Innovative Local-Global Methods for Wing Structural Design

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Only a couple months have elapsed since the last annual report, so there is nothing significant to report for those intervening months. The technical accomplishments and outcomes from the grant support have been documented in the three annual reports, which are attached as appendices. We provided here only summary statistics for the three year period, and a summary of the most significant accomplishments.
FINAL REPORT FOR AFOSR GRANT F49620-99-1-0128

INNOVATIVE LOCAL-GLOBAL METHODS FOR WING STRUCTURAL DESIGN


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Only a couple months have elapsed since the last annual report, so there is nothing significant to report for those intervening months. The technical accomplishments and outcomes from the grant support have been documented in the three annual reports, which are attached as appendices. We provide here only summary statistics for the three year period, and a summary of the most significant accomplishments.

Noteworthy accomplishments include

- development of genetic algorithms (GAs) involving both discrete and continuous variables for composite structure design;
- Fortran 90 templates and fully distributed massively parallel implementations for genetic algorithms;
- validation of the superiority of the normal flow homotopy algorithm for tracking nonlinear equilibrium curves;
- use of response surface approximations to optimal surfaces in global/local optimal design;
- use of compatibility metrics at the global level to improve blending between optimized local subsystems;
- binary tree memory and local approximations with continuous variables to improve GA efficiency;
- parallel GAs with migration for blending panels in the design of large composite aircraft structures;
- cellular automaton approaches to structural design;
- decomposition theory for quasi-separable multidisciplinary design optimization problems.

The grant supported the publication of 8 journal papers and 28 conference papers. During the grant period an additional 12 journal papers were accepted and 28 more journal papers were submitted for publication. Four graduate, four undergraduate, and six AASERT students were supported, with one MS thesis completed and three Ph.D. theses in progress. Three PIs—Zafer Gürdal, Raphael T. Haftian, Layne T. Watson—were partially supported by the grant. The project hosted five visiting scholars and involved three post-doctoral researchers. There were five distinct (different contacts) industrial technology transfers. The PI Watson received the Alumni Award for Excellence in Research in 2000.
APPENDIX I.

Innovative Local-Global Methods for Aircraft Structural Design

Progress Report

For the period
December 1, 1998 to August 31, 1999

Sponsored By
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September 1, 1999
Objectives

Global-local design of large structures with complex geometry presents a great challenge to structural designers. In particular, coordination between codes that optimize the structure or major structural components (such as entire wing or fuselage structures) and codes that optimize individual panel details requires substantial and careful effort to reach optimal or near optimal solutions. Moreover, the compatibility of the adjacent designs (or the lack of it) in terms of their geometric and material variables presents a serious manufacturing difficulty for the large structure. We are presently developing a computational infrastructure and algorithms with a sound theoretical basis to extend industrial ad hoc approaches to the global-local design and blending of local designs. We proposed a two-level optimization approach employing genetic algorithms tailored to panel design on the lower level. Response surface approximations to optimized panel failure loads are then used for the upper level wing or fuselage optimization. In addition, solutions to the blending problem at the local level using genetic algorithms will be investigated. We also proposed the development of a novel kind of population based cellular automata, similar to genetic algorithms, for enforcing construction compatibility between adjacent panels. Parallel computing and high-level object-oriented code development will be employed at every opportunity.

Status of Research

This document summarizes the work performed from September 1, 1998 to August 30, 1999. During this period, progress was made on the development of a global/local design capability for structures exhibiting geometrically nonlinear behavior and on refining capabilities for wing design under strength and buckling constraints.

The work on geometrically nonlinear behavior was motivated by engineers at the Boeing Company, who were interested in optimizing fuselage structures for postbuckling response. Traditional design procedures assume that a structure fails once buckling occurs. In reality, many structures, such as plates and stiffened panels, are able to safely carry a significant amount of additional load beyond the buckling load (see Figure 1). Structures that are capable of safely carrying this additional load are said to have postbuckling strength. If the structure is designed by taking into account this postbuckling strength, significant weight savings can be obtained compared to traditional buckling based designs. In weight critical applications, such as fuselage design, designers often attempt to utilize the structure's postbuckling strength to save weight. Unfortunately, no affordable optimization procedure incorporating postbuckling analysis currently exists to assist these designers in their task. Existing procedures are either computationally prohibitive or inaccurate. The goal of this effort is to use the global/local methodologies previously developed under this project to develop and demonstrate an affordable design methodology for designing structures for local postbuckling response.

First, the capability to perform affordable design optimization of postbuckled composite plates and stiffened panels was developed. This was achieved by connecting a computationally efficient geometrically nonlinear analysis code to a genetic algorithm computer code. Improvements in
the computational efficiency of the design process were obtained by implementing the normal flow algorithm for solving the nonlinear equations describing the response of the structure. Preliminary global/local results were generated for the design of a simple composite fuselage model.

Figure 1 - Postbuckling Response of Blade Stiffened Panel

The second thrust in the project was to refine the coordination process between global and local design in the linear range. This was applied to wing structural design, also motivated by interest from Boeing (formerly McDonald Douglas) engineers and researchers. One focus of the work was to improve the efficiency of the process of obtaining a large number of panel designs via genetic algorithms. This was achieved through introducing repair operators into the genetic algorithm used for panel design. The repair operators deal with constraints much more efficiently than the traditional treatment via penalty techniques.

A second focus in coordination was to improve the accuracy of the response surface of optimal panel failure loads used for communications between the global and local design optimization. A normalization procedure was implemented and currently we are exploring the use of derivatives of the panel failure loads for improving further the accuracy. Derivatives have not been traditionally used in response surface construction, because response surfaces were originally developed for fitting experimentally obtained data. Consequently, very little theory is available on efficient response surface construction using derivative data. One present effort is to develop the required theoretical basis.

Finally, students supported on the AASERT part of the research sought the application of the same local-global design philosophy to the design of Micro Aerial Vehicles (MAV). Previous
work under the project sought the coordination of wing and airfoil design. However, the coupling between these two aspects of the aerodynamic designs proved to be too weak to demonstrate the benefits of the approach. Current work is focused on coordination of the overall wing design and the design of the control system needed to provide the MAV with sufficient roll stability. MAVs can carry very little weight, and the design of the control system can substantially add to the required battery weight if the vehicle is difficult to control. Therefore, the design of the shape of the vehicle is closely coupled with the design of the control system.

Global/Local Design Formulation

The global/local design procedure for postbuckling response will be first demonstrated for a fuselage design example. At the global level, a finite element model of a fuselage segment is used (see Figure 2). The goal of the global optimizer is to minimize the fuselage weight without allowing structural failure to occur. Global design variables include the thickness of $0^\circ$, $\pm 45^\circ$, and $90^\circ$ plies, and constraints are imposed on the stresses in the skin and frame elements. The individual fuselage skin segments are each designed using a local postbuckling analysis. Local design variables control the stacking sequence of the skin, and constraints are imposed on the collapse load and material failure.

![Global Fuselage Model](image)

**Figure 2** - Global Fuselage Model

Design Capability for Local Postbuckling Response

Before the design of the overall structure begins, the local optimizer is used to construct a database of optimized local designs. Each design in the database is generated as a function of the global design variables and the applied loads. For a given set of global design variables and applied loads, the local stacking sequence will be designed to maximize the failure load. Since the global design variables and the internal loads vary as a function of the global optimization iterations, the local design database will be constructed for a range of these values. The range
will correspond to realistic combination of those global variables and loads that may be observed in a realistic structure.

Once the local design database is constructed, the design of the overall structure can begin. For any set of global design variables and applied loads, the global optimizer can consult the local design database for each skin panel to determine the failure load of the optimized panel. In this way, the global optimizer obtains information about the optimized local designs and can incorporate this information into the design of the global structure without actually doing the local optimization in real time.

In order to implement a global/local design methodology for postbuckled designs, the capability to optimize local panels in the nonlinear regime must be developed. This can be accomplished by linking a computationally efficient nonlinear analysis code with an appropriate optimization code. In this project, a nonlinear finite strip analysis was adapted to communicate with a genetic algorithm based optimization code. The finite strip analysis [1], which was developed as part of a previous research project, is capable of approximating the geometrically nonlinear response of compressively loaded prismatic plate structures such as plates and stiffened panels. The code includes the effects of geometric shape imperfections and is capable of identifying and traversing limit points in the response. The genetic algorithm [2], which was developed earlier under the present grant, allows us to design the stacking sequences of plate and stiffener elements while imposing realistic manufacturing constraints.

