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Surveillance Performance Requirements for Runway Incursion Prevention Systems

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16. Abstract

In response to concerns over the number of runway incursions and runway conflicts at U.S. airports, the Federal Aviation Administration is sponsoring research and development of safety systems for the airport surface. Two types of safety systems are being actively pursued, a tower cab alerting system and a runway status light system.

The tower cab alerting system, called the Airport Movement Area Safety System (AMASS) is currently undergoing initial operational evaluation at several major airports. It provides aural and visual alerts to the tower cab to warn the controllers of potential traffic conflicts. The runway status light system is currently in the development phase, with initial operational suitability demonstrations planned at Dallas/Fort Worth International Airport during FY2003. Intended to offer protection in time-critical conflict scenarios where there is not enough time to warn the aircrews indirectly via the tower cab, the runway status light system provides visual indication of runway status directly to the cockpit: runway entrance lights warn pilots not to enter a runway on which there is approaching high-speed traffic; takeoff-hold lights warn pilots not to start takeoff if a conflict could occur.

Both systems operate automatically, requiring no controller inputs. Activation commands for alerts and lights are generated by the systems' safety logic, which in turn receives airport traffic inputs from a surface surveillance and target tracking system. Accurate traffic representation is essential to meet system requirements, which include high conflict detection rate, prompt and accurate alerting and light activation, low nuisance and false alarm rates, and negligible interference with normal operations.

This report analyzes the effect of the two fundamental surveillance performance parameters – position accuracy and surveillance update rate – on the performance of three different surface safety systems. The first two are the above-mentioned tower cab alerting and runway status light systems. The third system is a hypothetical cockpit alerting system that delivers alerts directly to the cockpit rather than to the tower cab.

The surveillance accuracy and update rate requirements of these three systems are analyzed for three of the most common runway conflict scenarios, using realistic parameter values for aircraft motion. The scenarios are 1) a runway incursion by a taxiing aircraft in front of a departure or arrival, 2) a departure on an occupied runway, and 3) an arrival to an occupied runway.

The conclusion from this analysis is that a system incorporating runway status lights with tower cab alerting will be effective in preventing most runway incursion accidents with a surveillance system providing $\sigma_s = 20$ feet ($2\sigma_s = 40$ feet) and update interval $\Delta\tau = 1$ second. Runway status lights are especially effective at preventing incursions and accidents between takeoff or arrival aircraft and intersection taxi aircraft. Tower cab alerts are effective at alerting controllers to aircraft crossing or on a runway during an arrival. Runway status information provided directly to the cockpit will be required for the case where a previous arrival or a taxi aircraft fails to exit the runway as anticipated shortly before the arrival crosses the threshold.

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EXECUTIVE SUMMARY

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The runway status light system [1,2,3,4] is currently in the development phase, with initial operational suitability demonstrations planned at Dallas/Fort Worth International Airport during FY2003. Intended to offer protection in time-critical conflict scenarios where there is not enough time to warn the aircrews indirectly via the tower cab, the runway status light system provides visual indication of runway status directly to the cockpit: runway entrance lights warn pilots not to enter a runway on which there is approaching high-speed traffic; takeoff-hold lights warn pilots not to start takeoff if a conflict could occur.

Both systems operate automatically, requiring no controller inputs. Activation commands for alerts and lights are generated by the systems' safety logic, which in turn receives airport traffic inputs from a surface surveillance and target tracking system. Accurate traffic representation is essential to meet system requirements, which include high conflict detection rate, prompt and accurate alerting and light activation, low nuisance and false alarm rates, and negligible interference with normal operations.

To ensure that a true traffic picture is provided to the safety logic, the surface surveillance and target tracking system must meet stringent track integrity requirements. All traffic on, near, and on final approach to the runways must be tracked reliably and accurately. The incidence of track drops and false tracks must be low. Target position accuracy, surveillance update rate, and track synthesis must be sufficient to determine a target's dynamic state and future motion with the required accuracy.

This report analyzes the effect of the two fundamental surveillance performance parameters – position accuracy and surveillance update rate – on the performance of three different surface safety systems. The first two are the above-mentioned tower cab alerting and runway status light systems. The third system is a hypothetical cockpit alerting system that delivers alerts directly to the cockpit rather than to the tower cab.

The surveillance accuracy and update rate requirements of these three systems are analyzed for three of the most common runway conflict scenarios, using realistic parameter values for aircraft motion. The scenarios are 1) a runway incursion by a taxiing aircraft in front of a departure or arrival, 2) a departure on an occupied runway, and 3) an arrival to an occupied runway.

The assumptions made are: 1) that the safety system must operate solely from electronic surveillance with no controller input (except airport configuration); 2) that the safety system doesn't know controller or pilot intent; 3) that the system is fully automatic; and 4) that the system is designed to prevent accidents rather than "catch" operational errors. A consequence of these assumptions is that there is a delay (for example) between the time when an aircraft starts a takeoff roll and the time that the system detects that event and can act to prevent the accident.

The approach taken to analyzing the first category of incursion was to examine the events and actions that must occur after the start of an incursion before the braking of the taxiing aircraft can begin. A set of probability density functions for each of the required reaction events was convolved to provide a total system reaction time probability density function. Next, the additional delay introduced by the positional uncertainty and update interval of the surveillance system was convolved with the reaction time probability density function to determine the probability density function of total time required to start braking of the taxi aircraft. This is compared with the time available, which is the latest time after crossing the taxi-hold position that the taxi aircraft can begin braking and still avoid the wing tip of the aircraft on the runway. This provided a single number representing the percentage of the cases where the total time required was less than or equal to the time available. The surveillance parameters were then varied to measure the expected percentage of "saves" as a function of surveillance parameters. The analysis was then expanded to study the effects of different wingspans for aircraft on the runway and to see the effects of the taxiing aircraft starting from a stopped position at the taxi-hold position line instead of crossing the hold position without stopping. The results indicated that, depending on the safety alerting system chosen, this category of incursion may not be totally protected, even with "perfect" surveillance. Specifically, a taxi towards a "hot" runway can only be protected for relatively low taxi speeds with a tower cab alerting system. A taxi-from-a-stop profile was introduced without a significant increase in the degree of protection. A system using a direct cockpit alert or runway status lights proved much more effective. The surveillance requirements to fully support these systems is $\sigma_s = 20$ feet and update interval of 1 second.

The analysis of the departure with a blocked runway category of incursion investigated how much runway was used by a takeoff aircraft that was required to reject its takeoff and come to a stop due to a blocked runway. Safety systems cannot alert until it is apparent that the aircraft is a departure because of the nuisance alarm problem. A system that simply detects a blocked runway can convey that information to the controller in any number of ways, for example, by highlighting a bar on the takeoff end of the runway in a surface radar display. However, this is not an alert because the runway will be blocked on a regular basis, often with an aircraft in position at the departure end of the runway. An alerting system that can detect a departure with a blocked runway conflict must delay the alert until the aircraft is known to be a departure in order to reduce nuisance alarms. This will, depending on the algorithms, require that the departure at least begin to roll and accelerate to some velocity or travel some distance in an accelerating mode while the runway is blocked. How long it takes a system to correctly declare that the aircraft is departing depends on this threshold velocity and the surveillance parameters. A simple threshold velocity requirement was assumed that was then "padded" to eliminate nuisance alarms caused by the positional uncertainty of the surveillance system. In the case of a tower alerting system, the controller reaction probability density function (pdf), the VHF channel availability pdf, and the pilot reaction pdf must be taken into account before evasive braking will begin. In the case of direct cockpit alerts, only the pilot reaction pdf need be considered. The results of the analysis of this incursion category indicated that a σ_s positional uncertainty of 20 feet will be required with an update interval of about 1 second. This is coincidentally similar to the results for the taxi incursion. For this incursion, it is apparent that a tower cab alerting system will not protect any reasonable portion of the runway. A direct cockpit alerting system offers a significant improvement and will protect all but the near portion of the runway, however "near" is relative in 600-foot visibility takeoffs. The takeoff-hold lights are particularly effective in protecting the near portion of the departure runway, even with simultaneous taxi incursions.

The analysis for the arrival with a blocked runway category of incursion investigated the case of an arrival where the runway is blocked by an aircraft in position to depart or by an arrival or taxi aircraft that has not exited the runway. The controller's handbook [5] requires that if the arrival is a Category III aircraft (any aircraft other than small single or twin engine propeller aircraft) the preceding departure must be at least 6000 feet down the runway before the arrival crosses the threshold. This is only allowed between sunrise and sunset if the controller can determine distances by reference to suitable landmarks and the departing aircraft is airborne. It need not have crossed the runway end. The challenge for the surveillance is to determine whether or not the departing aircraft is indeed departing in time to alert the arriving aircraft of a blocked runway. If the aircraft does not depart but remains on the runway, there is a danger of the arrival landing on a blocked runway as was the case for the accident in Los Angeles on February 1, 1991. However, if the aircraft on the runway is departing in time to allow sufficient separation from the arrival aircraft but the surveillance system is too poor to allow the safety system to detect the departure, then a nuisance alarm will be issued.

The approach taken was to examine the case of a Category III aircraft landing at an approach speed of 150 knots with a departure starting a takeoff with a constant acceleration of 0.26 g's to 150 knots. The "window" between when the arrival aircraft reaches a point where it will violate separation, even if the departure begins to roll, and the point where a go-around must be initiated to avoid a collision was defined. The surveillance and safety system requirements are derived based on determining whether or not the departure aircraft has begun its departure in time to alert the arriving aircraft to go-around. The runway status lights play no role in preventing this category of incursion. There is no problem in detecting that there is an aircraft on the runway and alerting the tower in time to have a go-around of the arrival aircraft at the decision height of 200 feet above the ground. The problem is that the aircraft on the runway may be a normal departure with no separation violation and the alerts would routinely be false or nuisance alarms. This is because the departure aircraft can wait until the arrival is some 10,000 feet from the threshold before starting to accelerate and still have no separation violation.

A practical application of the analysis was performed for Dallas/Fort Worth International airport's runway 18L taxi placement using the specified surveillance parameter for the multilateration portion of the Airport Surface Detection Equipment X-band radar (ASDE-X).

The conclusion from this analysis is that a system incorporating runway status lights with tower cab alerting will be effective in preventing most runway incursion accidents with a surveillance system providing $\sigma_s = 20$ feet ($2\sigma_s = 40$ feet)[1] and update interval $\Delta\tau = 1$ second. Runway status lights are especially effective at preventing incursions and accidents between takeoff or arrival aircraft and intersection taxi aircraft. Tower cab alerts are effective at alerting controllers to aircraft crossing or on a runway during an arrival. Runway status information provided directly to the cockpit will be required for the case where a previous arrival or a taxi aircraft fails to exit the runway as anticipated shortly before the arrival crosses the threshold.

Track integrity must be very reliable for a safety system to be effective. The track integrity depends on the probability of detection and probability of false detection by the surveillance system as well as the tracker design. The probability of detection or false detection depends on the type of surveillance system implemented and the location on the airport. An operational system will need to assess the surveillance systems track performance for that individual installation.

[1] σ_s is the standard deviation of uncorrelated position reports. System performance is often specified in terms of 95% or $2\sigma_s$.

PREFACE

The material contained in this document is based on work performed at MIT Lincoln Laboratory under the sponsorship of the Federal Aviation Administration (FAA). Related FAA work being performed elsewhere includes the Airport Movement Area Safety System (AMASS) project intended to provide a near-term enhancement in airport safety by providing alerts directly to the tower cab when the surveillance system detects hazardous situations on the airport surface (including arrival aircraft). AMASS, which is based on concepts developed at the MITRE Corporation and at the Norden Systems Division of United Technologies (now part of Northrop Grumman), is being implemented as an add-on to the ASDE-3 surface radar.

The implementation of the capabilities embodied in AMASS is viewed within the FAA as an essential first step. Subsequent phases will introduce additional safety products as well as elements designed to improve airport capacity.

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1. INTRODUCTION

In response to concerns over the number of runway incursions and runway conflicts at U.S. airports, the Federal Aviation Administration is sponsoring research and development of safety systems for the airport surface. Two types of safety systems are being actively pursued, a tower cab alerting system and a runway status light system.

The tower cab alerting system, called the Airport Movement Area Safety System (AMASS) is currently undergoing initial operational evaluation at several major airports. It provides aural and visual alerts to the tower cab to warn the controllers of potential traffic conflicts.

The runway status light system [1,2,3,4] is currently in the development phase, with initial operational suitability demonstrations planned at Dallas/Fort Worth International Airport during FY2003. Intended to offer protection in time-critical conflict scenarios where there is not enough time to warn the aircrews indirectly via the tower cab, the runway status light system provides visual indication of runway status directly to the cockpit: runway entrance lights warn pilots not to enter a runway on which there is approaching high-speed traffic; takeoff-hold lights warn pilots not to start takeoff if a conflict could occur.

Both systems operate automatically, requiring no controller inputs. Activation commands for alerts and lights are generated by the systems' safety logic, which in turn receives airport traffic inputs from a surface surveillance and target tracking system. Accurate traffic representation is essential to meet system requirements, which include high conflict detection rate, prompt and accurate alerting and light activation, low nuisance and false alarm rates, and negligible interference with normal operations.

To ensure that a true traffic picture is provided to the safety logic, the surface surveillance and target tracking system must meet stringent track integrity requirements. All traffic on, near, and on final approach to the runways must be tracked reliably and accurately. The incidence of track drops and false tracks must be low. Target position accuracy, surveillance update rate, and track synthesis must be sufficient to determine a target's dynamic state and future motion with the required accuracy.

Both the Airport Surface Detection Equipment (ASDE-3) radar installed at approximately thirty of the nation's busiest airports and a lower cost alternative surface radar, ASDE-X, which is being installed at an additional 25 airports, are a part of the Federal Aviation Administration's plans for modernization of the nation's air traffic control system.

ASDE-X includes both surface primary radar and transponder multilateration and incorporate data from the Airport Surveillance Radar (ASR) and Automated Radar Terminal System (ARTS). Multilateration, which includes the capability of adding data tags to aircraft on the controller's display, is also being considered as an addition to ASDE-3. It is also possible to provide surveillance with a stand-alone multilateration system.

All of these are area surveillance systems. An area surveillance system tracks targets anywhere in the area under surveillance as opposed to a block or point surveillance system, that records when a target enters or exits a block or crosses a point or line. Any of these area surveillance systems could be coupled with logic designed to reduce incursions and prevent accidents. This could involve alerts in the tower cab, alerts or traffic information fed directly to the aircraft cockpit, or the activation of runway status lights on the airport surface. Whether such a system can be effective in preventing accidents and yet not create distracting nuisance alarms in the tower cab or cockpit is in the process of being tested. It may be that a combination of systems is needed to be effective in preventing surface accidents.

When specifying the design for a runway incursion prevention system, it is worthwhile to examine the requirements of the system in terms of its effectiveness in reducing runway incursions and preventing surface accidents. The purpose of this analysis is to determine the performance requirements of area surveillance systems to support runway incursion prevention systems.

The assumptions made are that:

1) the safety system must operate solely from electronic surveillance with no controller input (except airport configuration);
2) the safety system doesn't know controller or pilot intent;
3) the system is fully automatic; and
4) the system is designed to prevent runway incursions and conflicts rather than "catch" operational errors.

A consequence of these assumptions is that there is a delay (for example) between the time when an aircraft starts a takeoff roll and the time that the system detects that event and can act to prevent the accident.

This report analyzes the effect of the two fundamental surveillance performance parameters – position accuracy and surveillance update rate – on the performance of three different surface safety systems. The first two are the above-mentioned tower cab alerting and runway status light systems. The third system is a hypothetical cockpit alerting system that delivers alerts directly to the cockpit rather than to the tower cab.

The surveillance accuracy and update rate requirements of these three systems are analyzed for three of the most common runway conflict scenarios, using realistic parameter values for aircraft motion. The scenarios are 1) a runway incursion by a taxiing aircraft in front of a departure or arrival, 2) a departure on an occupied runway, and 3) an arrival to an occupied runway.

The organization of this paper details the technical approach in Section 2. Sections 3, 4, and 5 are the detailed analysis of the surveillance requirements to support safety systems to prevent three categories of incursion. In each of the sections, three different generic safety system designs are analyzed to see what effect the safety system design has on the surveillance requirements. In all cases the emphasis of the analysis is on defining surveillance parameters necessary in preventing the accident rather than the incursion. As will be seen, in many cases it is impossible to prevent the incursion. Section 6 provides a practical application of the analysis to a specific runway at Dallas/Fort Worth airport and assesses the effectiveness of the multilateration portion of the ASDE-X with a safety system that employs both tower cab alerts and runway status lights. Section 7 contains the summary and conclusions.

2. APPROACH

2.1 INTRODUCTION

The purpose of this analysis is to derive quantitative requirements for area surveillance systems to support runway incursion prevention safety systems. The approach taken is to determine surveillance accuracy and update interval required to support three types of safety systems for three specific instances of runway incursions. The effectiveness of a given safety system will necessarily start to fall off when the quality of the surveillance inputs degrades beyond a certain critical point. The goal in this analysis is to find that critical point for various safety systems and different categories of incursion.

Note that a given safety system approach may not be particularly effective for a specific category of incursion for reasons other than surveillance. It is still worthwhile to determine the required accuracy and update interval of the surveillance system to support that safety system. In the course of the analysis, it became apparent that the approach taken was not conducive to making direct comparisons in effectiveness of safety systems although some attempt to do so is included in the results.

The analysis and measurements of effectiveness are based on preventing the accident in the category of incursion being analyzed as opposed to preventing the incursion. This is because in many cases it is impossible to prevent the incursion and yet the safety system could still prevent the accident.

Because this analysis is for generic area surveillance systems it is necessary to use the generic surveillance parameters of accuracy and update interval. In order to keep the analysis manageable, three types of safety systems were analyzed for three representative challenging categories of incursions. The safety systems analyzed were controller alerts, cockpit alerts, and runway status lights. The incursions analyzed were taxi conflicts with an arrival or departure, a departure with a blocked runway, and an arrival to a blocked runway.

2.2 SURVEILLANCE PARAMETERS

The two metrics chosen for measuring an area surveillance system are positional uncertainty and target position update interval. In a radar system, update interval is a function of the antenna rotation rate or scan interval. Some systems, such as multilateration systems receiving transponder replies, will have statistical update intervals. A single measure of positional uncertainty is somewhat of an oversimplification for most surveillance systems since the positional uncertainty will almost certainly be due to more that one underlying factor. The positional uncertainty might be more accurately characterized in terms of bias, scan-to-scan error or "jitter," and azimuth and range uncertainties. Also of concern are the probabilities of losing a target (dropping track) or failing to identify a new target. However, this investigation is intended to cover area systems in general and the simpler the metrics the more general the application of the results.

2.3 TRACK INTEGRITY

A safety system cannot be effective in preventing runway incursions or runway conflicts involving an aircraft that is not being tracked. High track integrity in the difficult environment of the airport surface is required for an accurate representation and prediction of the airport traffic situation. Without high track integrity, neither alerts nor lights will perform as required.

Track integrity must be very reliable for a safety system to be effective. The track integrity depends on the probability of detection and probability of false detection by the surveillance system as well as the tracker design. The probability of detection or false detection depends on the type of surveillance system implemented and the location on the airport. An operational system will need to assess the surveillance systems track performance for that individual installation.

In the case of alerting systems, an aircraft not in track may go unnoticed, thus the fact that the safety system is offering no protection may go unnoticed. However, in the case of a runway status light system, the fact that an aircraft is not in track is likely to have a deleterious effect on system operation, thus degrading users' confidence in the system. For instance, an aircraft waiting to taxi across an active runway that observes an aircraft taking off without having the runway entrance lights turn red may lose confidence in the effectiveness of the system or be confused as to the principle of operation of the status lights.

2.4 INCURSION CATEGORIES

Specific instances of three representative categories of runway incursion were chosen for investigation. One way to classify incursions is to divide them into those involving aircraft traveling along the same track parallel to the runway and those that involve one aircraft traveling parallel to the runway while another aircraft crosses that runway. The most time critical incursions are those involving crossing tracks at taxiway/runway intersections. The parallel path incursions tend to involve one aircraft overtaking another aircraft (e.g., an arriving aircraft overtaking a previous arrival or preceding departure) and, in general, offer more time for the detection of the conflict and its subsequent resolution. The most challenging commonly occurring categories of incursion include the one in which a taxiing aircraft enters an active runway from an intersecting taxiway in front of a fast moving arrival or departure aircraft on the runway. The most time critical demand on a safety system results from the set of dynamics where the only evasive action possible is the braking of the taxiing aircraft. This offers the least time for reaction. A safety system cannot alert before it is certain that the taxiing aircraft will cross the taxi-hold position and yet there are typically only 280 feet from the taxi-hold position to the center of the runway. This category of incursion is labeled Category 1: Intersection Taxi-Takeoff/Landing in this analysis.

