream velocity was 4.1 ft/sec and the free-stream and surface temperatures were, respectively, 118 and 92 deg F. Under such conditions the flame stood still at a distance of 0.7 inch from the suction slot with a corresponding length Reynolds number of 1270. Since there is no provision for cooling the flat plate in the present apparatus, it is difficult to obtain a stationary flame under widely different test conditions. However, such stationary flames have been observed more readily in the apparatus of Hottel, Toong and Martin\textsuperscript{5} by the use of a water-cooled solid boundary as a flame holder.

Another series of experiments has been performed in which disturbances of high-enough intensity were triggered and maintained by placing a wire upstream of the flame. The wire was first introduced against the isothermal surface, normal to the flow direction and downstream of an oscillating flame. It was then moved slowly upstream. As soon as the flame touched the wire, it immediately attached.
itself to the wire and would no longer oscillate. As the wire moved further upstream, the flame first remained attached to the wire, then developed seemingly a tendency to detach itself from the wire and finally jumped back to a position about 3/8 to 1/2 inch downstream of the wire. The most interesting observation was that the flame was very steady at this location. This evidence demonstrates vividly that the oscillation mechanism has been destroyed by the disturbances generated due to the presence of the wire. As the wire was moved further upstream, localized turbulence was damped due to the low value of the Reynolds number in this region and the flame resumed its oscillations. Wires of different sizes were tried and the same general phenomena were observed. When the wire diameter was very small, however, it was not possible to maintain a stationary flame immediately downstream of the wire, because the disturbance so generated was too small to be effective.

The above two series of experiments (involving the use of a cooled wall and a wire) indicate that a stationary flame can be obtained if some links in the oscillation mechanism are broken. These results seem to lend strong support to the validity of the postulated mechanism. Furthermore, the dependence of the advance speed of the oscillating flame on the plate temperature and that of the retreat speed on the free-stream velocity of the combustible mix-
ture and the relative flow confinement as controlled by the length (L) of a plate opposite to the isothermal surface (as shown in Table 1) can all be explained by the use of the postulated mechanism. Interested readers may find such detailed explanations in the afore-mentioned paper of Wu and Toong.

SELF-SUSTAINED OSCILLATIONS OF A DIFFUSION FLAME IN A NATURAL-CONVECTIVE FLOW

The mechanism which leads to the onset of the self-sustained oscillations of a diffusion flame around a simulated fuel droplet has also been investigated. To permit rather detailed studies of the instability mechanism and a comparison of the theoretical and experimental results, a two-dimensional (cylindrical) model was chosen to simulate a fuel droplet. Fuel (either liquid ethanol or gaseous propane) is supplied under constant pressure to the interior of a hollow, porous, ceramic cylinder such that it passes through the porous wall and burns (in a natural-convective flow field) in a manner similar to a fuel droplet. Details of this study will be presented in another paper. Again, for reasons of space, only the important findings will be given in this review paper.
Table 1. Flame Oscillation Characteristics Under Different Conditions.

<table>
<thead>
<tr>
<th>Case</th>
<th>$T_w$, °F</th>
<th>$U_\infty$, ft/sec</th>
<th>Adv. Speed ft/sec</th>
<th>Retreat Speed ft/sec</th>
<th>Osc. Freq. cps</th>
<th>X min. in inches</th>
<th>X max. in inches</th>
<th>Osc. Ampl. in inches</th>
<th>L in inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>632</td>
<td>10.25</td>
<td>3.96</td>
<td>2.50</td>
<td>11.49</td>
<td>1.10</td>
<td>2.00</td>
<td>0.90</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>632</td>
<td>10.25</td>
<td>4.06</td>
<td>2.81</td>
<td>8.33</td>
<td>1.33</td>
<td>2.80</td>
<td>1.53</td>
<td>3.0</td>
</tr>
<tr>
<td>3</td>
<td>632</td>
<td>10.25</td>
<td>4.16</td>
<td>3.22</td>
<td>6.13</td>
<td>1.10</td>
<td>4.10</td>
<td>3.00</td>
<td>4.0</td>
</tr>
<tr>
<td>4</td>
<td>829</td>
<td>10.31</td>
<td>6.67</td>
<td>3.50</td>
<td>12.5</td>
<td>0.50</td>
<td>2.00</td>
<td>1.50</td>
<td>4.5</td>
</tr>
<tr>
<td>5</td>
<td>413</td>
<td>4.10</td>
<td>1.47</td>
<td>1.30</td>
<td>19.8</td>
<td>0.41</td>
<td>0.66</td>
<td>0.25</td>
<td>2.0</td>
</tr>
</tbody>
</table>
The oscillations observed are characterized by periodic lengthening of the flame, necking in the region below the flame tip and subsequent separation of the flame tip from the main body of the flame surrounding the fuel cylinder. Such a sequence is shown in the direct photographs of Fig. 3. Schlieren motion pictures taken with a stroboscopic light source and a streak camera are reproduced in Fig. 4. These pictures show clearly the periodic formation of "cusps" at the outer edge of the thermal boundary layer surrounding the flame and the continued movement of these cusps downstream after they are formed.