During the development of the nonlinear design capability, the design process was made as computationally efficient as possible. To this end, a new algorithm for solving the nonlinear system equations was implemented in the finite strip analysis. This algorithm, the normal flow algorithm, was found to yield significant computational savings compared to the previously implemented solution algorithm (a variant of the Crisfield method). For several of the test problems, the normal flow algorithm required fewer than half the number of matrix factorizations in order to trace the nonlinear response up to the point at which the structure collapsed. In an effort to call more attention to the normal flow algorithm in the structural mechanics community, a technical paper was written and submitted to the International Journal of Solids and Structures [3].

In order to increase further the computational efficiency of the nonlinear design procedure, the combined analysis/design code was modified so that multiple local optimization runs could be performed in parallel. This capability, which is still being tested, will be of great benefit in the generation of the data required for building the local response surfaces.

**Design Study**

In order to demonstrate the global/local methodology and the potential benefits of designing for postbuckling response, a simple design study will be performed. The fuselage segment illustrated in Figure 2 will be designed for several different value of the maximum postbuckling load factor (PFBmax). The postbuckling load factor (PF) is defined as follows:

\[
P F = 1 - \frac{P_b}{P_f},
\]
where $P_b$ is the skin buckling load and $P_f$ is the skin failure load. The maximum postbuckling load factor, $PF_{\text{max}}$, is the maximum value of the postbuckling load factor that is allowed to occur for a given design. This factor governs how much of the total applied load is allowed to be carried in the postbuckling regime. For example, if $PF_{\text{max}}$ is 0.33, the skin panels will only be allowed to carry 33% of the total load in the postbuckling regime. If $PF$ is 0.0, none of the applied load is allowed to be carried in the postbuckling regime (this corresponds to a traditional buckling based design). In this project, fuselage designs will be generated corresponding to several different values of $PF_{\text{max}}$ in order to demonstrate the potential advantages of designing for postbuckling response.

Preliminary results corresponding to $PF_{\text{max}} = 0.33$ were generated for several portions of a fuselage segment obtained from Boeing. For the purposes of the design study described here, however, a simplified fuselage model is being used (Figure 2). This simplified model will allow the results of the design study to be published in the open literature, and will allow other researchers to duplicate our results. This simplified fuselage model is assumed to be loaded by bending loads only, resulting in a simple biaxial stress state for each skin panel. Locally optimized designs for the simplified model are currently being generated. Initial designs will neglect the effects of load redistribution at the global level due to postbuckling deformations. If these effects are significant, they will be accounted for later in this project.

Global-Local Wing Design Coordination

The improved global-local design coordination was investigated in the context of global-local wing design. The overall wing modeled was analyzed and designed using the GENESIS finite element based structural optimization code. GENESIS is a commercial code, in use in both the automotive and aerospace industries. It employs continuous design variables, so one of the hurdles that we needed to address was the process of rounding the continuous designs obtained from GENESIS for use in the panel design optimization that employs discrete variables. It was found that simple rounding, followed by minor adjustments in the design were sufficient to address this problem.

The global-local design procedure includes the following stages: (i) generation of large number of local panel designs; (ii) fitting a response surface to the failure loads of the optimal designs; (iii) overall wing design employing the response surface; and (iv) panel design using loads and overall thicknesses obtained from the overall wing design. The procedure was successfully demonstrated on a simple wing structure with up to 54 independent global design variables.

The next step, now in progress calls for integrating derivative information in the creation of the response surface. Derivatives of the failure loads of the optimal panel with respect to the global design variables and loads are quite inexpensive to obtain. Therefore, calculating these derivatives and using them in the creation of the response surface can be used to improve accuracy at low cost or to reduce computational cost at comparable accuracy. The improved accuracy was demonstrated derivatives of panel loads, and work is in progress on obtaining and using derivatives with respect to global thickness variables.
Parallel GAs for Blended Local Optimal Designs

In previous work a Fortran 90 GA framework was developed for use with composite laminate design optimization. The framework included a GA module, encapsulating GA data structures and basic operations, and a package of GA operators, which worked with the module. An extension to the framework was then introduced. The distributed GA extension implemented a migration algorithm to work with a group of sub-populations evolving in parallel. The goal of the migration algorithm was to improve the normalized cost and reliability of a set of GA optimization runs without significantly impacting the time it takes to make those runs. We are building on this research in a new direction now, considering the benefits of communication between populations of different optimizations running in parallel potentially to improve the blending of the local structural components.

An MPI based master-slave Fortran 90 program was developed to allow us to collect response surface data for the local/global optimization in parallel. This simple program is the first step in developing a far more useful tool to study how to promote continuity of properties between GA optimized adjacent panels. The global optimization provides constraints on the lower-level design allowing the local GA's to operate on a fixed number of plies of each orientation. The problem then becomes reconstructing the stacking sequence for each local optimization as a permutation of these plies. Currently, each GA run is conducted locally with no knowledge of the structure of the surrounding panels. Local designs created in this way often have little in common with adjacent panels causing manufacturing costs to rise. Fortunately GA's can develop many near optimal solutions with widely different stacking sequences for the same local design. We are exploring methods for these local optimizations to run in parallel, communicating through structure inherited topologies and migration, to construct a design that is optimized for both weight and manufacturing cost.

Personnel Supported

Faculty supported by the grant are Z. Gürdal, R. T. Haftka, and L. T. Watson. Dr. Scott Ragon, who completed his Ph.D. Program participated in the program as a part time Post Doctoral Research Associate. Graduate students supported by the grant are David Adams (Va Tech), Douglas Slotta (Va Tech, partial support), and Boyang Liu (UF). Undergraduate students associated with the grant include Mike Davis, Kevin Fowlks, and Theresa Olson.

The AASERT portion supported graduate students Jason Sloan, and Carlos Fuentes, and currently supports Jason Harper. Undergraduate assistants supported included George Jacobs and Tara Segall, and currently include Domenico Ruggiero.

Visiting Scholars associated with the grant include Maria Sosonkina from the University of Minnesota, Professor Akira Todoroki of the Tokyo Institute of Technology, Professor Mehmet Akgun of Middle Eastern Technical University in Turkey, and Professor Alfred van Keulen of the University of Delft in Holland. The AASERT part benefited from collaboration with Visiting Scholar Dr. Itsuro Kajiwara of Tokyo Institute of Technology.
Publications

Journal articles accepted during the grant period are:


Refereed conference papers published during the grant period are:


Journal articles submitted during the grant period are:


Technology Transfer/Interactions

In addition to the typical faculty interactions with industry, this reporting period has an interaction item that is accomplished through a small business located at the Va Tech Corporate Research Park. Two of the Virginia Tech students formerly funded by the AFOSR Grant, Mr. Grant Soremekun and Dr. Scott Ragon, are partners in a small business that was established two years ago. During the Grant period, Dr. Ragon submitted and won a Phase-I STTR Project on the use of global/local design methodology for the design of an advanced amphibious assault vehicle (AAAV) through the DoD - ONR Program. Currently, Dr. Ragon works as a 50% Va Tech employee (Post Doctoral Research Associate) and 50% for ADOPTECH. The Phase-I STTR has substantial industrial connection with one of the ONR contractors, namely General Dynamics Land Systems (GDLS). Dr. Ragon is in the process of putting together the Phase-II proposal to commercialize the global/local design methodology for general DoD and other industrial design problems.