Two other categories of time critical commonly occurring incursions are studied. Category 2 is the case of a departing aircraft with a blocked runway that requires a rejected takeoff. This is related to Category 1 except that now the runway is blocked (either from an intersection taxi or previous arrival) and the departure aircraft must brake to a stop before hitting the obstruction.

Category 3 is the case of an arriving aircraft with a blocked runway that requires a go-around of the approaching aircraft. The challenge here is to determine whether or not a runway is really blocked because normal operations include approaches with aircraft on the runway in position to depart.

2.5 DESCRIPTION OF THE SAFETY SYSTEMS

In each of the three categories of incursion, the surveillance requirements to support three types of safety systems are analyzed: 1) safety systems utilizing direct alerts to the tower cab that require interpretation and resolution of the conflict by the controller with subsequent voice commands to the pilot via the VHF voice channel, 2) systems utilizing direct alerts to the cockpit, and 3) systems using runway status lights. Each of these systems has different consequences with regard to timing analysis and effectiveness.

A safety system utilizing alerts in the tower cab or direct alerts to the aircraft cockpit is based on surveillance and tracking algorithms that detect a hazardous situation and deliver an audible alert to the tower cab or cockpit. Depending on the sophistication of the system, the alert may substantially increase the situational awareness of the controller or pilot. These systems depend on the controller and pilot reacting to unexpected situations and thus are treated with a probabilistic approach.

The runway status lights system involves two types of status lights. Runway entrance lights are located at all entrances to the runway at the edge of runway. These lights turn red when a runway is "hot," that is when the surveillance system detects that a high speed arrival or departure is traveling down the runway and it is unsafe to enter the runway. The system may turn the lights amber when a runway is "active" but not "hot." This would reinforce the amber "wig-wag" lights at the taxi-hold positions at many airports. In addition, there are takeoff-hold lights located ahead of the points where aircraft begin their takeoff roll that are red if the runway ahead is unsafe for departure. No lights are shown to arriving aircraft in the current concept. A more detailed description of the system is available from Lyon et al. [1,2,3,4].

3. CATEGORY 1 INCURSION: INTERSECTION TAXI-TAKEOFF/LANDING

3.1 APPROACH

The intersection taxi-takeoff/landing conflict is concerned with the case where an aircraft at a taxiway entrance to a runway taxis past the taxi-hold position while the runway is hot with a landing or departing aircraft. The geometry with a takeoff aircraft is depicted in Figure 1. The presumption in the analysis is that the takeoff or landing aircraft cannot or does not brake and that the only evasive action that will prevent the accident is the braking of the taxi aircraft. The challenge to the surveillance system is that it is normal for an aircraft to taxi up the taxi-hold position with a hot runway so care must be taken not to have nuisance alarms; the system cannot alarm until it is certain that the taxi aircraft has violated the taxi-hold position.

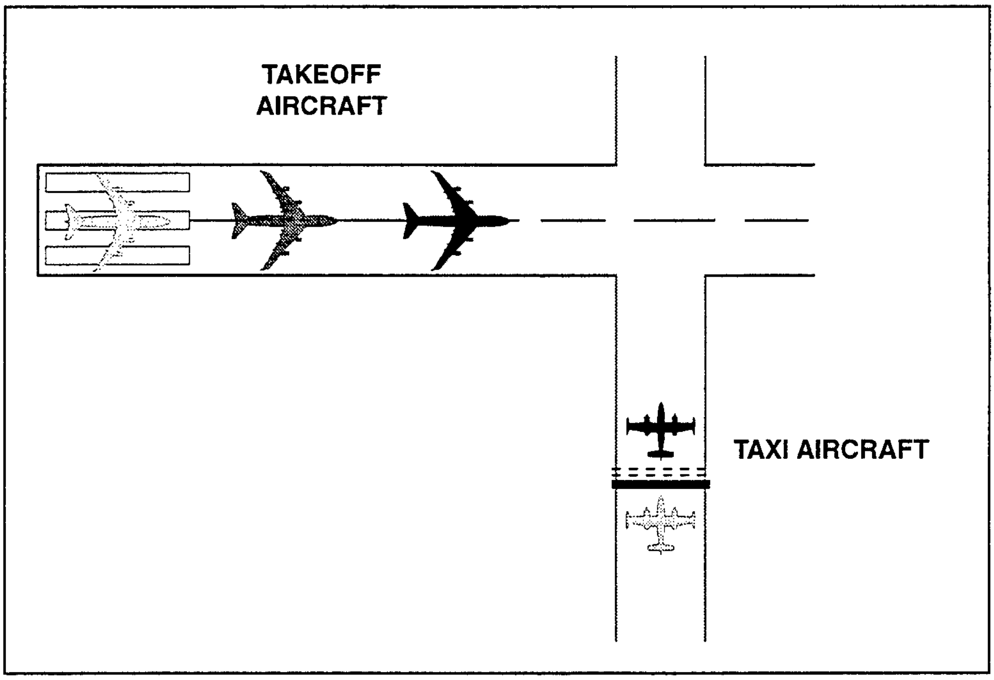


Figure 1. Category 1 incursion.

Two cases are examined for the alerting safety systems, the first labeled a full speed taxi towards a Boeing 747. In this case, it is assumed that the taxi aircraft approaches the taxi-hold position at normal taxi speed (which is a parameter in the analysis) and does not stop, but continues through at a constant velocity. The Boeing 747 defines the wingspan of the takeoff aircraft, which determines how far the taxi aircraft travels before entering the region where a collision is inevitable[2]. The second case assumes that the taxi aircraft is stopped at the taxi-hold position and then begins its taxi. The wingspan of a Boeing 727 is used for the aircraft on the runway in the

[2] Throughout this analysis, it is assumed that a collision will occur if the nose of the taxi aircraft is within the dimension of the wingspan of the takeoff aircraft. In actuality, it is possible for the nose of the taxi aircraft to pass beneath the wing of the takeoff aircraft in some instances.

second case. In an alerting system, it is necessary to detect that the taxi aircraft has crossed the taxi-hold position before alerting. However, in a runway status light system, this is not required since the runway entrance lights will be illumined red if the runway is hot, and the taxi aircraft will not enter. In order to develop the surveillance requirements for a runway safety system, a more intricate analysis is necessary that involves examining the relative timing of the motion of the two aircraft and developing requirements to protect a potential collision region. This alternate analysis is presented in Section 3.5.

The approach taken to analyze the taxiway incursion is first to examine the events and actions that must occur after the start of an incursion before the braking of the taxiing aircraft can begin. In the timing analysis of Section 3.2, a set of probability density functions (pdfs) for each of the events is convolved to provide a total system reaction time probability density function. Next, the additional delay determined by the positional uncertainty and update interval of the surveillance system is convolved with the reaction time distribution to determine the distribution of total time required before taxi aircraft braking can begin. A cumulative distribution is calculated from the total time required probability density function and this is compared with the time available, defined as the latest time after crossing the taxi-hold position that the taxi aircraft can begin braking and still avoid the wing tip of the aircraft on the runway. This will provide a single number representing the percentage of the cases where the total time required (both the reaction time of the events that comprise the reaction time probability density function and the probability density function for the detection time given the particular surveillance parameters specified) is less than or equal to the time available. The surveillance parameters are then varied to measure the expected percentage of accident preventions as a function of surveillance parameters.

The analysis is then expanded to investigate the effect of different wingspans for aircraft on the runway and to see the effect of the taxiing aircraft starting from a stopped position at the taxi-hold position line.

3.2 ANALYSIS FOR TOWER CAB ALERTS

3.2.1 Timing Analysis for Tower Cab Alerts and Direct Cockpit Alerts

3.2.1.1 Event Probability Density Function

There are five events that must take place before the taxi aircraft begins braking. First, the surveillance system must detect that the taxi aircraft has crossed the taxi-hold position while the runway is hot. This will initiate an alert in the tower cab. Second, the controller must react to the alert. Third, the controller must have access to a VHF channel. Fourth, the controller must issue the warning to the pilot. And fifth, the pilot must react to the alert and begin braking the aircraft. The time required for each of these events can be portrayed as a probability density function (pdf) and all of the pdfs convolved to create a probability density function representing the time from the start of the incursion to the start of aircraft braking.

The authors know of no specific studies that have measured pdfs for the events described above. However, the Precision Runway Monitor (PRM) program [6] made extensive measurements on a system that alerted the final approach controller of the potential deviation of one aircraft on a final approach towards another aircraft on a parallel approach. A warning or caution alert indicated to the controller that an aircraft was about to enter the "no transgression zone." The controller, reacting to the alert, had to have access to a clear VHF channel, issue a brakeout instruction to the parallel aircraft, and that aircraft had to react by beginning a turn. These scenarios are considered similar enough that the probability density functions measured in those studies are adopted here.

The probability density function for the surveillance depends on the accuracy and update interval of the surveillance system. Since these are the variables of interest, the surveillance pdf is a variable that is computed last and described below.

Figure 2 is the pdf for controller reaction time and is based on the studies in the PRM program. Figure 3 represents the VHF channel availability taken from the PRM report and based on extensive data taken in Memphis. Most of the time (86.6%) the channel is available because the controller is already talking on the channel or no pilot is transmitting. The remaining 13.3% of the time the controller must wait until the pilot completes a transmission in order to have access to the channel. Figure 4 is the pdf for pilot reaction time and represents the time from the start of the message from the controller until the start of evasive action, thus the time to speak the alerting message is included.

These pdfs are based on studies from the PRM program because that data is from the closest safety system available. However, there are noticeable differences. In the PRM system the controller is closely monitoring aircraft on parallel tracks with an audible warning before an aircraft flies into a no-transgression zone. In a runway incursion alerting system, the controller is more likely to have lost situational awareness and the reaction times may be longer on average with longer tails. The pilot reaction times in the PRM study are for pilot reactions while flying the airplane and require a change in aircraft flight trajectory. Pilot reaction times for braking a taxi aircraft may be shorter on average and have shorter tails. Nonetheless, these are the most representative data available and should provide representative results. Charts are derived in Appendix A to allow the reader to see the effects of different reaction times.

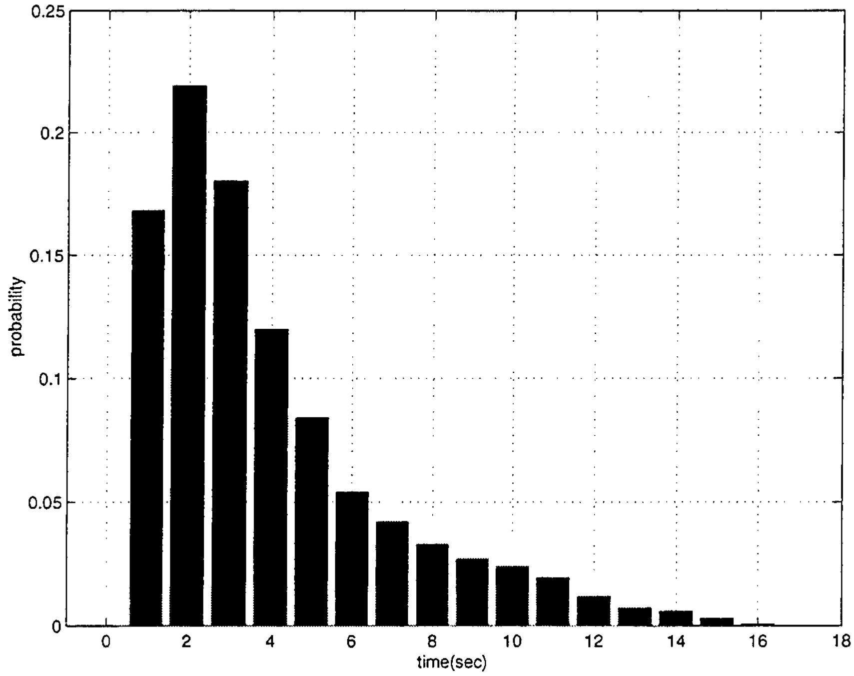


Figure 2. Controller reaction time pdf.

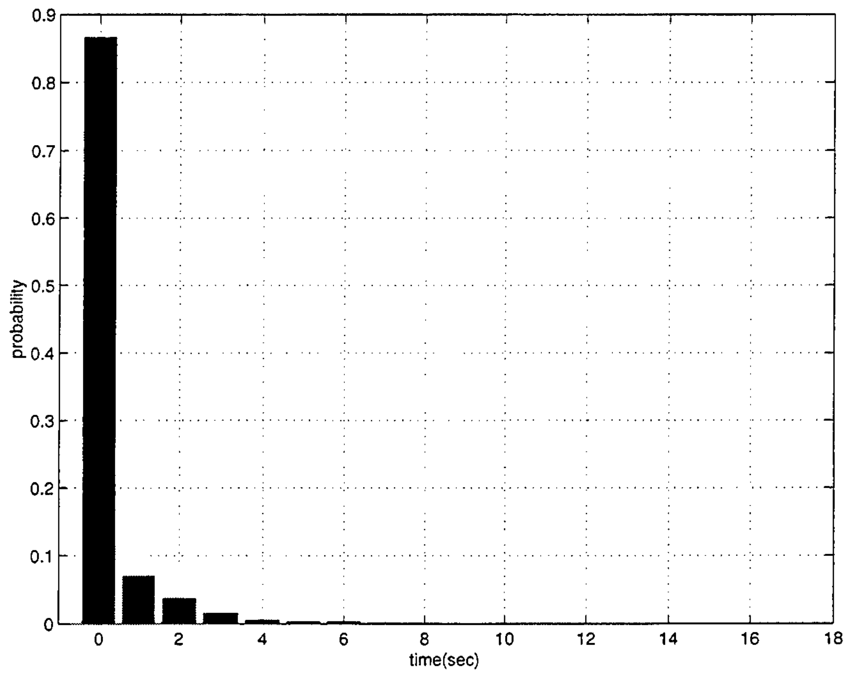


Figure 3. VHF channel availability pdf.

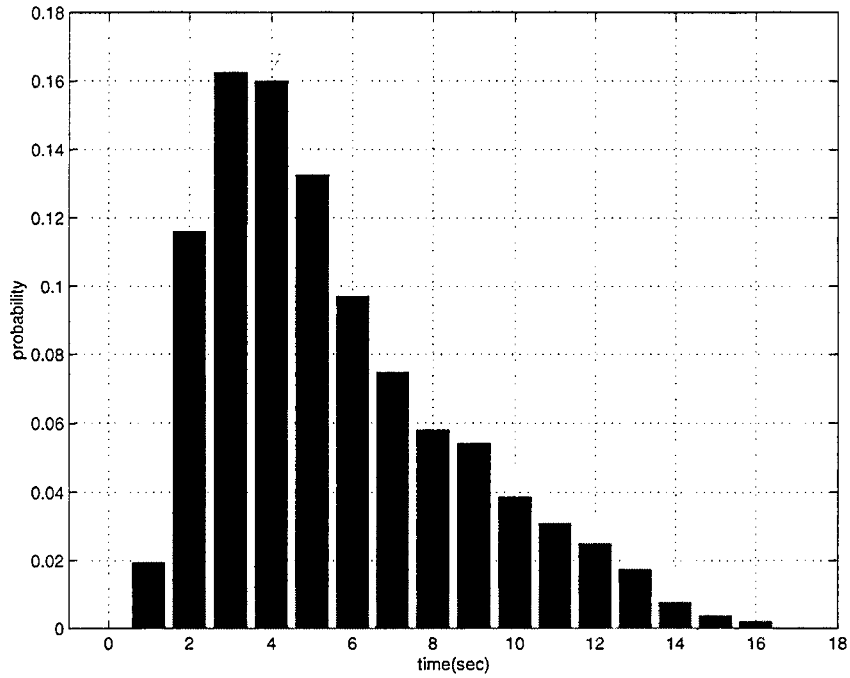


Figure 4. Pilot reaction time pdf.

A total reaction time pdf is calculated by convolving these three density functions. The result is plotted as a bar chart in Figure 5. A cumulative distribution function is also calculated and presented as a bar chart in Figure 6.

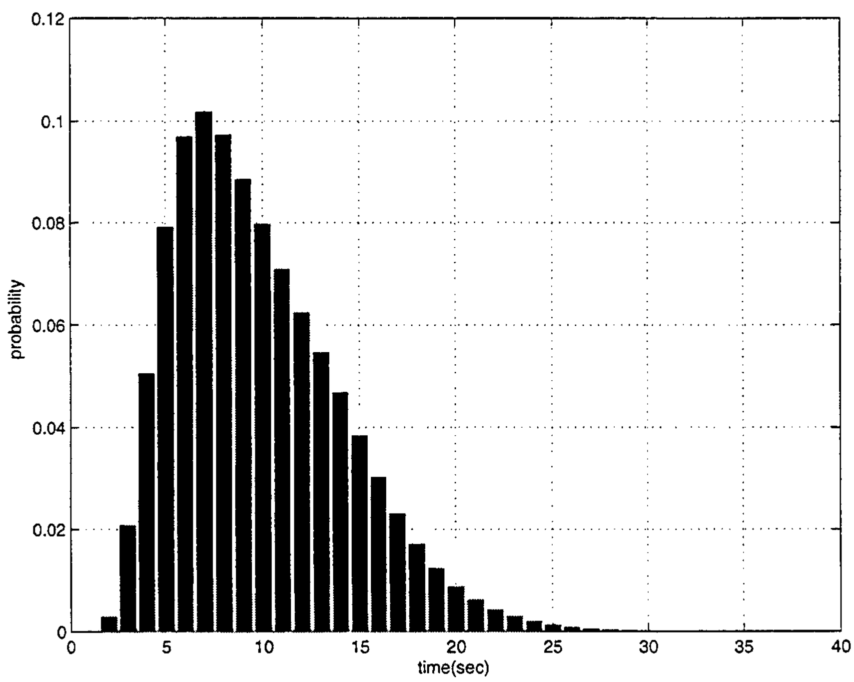


Figure 5. Total reaction time pdf.

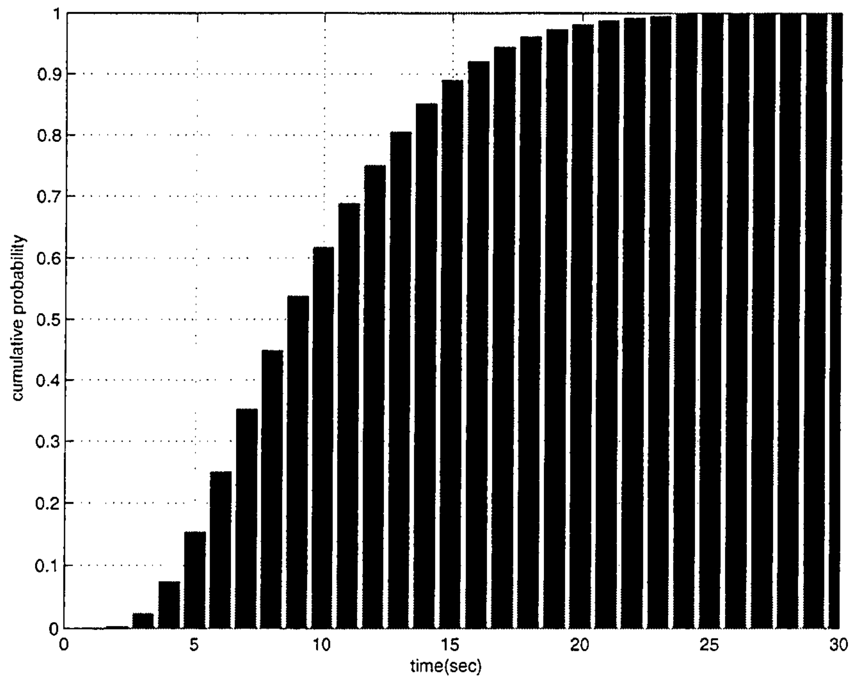


Figure 6. Cumulative distribution function for total reaction time.

3.2.1.2 Surveillance Delay Probability Density Function

Next, a probability density function is calculated that represents the time delay due to surveillance uncertainty. This will depend on the update interval and the positional uncertainty. For purposes of this analysis, positional uncertainty will be represented by the standard deviation in position measurements, σ_s, and update interval by $\Delta\tau$. It is assumed that an aircraft must taxi a distance of $2\sigma_s$ before it can be detected as having passed the taxi-hold position without producing nuisance alarms. A worst-case assumption is that the taxiing aircraft moves at a constant velocity through the hold position. An aircraft starting from a stopped position would afford more time for the system to react. The assumption is made that the surveillance system will use an estimate of the target extent, or size, to estimate the position of the aircraft's nose. In this analysis it is assumed that the surveillance knows the position of the aircraft's nose and uses that rather than the centroid to determine when the taxi-hold position has been crossed. If the target centroid was used for the taxi aircraft position without regard for the aircraft's extent then the distance traveled before detection must be further increased by half of the length of the taxi aircraft. This will degrade the safety system further.