A Mechanism of Flame Oscillations

The cusps noted above in the schlieren motion pictures resemble closely the disturbance waves associated with the transition of a laminar boundary layer to turbulence and seem to indicate the presence of vortices. (See, for example, the interferograms of Eckert and Sohngen in their study of the stability of a free-convection laminar boundary layer next to a vertical heated plate.) It is well known from the theory of boundary-layer instability (as given, for example, in the work of Kurtz) that a free-convection laminar boundary layer surrounding the diffusion flame is highly unstable be-
cause of the presence of an inflection point in the velocity profile. The critical Reynolds number would be further reduced due to the blowing effect at the cylinder surface as a result of injection of gaseous propane or evaporation of liquid ethanol. Thus, it is quite likely that the onset of the self-sustained flame oscillations is due to the amplification of the Tollmien-Schlichting disturbance waves in the region where the Reynolds number is greater than the critical value. The final finite amplitude of oscillation is arrived at, of course, as a result of non-linear effects.

Supporting Evidences

Evidence to support the mechanism of self-sustained flame oscillations postulated above has been provided by studying the wave forms, amplitudes and phase differences of the velocity oscillations at various locations around a horizontal burning fuel cylinder with the aid of a hot-wire anemometer. Fig. 5 shows a representative map of constant-amplitude and constant-phase contours, together with a profile of the steady component of the velocity in the natural-convolution boundary layer and the outermost outlines of the thermal boundary layer at various times of an oscillation cycle as taken from Fig. 4. The phase differences presented in Fig. 5 clearly show the downstream (or upward) propaga-
tion of a disturbance, the wave length of which (approximately 2-1/4 to 3-1/2 inches) is much smaller than that associated with an acoustic disturbance of the observed frequency of about 10.2 cps. For this case, the phase velocity of the disturbance wave is about 2 to 3 ft/sec rather than the speed of sound. Furthermore, as suggested by the theory of boundary-layer instability, this phase velocity is roughly equal to the maximum steady velocity in the boundary layer.

Approaching the cylinder from far upstream, the amplitude of the velocity oscillations first increases slowly as shown in Fig. 5. Close to the flame, however, and especially near the upstream half of the cylinder, the amplitude increases very rapidly. In the neighborhood of a location corresponding to 0.3 inch to the right and below the origin of Fig. 5, the amplitude increases by a factor of 6 in less than 1/8 inch. Such rapid growth may be an indication that the disturbance wave starts to be amplified somewhere near the upstream half of the cylinder.

It is interesting to note also the rapid decrease of the amplitude in a direction away from the cylinder normal to the propagation in a manner closely resembling that suggested by the theory of boundary-layer instability.
By the use of a particle-track technique, the mean velocities induced by natural convection in the neighborhood of the oscillating flame have been obtained. A typical profile of such velocities is shown in Fig. 5 at 90 degrees from the upstream stagnation point of the cylinder. The velocity profiles are compared with those in the neighborhood of a horizontal heated cylinder and of a vertical heated flat plate in a gravitational field. Although the magnitudes of the velocities are greatly increased as the result of combustion, the profiles are almost identical when they are plotted in a non-dimensional manner. As the instability characteristics of a boundary layer depends very much on the shape of the velocity profile, the similarity of these profiles with and without combustion suggests that it might be a good approximation to predict the wave length and the frequency of the disturbance wave which would be observed for the case with combustion on the basis of a theoretical study of the stability of a natural-convective boundary layer adjacent to a heated vertical plate. Indeed, the predicted and the observed values agree quite well. Interested readers may find such detailed comparisons in a paper by Toong, Anderson and Stopford.

The effects of the cylinder diameter, the type of fuel and the superimposed forced-convection velocity onto the
natural-convective flow) on the oscillation frequency of a diffusion flame surrounding a single, horizontal, burning fuel cylinder have also been investigated. The results are summarized in Fig. 6. The frequency is found to increase with decreasing cylinder diameter and with increasing mean free-stream velocity, and to remain the same whether the fuel used is liquid ethanol or gaseous propane. As the use of a gaseous fuel eliminates the coupling between mass and heat transfer through evaporation at the liquid-vapor interface near the cylinder wall for the case of a liquid fuel, the last observation mentioned above immediately rules out the possibility that the onset of the self-sustained oscillation is due to such a coupling.