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GDLS Contact: Tom Stoubos, (703) 492-3112

RESULT
STTR Phase-I Feasibility of the Global/Local design methodology for complex structural design

APPLICATION
Design optimization of Advanced Amphibious Assault Vehicle for weight minimization

PERFORMER
Layne T. Watson, Virginia Polytechnic Institute & State University
Telephone: 540-231-7540

CUSTOMER
Lucent Technologies
Murray Hill, NJ
Contact: Robert Melville, 908-582-2420

RESULT
Homotopy algorithms; mathematical software
APPLICATION
Circuit design and modelling

PERFORMER
Layne T. Watson, Virginia Polytechnic Institute & State University
Telephone: 540-231-7540
CUSTOMER
Michelin Americas
Greenville, SC
Contact: John Melson, 864-422-4246
RESULT
Adaptive GMRES algorithm; mathematical software
APPLICATION
Iterative solution of large linear systems arising from tire modelling

PERFORMER
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Telephone: 540-231-5905
CUSTOMER
Sikorsky Aircraft
Stratford, Connecticut, 06615-9129
Contact: Christos Kassapoglou, 203-386-3292
RESULT
Design optimization of composite laminates for weight minimization. Achieved %20 reduction in structural weight.
APPLICATION
Design of a new generation helicopter fuselage.

PERFORMER
Raphael T. Haftka, University of Florida
Telephone: 352-392-9595
CUSTOMER
Ford Motor Company
Dearborn, MI
Contact: Dr. Mehran Chirehdast, 313-390-5201
RESULT
Demonstrated application of response surface technology to automotive problems
APPLICATION
Optimal design of automotive structures for increased fatigue life
Inventions of Patents

None

Honors/awards

None
APPENDIX II.

ANNUAL REPORT FOR AFOSR GRANT F49620-99-1-0128

INNOVATIVE LOCAL-GLOBAL METHODS FOR WING STRUCTURAL DESIGN


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December 5, 2001
Objectives.

Global-local design of wing and fuselage structures presents a great challenge to structural designers. In particular, coordination between codes that optimize the entire wing or fuselage structure and codes that optimize individual panel details requires substantial and careful effort to reach optimal or near optimal solutions. We are presently working with a group at Sikorsky Aircraft and another group at Boeing Long Beach (formerly McDonnell Douglas) on solutions to this problem. This proposal calls for developing a computational infrastructure and algorithms with sound theoretical basis to extend industrial ad hoc approaches. We propose a two-level optimization approach employing genetic algorithms tailored to panel design on the lower level. Response surface approximations to optimized panel failure loads are then used for the upper level wing or fuselage optimization. We also propose the development of a novel kind of population based cellular automata, similar to genetic algorithms, for enforcing construction compatibility between adjacent panels. Parallel computing and high-level object-oriented code development will be employed at every opportunity.

Status of research.

Global/local optimization and blending using genetic algorithms with migration:

Aircraft stiffened composite panels are typically designed by optimizing them locally, resulting in incompatibilities between the stacking sequence of the adjacent panels that form the entire structure. Such incompatibilities often result in structures that are difficult to manufacture, or in some cases impossible to manufacture. Although the differences are driven by local loads and constraints that are applied across the structure at different panels differently, it may be possible to improve the manufacturability by directly, or indirectly, enforcing the adjacent laminates to have similar stacking sequences. In particular, when genetic algorithms are used to optimize the local panel stacking sequences, often multiple near optimal solutions with minor differences in the laminate stacking sequences are obtained for each panel. Therefore, it may be possible to accept small losses from optimality of the adjacent panels but achieve substantial gains in the continuity of the laminate stacking sequences between adjacent panels may be possible. In implementing the genetic algorithm optimization, by allowing local designs to evolve in parallel, sending migrants to adjacent panels, the objective functions may be intelligently modified to favor a globally blended design. During the present effort the blending of the laminate stacking sequence is accomplished using the string edit distance between individuals and the set of emigrants from adjacent panels. Parallel results are presented for multiple communication topologies as well as varying migration and blending rates.

In the present research design of composite material aircraft structures, particularly designing the local details of stiffened panels in the wing and fuselage. These panels are made of composite laminates, using fiber-reinforced layers. The design of the fiber-reinforced composite laminate requires the specification of the stacking sequence with orientation and material type, creating a discrete optimization problem. It is computationally expensive to design an entire wing or fuselage structure with the panels optimized simultaneously. Instead, local panels are commonly optimized (failure load is maximized given a fixed number of plies of each orientation) for the specified local loads. An improvement to this process, a global/local procedure, fits a response surface to the maximal failure load of the local panel configuration as a function of the local loads and the number of plies of each orientation. The response surface is then used in a global wing optimization (with numbers of plies of each orientation as design variables) providing constraints on
panel load carrying capacity while taking into account the variation of the local load redistribution as a function of the design changes. The global solution (the load distribution) is then used one final time to construct locally optimal panels that match the overall wing design. The stacking sequences of the final panel designs in this scenario are usually incompatible across the overall structure.

The global/local optimization strategy briefly described above yields designs that are locally optimal but potentially infeasible as a final solution due to manufacturing incompatibilities between adjacent panels. The incompatibilities themselves result from the large differences in stacking sequence (the difference in the fiber orientation angle at different through-the-thickness locations in the laminate) across the overall structure. The research conducted during the present reporting period resulted in the development of an algorithm that takes advantage of the parallel implementation of the genetic algorithm optimization to enforce some level of continuity in the laminate stacking sequence between adjacent panels.

The advantage of the algorithm proposed here lies in the nature of multiple elitist selection in genetic algorithms themselves. In a standard elitist selection strategy only a single member of a parent population can survive the selection process and be placed in the child population. In a multiple elitist selection strategy the genetic algorithm allows for a greater number of high fitness members to survive the selection process at each generation. Building on the fact that a local panel optimum (or near optimum) can appear in several distinct stacking sequences, perhaps a globally blended solution can be derived via intelligent selection from the elite of a population.

The particular focus of this work is in using genetic algorithms in parallel to achieve compatible local panels. Each individual panel is designed by a different processor in a parallel processing environment. The populations representing each panel evolve in parallel and send migrant individuals to adjacent populations. The migrant population carries information about the best individuals from adjacent populations to compute string edit distances from the working population to the migrant population. The computed distance value is used to reward those individuals in a population that are evolving closely to those found in the migrant populations from adjacent panels. This rewarding process is a small modification to the objective function fitness value and serves to blend panel designs of the population balancing the optimization of weight with stacking sequence compatibility. Comparative results are shown for a base line of oblivious unblended panels with those panels obtained from multiple migration and blending variations as well as the effect of varied communication topologies on the propagation of information and communication time.

**Geometrically nonlinear analysis of local panels:** During local panel optimization, it is necessary to compute the nonlinear relationship between the applied loads and the resulting structural deformations. By tracking this nonlinear equilibrium path, it is possible to identify critical buckling and collapse phenomena. In structural mechanics, the nonlinear equilibrium equations are often solved using the Newton Raphson method in conjunction with an incremental/iterative solution method (Haisler and Stricklin, 1972). In order to trace the equilibrium paths through limit points in the response, the "arc length" methods of Riks/Wempner (Wempner, 1971 and Riks, 1979) or Crisfield (Crisfield, 1981) are commonly employed. A closely related method, which is based on a "normal flow" algorithm (Watson et al., 1987 and Watson et al., 1997), is less popular despite the fact that it may have advantages in terms of computational efficiency and robustness. During the reporting period, an implementation of the normal flow algorithm to nonlinear local composite panel analyses is investigated, and the potential advantages of the method for the solution of structural mechanics problems is demonstrated.
In the following, a brief description of the Riks/Wempner, Crisfield, and normal flow solution methods is provided. Numerical results are presented for the collapse of a blade stiffened panel. The relative efficiencies of each method are compared and the advantages of the normal flow algorithm in this case are illustrated.