The pdf is a uniform distribution spread from the time the aircraft reaches the prescribed distance past the taxi-hold position until the maximum update interval of the surveillance system. In other words, no detection is possible until the aircraft reaches a point corresponding the positional uncertainty past the taxi-hold position. Then there is an equal probability of detection from that point until the maximum update interval. This uniform distribution is distributed over the "bins" representing tenths of seconds, i.e., the two-second "bin" corresponds to the probability that a detection occurs between 1.95 and 2.05 seconds.

A sample calculation for a single case serves as an example. First the position uncertainty, update interval and taxi velocity are defined.

σ_s = 25 feet

$\Delta\tau$ = 2 seconds

v_{taxi} = 10 Knots (16.878 feet/second)

The earliest time that a detection can take place is the time it takes to taxi the distance represented by $2\sigma_s$ past the taxi-hold position. The maximum time it takes for a detection assumes that the aircraft crosses the uncertainty distance at one update and it takes another full update interval to detect the crossing.

tdetmin = $2\sigma_s / v_{taxi}$ = 2.96242 seconds

tdetmax = tdetmin+ $\Delta\tau$ = 4.96242 seconds

The probability density function for surveillance detection is created by distributing the uniform distribution from **tdetmin** to **tdetmax** into the "bins" representing tenths of seconds, each element representing the probability that detection will occur between .05 second less to .05 second more than the tenth of a second bin. The surveillance detection pdf for this example is plotted as a bar chart in Figure 7.

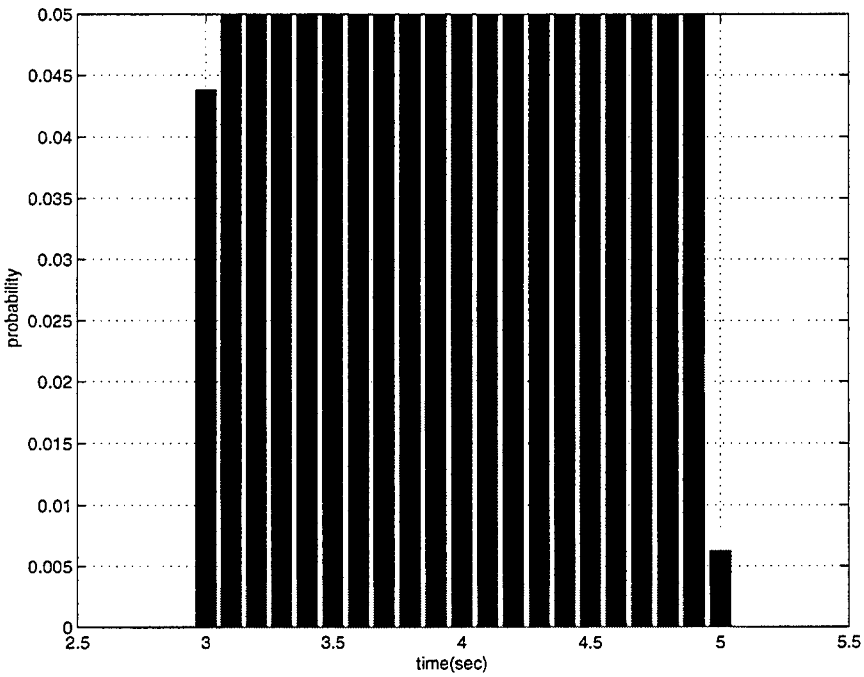


Figure 7. Surveillance detection time pdf for example with positional uncertainty of σ_s of 25 feet and update interval of 2 seconds.

3.2.1.3 Total Time Required to Begin Evasive Action

Convolving the surveillance detection probability density function with the reaction time probability density function yields the probability density function for the time required before evasive action can begin. The reaction time pdf is interpolated to match the tenth of a second intervals used for the surveillance pdf. This is expressed as a probability density function and a cumulative distribution function in Figures 8 and 9 for the example case described above.

The total cumulative distribution can be interpolated to determine the probability of all of the required events occurring before any time as illustrated in Figure 10. If we know when the aircraft must begin braking in order to avoid the accident, then the total cumulative distribution can be interpolated at this time to determine the probability that this will be successful.

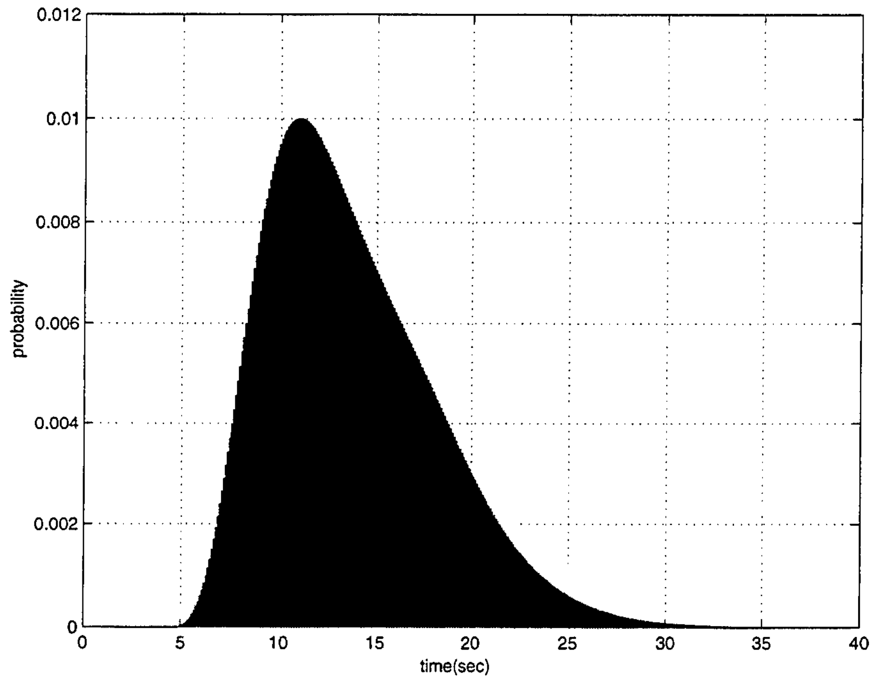


Figure 8. Total time required pdf for example with positional uncertainty σ_s of 25 feet and update interval of 2 seconds.

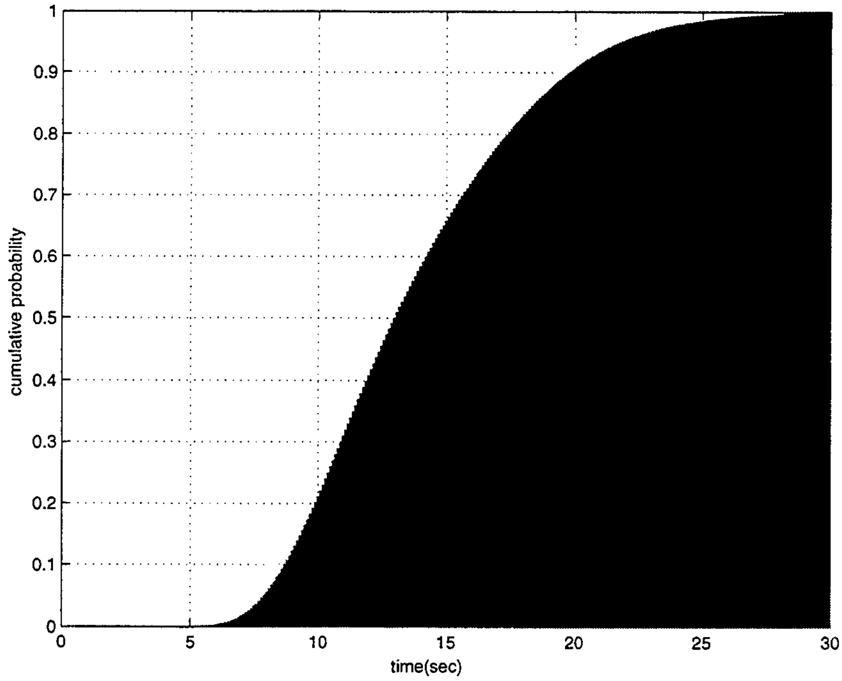


Figure 9. Total time required cdf for example with positional uncertainty σ_s of 25 feet and update interval of 2 seconds.

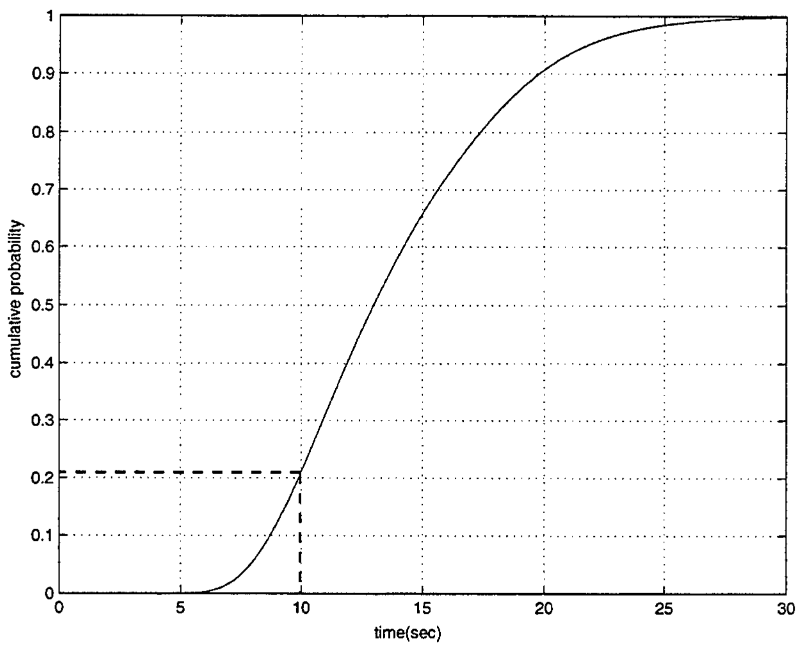


Figure 10. Total time cdf.

3.2.1.4 Time Available for Evasive Action

The assumed incursion is a taxiing aircraft not holding short at the hold position but continuing to taxi onto a runway with a departing or arriving aircraft crossing at the intersection. The assumption is that the only evasive action possible in this case is the braking of the taxi aircraft to stop short of the portion of the runway that is occupied by the wing-tip of the aircraft on the runway. The question is, how long after the taxi aircraft crosses the taxi-hold position must it begin braking to avoid a collision? This will depend on the velocity of the taxi aircraft, the distance from the taxi-hold position line to the runway, and the wingspan of the aircraft on the runway. The size of the taxi aircraft does not affect the results if the assumption is made that the surveillance system measures the length (extent) of the taxi target and estimates the position of the nose of the aircraft rather than tracking only the centroid of the aircraft.

In the case above, the taxi aircraft was assumed to taxi at 10 knots. We will later analyze the results as a function of taxi velocity noting that changing the taxi velocity will change the surveillance detection pdf and the resulting total time required pdf. For the continuation of the analysis of this sample case we assume a Boeing 747 with a wingspan of 195.7 feet for the aircraft on the runway. The surveillance system is assumed to be designed to alert when the taxi aircraft's nose crosses the taxi-hold position. The standard distance from the taxi-hold position to the runway centerline is 280 feet. Finally, we assume a maximum deceleration of .33 g's. This allows the calculation of the maximum time after the aircraft crosses the taxi-hold position before it must start braking to avoid a collision. The time and distance required for the taxi aircraft to brake to a full stop from the taxi speed of 10 knots (16.88 ft/sec) with a braking of .33 g's are 1.59 seconds and 13.4 feet.

The maximum time available (the time between when the taxi aircraft crosses the taxi-hold position until the moment it must begin braking to avoid the collision) is the distance the aircraft taxis from the taxi-hold position to the point at which is must begin braking divided by the taxi velocity. The point at which the taxi aircraft must begin braking is the 280 feet to the runway centerline less half of the takeoff aircraft's wingspan less the taxi braking distance (280-(195.7/2)-13.4) or 168.75 feet. The taxi velocity in this case is 10 knots. In this sample case, the result is 9.99 seconds.

This time can then be compared with the cumulative distribution of the total time required to complete all required actions (surveillance detection, controller reaction, channel availability, and pilot reaction) to determine that portion of the incidents that can be prevented given the reaction time pdfs and surveillance uncertainty parameters. Referring to Figure 10, entering at 9.99 seconds we find the cdf is at approximately 0.21 or 21%.

In this case, an aircraft taxiing at 10 kts that will brake at .33 g's will receive an alert from the controller in time to stop short of the wing tip of a B-747 approximately 21% of the time based on the assumed reaction time pdfs. If we assume the worst-case scenario involving a B-747, we keep the braking deceleration at .33 g's, and we keep the individual reaction time pdfs constant, then the three variables that will effect this result are the two surveillance performance parameters, update interval and positional uncertainty, and the taxi velocity. Any change in the surveillance performance parameters will change the detection pdf and therefore the total time required cumulative distribution. Any change in the taxi velocity will also affect this distribution as well as the braking time and distance.

3.2.2 Case 1-Full Speed Taxi Towards Boeing 747

The analysis presented above for the example case of a taxi speed of 10 knots, a positional uncertainty σ_s of 50 feet, and an update interval $\Delta\tau$ of 2 seconds can be replicated for various taxi speeds, positional uncertainties, and update intervals. The deceleration rate of .33 g's, the distance from the taxi-hold position to the center of the runway of 280 feet, and the wingspan of the B-747 on the runway (195.7 feet) are held constant.

The results presented are from repeated calculations performed as shown above for taxi speeds of 5, 10, 15 knots, for positional uncertainties σ_s from 0 to 50 feet, and for update intervals $\Delta\tau$ from 0 to 10 seconds.

Figures 11-13 are the results, plotted as probability of detection in time to prevent the accident, as a function of positional uncertainty and update interval. Surface plots and contour plots are provided for taxi speeds of 5 kts, 10 kts, and 15 kts.

The three surface plots are overlaid in Figure 14 to compare the effects of changing taxi speed. The grid scales have been moved for clarity; the scales are available in the individual figures above. Clearly, taxi speed in this scenario has a large effect on the probability of getting an alert to the pilot in time to prevent a collision.

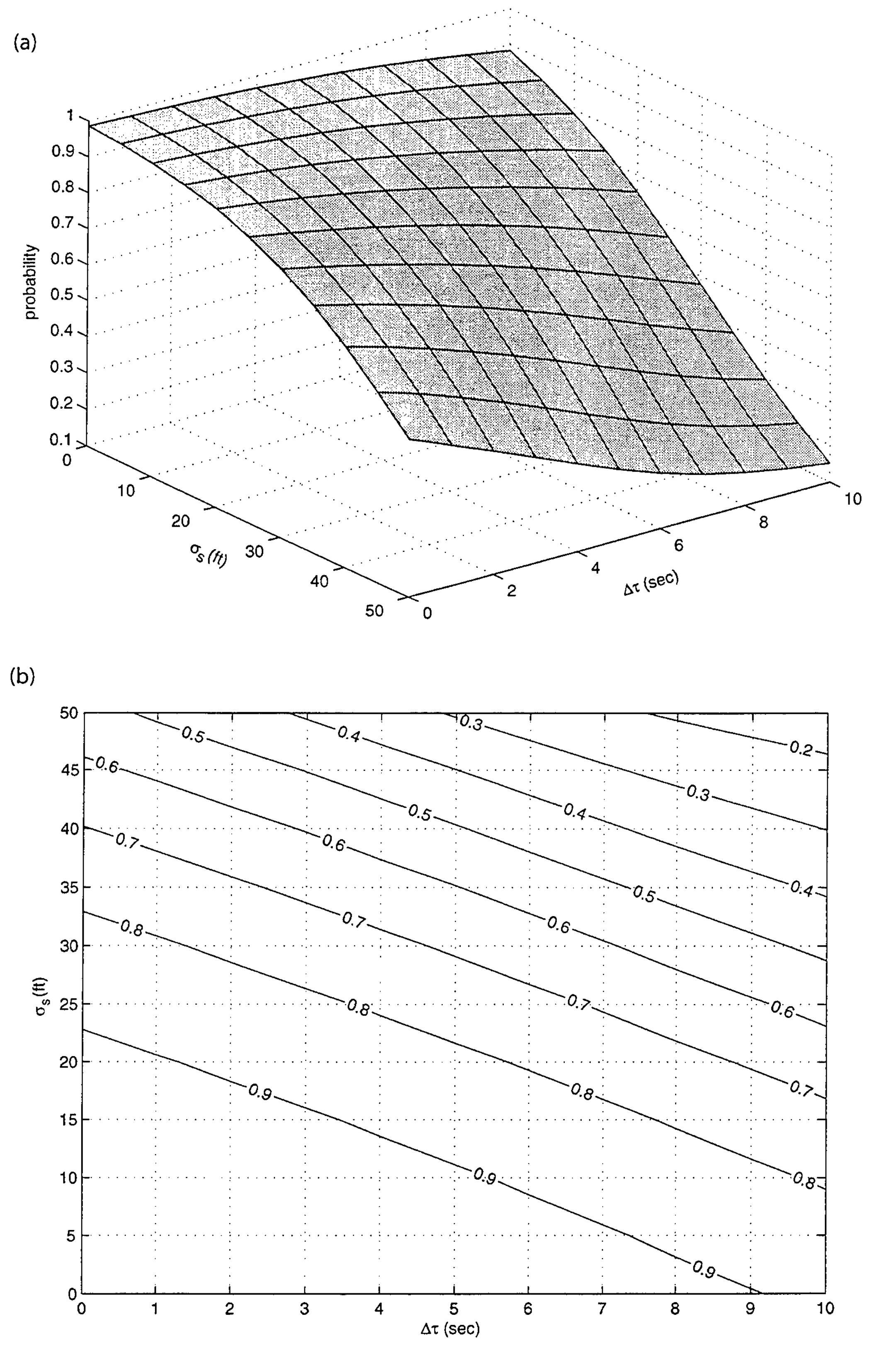


Figure 11. Save probability surface (a) and contour (b) plots for a taxi speed of 5 kts, B-747 on runway.

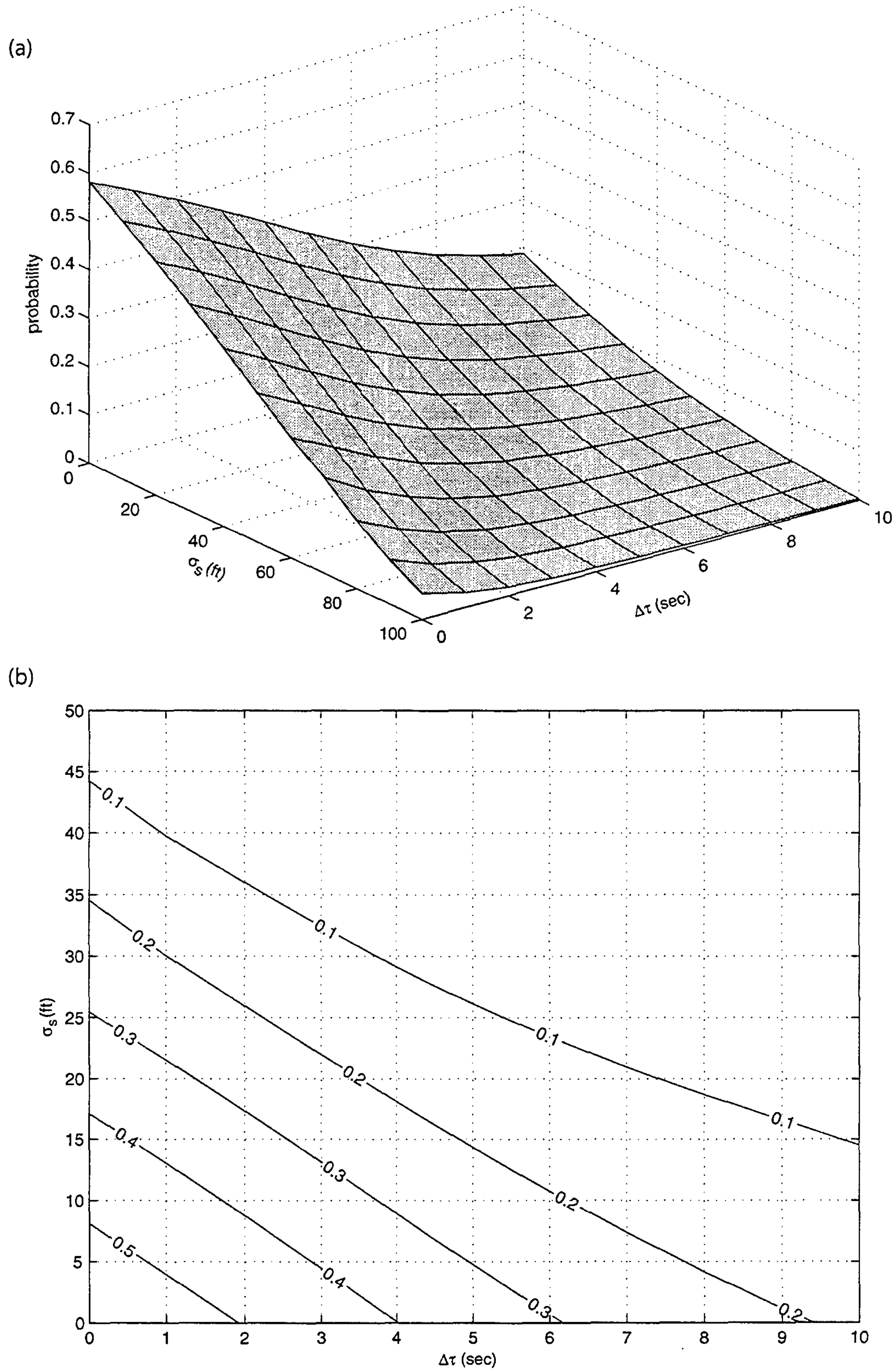


Figure 12. Save probability surface (a) and contour (b) plots for a taxi speed of 10 kts B-747 on runway.