The effects of the cylinder diameter and the super-imposed forced-convection velocity can also be explained qualitatively by the use of the postulated mechanism of instability. However, the strong support of the validity of this mechanism comes from the results shown in Fig. 5.

Interactions Between Two Fuel Cylinders

As a first step leading toward a study of the triggering of acoustic oscillations by flame oscillations and of the effects of acoustic oscillations on the triggering mechanisms, the interactions between two burning fuel cylinders have been investigated. Fig. 7 shows the effect of spacing be-
between two parallel horizontal cylinders on the frequency of
the self-sustained flame oscillations. At large spacing-to-
diameter ratios, (say, D/d > 10,) the flames surrounding the
two cylinders oscillate independent of each other. The osc-
cillations have random phase difference and may differ slight-
ly in frequency. When a spacing of approximately ten diam-
eters is reached, the flames begin to interact and their
oscillations occur out of phase. With further decrease in
spacing, the oscillation frequency rises, reaching a maximum
at D/d of about 3.7. However, for 3.7 < D/d < 3.9, two
distinct modes of oscillation have been observed. At smaller
spacings, the higher-frequency mode is no longer observed
and the frequency becomes roughly independent of the spacing.
It is interesting to note that the visible flame fronts
surrounding the two cylinders begin to merge together for
spacing-diameter ratio of less than 1.9.

The two frequencies observed over the narrow range of
critical spacings are associated with two distinct modes of
oscillation. The lower frequency occurs when the two flames are
oscillating in the same mode as that for the flame surrounding
a single cylinder. The oscillations are in phase, and by ex-
tinguishing and relighting the flames in varying time sequences,
it has been demonstrated that the interactions in the flow
field force the flames to oscillate together. Since the
frequency of this mode is rather insensitive to further decrease in spacing, the stability characteristics of the boundary-layer flow on that side of the cylinder farther from its neighbor are probably controlling the oscillation. When the two cylinders are very close together and the flames become merged, the frequency is lower than that for a single cylinder. Under such conditions, the two cylinders burn like one of larger diameter. The effect of interaction on frequency can thus be explained by the use of Fig. 6a.

The mode of higher frequency occurs when the two flames become much shorter and deflected towards each other, tending to merge. Direct photographs taken under these conditions show that they are oscillating out of phase. The increase in frequency for this mode of oscillation as the two cylinders approach each other from large spacings can be explained by means of Fig. 6b as the consequence of increased velocity between the cylinders.

CONCLUSIONS

The results presented in this paper indicate that combustion in laminar boundary layers is inherently unstable in at least two cases. In both cases, the onset of the self-sustained flame oscillations has been shown to be due to
the amplification of travelling Tollmien-Schlichting waves —
the same mechanism which leads to the onset of transition
from laminar to turbulent flows.
REFERENCES


Discussion on Heat Transfer, the Institution of Mechanical Engineers and the American Society of Mechanical Engineers, 1951, pp. 321 - 323.

$U_0 = 10.25 \text{ ft/sec}$  Frame 1-13 1.96 m. sec between frames
$T_w = 632^\circ F$  13-27 7.89 m. sec between frames
$L = 2 \text{ in.}$

FIG. 1 MOTION-PICTURE STUDY OF A PREMIXED FLAME IN SELF-SUSTAINED OSCILLATIONS
$U_\infty = 10.25 \text{ FT/SEC.}$

$T_w = 632 \degree \text{ F}$

$L = 2 \text{ IN.}$

$f = 11.5 \text{ CYCLE/SEC.}$

**FIG. 2 FLAME POSITION VERSUS TIME**
FIG. 3 DIRECT PHOTOGRAPHS OF A DIFFUSION FLAME IN SELF-SUSTAINED OSCILLATIONS
VERTICAL KNIFE-EDGE
TIME BETWEEN EXPOSURES: 9.4m sec
PROPANE FUEL
EXPOSURE TIME: c.2μ sec
0.377" O.D. CYLINDER

FIG. 4 SCHLIEREN MOTION-PICTURE STUDY OF A DIFFUSION FLAME IN SELF-SUSTAINED OSCILLATIONS
FIG. 5 PHASE AND AMPLITUDE CONTOURS OF VELOCITY OSCILLATIONS NEAR A DIFFUSION FLAME IN SELF-SUSTAINED OSCILLATIONS.
**FIG. 6a** EFFECT OF CYLINDER DIAMETER ON OSCILLATION FREQUENCY

**FIG. 6b.** EFFECT OF A SUPERIMPOSED FORCED-CONVECTIVE VELOCITY ON OSCILLATION FREQUENCY
Fig. 7 Effect of Cylinder Spacing on Oscillation Frequency

Oscillation Frequency vs. D/d

Symbols:
- ■ Thermocouple
- ○ Anemometer

Legend:
- Flow
- Free Convection
- Thermocouple
- Anemometer

END