The discrete set of nonlinear equilibrium equations representing the response of a structural system may be written in the following general form:

\[ f(\lambda, d) = 0, \]  

(1)

where \( f \in \mathbb{R}^m \) is a nonlinear function of a scalar loading parameter \( \lambda \) and the displacement vector \( d \in \mathbb{R}^m \). In an incremental/iterative solution method, these equations are solved in a series of steps or increments, usually starting from the unloaded state \( (\lambda = 0) \). The \( n \)-th step begins from a known solution on the equilibrium path, \( z_n^* = (\lambda_n^*, d_n^*)^T \), and consists of a prediction phase and a correction phase. In the prediction phase, an estimate for the next point on the equilibrium path, \( z_{n+1}^0 = (\lambda_{n+1}^0, d_{n+1}^0)^T \), is generated. Beginning at this point, Newton Raphson iterations are employed during the correction phase to find a new point on the equilibrium curve. The Riks/Wempner, Crisfield, and normal flow algorithms discussed here all utilize an approximation to the arc length of the equilibrium path, \( \Delta l \), to fix the size of each step. The \( i \)-th iteration executed by each algorithm at step \( (n + 1) \) can be described using the same general set of equations:

\[ Df(z_{n+1}^{i-1}) \Delta z = -f(z_{n+1}^{i-1}), \]  

(2)

\[ c(z_{n+1}^{i}) = 0, \]  

(3)

where \( \Delta z = z_{n+1}^i - z_{n+1}^{i-1}, \quad i = 1, 2, \ldots, \)

and

\[ Df(z_{n+1}^{i-1}) = \begin{bmatrix} \frac{\partial f(z_{n+1}^{i-1})}{\partial \lambda} & \frac{\partial f(z_{n+1}^{i-1})}{\partial d} \end{bmatrix} \]

is the \( m \times (m + 1) \) Jacobian matrix of the system (2). Equation (3) is an auxiliary "constraint" equation that is different for each algorithm. Upon convergence of the Newton Raphson iterations, a new solution, \( z_{n+1}^* \), is obtained and the prediction/correction process is continued for subsequent steps.

It should be noted that the system of equations (2)–(3) are not usually solved simultaneously, as equation (3) destroys the symmetric and banded qualities that equation (2) usually possesses. Instead, a procedure similar to that first proposed by Batoz and Dhatt (1979) is often employed (Ramm, 1980). If the modified Newton Raphson algorithm is used, \( Df(z_{n+1}^{i-1}) \) and \( f(z_{n+1}^{i-1}) \) are updated only at the beginning of each step.

Riks/Wempner algorithm: In the Riks/Wempner algorithm, the auxiliary equation (3) takes the following form:

\[ c(z_{n+1}^i) = (\dot{z}_n^*)^T (z_{n+1}^i - z_n^*) - \Delta l = 0, \]  

(4)

where \( \dot{z}_n^* \) is the unit tangent to the converged solution at step \( n \). This equation defines a hyperplane that is normal to \( \dot{z}_n^* \) and which is at a "distance" \( \Delta l \) from the previously obtained solution at step
During the Newton Raphson iterations, the successive iterates are forced to return to the equilibrium path along this hyperplane.

**Crisfield algorithm:** In the Crisfield algorithm, the auxiliary equation defines a hypersphere of radius $\Delta l$ centered on the converged solution at step $n$:

$$
\mathbf{c} (\mathbf{z}_{n+1}^i) = (\mathbf{z}_{n+1}^i - \mathbf{z}_{n}^*)^T (\mathbf{z}_{n+1}^i - \mathbf{z}_{n}^*) - \Delta l^2 = 0.
$$

(5)

Successive iterates are confined to the surface of this hypersphere as they converge to the equilibrium solution.

In practice, $\lambda$ is sometimes omitted from equation (5), and the constraint is instead imposed in $m$-dimensional space (Crisfield, 1981):

$$
\mathbf{c} (\mathbf{z}_{n+1}^i) = (\mathbf{d}_{n+1}^i - \mathbf{d}_{n}^*)^T (\mathbf{d}_{n+1}^i - \mathbf{d}_{n}^*) - \Delta l^2 = 0.
$$

(6)

This confines the iterates to lie on a cylinder in $(\lambda, \mathbf{d})$ space parallel to the $\lambda$ axis.

**Normal flow algorithm:** In the normal flow algorithm, successive Newton Raphson iterates converge to the equilibrium solution along a path which is normal (in an asymptotic sense) to the so-called Davidenko flow. The Davidenko flow can be described by considering a small perturbation $\delta$ to the nonlinear system equations:

$$
f(\lambda, \mathbf{d}) = \delta.
$$

(7)

As the perturbation parameter varies, small changes will occur in the solution curve for equation (7). The family of curves generated by varying $\delta$ is known as the Davidenko flow (Allgower and Georg, 1990).

The Newton Raphson iterate $\Delta \mathbf{z}$ for the normal flow algorithm is the unique minimum norm solution of the $m \times (m + 1)$ equations (2). This solution may be obtained in two steps. First, a particular solution $\mathbf{v}$ to the equations can be found by selecting an auxiliary equation (3) and solving the resulting system of equations (2)–(3). As long as this system of equations has rank $m + 1$, the auxiliary equation may be chosen arbitrarily. Once a particular solution is obtained, the minimum norm solution $\Delta \mathbf{z}$ is

$$
\Delta \mathbf{z} = \mathbf{v} - \frac{\mathbf{v}^T \mathbf{u}}{\mathbf{u}^T \mathbf{u}} \mathbf{u},
$$

where $\mathbf{u}$ is any vector in the kernel of $[Df]$. A convenient choice for $\mathbf{u}$ in this case is the unit tangent vector $\mathbf{z}_{n}^*$, which has usually already been calculated.

**Numerical results:** In order to quantitatively compare the relative performances of the solution algorithms, a sample problem was solved using each method. This sample problem consisted of computing the response of a compressively loaded graphite-epoxy blade stiffened panel loaded into the postbuckling regime. A "stiffener-unit" representation of the panel was utilized, wherein the complete panel was modeled using a single repeating element of the structure comprised of a single stiffener and the adjacent skin (Figure 1). The stiffener unit was 48.1in long, 6.28in wide, and had a stiffener height of 1.10in. The skin layup was $(\pm 45/0_3)$, the stiffener layup was $(90/0)_s$, and the ply thickness was 0.006in. The skin laminate was assumed to have an imperfection in the
shape of seven half-waves in the longitudinal direction with a maximum imperfection amplitude of 0.0001in.

Uniform end shortening was imposed at the longitudinal edges of the structure, and symmetry boundary conditions were imposed at the lateral edges. At the longitudinal edges, the skin was simply supported out of plane but the stiffener was not allowed to rotate in its own plane. The panel was analyzed using a finite strip analysis of the semi-analytical, multi-term type (Dawe, 1995 and Ragon, 1998). The finite strips were modeled as balanced and symmetric laminated composite materials which were assumed to behave orthotropically in bending. Material properties were as follows: $E_1 = 20.0 \cdot 10^6 lb/in^2$, $E_2 = 1.30 \cdot 10^6 lb/in^2$, $G_{12} = 1.03 \cdot 10^6 lb/in^2$, and $\nu = 0.30$.

All three of the solution algorithms were implemented using the full Newton Raphson method. The prediction phase of the Riks/Wempner and Crisfield algorithms (including the determination of the arc length step size at each increment) were implemented as described in Crisfield, 1981 under the heading "A Modified Riks Method". This prediction scheme is briefly described here.

The first step is the selection of an appropriate value for the arc length, $\Delta l$. The arc length is adjusted from one step to the next using the following simple formula:

$$\Delta l_{n+1} = \Delta l_n \frac{\tilde{m}}{m},$$

where $m$ is the number of iterations that were required at the previous step and $\tilde{m}$ is the (user specified) desired number of iterations at each step. This procedure allows larger steps to be taken
when the solution is converging easily, and forces the solver to take smaller steps when convergence is more difficult. For the present work, \( \bar{\hat{m}} \) was selected between 3 and 5 so as to achieve the most favorable results. Once a value for the arc length has been computed at step \( n + 1 \), the following equation is used to predict \( \Delta \lambda_{n+1}^0 \):

\[
\Delta \lambda_{n+1}^0 = \pm \Delta \ell / ((\Delta d_{n+1}^0)^T \Delta d_{n+1}^0),
\]

where \( \Delta d_{n+1}^0 \) is taken to be the tangential solution from the previously converged point.