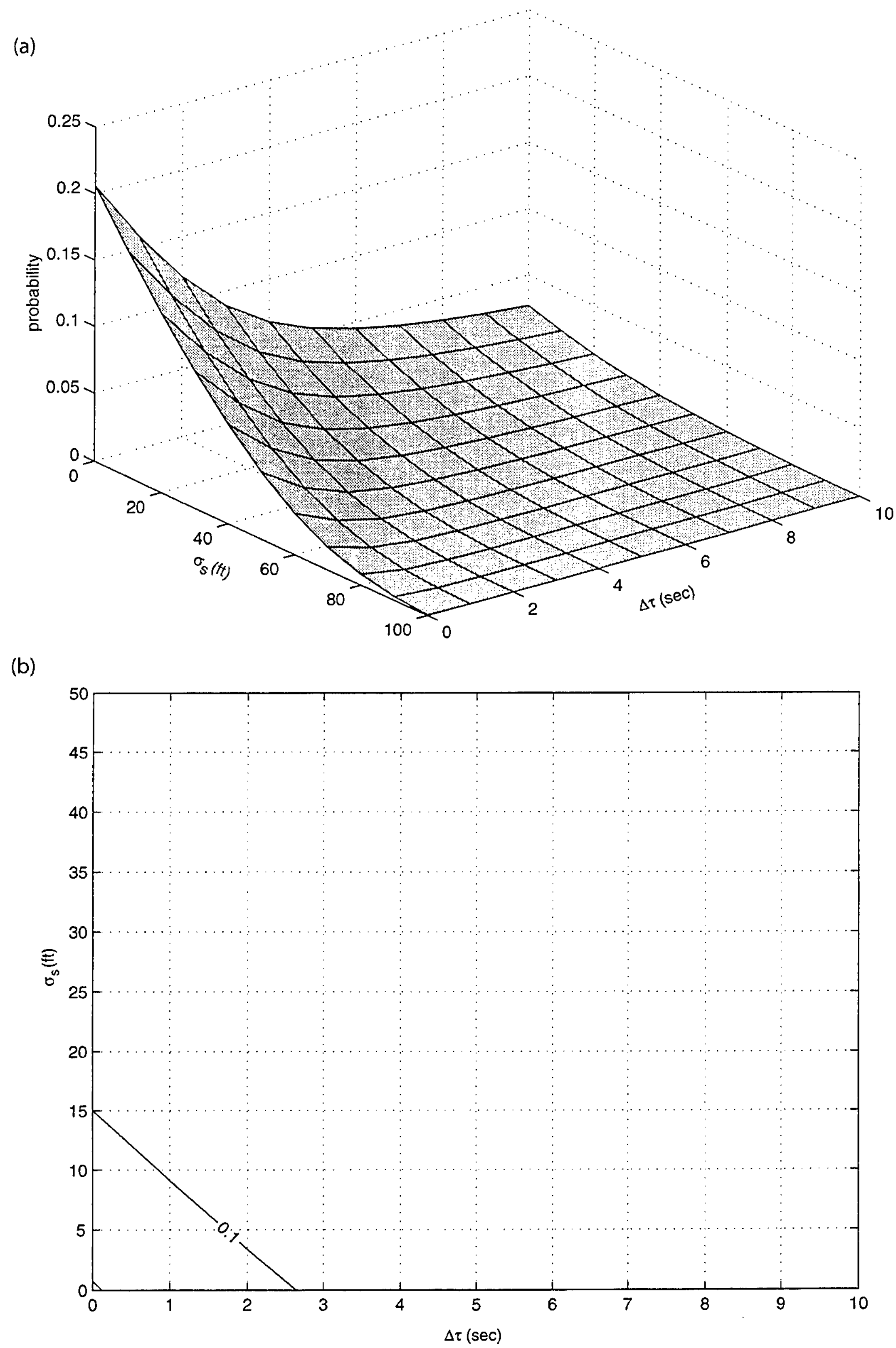
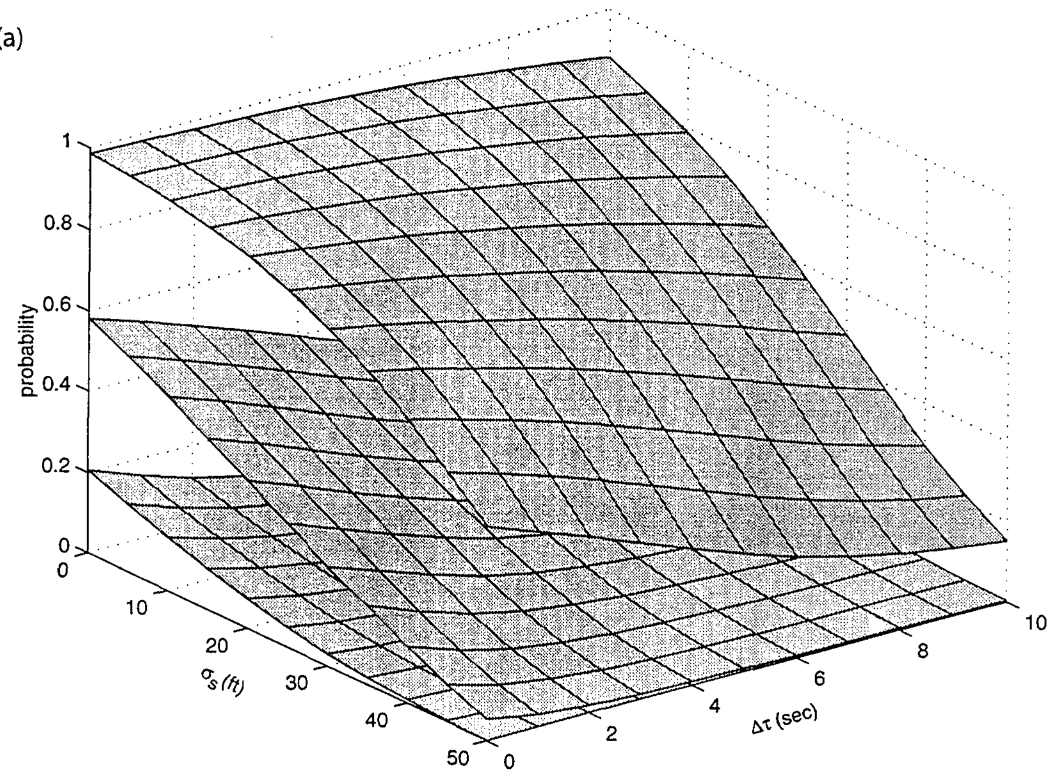


Figure 13. Save probability surface (a) and contour (b) plots for a taxi speed of 15 kts, B-747 on runway.

(a)



(b)

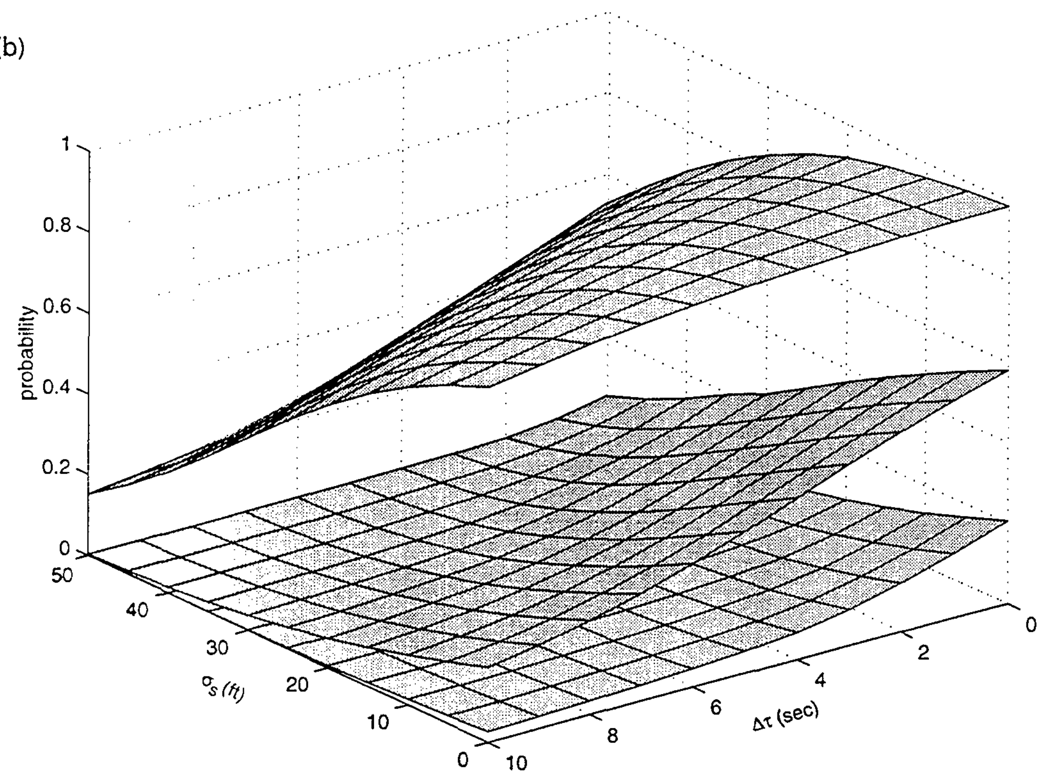


Figure 14. Two different axes views of save probability plots for a taxi speed of 5, 10, and 15 kts, B-747 on runway.

Reviewing the results presented in Figures 11-14 leads to the conclusion that no surveillance performance criteria will be effective in preventing this case study of this category of incursion. This conclusion is valid over a range of expected taxi speeds given the reaction time pdfs assumed for this exercise. About the only quantitative conclusion that can be drawn is that the surveillance effectiveness seems to fall off rapidly once positional uncertainty σ_s goes above 10 feet for low taxi speeds (5 kts or less). At higher taxi speeds, (10 kts or above), the save probability is lower than 50% for zero positional uncertainty. These results are partially the consequence of choosing a challenging version of this scenario, i.e., full speed taxi and the wingspan of a Boeing 747. Given the case that this version of the scenario cannot be prevented by tower cab alerts, the next step is to gain some insight into the relative sensitivity of the surveillance performance parameters in preventing a less challenging version of this scenario. This is presented in the following section.

3.2.3 Case 2-Stopped Taxi Towards Boeing 727

The approach taken in this section is to choose a less challenging taxi profile and a Boeing 727 wingspan. The next step is to map out the update interval and positional uncertainty requirements (tradeoffs) necessary to save given percentages of the cases assuming the same reaction time pdfs used above. As before in Section 3.2.2, convolving the four reaction pdfs produces the same cumulative distribution function represented shown again as Figure 15.

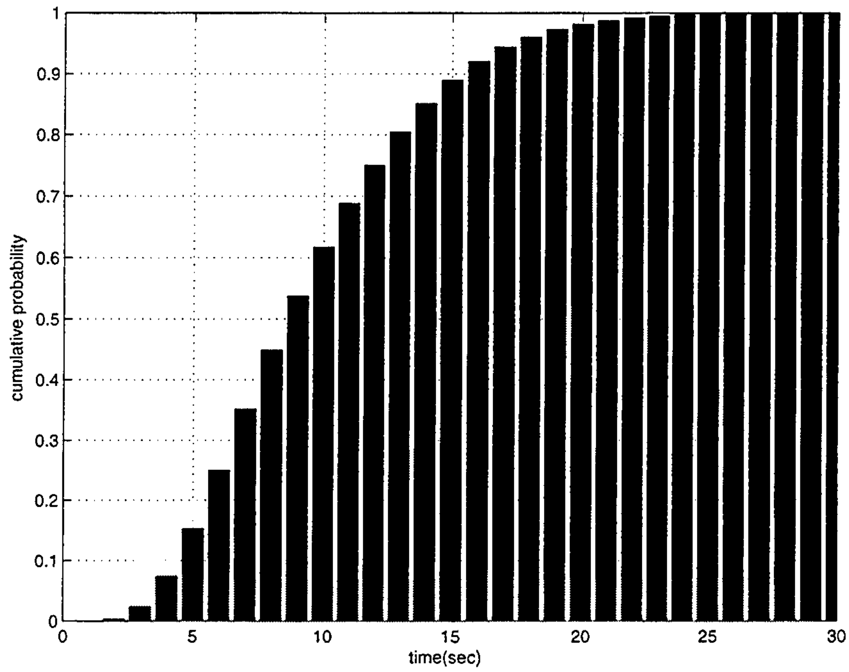


Figure 15. Cumulative distribution function for total reaction time.

The values used for the nominal taxi profile are based on observations of several aircraft that were tracked with a surface radar at Boston Logan. The profile consists of an acceleration at .09 g's to a maximum velocity of 10 knots. Maximum braking is assumed to be .33 g's. The distance from the taxi-hold position to the center of the runway is 280 feet and the takeoff aircraft is assumed to be a Boeing 727 with a wingspan of 108 feet. The taxi velocity and distance profiles for the assumed nominal taxi are shown in Figures 16 and 17 which assume a maximum braking effort that results in a stop of the taxi aircraft's nose just short of the wing tip of the Boeing 727. Also shown in Figures 16 and 17 is that the braking must begin at 15.51 seconds after the start of taxi in order to stop short of the B-727 wing tip. This is the time available and is the time compared with the cumulative distribution function for total time required in order to determine the probability that the collision will be prevented.

Figure 18 is an enlargement of the early part of taxi shown in Figure 17 but with the axis inverted. This is used to determine the minimum time required for detection depending on the uncertainty in position. Entering the x axis at the taxi distance equivalent to the positional uncertainty $2\sigma_s$, the time is read off of the y axis. As before, this is used to create a surveillance detection probability density function. For instance, if $2\sigma_s$ is 50 feet and the update interval is 2 seconds, the probability for detection is uniform between the minimum and maximum times. The minimum time, as shown in Figure 18 is 5.876 seconds. The maximum time in this case is 7.876 seconds.

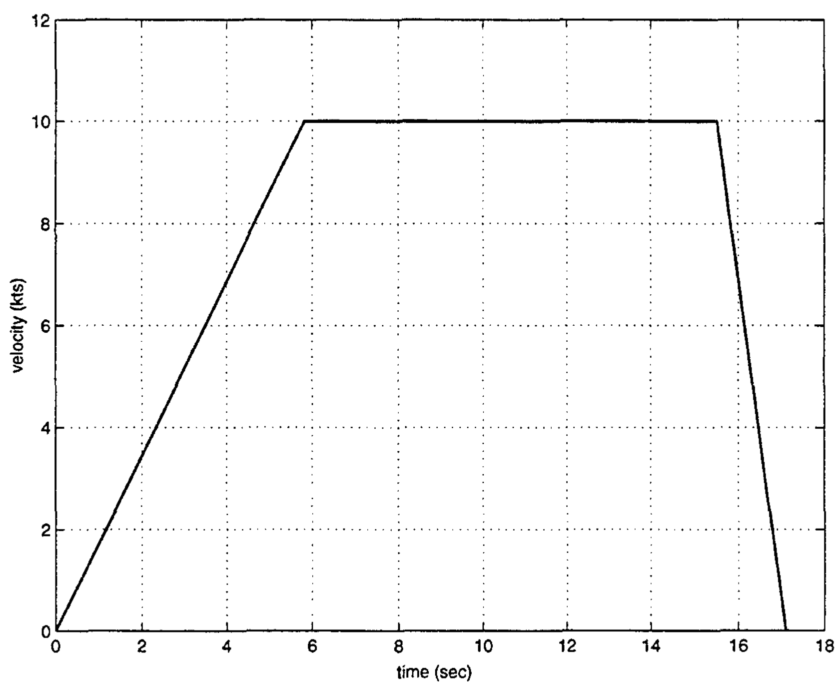


Figure 16. Taxi velocity profile for nominal taxi.

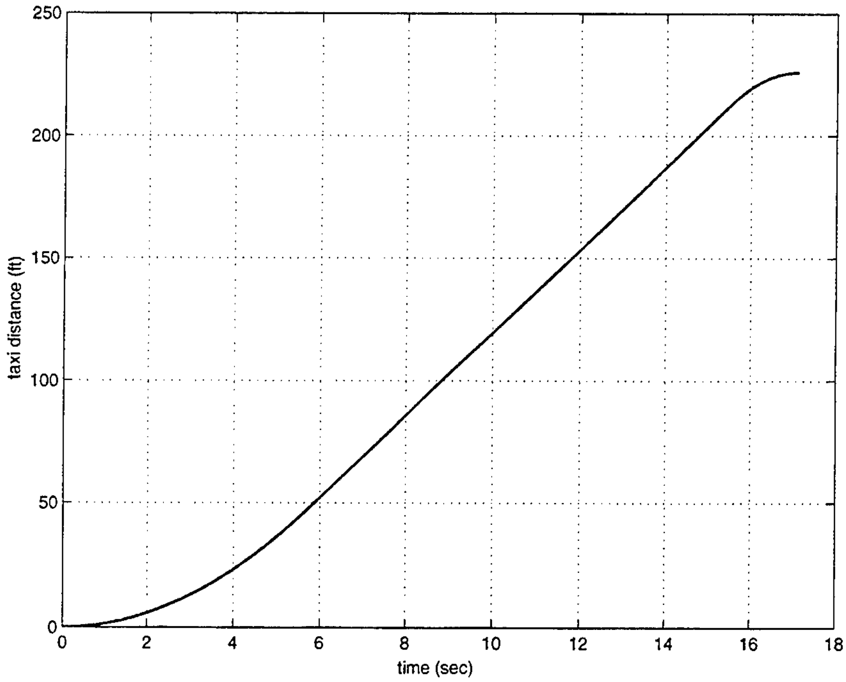


Figure 17. Taxi distance profile for nominal taxi.

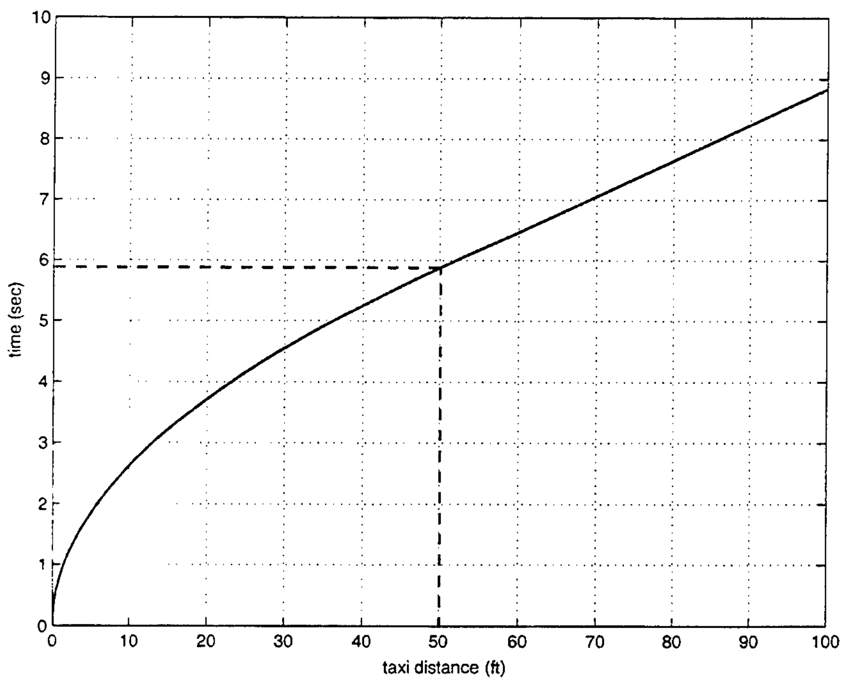


Figure 18. Taxi time profile for nominal taxi.

This produces a surveillance detection pdf illustrated in Figure 19. When this is convolved with the cumulative reaction time distribution illustrated in Figure 15, a cumulative distribution function for the total time required to begin evasive action results shown in Figure 20 for this example.

