We examined small deviations from axial symmetry in a solid-propellant rocket motor, and describe a "bathtub-vortex" effect, in which substantial axial vorticity is generated in a neighborhood of the chamber centline. The unperturbed flow field is essentially inviscid at modest Reynolds numbers, even at the chamber walls, as has long been known, but the inviscid perturbed flow is singular at the centerline, and viscous terms are required to regularize it. We examine perturbations sufficiently small that a linear analysis is valid everywhere and larger perturbations in which a nonlinear patch is created near the centreline of radius. Our results provide and explanation of swirl experimentally observed by others, and a cautionary note for those concerned with numerical simulations of these flows, whether laminar or turbulent.
FINAL TECHNICAL REPORT FOR ASSERT/97 MODELLING OF COMPOSITE-PROPELLANT FLAMES
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These supplementary funds were used to support a graduate student, James Dow Chenoweth, who completed the thesis 'Porous wedge flows in a solid rocket motor' in 2001.

This work is closely related to AFOSR funded work that we published in the Journal of Fluid Mechanics, vol. 429, 2001, pp. 283-305: 'The generation of axial vorticity in solid-propellant rocket-motor flows', by S.Balachandar, J.Buckmaster, M.Short. We record the abstract of that paper as it conveys something of the general nature of the problem addressed by Chenoweth.

_We examine small deviations from axial symmetry in a solid-propellant rocket motor, and describe a "bath-tub-vortex" effect, in which substantial axial vorticity is generated in a neighborhood of the chamber centerline. The unperturbed flow field is essentially inviscid at modest Reynolds numbers, even at the chamber walls, as has long been known, but the inviscid perturbed flow is singular at the centreline, and viscous terms are required to regularize it. We examine perturbations sufficiently small that a linear analysis is valid everywhere (\(\epsilon Re\) small, where \(\epsilon\) is a measure of the perturbation amplitude and Re is a Reynolds number), and larger perturbations in which a nonlinear patch is created near the centreline of radius \(O(\epsilon^{3})\). Our results provide an explanation of swirl experimentally observed by others, and a cautionary note for those concerned with numerical simulations of these flows, whether laminar or turbulent._

Chenoweth looked at a similar problem, but one in which the propellant geometry is defined by a wedge. There are two applications for such a study.

1. The grain in solid rockets is sometimes star-shaped, to increase the burning surface. If the arms of the star are slender, it may be shown that the flow in an arm has a radial component, orthogonal to the rocket axis, that is defined by the plane-flow equations in a wedge.

2. The formation of cracks in solid propellants is a serious safety issue, and knowledge of the burn rate, pressure, and flow-field in the crack is of great importance. The wedge geometry studied by Chenoweth provides a simple model of a crack.

Fundamentally, Chenoweth did two things.

1. He constructed a similarity solution for the inviscid flow in the wedge, a variation on the famous Taylor-Proudman-Culick solution for a constant diameter configuration, one that we perturbed in the Journal of Fluid Mechanics paper cited above. These flows are essentially inviscid, as any boundary layer postulated for the walls is necessarily blown off by the strong injection from the propellant surface.

   However, it is generally the case that the inviscid solution breaks down in some interior domain. In the JFM paper, small asymmetries lead to algebraic singularities at the axis. In the case of the wedge, there is a logarithmic singularity at the center-line of the wedge. These singularities signal that viscous terms must be introduced locally, either via asymptotic analysis or a numerical treatment. Both are done in the JFM paper.

2. Chenoweth examined the viscous domain in the wedge numerically, to resolve the singularity. Unexpectedly, this turned into a serious computational effort at the large Reynolds
numbers we were interested in (Reynolds number based on distance from the apex). For a slender wedge, parabolized equations are appropriate, and so a simple marching scheme can be adopted, but the large inviscid domain in which is embedded a thin viscous layer is not a trivial problem. It became a valuable exercise for Chenoweth to examine and test a variety of numerical strategies, to see which worked well and which did not. In this enterprise he interacted with members of my research group in the Center for the Simulation of Advanced Rockets, and thereby received some sound numerical training. Upon completing his degree he obtained employment with a Defense Industry company located in Huntsville Alabama, where he is now engaged in numerical studies.