The prediction strategy used for the normal flow algorithm was that implemented in subroutine \textsc{SteveN} in \textsc{HOMPAC}K90. This strategy utilizes a Hermite cubic interpolating polynomial through previously converged points and a sophisticated step size estimation algorithm. The load/end-shortening behavior of the structure as obtained using the normal flow algorithm is illustrated in Figure 2, where \( N_x \) is plotted against a scalar loading parameter, \( \lambda \). A value of \( \lambda = 1.0 \) corresponds to a uniform axial strain \( \varepsilon_x = -2.34 \times 10^{-4} \) and a uniform transverse strain in the skin of \( \varepsilon_y = +1.40 \times 10^{-4} \). The response is linear up until \( N_x \approx 1130 \text{ lb} \), where the skin buckles into seven half-waves along the length of the stiffener-unit. Beyond this point, the panel continues to carry load up to a maximum of \( N_x \approx 1620 \text{ lb} \), where the panel collapses. The normal flow algorithm was able to continue tracking the equilibrium path through the collapse point with no difficulty; the algorithm was eventually terminated at the user's request at the point shown (the algorithm did not terminate because of convergence difficulties).
The Riks/Wempner and Crisfield algorithms computed the same load/end-shortening curve as did the normal flow algorithm, but they both encountered difficulties in tracing the complete curve. In Figure 3, a zoomed in portion of the load/end-shortening curve in the vicinity of the critical point is illustrated and the solutions obtained using the Riks/Wempner algorithm and the normal flow algorithm ($\bar{m} = 4$) are compared. The normal flow algorithm was able to traverse this portion of the curve using a relatively small number of steps and continue on with the remainder of the curve. In contrast, the Riks/Wempner algorithm was able to make forward progress only by taking significantly smaller steps. Furthermore, the Riks/Wempner algorithm was unable to continue tracing the curve past a certain point; at this point the algorithm began chattering back and forth over already computed portions of the curve, perhaps due to the step size control rules. A Crisfield solution obtained using equation (6) ($\bar{m} = 4$) and the normal flow solution on the same portion of the curve are compared in Figure 4. The Crisfield algorithm is able to compute more of the curve than the Riks/Wempner algorithm, but it also failed to make forward progress after reaching a certain point on the curve. As with the Riks/Wempner algorithm, it is forced to take significantly smaller (and therefore more) steps, as compared to the normal flow algorithm, in order to obtain converged solutions. Equation (5) was also used to obtain solutions the problem, but this algorithm performed no better than the equation (6) algorithm.

Using the normal flow algorithm, only one or two iterations were required to obtain a converged solution at each step, whereas the Riks/Wempner and Crisfield algorithms typically required from three to five iterations at each step. Depending on the value of $\bar{m}$, the Riks/Wempner and Crisfield
algorithms each required from 250 to 400 matrix formations/factorizations in tracing the load/end-shortening curve up to the points where the algorithms failed; the normal flow algorithm reached the same portion of the curve with only 78 formations/factorizations.

The increased computational efficiency that can be achieved using normal flow algorithm can be particularly advantageous in an optimization environment, where the computational efficiency of a single analysis is of utmost importance. If the equilibrium path must be traced with a higher degree of fidelity (compared to that shown in Figures 3 and 4), the normal flow algorithm can be forced to take smaller steps along the path. This will, of course, result in a degradation of computational efficiency.

**Genetic algorithms for local panel design:** The basic genetic algorithm designed earlier in the research is capable of dealing with continuous design variables as well as discrete variables. Most composite structural design problems deal with determining the laminate stacking sequences, which is a discrete problem. However, continuous variables, such as stiffener height and core thickness for sandwich laminates, are often important elements of a design problem. Therefore, inclusion of the continuous design variables into the algorithm was implemented by using a state-of-the-art approach; the continuous variables are handled as real numbers rather than discretizing them into binary (or any other) alphabet strings. In addition, special operators for crossover and mutation of the continuous numbers were used to improve the efficiency of handling continuous variables.

Despite the improved treatment of the continuous variables, recent studies suggested that the efficiency of the genetic algorithm might be increased for certain problems that involve mixed
discrete/continuous variables. This is especially true when there are a large number of near optimal designs in the vicinity of the global optimal design. This was the case for a sandwich composite laminate design that was performed for Sikorsky Aircraft. The problem was to determine the stacking sequence of the face-sheets as well as the thickness of the honeycomb core of the laminate. It was observed that a certain face-sheet laminate design appeared repeatedly with different values of the continuous core thickness. In order to improve the efficiency of the algorithm, we decided to introduce local improvements into the search in the form of local approximations to the response of the panels. The approach used is to monitor and store the analysis information during the search into a binary tree structure, which serves as memory for the optimization.

For a purely discrete variable formulation, a binary tree memory structure is easy to implement, and has been used earlier by the present investigators to improve the efficiency of search as much as 30%. The improvement is due to the fact that once the analysis is performed for a specified discrete combination of design variables, the next time the same combination appears the analysis information can be obtained from the memory instead of performing the analysis. In the case of mixed problems, storing the analysis information for every value of the continuous design variables would make little sense, because even slight perturbations in the continuous values would make a design appear like one that has never been evaluated. However, by monitoring the distribution and proximity of the continuous design variables to previously evaluated values, we should be able to construct local low order polynomial approximations. We are currently in the process of designing a procedure to implement memory for the mixed discrete/continuous design variable problems.

**Accomplishments/new findings.**

**Genetic algorithms for local panel design:** The basic genetic algorithm data structures were extended to use multiple chromosomes (for structures with several different laminates such as wing boxes) and real variables (such as geometrical dimensions). The approach is implemented in the new object-oriented Fortran 90 code for the design of laminated composite panels. The multiple chromosome approach yields efficient and elegant code, and implementing crossover and mutation directly with real numbers is more effective than working with discrete binary approximations of real numbers. Fortran 90 modules and GA operators continue to be refined and added to the package, and new capabilities such as memory (storing old designs in a balanced binary tree to avoid reanalysis) and local improvement (using local approximations to replace an individual by the best nearby individual) are being added.

**Nonlinear local panel analysis:** The relative efficiencies of the Riks/Wempner, Crisfield, and normal flow solution algorithms for tracking nonlinear equilibrium paths of structural systems are compared. It is found that the normal flow algorithm may be both more computationally efficient and more robust compared to the other two algorithms when tracing the path through severe nonlinearities such as those associated with structural collapse. This is demonstrated by comparing the relative behaviors of each algorithm in the vicinity of a severe nonlinearity. Results for computing the collapse load of a blade stiffened panel demonstrated substantial computational time savings during the local design of those panels.

**Parallel genetic algorithms:** The parallel GA code implemented migration between subpopulations (evolving independently on different processors), and the parallel, dynamically adapting algorithm resulted in both better reliability and reduced normalized cost compared to a static serial algorithm with migration. The parallel implementation used dynamic load balancing, fully distributed control, and a sophisticated termination detection algorithm. The surprising finding
was that nondeterminism in the parallel task management significantly enhanced the performance of the (already nondeterministic) GA and migration. One interpretation of this finding is that random migration is superior to a fixed migration pattern. The parallel code also scaled very well, showing no significant communication penalty with 64 processors. Current work involves improving the parallel GA code, exploring the effect of different migration patterns (corresponding to different parallel connection topologies), and blending local panel designs to meet manufacturing constraints.

**Personnel supported.**

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**Publications.**

**Journal articles published during the grant period are:**


**Refereed conference papers published during the grant period are:**


Journal articles accepted during the grant period are:


Refereed conference papers accepted during the grant period are:

Journal articles submitted during the grant period are:


13
Interactions/transitions.

Conference presentations were:


Technology transitions or transfer:

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CUSTOMER
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RESULT
Homotopy algorithms; mathematical software

APPLICATION
Linkage mechanism design; combustion chemistry; robotics; CAD/CAM

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RESULT
Homotopy algorithms; mathematical software

APPLICATION
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RESULT
Adaptive GMRES algorithm; mathematical software
APPLICATION
Iterative solution of large linear systems arising from tire modelling

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RESULT
Multiobjective genetic algorithms for composite laminate design
APPLICATION
Design of helicopter frame structures for minimum weight and cost

Inventions or patents.
None.

Honors/awards.

- Alumni Award for Excellence in Research (VPI&SU), Layne T. Watson.
APPENDIX III.

ANNUAL REPORT FOR AFOSR GRANT F49620-99-1-0128

INNOVATIVE LOCAL-GLOBAL METHODS FOR
WING STRUCTURAL DESIGN

Period: 12/01/2000 – 11/30/2001

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December 5, 2001
Objectives.