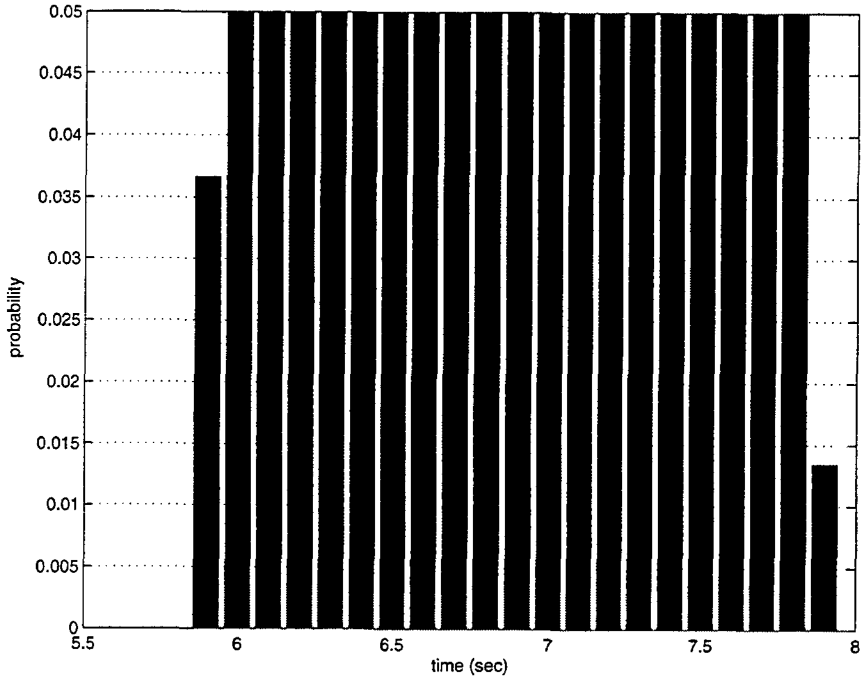


Figure 19. Surveillance detection time pdf.

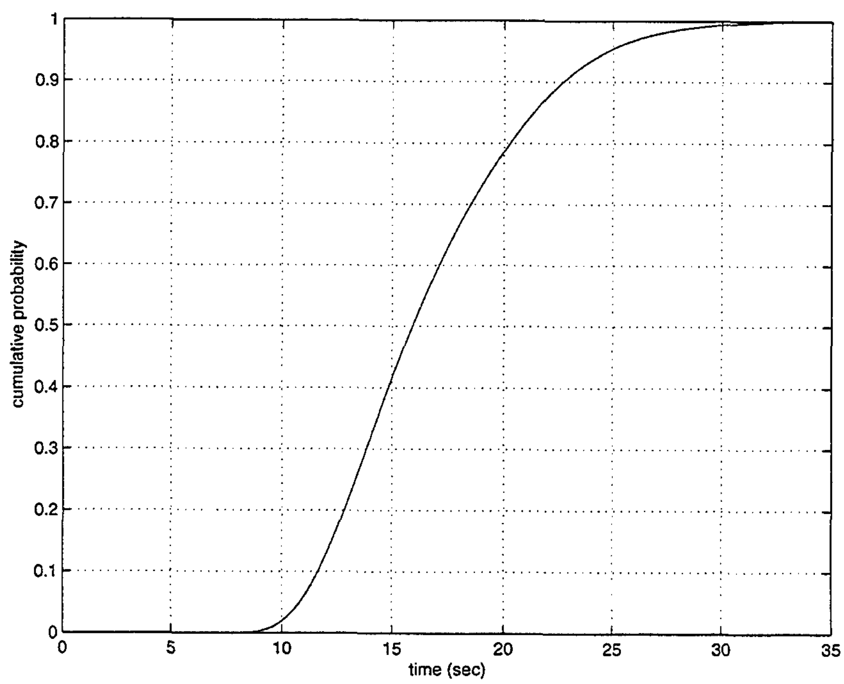


Figure 20. CDF for total time required.

From this point on, the analysis is the same as in Section 3.2.2 above except that the taxi velocity is not varied. The results of this analysis are the probability of accident prevention as a function of update interval and positional uncertainty for the nominal taxi profile. The results are plotted in Figure 21a and b which is a surface map of the probability of accident prevention with the other two axis being update interval and positional uncertainty and a contour plot of the same data.

Again, even for this "easier" nominal taxi profile, the results are disappointing in that there is no range where all events can be prevented. The reason the results are not significantly more encouraging for the stopped taxi case is that even though it takes longer for the taxiing aircraft to reach the danger region, most of that extra time is during the start of taxi and it therefore takes correspondingly longer to detect that the aircraft has taxied a distance corresponding to $2\sigma_S$. By referring to the cumulative distribution for reaction time in Figure 15, we can see that even with "perfect" surveillance of 0 feet positional uncertainty and instantaneous update interval, the probability of the collision being prevented with a maximum time available in this nominal taxi profile of 15.51 seconds is about 90%. This, of course is a consequence of the convolved reaction time pdfs. The tower cab alert system requires the controller to regain situational awareness, resolve the conflict, gain access to a voice channel, and deliver the correct command to the correct aircraft.

3.2.4 Results for Tower Cab Alerts

For a full speed taxi across the taxi-hold position towards a Boeing 747 the tower cab alerting system can only protect aircraft taxiing at speeds of about 5 knots or less. The positional uncertainty requirement is very sensitive to taxi speed. At a speed of 5 knots, the positional uncertainty must be near zero. The update interval is relatively less important with an interval of 2 seconds deemed sufficient.

For the case of a taxi from a stop towards a Boeing 727, the tower cab alert system can protect 80% of the cases with a $2\sigma_s$ of 5 feet, 77% with 10 feet, and 68% with 20 feet. This is at an update interval of 1 second. There is some sensitivity to update interval. At an update interval of 2 seconds, the corresponding $2\sigma_s$ requirements are: 80% with a $2\sigma_s$ of 4 feet, 70% with 20 feet, and 60% with 26 feet.

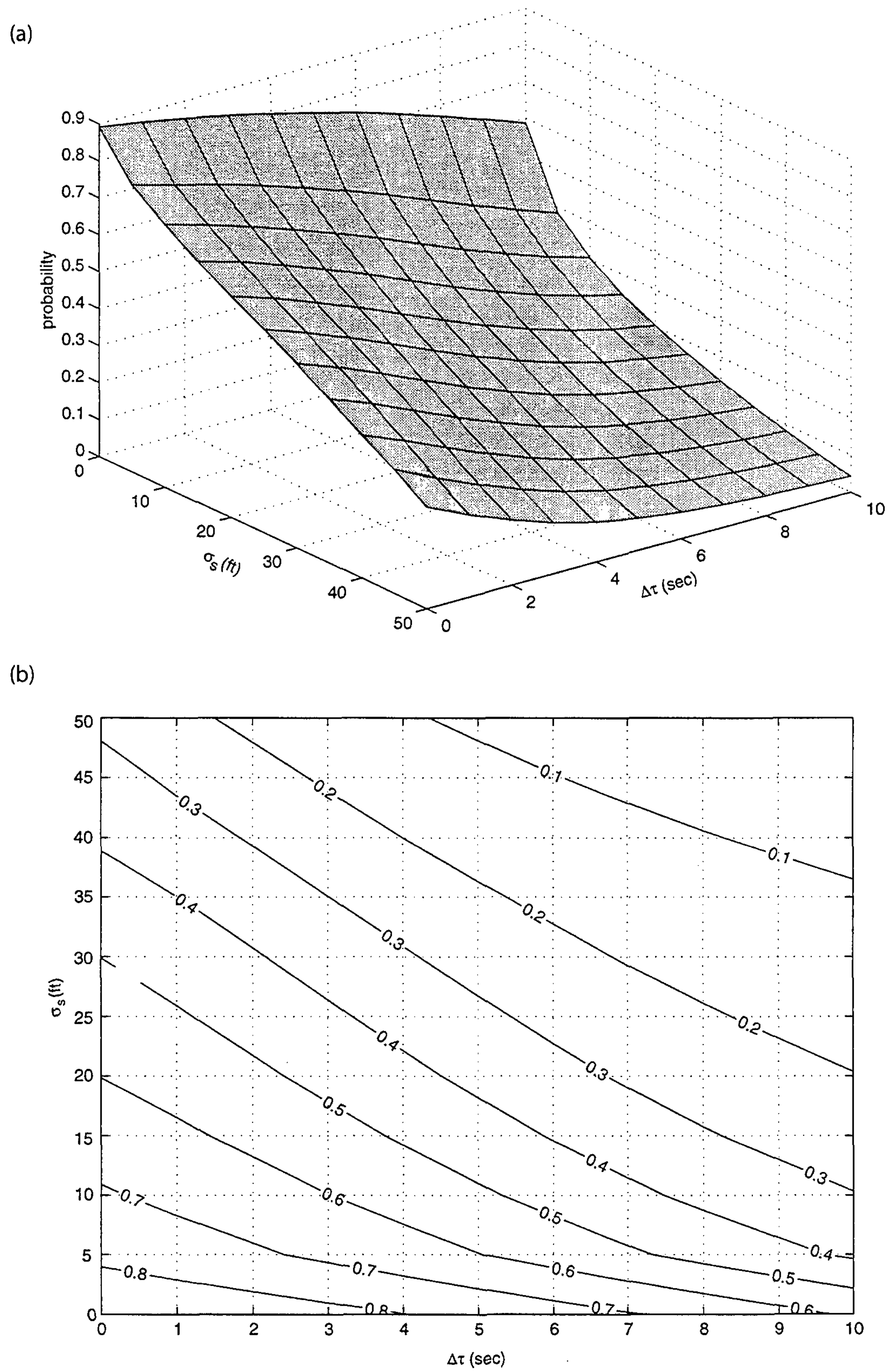


Figure 21. Save probability surface for surface for nominal taxi profile (a) and contour plot of save probabilities for nominal traffic profile (b).

3.3 ANALYSIS FOR DIRECT COCKPIT ALERTS

3.3.1 Case 1-Full Speed Taxi Towards Boeing 747

The effect of having the alert go directly to the cockpit is to remove three of the four reaction time pdfs used in the analysis above leaving only the pilot reaction time. When this is convolved with the surveillance detection pdf, the resulting cumulative distribution for total time required is used as before to ascertain the probability of preventing a collision by using the time at which braking must occur to enter the cdf. In the case of the full speed taxi, this value depends on the taxi velocity; in the case of the nominal taxi profile, this is 15.51 seconds. In any case the cdf resulting from the pilot reaction time alone convolved with the surveillance detection vector will produce a greatly increased cdf and should result in a larger portion of the surveillance parameters resulting in saves.

The approach is simply to repeat the analysis done in Section 3.2 for the new pdfs and cdfs. Figures 22 through 25 present these results and can be compared with Figures 11 through 14 for the tower cab alerting results. From these results, taxi velocities of up to 15 knots can be handled with positional uncertainties of 25 feet and update intervals of 1 or 2 seconds.

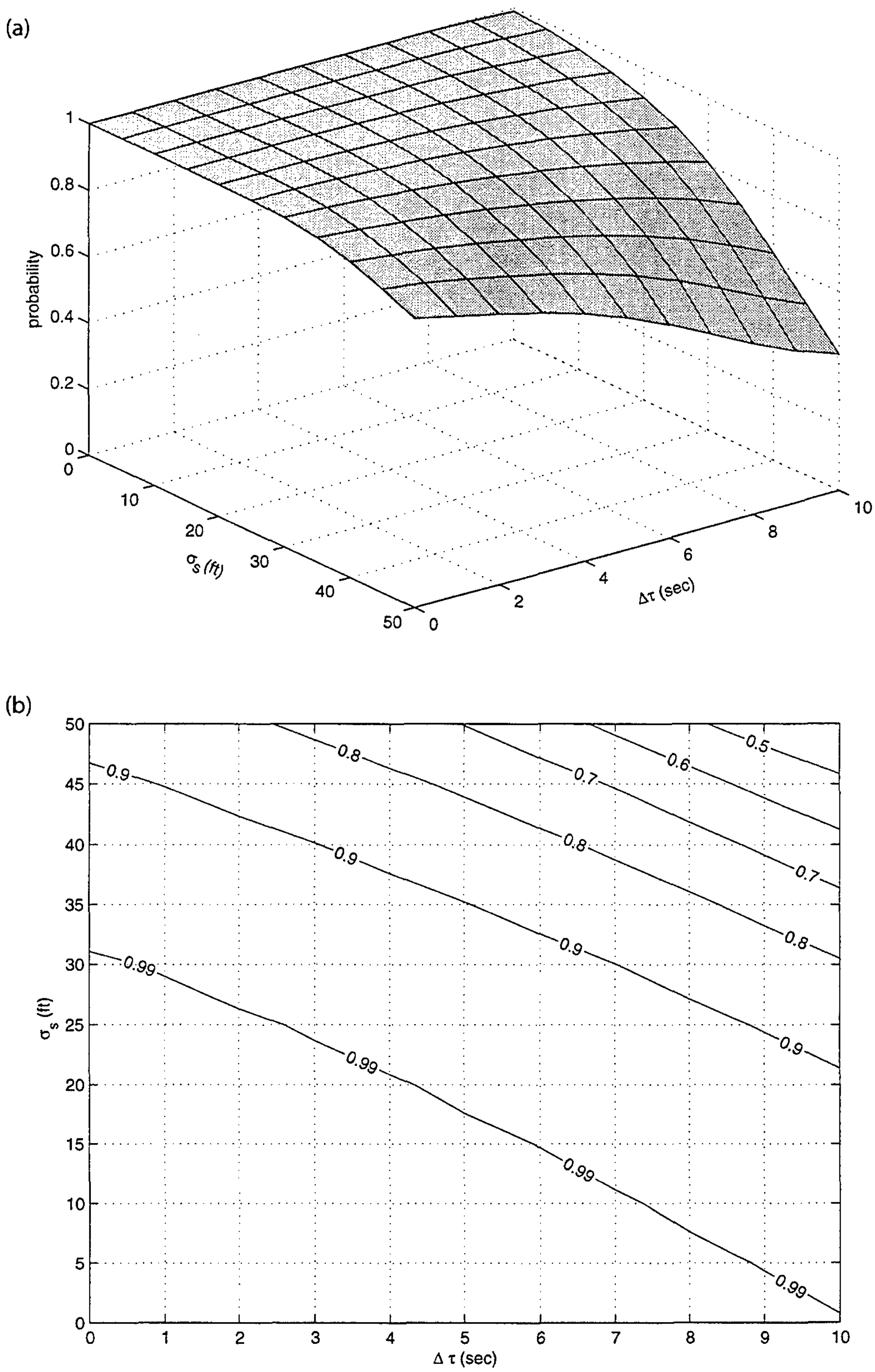


Figure 22. Save probability surface for taxi, 5 kts, direct clock alert with B-747 on runway.

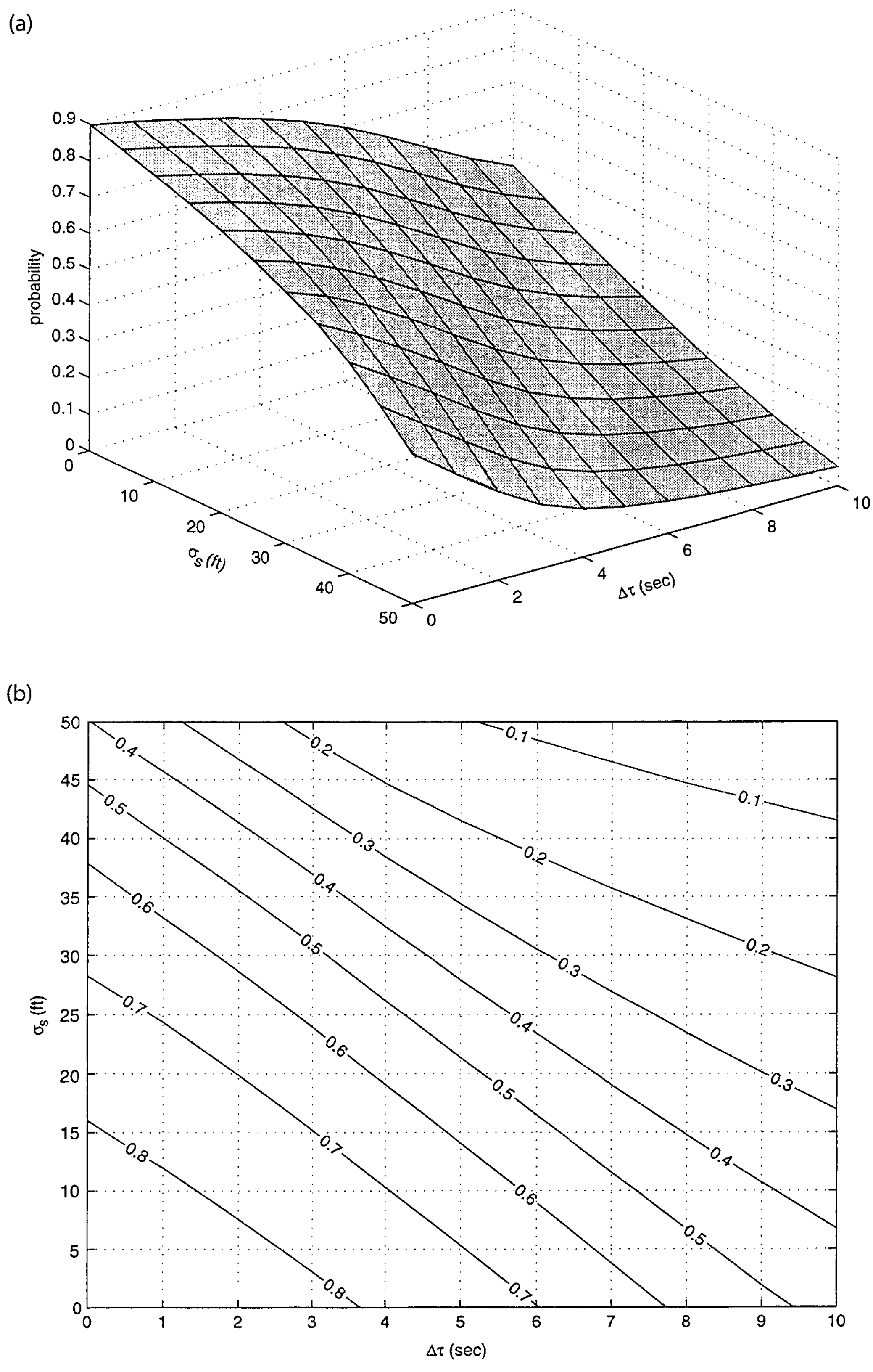


Figure 23. Save probability surface for taxi, 10 kts, direct cockpit alert with B-747 on runway.

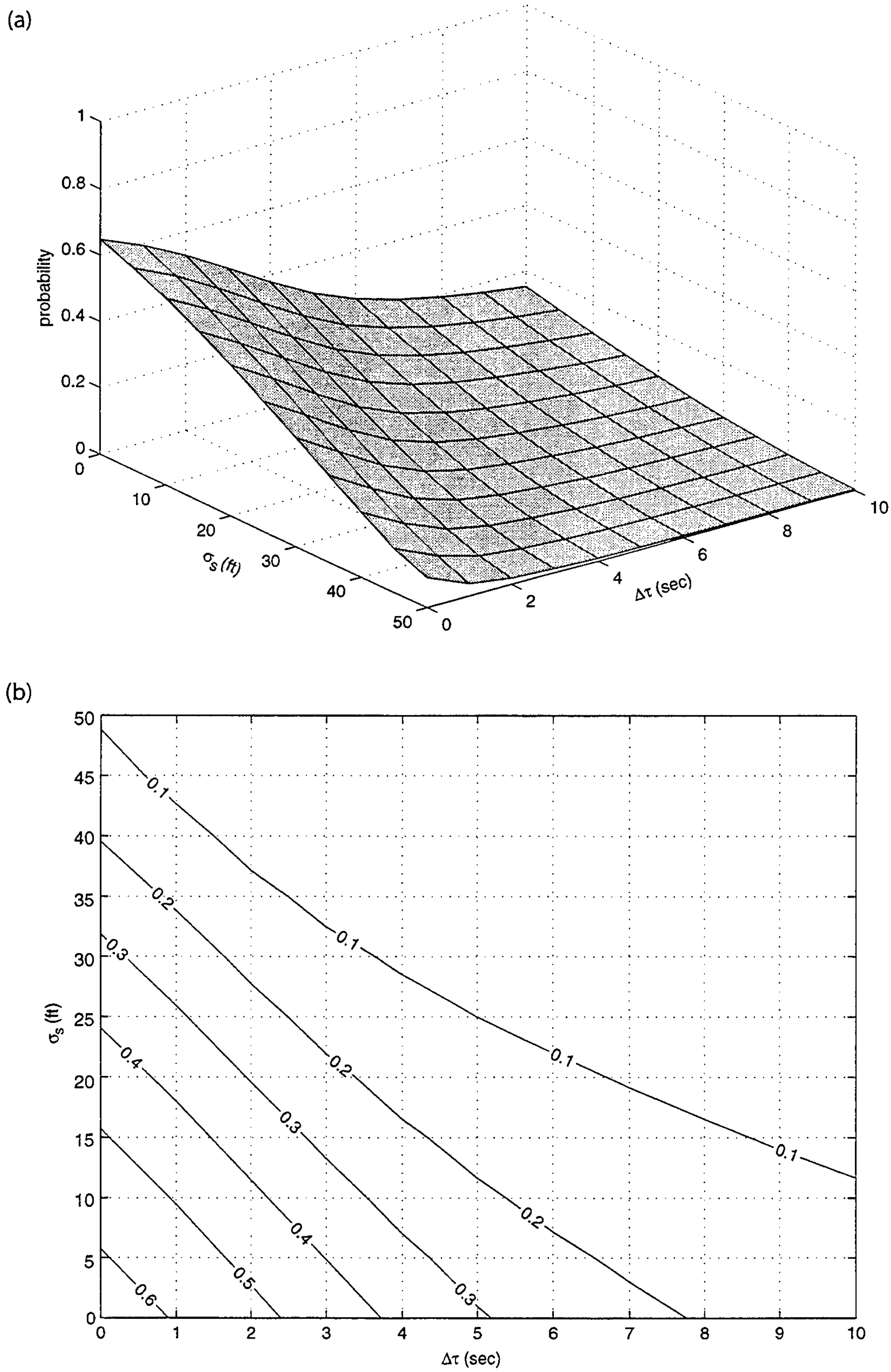


Figure 24. Save probability surface for taxi, 15 kts, direct cockpit alert with B-747 on runway.