Design and optimization of large wing and fuselage structures presents a great challenge to structural designers. Complex and highly local details in these structures preclude the use of a single level design model for the entire structure. We have successfully introduced a two-level global-local design methodology to provide designers with the capability of accounting for local details (panel geometric variables, stacking sequence) associated with local configurations (laminated composite sandwich panels). At the local level of the two-level optimization approach we employ genetic algorithms to tailor the panel designs. Response surface approximations to optimized panel failure loads are then used for the upper level wing or fuselage optimization. Much of the effort during the present reporting period was invested in the coordination between codes that optimize the entire wing or fuselage structure and codes that optimize individual panel details, so that optimal or near optimal designs could be obtained. In particular, we introduced new schemes for blending of local laminate stacking sequences across multiple panels using parallel computing and high-level object-oriented code development. We also improved the efficiency of the genetic algorithm for local panel designs by introducing a new local improvement strategy capable of handling continuous design variables without discretization. Also, development of a novel kind of design approach based on cellular automata was initiated.

Status of research.

Global/local optimization and blending using genetic algorithms with migration: The present research concerns the design of composite material aircraft structures, particularly designing the local details of stiffened and sandwich panels in the wing and fuselage. These panels are made of composite laminates, using fiber-reinforced layers. The design of the fiber-reinforced composite laminate requires the specification of the stacking sequence with orientation and material type, creating a discrete optimization problem. It is computationally expensive to design an entire wing or fuselage structure with the panels optimized simultaneously. Instead, local panels are commonly optimized (failure load is maximized given a fixed number of plies of each orientation) for the specified local loads. An improvement to this process, a global/local procedure, fits a response surface approximation to the maximal failure load of the local panel configuration as a function of the local loads and the number of plies of each orientation. The response surface approximation is then used in a global wing optimization (with numbers of plies of each orientation as design variables) providing constraints on panel load carrying capacity while taking into account the variation of the local load redistribution as a function of the design changes. The global solution (the load distribution) is then used one final time to construct locally optimal panels that match the overall wing design.

The global/local optimization strategy briefly described above yields designs that are locally optimal. However, process often results in designs that have incompatibilities between the stacking sequence of the adjacent panels that form the entire structure. Such incompatibilities often result in structures that are difficult to manufacture, or in some cases impossible to manufacture. Although the differences are driven by local loads and constraints that are applied across the structure at different panels differently, it may be possible to improve the manufacturability by directly, or indirectly, coercing the adjacent laminates to have similar stacking sequences. In particular, when genetic algorithms are used to optimize the local panel stacking sequences, often multiple near optimal solutions with minor differences in the laminate stacking sequences are obtained for each panel. Therefore, it may be possible to accept small losses from optimality of the adjacent panels.
but achieve substantial gains in the continuity of the laminate stacking sequences between adjacent panels. In implementing the genetic algorithm optimization, by allowing local designs to evolve in parallel, sending migrants to adjacent panels, the objective functions may be intelligently modified to favor a globally blended design.

During the present effort the blending of the laminate stacking sequence is accomplished using the string edit distance between individuals and the set of emigrants from adjacent panels. That is, instead of optimizing local panels in isolation, each individual is designed by a different processor in a parallel environment. The populations representing each panel evolve in parallel and periodically send migrant individuals to adjacent populations. A metric is applied, in this case the string edit distance, to evaluate the similarities in the current population with members stored as migrants. The computed distance value is used to reward (increase the fitness of) those individuals in a population that are evolving closely to those found in the migrant populations from adjacent panels. This rewarding process is a small modification to the objective function fitness value and serves to blend the panel designs of the population balancing the optimization of weight with stacking sequence compatibility.

Early tests this year showed that applying the blending metric to multiple migrants from a single neighbor created a theoretical deadlock situation. Although it is necessary to use only a single migrant from neighboring populations in the blending metric to avoid possible deadlock, this reduces genetic diversity and often results in convergence to an inferior local minimum. The first barrage of tests used an unmodified version of the edit distance metric. No algorithmic modifications were made to match common heuristics used in evaluating the quality of a blended design. This would include adding machinery that could evaluate an internal mismatch in ply angles as a worse design than an exterior mismatch. Similarly it could include the matching of an empty ply as more desirable than a physical ply angle disruption. Promising results for the method were gathered on both two-panel and four-panel ring topology test examples.

At this point well blended designs became much heavier as required plies were added to satisfy the blending requirements. To combat this trend, heuristic machinery was added to the blending algorithm to allow empty ply encodings to perfectly match any ply angle encoding. A more complex and irregular grid example with eighteen panels was adopted for more rigorous testing of the methodology and is shown in Figure 1. Ignoring the topology at first, tests were run on the eighteen panels in a ring formation with again promising results and a convergence to perfectly blended panels using the naive edit distance metric. Using the modified metric, allowing empty plies to match anything, promising results were gained even in a ring topology, but by ignoring the true topology in this way a perfectly blended design could not be derived.

Current tests are running to evaluate a communication structure mapped directly from the physical topology of the panel layout. Initial runs show a much larger iteration count towards convergence but should be able to converge to a perfectly blended design with respect to the actual grid panel connections.

Geometrically nonlinear analysis of local panels: During local panel optimization, sometimes it is necessary to compute the nonlinear relationship between the applied loads and the resulting structural deformations. By tracking this nonlinear equilibrium path, it is possible to identify critical buckling and collapse phenomena. In structural mechanics, the nonlinear equilibrium equations are often solved using the Newton Raphson method in conjunction with an incremental/iterative solution method. In order to trace the equilibrium paths through limit points in the response, the "arc length" methods of Riks/Wempner or Crisfield are commonly employed.
A closely related method, which is based on a “normal flow” algorithm (from the HOMPACK software package by Watson, Billups, and Morgan), is less popular despite the fact that it may have advantages in terms of computational efficiency and robustness. During the previous reporting period, an implementation of the normal flow algorithm to nonlinear local composite panel analyses was investigated, and the potential advantages of the method for the solution of structural mechanics problems were demonstrated. The normal flow algorithm for tracing nonlinear equilibrium paths, developed under the TAP project, is now in production use for composite panel analysis in other projects.

**Genetic algorithms for local panel design:** The basic genetic algorithm designed earlier in the research is capable of dealing with continuous design variables as well as discrete variables. Most composite structural design problems deal with determining the laminate stacking sequences, which is a discrete problem. However, continuous variables, such as stiffener height and core thickness for sandwich laminates, are often important elements of a design problem. Therefore, inclusion of the continuous design variables into the algorithm was implemented by using a state-of-the-art approach; the continuous variables are handled as real numbers rather than discretizing them into binary (or any other) alphabet strings. In addition, special operators for crossover and mutation of the continuous numbers were used to improve the efficiency of handling continuous variables.

Despite the improved treatment of the continuous variables, recent studies suggested that the efficiency of the genetic algorithm might be increased for certain problems that involve mixed discrete/continuous variables. This is especially true when there are a large number of near optimal designs in the vicinity of the global optimal design. This was the case for a sandwich composite laminate design that was performed for Sikorsky Aircraft. The problem was to determine the stacking sequence of the face-sheets as well as the thickness of the honeycomb core of the laminate.
It was observed that a certain face-sheet laminate design appeared repeatedly with different values of the continuous core thickness. In order to improve the efficiency of the algorithm, we decided to introduce local improvements into the search in the form of local approximations to the response of the panels. The approach used is to monitor and store the analysis information during the search into a binary tree structure, which serves as memory for the optimization.

For a purely discrete variable formulation, a binary tree memory structure is easy to implement, and has been used earlier by the present investigators to improve the efficiency of search as much as 30%. The improvement is due to the fact that once the analysis is performed for a specified discrete combination of design variables. The next time the same combination appears, the analysis information can be obtained from the memory instead of performing the analysis. In the case of mixed problems with discrete and continuous variables, storing the analysis information for every value of the continuous design variables would make little sense, because even slight perturbations in the continuous values would make a design appear like one that has never been evaluated. However, by monitoring the distribution and proximity of the continuous design variables to previously evaluated values, accurate piecewise polynomial approximations to the analysis data can be constructed. An algorithm to effect memory for mixed discrete/continuous design variable problems follows.

Let \( v \) be a discrete vector variable, \( x \) a scalar real variable, \( f(v, x) \) the fitness of the individual defined by \((v, x)\), and \( d \) a real number. Each node in the binary tree memory structure records the tuple \((v, x, f(v, x), d)\).