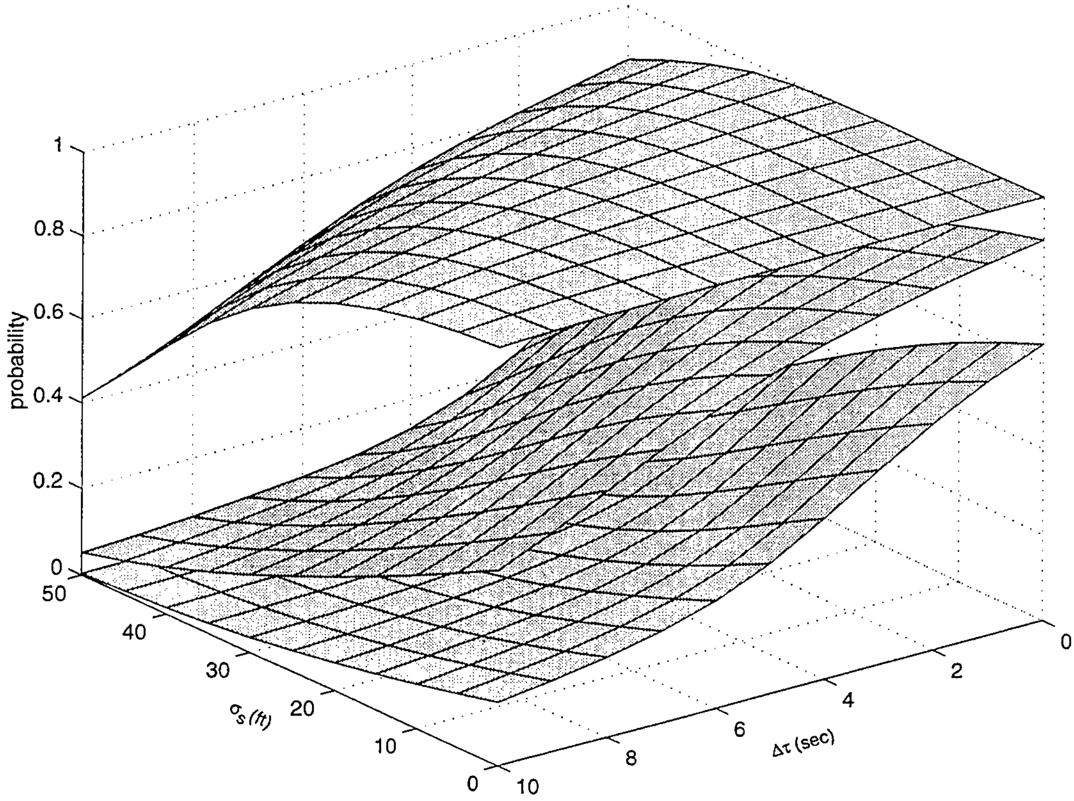


Figure 25. Save probability surface for taxi, 5, 10, and 15 kts, direct cockpit alert with B-747 on runway.

3.3.2 Case 2-Stopped Taxi Towards Boeing 727

Again, the analysis of Section 3.3.1 is repeated except with the nominal taxi profile from a stop towards a Boeing 727. Figure 26 present the results corresponding to Figure 21 for the tower cab alert analysis in Section 3.2.3. From these figures it appears that σ_s positional uncertainties of up to 15 feet with update intervals as slow as 2 seconds will be sufficient with over a 90% confidence in prevent accidents resulting from the more commonly expected nominal taxi profile if a system employing direct cockpit alerts is used. An update interval of 1 second would allow a positional uncertainty of up to 18 feet at the 90% confidence level.

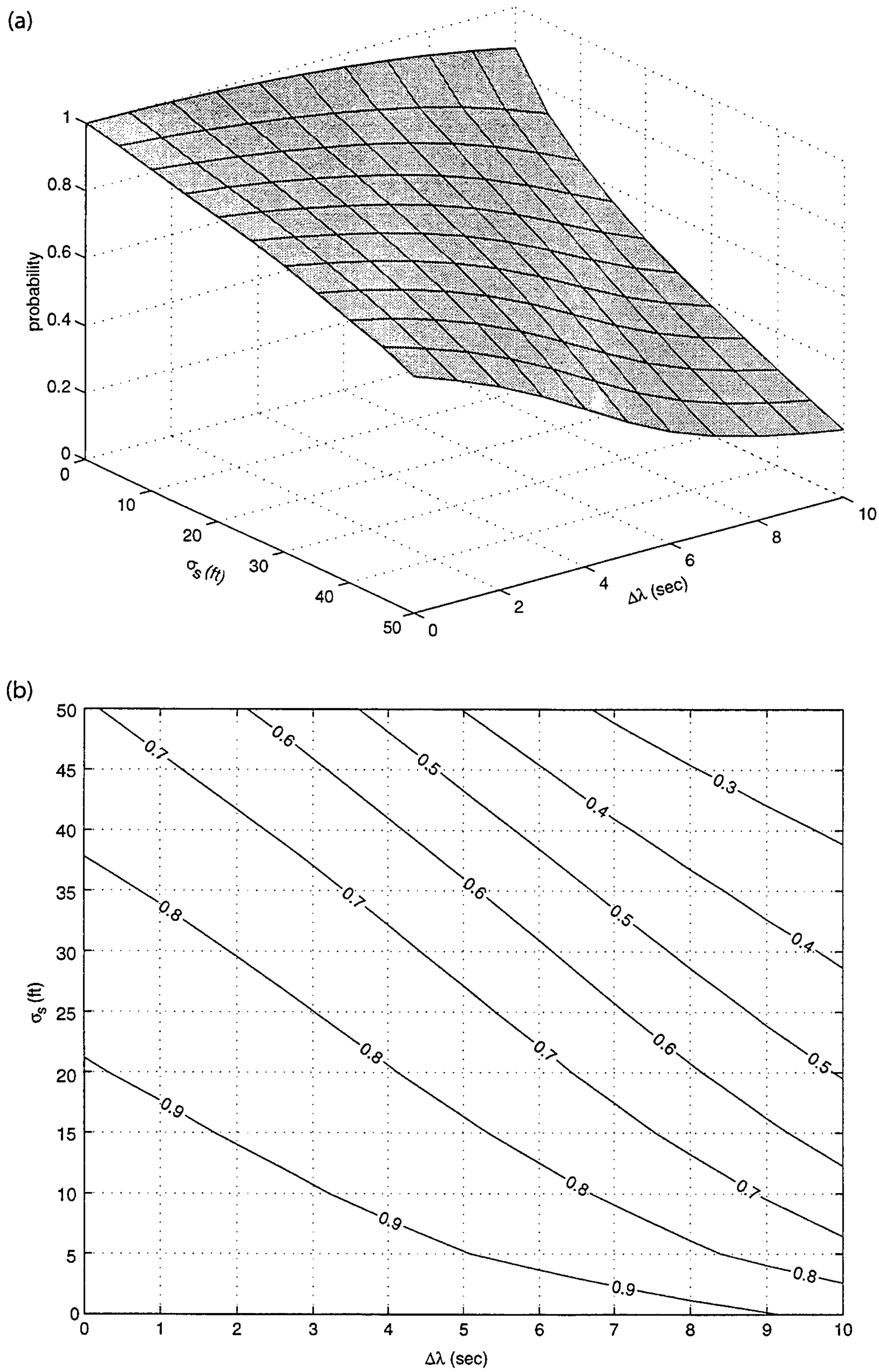


Figure 26. Save probability surface for nominal taxi profile, direct cockpit alerts with B-727 on runway.

3.3.3 Results for Direct Cockpit Alerts

The direct cockpit alerting system can protect the runway from the full speed taxi towards a Boeing 747 for this category of incursion at taxi speeds of up to 5 knots with a surveillance Positional uncertainty σ_s of 15 feet and an update interval of 2 seconds. A σ_s = 12 feet with a $\Delta\tau$ = 1 second will protect the 10 knot taxi with an 80% confidence level.

For the case of a taxi from a stop towards a Boeing 727 the positional uncertainty required is 17 feet or less at a 1-second update interval and a positional uncertainty of 15 feet or less at a 2-second update interval in order to achieve a 90% confidence level.

3.4 ANALYSIS FOR RUNWAY ENTRANCE LIGHTS

3.4.1 Collision Region

An analysis of the two cases of this category of incursion described above results in the status light system being effective in preventing an incursion in both cases as long as the surveillance system can detect a hot runway. The surveillance of the taxi aircraft motion that was analyzed above for the alerting safety systems is not necessary for the runway entrance light safety system. This is because when the runway is detected as hot, the runway entrance lights will be red and the taxi aircraft will see the lights before entering the runway.

In order to derive surveillance requirements, the relative timing of the motion of the aircraft on the runway and the taxi aircraft must be introduced. This results in a much more detailed analysis. Because the goal is to derive surface surveillance requirements, it is assumed that the runway is hot because of a departure. An arrival will be detected by terminal beacon radar. The geometry of the case is described in Section 3.4.1.1 and the motion of the aircraft described in Section 3.4.1.2.

The concept of a collision region is introduced in Section 3.4.1.3. This is the combination of relative taxi and takeoff motion times and distances down the runway that result in a collision if the taxi aircraft taxis across the runway. Areas outside of the collision region represent timing and distance situations where the taxi aircraft would taxi safely across the runway ahead of or behind the departing aircraft on the runway.

In order to understand the surveillance requirements for automatic status lights (runway entrance lights in this case), three representative cases are studied. The thrust of the analysis is that the surveillance system must be able to detect that the runway is hot (that the aircraft in position to depart is indeed departing) in time to stop the taxi aircraft by turning the entrance lights red. This must be accomplished while the taxi aircraft can still see the lights and still has the distance required to brake.

3.4.1.1 Runway Taxiway Geometry

The runway/taxiway geometry used in the analysis is illustrated in Figure 27. The length and span of the taxi and takeoff aircraft are specified parameters. The distance d_{taxi} refers to the distance from the taxi-hold position to the runway centerline which is nominally 280 feet but increases slightly with airport elevation. In some cases the hold line may be further back, but 280 feet was used throughout this analysis. The distance $d_{effective}$ refers to the distance over which the runway entrance lights are still effective. The placement of the status lights is specified with d_{lights} and d_{offset}, and the effectiveness angle α_{max}. $d_{effective}$ is computed from d_{lights} , d_{offset} and α_{max}. If the runway entrance lights are installed across the taxiway, the lights are effective up to the point where the taxi aircraft crosses them. d_{in} and d_{out} are the taxi distances into and out of the danger zone and are computed from the specified parameters.

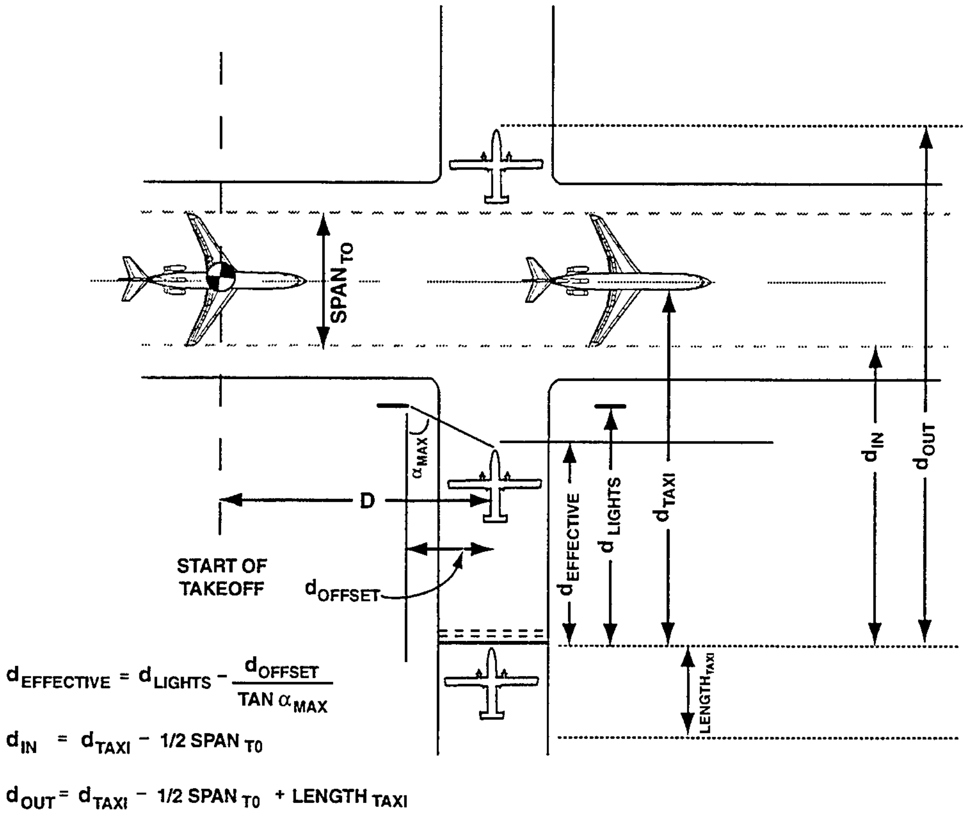


Figure 27. Runway/taxiway geometry for the intersection taxi-takeoff conflict.

3.4.1.2 Aircraft Motion

The parameters required to define the aircraft motion are the takeoff acceleration (a_{to}) and takeoff distance ($d_{takeoff}$) and the taxi aircraft acceleration (a_{taxi}), braking (b_{taxi}), and taxi velocity (v_{taxi}). The aircraft motion profiles are computed from these input parameters. First the maximum taxi velocity achievable is calculated assuming a constant acceleration at a_{taxi} up to the point where a constant braking at b_{taxi} is required to stop just short of the danger zone (d_{in}). This is compared to the specified v_{taxi} to see if the taxi aircraft ever reaches taxi velocity. v_{max} is defined as the greater of v_{taxi} or the velocity achieved with a constant acceleration and braking to stop short of the danger zone. Using v_{max}, the time and distance required for acceleration and braking and the time and distance at taxi velocity are calculated. This assumes that the taxi aircraft will brake to stop just short of the danger zone. The time after the start of taxi at which braking must begin in order to stop short of the danger zone is computed as τ_b. Finally, the times that the taxi aircraft will pass the point where the status lights are effective (which may be before τ_b) and the times that the aircraft will enter, τ_{in} and exit, τ_{out}, the danger zone (assuming no braking) are calculated.

Figures 28-34 are representative plots for the aircraft motion. These plots use the values for the nominal case (Case 1). The motion of the aircraft is plotted with and without braking. The solid line indicates motion with no braking and the dashed line motion with braking required to stop short of the danger zone.

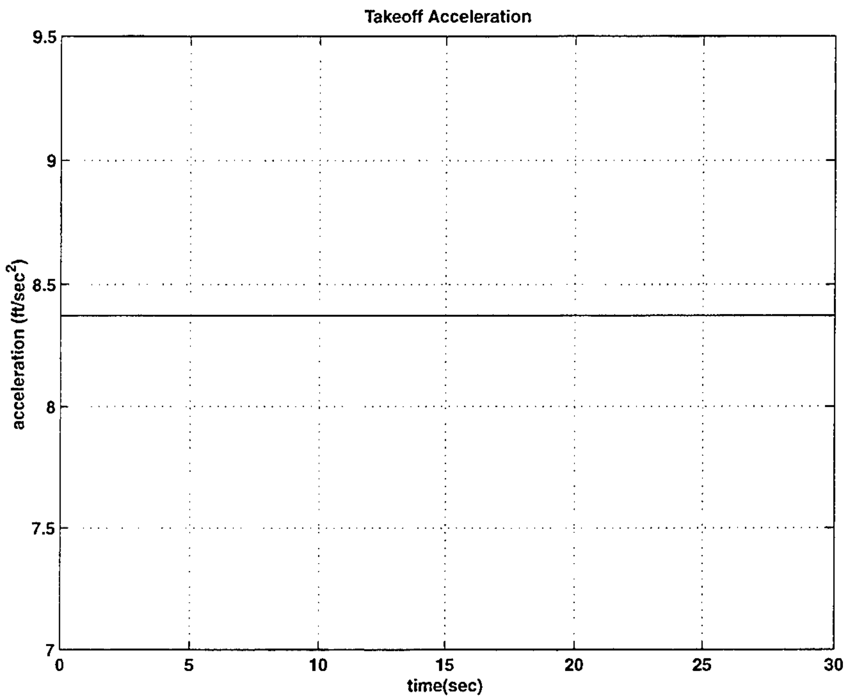


Figure 28. Nominal takeoff aircraft acceleration.

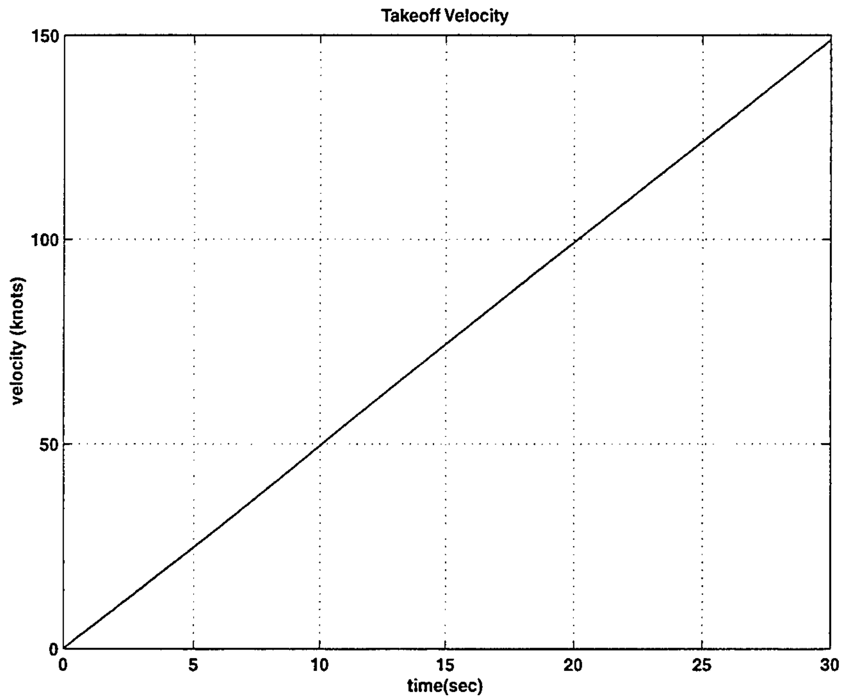


Figure 29. Nominal takeoff aircraft velocity.