Consider a candidate individual \((v, x)\) for insertion in the tree.

If \( v \) is not found in the tree, then

1. evaluate \( f(v, x) \);
2. add a node corresponding to \((v, x, f(v, x), 0)\);
3. return with value \( f(v, x) \) to the GA;

else

1. let \( \{(v, x_i, f_i, d_i)\}_{i=1}^n \) be all the node data matching \( v \);
2. compute a spline \( S(\cdot) \) of order \( \leq 4 \) interpolating the data \((x_i, f_i), \) \( i = 1, \ldots, n; \)
3. evaluate the spline \( S(x) \);
4. if \( \max_{1 \leq i \leq n} |d_i - |x - x_i|| \geq 0 \), then
5. return with value \( S(x) \) to the GA;
else
6. evaluate \( f(v, x) \);
7. if \( |f(v, x) - S(x)| > \epsilon \), then
8. add a node corresponding to \((v, x, f(v, x), 0)\);
9. return with value \( f(v, x) \) to the GA;
else
10. define \( k \) and \( d \) by \( d = |x - x_k| = \min_{1 \leq i \leq n} |x - x_i|; \)
11. change the data at node \((v, x_k, f_k, d_k)\) to \((v, x_k, f_k, d)\);
12. add a node corresponding to \((v, x, f(v, x), d)\);
13. return with value \( f(v, x) \) to the GA;
end if
end if
end if
During the last activity period, an additional use of the spline approximation for the continuous variables was developed. Rather then using the spline passively just as a memory (or surrogate analysis) for new design points, we have started using it actively in generating new points that will be included into the next generation. That is, knowing the spline curve in a given generation we were able to determine the value of the continuous variable that provided the best value of the fitness function for a particular value of the discrete variable associated with the spline (note that there is a spline approximation for each discrete node in the binary tree if that node is visited more than twice). When creating children from the parent generation using cross-over, if the discrete part of the child was associated with a spline that stored the best value of the continuous variable, then this value is used to replace the value of the continuous cross-over value for the child design. Results generated so far indicate substantial improvement in the performance of the genetic algorithm in terms of the number of function evaluations to reach an optimal design.

**Structural Design Using Cellular Automata:** During the present performance period, a new task was initiated. While the traditional method of structural design using finite element based numerical analysis programs works well for many problems, it does not parallelize efficiently on massively parallel processors (MPPs), thus limiting the size and complexity of the structures that can be designed. The new task demonstrates the use of a new implementation of the cellular computational paradigm for simultaneous analysis and design of two-dimensional structural domains. In the following, a brief description of the method and its adaptation to structural analysis and design is provided. An example is provided, and its convergence properties are discussed.

**Method Description:** Cellular automata (CA) were used at least as early as 1946 by Wiener [4] to describe the operation of heart muscle, even though their use was not computationally feasible at the time. CA tiles a problem domain into cells of equal size. Each cell has the same set of simple rules that dictate how it behaves and interacts with its neighboring cells. Each cell is a fixed point in a regular lattice. The state of each cell is updated at discrete time steps, based upon conditions in previous time steps. All of the cells are updated every time step, thus the state of the entire lattice is updated every time step. The basic principle behind the method is that an overall global behavior can be computed by a group of cells that only know local conditions [5]. If each cell only needs to know local conditions, then this minimizes the communication requirements and therefore the problem scales well on a MPP. A CA is the archetypical algorithm for the SIMD parallel architecture [3].

CA have recently been used to perform structural analysis and design [1, 2]. In this particular implementation, the intention is to describe the static equilibrium of a two-dimensional domain under a system of forces acting on it. In this sense, time is not being simulated, rather each step of the automaton is used to propagate (local) stresses and strains through out the domain to allow it to reach equilibrium state while simultaneously determining the shape and/or dimensions of the cells associated with this equilibrium state. This is continued until the entire process converges (ideally) to a global state where there is no significant change in the structure for every subsequent iteration, corresponding to a static equilibrium state. Note that, as will be described later, analysis and design are done simultaneously and locally by each cell.

**Domain Definition:** Each cell of this CA is an eight-beam truss where each beam starts at the center of the cell and connects to its opposite member in an adjacent cell as illustrated in Figure 2. This type of structure is known as a ground truss. Those cells that fall on the border of the rectangular domain are not partial cells requiring special rules, but are complete cells with the area of the beams that fall outside the computational domain set to zero. In addition, they are
connected to an invisible set of surrounding cells that are turned off and that also have all of their beam areas set to zero. Cells that are turned off are not part of the computation, being used only to make the rules for the border and non-border cells consistent, since the stress analysis rules require the displacements of all eight surrounding cells.

The actual border of the computational domain of the CA need not be rectangular. Any shape can be defined for the truss by turning off any cells that are not within the computational domain, as illustrated in Figure 2. A simple method to define a shape for the truss is to define an enclosing polygon, and then turn off every cell that does not fall within the polygon. The "edge crossings" algorithm to determine those points within the polygon can be used; it is simple and parallelizable well. A more sophisticated method could be used to allow for holes, circular regions, or other, nonstraight boundaries. In addition, a better resolution can be obtained by decreasing the cell size in the domain, thereby increasing the number of cells that form the shape. The amount by which the original cells have been subdivided to increase the resolution is known as the cell density factor (CDF).

**CA Rules:** There are two sets of rules employed to compute the response of the truss to applied loads or displacements, and the change in the cross sectional areas of the truss members to fully utilize the members. The first set of rules is (normally) executed at every iteration to determine the displacements of a cell associated with deformations. The cell attempts to reach equilibrium with the surrounding cells by displacing itself to minimize the potential energy.

Within a cell, each truss member (indexed relative to the cell by \( k = 1, \ldots, 8 \)) has an elastic modulus \( E \), length \( L_k \), a cross-sectional area \( A_k \), and an orientation angle \( \theta_k \) from the cell center. Denote the displacement of the \( k \)-th truss member's near end from the original cell center by \( u, v \), and the displacement of the neighboring cell's center by \( u_k, v_k \). These neighboring displacements are assumed to be specified when the CA calculates the displacements of a cell. The extension of
a member between two cells $\Delta_k$, strain $\varepsilon_k$, and force $F_k$ within each member are calculated from these properties and displacements by

\begin{align}
\Delta_k &= (u - u_c) \cos \theta_k + (v - v_c) \sin \theta_k, \\
\varepsilon_k &= \Delta_k/L_k, \\
F_k &= EA_k\varepsilon_k.
\end{align}

Taking into account the applied external forces $F_x, F_y$, the total (internal strain plus external) potential energy $V$ for a cell is given by

$$V = \sum_{k=1}^{8} EA_k L_k \varepsilon_k^2 - F_x(u - u_c) - F_y(v - v_c).$$

Setting the partial derivatives of the potential energy with respect to the cell displacements to zero gives the equilibrium equations that can be solved for the cell displacements. In general, this is a system of two equations with two unknowns. If there is an (externally) applied displacement along one or both of the axes, then the equations will be solved for forces, $F_x$ and/or $F_y$, instead of the displacements.

Designing the structure requires resizing the beams in the cells. If displacements have already been calculated then some scheme for changing the cross sectional areas $A_k$ is required. In terms of allowable stress $\sigma_{allow}$, which is specified as the maximum stress that any given beam should endure, one scheme for computing a new cross sectional area $A_k^{new}$, based upon the previous cross sectional area $A_k^{old}$, is

$$A_k^{new} = \frac{E|\varepsilon_k|}{\sigma_{allow}} A_k^{old}.$$

If the displacement calculation and sizing are done sequentially, the sizing period (how often sizing is done) depends on many factors: the number of cells in the domain, the locations and relative displacements of the applied forces and displacements, the iteration method (Jacobi vs. Gauss-Seidel), and for Gauss-Seidel implemented in parallel, the number of processors used.

**Example:** Consider the problem of a simple bridge truss. The first image in Figure 3 shows a CA with six cells. The bottom two corner cells have an applied displacement of $(0,0)$ so they are fixed in place. The bottom middle cell has an applied force of $100kN$ downward. The width of the bridge is 50 meters and the height is 25 meters. The bars are composed of medium steel with $E = 200GPa$ and $\sigma_{allow} = 250MPa$. Each beam has an initial area of $0.0175m^2$.

Running the CA on the bridge problem using the Gauss-Seidel iteration method for displacements and applying the sizing rules every sixth iteration until it converges at iteration 253, the result shown in the second image of Figure 3 is obtained. Since the bridge is 50m across and the steel beams are no more than a few cm thick, the areas in this view are exaggerated by a factor of 3000 to show the differences in the beam sizes.

The bridge in Figure 3 is only composed of 6 cells, and the solution could easily have been computed by hand. By increasing the cell density, the complexity of the problem rises. Figure 4 shows the final configuration of a problem of the exact same dimensions, where each cell is 40 times smaller than previously. Each horizontal and vertical beam is $0.625m$ long, rather than $25m$. 

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Figure 3. Simple bridge truss, before and after running CA.

Figure 4. Bridge truss with 3321 cells after convergence.

Convergence Analysis: To analyze the efficacy of the iteration method used, it is useful to transform the CA into an equivalent system of linear equations. Recalling that each cell is computing its position $u, v$ based upon the position of the surrounding cells. If each cell were assigned unique variables for its position, such that cell 1 has $u_1$ and $v_1$, cell 2 has $u_2$ and $v_2$, and so forth, then the equations for each cell can be expressed in terms of the variables for the surrounding cells. For a CA structure composed of 6 cells, this will form a linear system of 12 equations and 12 unknowns. This standard system of linear equations, $Ax = b$, can be solved by the Jacobi and Gauss-Seidel fixed-point iteration methods or block versions thereof, which are the exact mathematical formulations of the local cell calculations. For the Jacobi, $A$ is split into its strictly $2 \times 2$ block lower triangular (L), $2 \times 2$ block diagonal (D), and strictly $2 \times 2$ block upper triangular (U) parts,

$$A = L + D + U$$

The system is then rewritten as a fixed point iteration where the next iterate $x^{(n+1)}$ is computed from the previous iterate $x^{(n)}$ via

$$x^{(n+1)} = Bx^{(n)} + C,$$

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<table>
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<th>CDF</th>
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<th>Gauss-Seidel</th>
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<tr>
<td>10</td>
<td>458</td>
<td>0.998765</td>
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</tr>
</tbody>
</table>

Table 1. Spectral radius of the bridge truss for various CDFs.

where

$$B = -D^{-1}(L + U), \quad C = D^{-1}b.$$ 

Note that $Ax = b$ if and only if $x = Bx + C$, assuming $D^{-1}$ exists. For Gauss-Seidel the iteration $x^{(n+1)} = Bx^{(n)} + C$ has

$$B = -(D + L)^{-1}U, \quad C = (D + L)^{-1}b,$$

assuming $(D + L)^{-1}$ exists.

The fixed-point iteration converges for any starting point $x^{(0)}$ if and only if all of the eigenvalues of $B$ are less than one in absolute value. The maximum absolute value of the eigenvalues for a matrix is called the spectral radius. The spectral radius for the bridge structure at various cell density factors (CDF's) is shown in Table 1.

This table shows that the Jacobi or Gauss-Seidel CA iteration for analysis does converge, but extremely slowly. For larger CDFs, the improvement of Gauss-Seidel over Jacobi is marginal. Even with massive parallelism, any competitive advantage of CA (over solving the linear system with standard iterative numerical methods) must come by combining analysis with sizing.


Accomplishments/new findings.

Genetic algorithms for local panel design: The basic genetic algorithm data structures were extended to use multiple chromosomes (for structures with several different laminates such as wing boxes) and real variables (such as geometrical dimensions). The approach is implemented in the new object-oriented Fortran 90 code for the design of laminated composite panels. The multiple chromosome approach yields efficient and elegant code, and implementing crossover and mutation directly with real numbers is more effective than working with discrete binary approximations of real numbers. Fortran 90 modules and GA operators continue to be refined and added to the package, and new capabilities such as discrete memory (storing old designs in a balanced binary tree to avoid reanalysis) and continuous memory (extending the binary tree via spline approximations in the continuous variable) were added. Local improvement (using local approximations to replace an individual by the best nearby individual) is also being added.

Laminate blending with Genetic Algorithms: In parallel with the research carried out at Virginia Tech, a simple blending algorithm that is based on an automated 2-step optimization process was incorporated by ADOPTECH Inc. into a commercial genetic algorithm software code called DARWIN. The approach is based on the observation that if the individual panels of an assembly of panels are optimally designed by completely ignoring the need for blending, the total weight of the panel system will be the lowest possible weight. However, the stacking sequence of the individually designed panels will rarely have any continuity across the panels, potentially depicting the least amount of blending characteristics. Alternatively, a single sub-laminate could be defined for all panels without any prior knowledge about the characteristics of the individually optimized designs. In this case the panels will have the best blending characteristics, and the weight will constitute an upper bound weight since the most critical panel will dictate the laminate stacking sequence of the entire panel system. Between these two extreme cases, other solutions exist that produce a total weight closer to that of the lower bound weight and display good blending characteristics. The approach used by ADOPTECH is a systematic procedure that searches for a blended system of panels by starting from the lower bound design.

The methodology is tested using a 3×3 array of sandwich panels, and an 18-panel group (Figure 1) arranged in a horseshoe pattern provided to ADOPTECH by Sikorsky Aircraft. Results for each design problem are generated using ADOPTECH’s Java-based composite laminate design software called OLGA, which utilizes the DARWIN optimization engine. Those results are presented in a paper jointly authored by ADOPTECH, Sikorsky, and Virginia Tech personnel.

Nonlinear local panel analysis: Earlier work under this grant compared the relative efficiencies of the Riks/Wempner, Crisfield, and normal flow solution algorithms for tracking nonlinear equilibrium paths of structural systems. It was found that the normal flow algorithm was both more computationally efficient and more robust than the other two algorithms when tracing the path through severe nonlinearities such as those associated with structural collapse of a blade stiffened panel. Consequently the normal flow algorithm is being routinely used in other projects for tracking equilibrium curves.

Parallel genetic algorithms: The parallel GA code implemented migration between subpopulations (evolving independently on different processors), and the parallel, dynamically adapting algorithm resulted in both better reliability and reduced normalized cost compared to a static serial algorithm with migration. The parallel implementation used dynamic load balancing, fully distributed control, and a sophisticated termination detection algorithm. The surprising finding
was that nondeterminism in the parallel task management significantly enhanced the performance of the (already nondeterministic) GA and migration. One interpretation of this finding is that random migration is superior to a fixed migration pattern. The parallel code also scaled very well, showing no significant communication penalty with 64 processors. Current work involves improving the parallel GA code, exploring the effect of different migration patterns (corresponding to different parallel connection topologies), and blending local panel designs to meet manufacturing constraints.

Mathematical theory for global/local decomposition: A rigorous decomposition theory has been developed for a class of optimization problems under mild smoothness assumptions. Necessary and sufficient conditions under which the decomposition reformulation is equivalent to the original problem have been derived. The reformulation can potentially remove a large number of local optima from the original problem, making the decomposition computationally attractive. The next task is to extend these results to mixed continuous/discrete problems, so as to include inherently discrete panel design.

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Publications.

Journal articles published during the grant period are:


Refereed conference papers published during the grant period are:


Journal articles accepted during the grant period are:


Journal articles submitted during the grant period are:


**Interactions/transitions.**

**Technology transitions or transfer:**

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Warren, MI
Contact: Alexander P. Morgan, 810-986-2157
RESULT
Homotopy algorithms; mathematical software
APPLICATION
Linkage mechanism design; combustion chemical, robotics; CAD/CAM

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RESULT
Homotopy algorithms; mathematical software
APPLICATION
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RESULT
Adaptive GMRES algorithm; mathematical software
APPLICATION
Iterative solution of large linear systems arising from tire modelling

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RESULT
Multiobjective genetic algorithms for composite laminate design
APPLICATION
Design of helicopter frame structures for minimum weight and cost

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RESULT
Blending algorithm for multipanel composite structural design
APPLICATION
Design of helicopter skin structures for minimum weight and cost

Inventions or patents.
None.

Honors/awards.
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