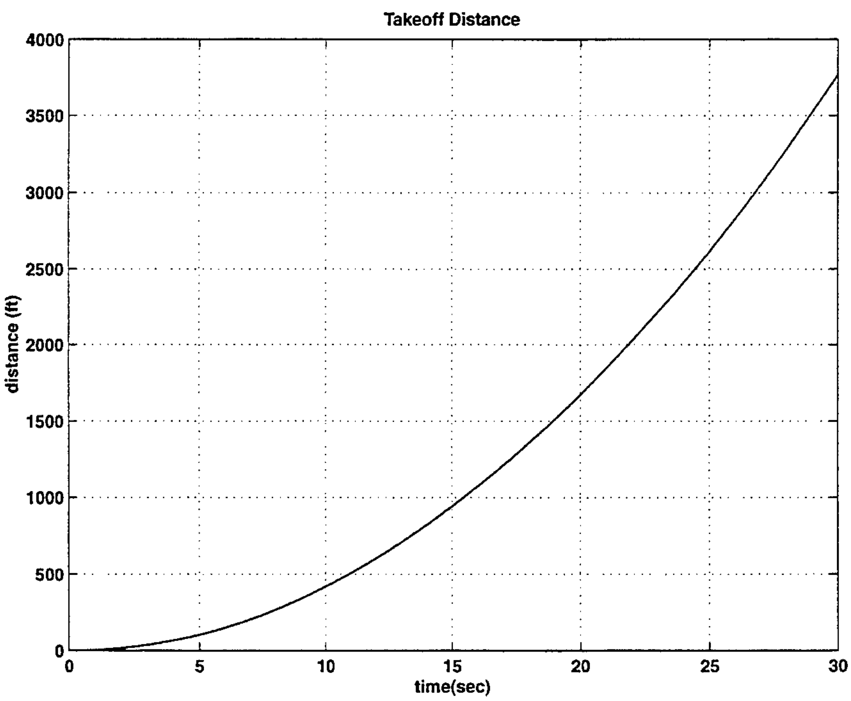


Figure 30. Nominal takeoff aircraft takeoff distance.

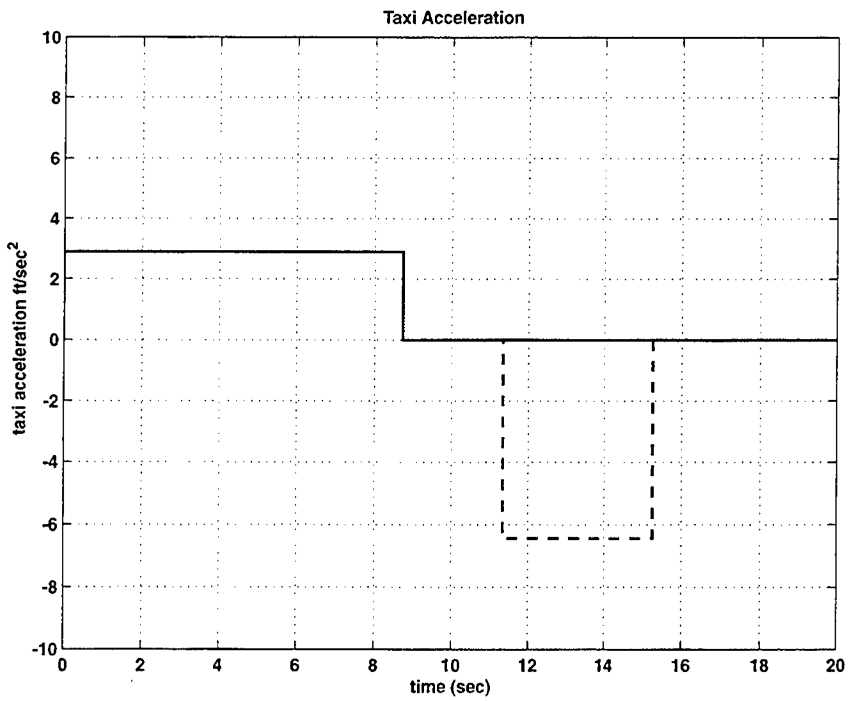


Figure 31. Nominal taxi aircraft acceleration.

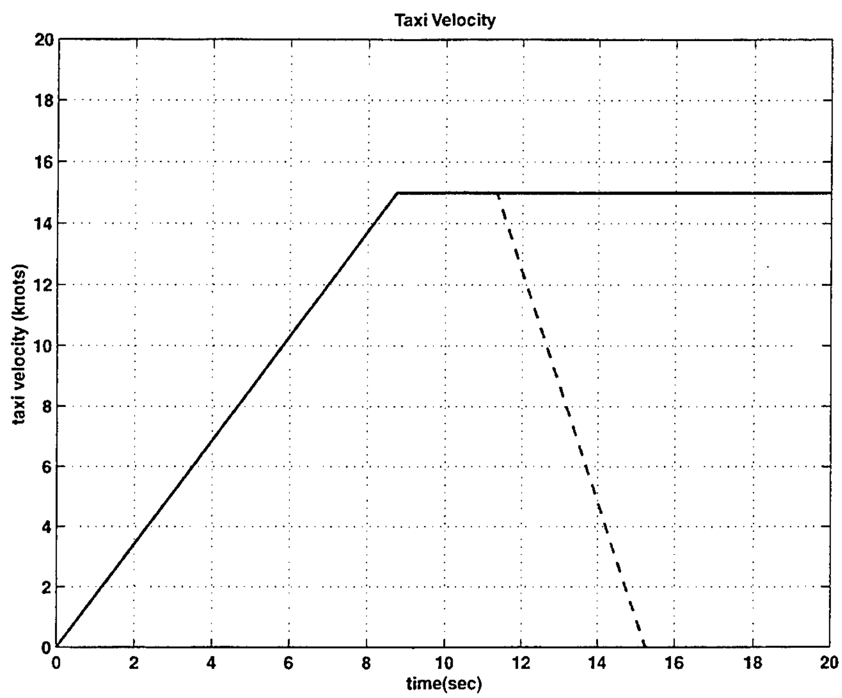


Figure 32. Nominal taxi aircraft velocity.

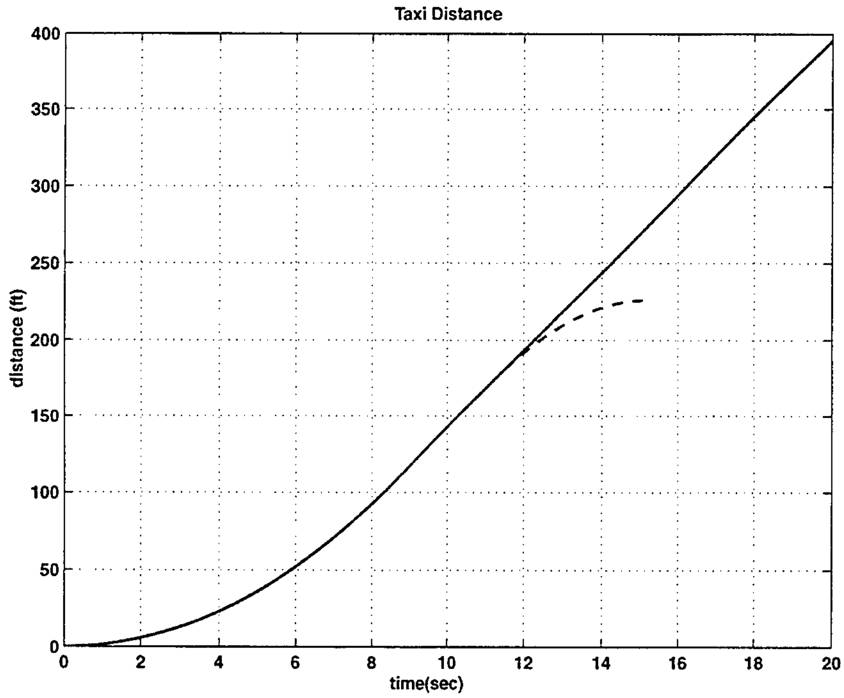


Figure 33. Nominal taxi aircraft taxi distance.

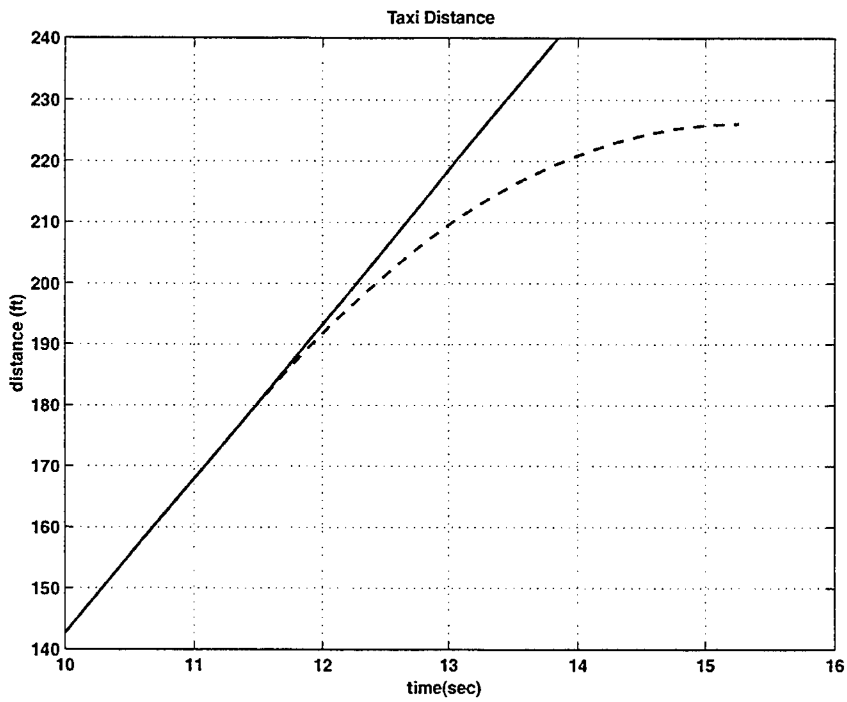


Figure 34. Nominal taxi aircraft taxi braking.

3.4.1.3 Collision Region

From the motions of the taxi aircraft and takeoff aircraft and the times that the taxi aircraft will take to enter and exit the danger zone without braking we can compute a potential collision region. This region is a function of the distance **D** of the taxiway from the start of the takeoff run (see Figure 27) and the relative timing between the start of the takeoff aircraft's motion and the start of the taxi aircraft's motion. We define **Δt** as the difference between the time the takeoff aircraft starts its motion and the time the taxi aircraft starts to move (positive **Δt** means the taxi starts after the takeoff). We then can plot the region in which a collision would occur without evasive action. In this section we are only considering braking of the taxi aircraft as the possible evasive action but in later sections we will examine the effectiveness of the takeoff-hold lights in stopping the takeoff aircraft. Knowledge of the collision region is important because it is this combination of distances and times over which the incursion prevention system must be effective. The collision region for the nominal case (Case 1) is illustrated in Figure 35. The interpretation is that the lower line represents the locus of **Δt** and **D** combinations for which the taxi aircraft exits the danger zone before the takeoff aircraft arrives if it continues to taxi across the runway. The upper line is the locus of points at which the taxi aircraft enters the danger zone just as the takeoff aircraft reaches that point. The lines end at the takeoff distance, which is defined as the point at which the takeoff aircraft clears (flies over) the taxi aircraft[3].

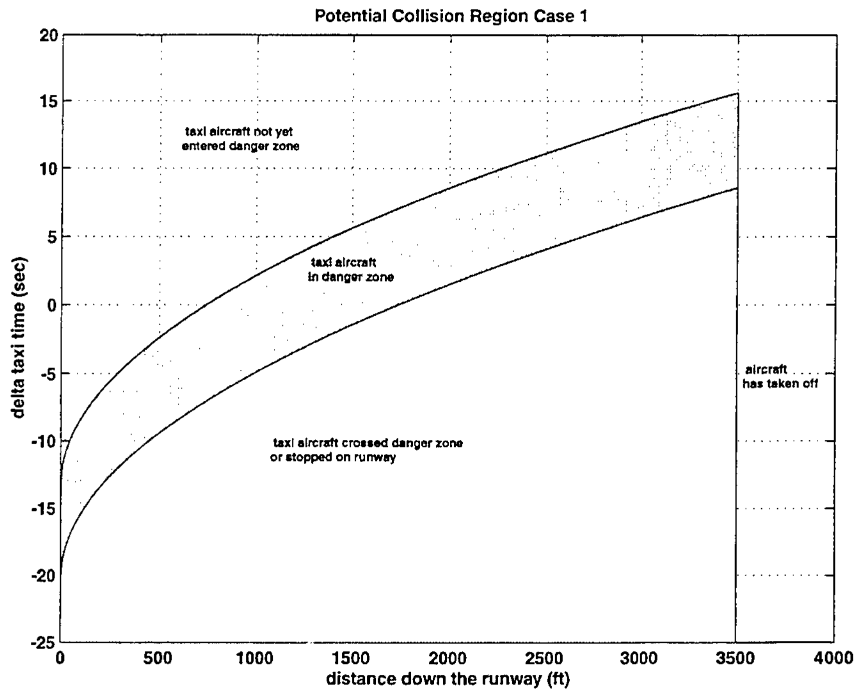


Figure 35. Collision region for nominal case.

[3] The takeoff distances are representative of the aircraft modeled in each of the cases considered. However, the takeoff distance will vary with aircraft weight, density altitude, flap settings, and takeoff power management.

3.4.2 Timing Analysis for Runway Status Light System

The events that must take place before the taxi aircraft begins to brake are: 1) the surveillance and tracking algorithms must determine that the aircraft moving at the end of the runway is, in fact, a take-off aircraft; 2) the lighting system must respond and turn on the runway status lights; and 3) the pilot must react to the lights. For the purposes of this analysis it is assumed that the delay time of the lighting system after input from the safety system is one second and the reaction time of the pilot is one second. The design of the runway status lights is a human factors issue. Obviously, the pilot might not see the lights in a poorly designed system. For the purposes of this analysis, it is assumed that the lights can be designed so that the pilots will easily see them and that after being observed, the pilot will begin braking in one second.

The major uncertainty in the timing analysis is how long it will take the system to determine that an aircraft is in the take-off state without false alarming (turning on the runway entrance lights when the aircraft is not taking off). This depends on the quality of the surveillance and the algorithms used to determine aircraft state.

In this analysis, it is assumed that the aircraft will be declared as in the take-off state when it is observed to be in an area of the runway consistent with a take-off and is accelerating and past a threshold velocity. In this analysis, the threshold velocity chosen is 30 knots. This is the "absolute" threshold velocity assuming perfect surveillance. Errors in velocity estimates due to surveillance will require this to be increased.

If the surveillance system were perfectly accurate and instantaneous, then the system would determine that the takeoff aircraft described above as the nominal case (0.26g acceleration) would reach 30 knots approximately 6 seconds after the start of the takeoff roll. The distance traveled would be approximately 150 feet.

If the surveillance is less than perfectly accurate and instantaneous, then a degree of uncertainty is introduced by the surveillance and the safety system must accommodate for the uncertainty, which introduces an additional delay.

There are two components that contribute to this delay. One is the surveillance interval $\Delta\tau$. For the purposes of this analysis the maximum time it can take to detect that the takeoff aircraft has reached the velocity threshold is used; i.e., $\Delta\tau$ past the first possible determination.

The other component that contributes to delay is due to the uncertainty in estimated velocity introduced by the uncertainty in position estimates of the surveillance system. For the purposes of this analysis, it is assumed that the declaration of a takeoff must be delayed until the estimated aircraft velocity is $2\sigma_v$ above the threshold velocity. The standard deviation in velocity estimate, σ_v, is a function of σ_s, the standard deviation of the one-dimensional estimate of the position of the aircraft down the runway.

The relationship between σ_v and σ_s depends on the method used to produce the velocity estimates. Surveillance systems will typically use a tracker and a filter, such as a Kalman filter, to produce velocity and position estimates. The output of the filter will depend on the weighting factors used in the filter.

A simulation was developed to study the effects of three different methods of estimating velocity. These methods can be viewed as the results of different weighting schemes in a filter.

The first method is to estimate the velocity as simply the difference in position estimates divided by the update interval:

$$E(v(t_i)) = \frac{E(s(t_i)) - E(s(t_{i-1}))}{\Delta\tau}$$

This is sufficient for accurate estimates of position and longer update intervals, but for larger position errors and short update intervals, this method produces large errors in velocity estimates as will be demonstrated below. An additional bias or lag is introduced during periods of acceleration because the estimate will always be of the average velocity between the two position estimates.

Another method is to fit a line through N estimates of the velocity. In this analysis, the last six seconds of velocity estimates were used for a linear regression to estimate the current velocity. This will reduce the large errors introduced by small update intervals but will not change the bias or lag during periods of acceleration. Six seconds was chosen because that is the approximate time it takes for an aircraft accelerating at a constant 0.26g's to reach 30 knots.

The third method used in this analysis was to least squares fit a second degree polynomial through the previous 6 seconds of position estimates to produce a position estimator:

$$E(s(t)) = \beta_2 t^2 + \beta_1 t + \beta_0$$

The estimate of the current velocity is then the derivative at time t_i:

$$E(v(t_i)) = 2\beta_2 t_i + \beta_1$$

This proved most successful and was the method used to estimate delays in determining when an aircraft under surveillance had reached the take-off state. This is to be expected since we are tracking an aircraft with constant acceleration that reaches the absolute threshold velocity in six seconds.

Sample results of the simulation are shown in Figures 36 through 44 for σ_s of 5, 10, and 20 feet and update interval of 0.5, 1, and 2 seconds. Plots of "truth" for position and acceleration are computed for a 0.26g acceleration starting at time zero. Surveillance estimates are generated by adding a normally distributed random error to the true position for the appropriate σ_s and update interval. Velocity estimates are generated as described above. The figures give an insight into the consequences of estimating velocity by the various methods. The simple estimator using the difference in position divided by update interval becomes unusable at small update intervals. This method and the least squares linear fit through six seconds of data both suffer from the lag effect of averaging the velocity estimates during periods of acceleration. This effect is most noticeable at larger update intervals. The method of least squares fitting a polynomial through six seconds of position estimate data and taking the derivative shows some lag at the start of acceleration as expected but serves well as an estimator at the time of interest as the aircraft approaches thirty knots.

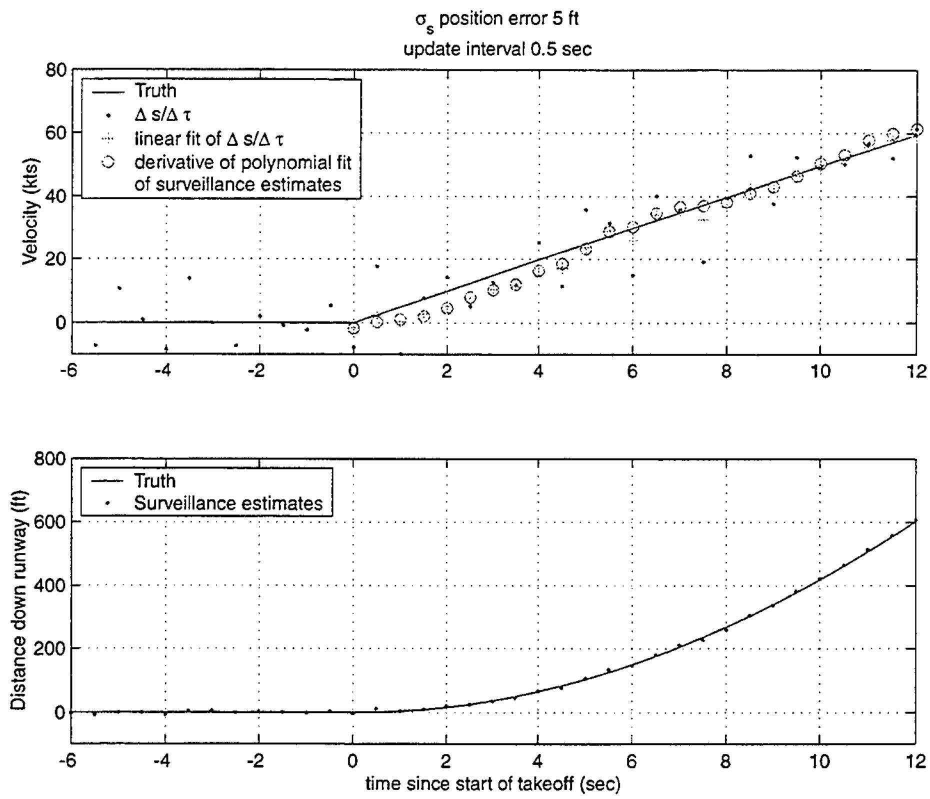


Figure 36. Monte Carlo trial of velocity estimates with σ_s = 5 feet and $\Delta\tau$ = 0.5 seconds.

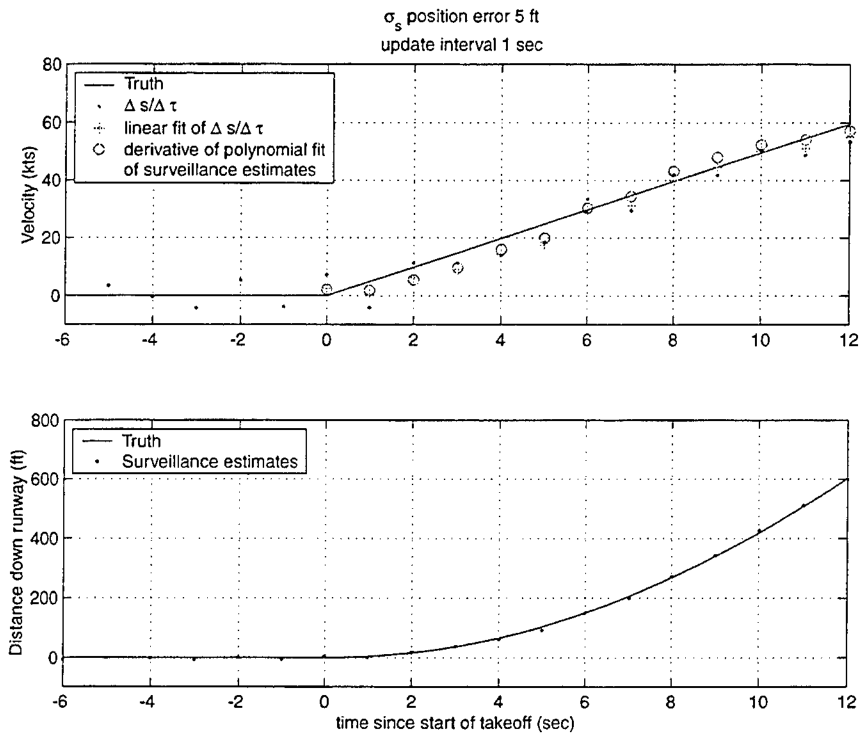


Figure 37. Monte Carlo trial of velocity estimates with σ_s = 5 feet and $\Delta\tau$ = 1 seconds.

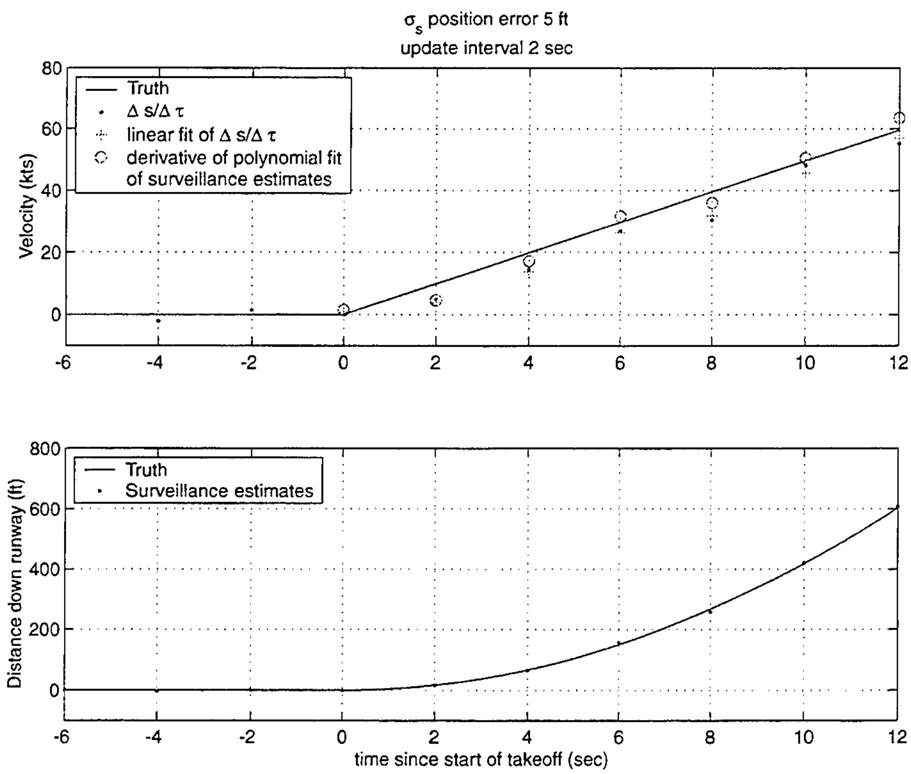


Figure 38. Monte Carlo trial of velocity estimates with σ_s = 5 feet and $\Delta\tau$ = 2 seconds.

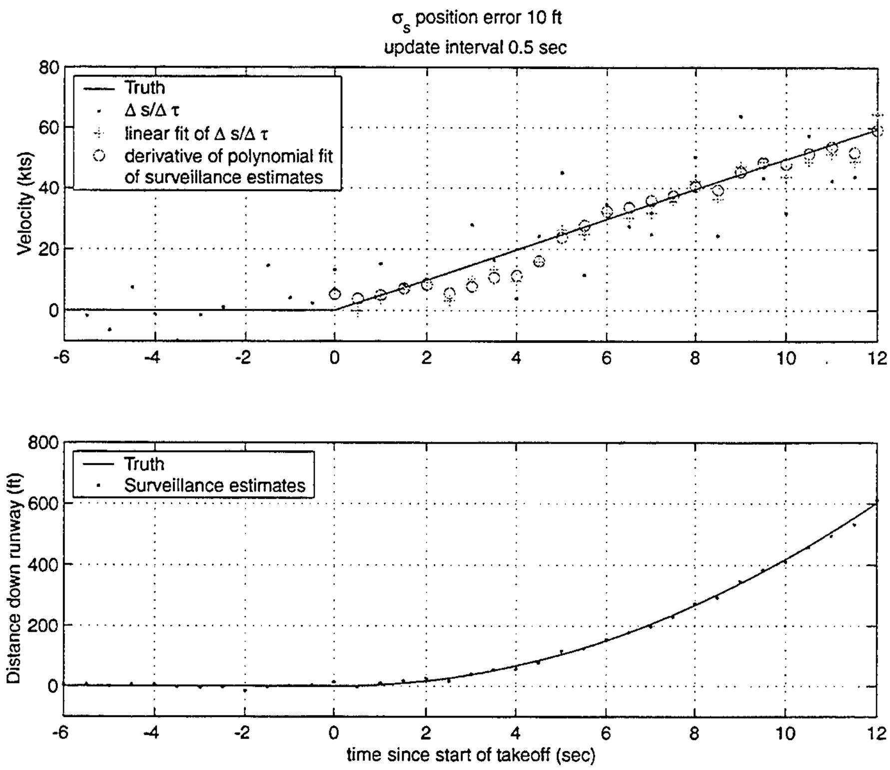


Figure 39. Monte Carlo trial of velocity estimates with σ_s = 10 feet and $\Delta\tau$ = 0.5 seconds.

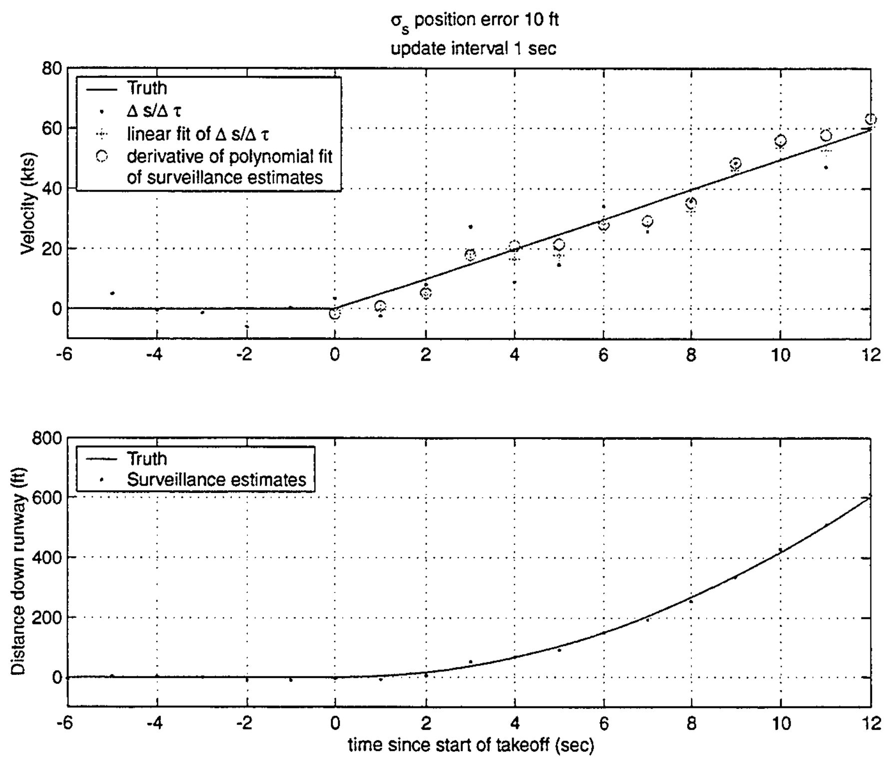


Figure 40. Monte Carlo trial of velocity estimates with σ_s = 10 feet and $\Delta\tau$ = 1 seconds.

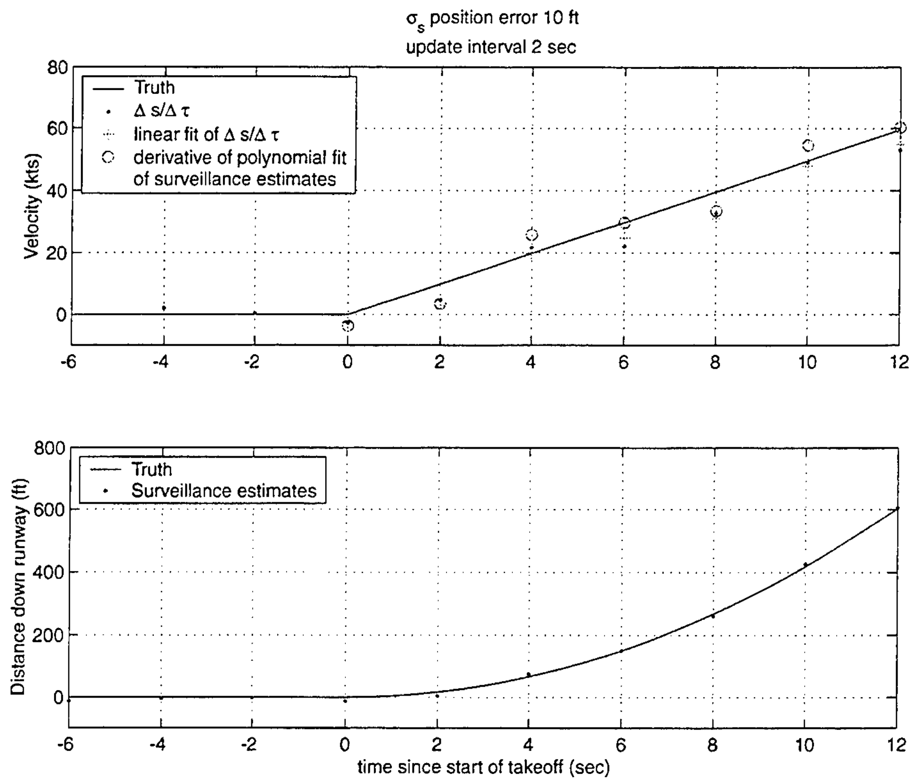


Figure 41. Monte Carlo trial of velocity estimates with σ_s = 10 feet and $\Delta\tau$ = 2 seconds.

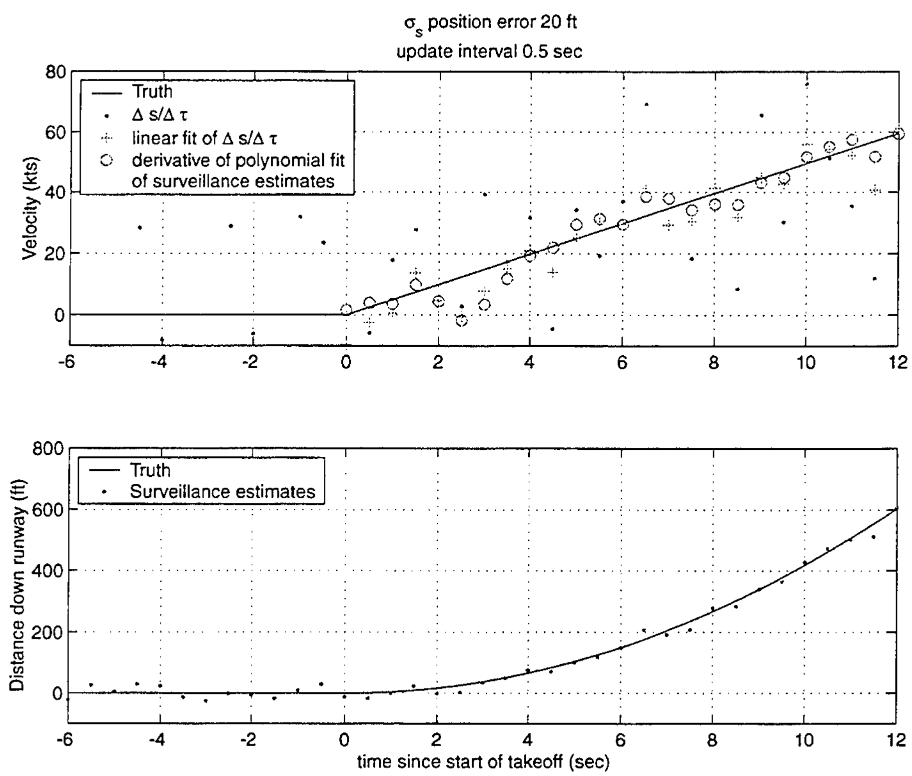


Figure 42. Monte Carlo trial of velocity estimates with σ_s = 20 feet and $\Delta\tau$ = 0.5 seconds.

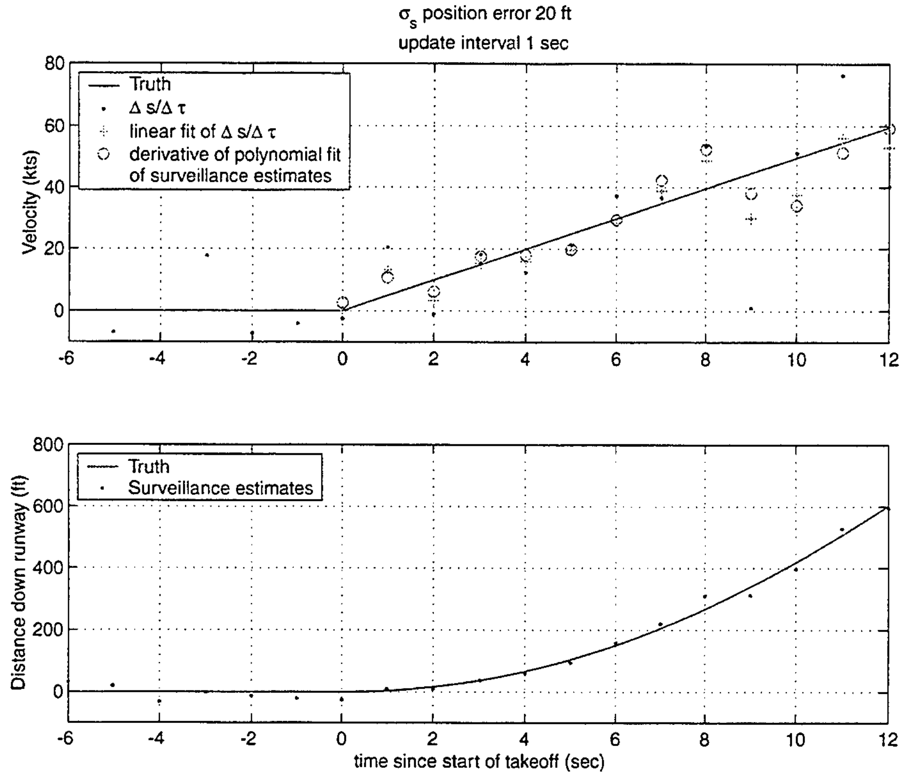


Figure 43. Monte Carlo trial of velocity estimates with σ_s = 20 feet and $\Delta\tau$ = 1 seconds.

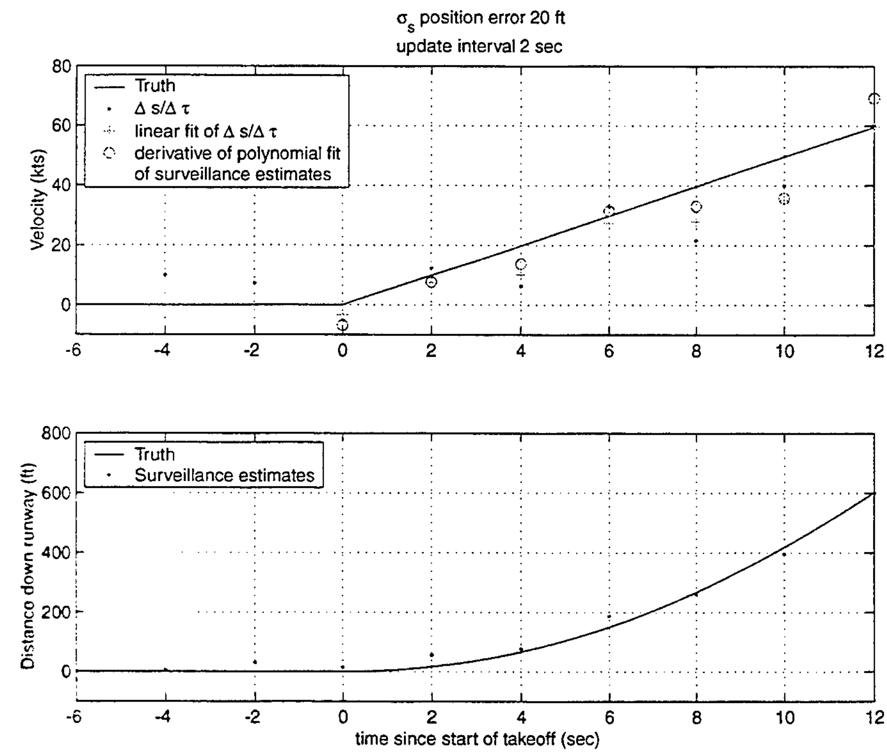


Figure 44. Monte Carlo trial of velocity estimates with σ_s = 20 feet and $\Delta\tau$ = 2 seconds.

Monte Carlo simulation runs of 100,000 trials were run for each of the velocity estimators and the mean and standard deviations of the velocity estimate at six seconds were computed for several cases of update interval $\Delta\tau$ and standard deviation of position estimate σ_s. The results of these trials are presented in Tables 1, 2, and 3. The true velocity at six seconds is 29.7616 knots. Table 1 shows the Monte Carlo simulation results for the mean μ_v and standard deviation σ_v of velocity estimates at t = 6 seconds for a takeoff aircraft at .026 g and 29.7616 knots as a function of position uncertainty standard deviation σ_s and update interval $\Delta\tau$ for an estimate of velocity computed by $E(v(t_i)) = \frac{E(s(t_i)) - E(s(t_{i-1}))}{\Delta\tau}$. Note the bias in μ_v in Tables 1 and 2 and the large σ_v at small $\Delta\tau$ in Table 1. Table 2 shows Monte Carlo simulation results for the mean μ_v and standard deviation σ_v of velocity estimates at t = 6 seconds for a takeoff aircraft at .026 g and 29.7616 knots as a function of position uncertainty standard deviation σ_s and update interval $\Delta\tau$ for an estimate of velocity computed by a linear least squares fit of the previous six seconds of velocity estimates computed by $E(v(t_i)) = \frac{E(s(t_i)) - E(s(t_{i-1}))}{\Delta\tau}$. Also note that σ_v increases with decreasing $\Delta\tau$ even for the linear least square fit estimator in Table 2. Although the σ_v is lower for the least square fit of velocities in Table 2 than for the polynomial fit estimator in Table 3 for large $\Delta\tau$, the bias is so much greater for a linear fit through the velocity estimates that the polynomial fit through position estimates is considered superior. Finally, Table 3 shows Monte Carlo simulation results for the mean μ_v and standard deviation σ_v of velocity estimates at t = 6 seconds for a takeoff aircraft at .026 g and 29.7616 knots as a function of position uncertainty standard deviation σ_s and update interval $\Delta\tau$ for an estimate of velocity computed as the derivative $E(v(t_i)) = 2\beta_2 t_i + \beta_1$ of a second order least squares fit through six seconds of position estimates $E(s(t)) = \beta_2 t^2 + \beta_1 t + \beta_0$.

Table 1. Monte Carlo Simulation Results for Point Estimates of Velocity

$\Delta\tau$ (sec)

σ_s (ft)	0.5	1.0	2.0	3.0	
0	28.5216	27.2815	24.8014	22.3212	
10	28.6351	27.4121	24.7597	22.3379	values of μ_v (kts)
20	28.3742	27.4604	24.9153	22.3391	mean of velocity estimate
50	26.8641	26.9371	24.7694	22.2808	at t = 6 sec.
100	30.2414	28.2909	25.3307	22.2213	
0	0.0000	0.0000	0.0000	0.0000	
10	16.7262	8.4741	4.2340	2.8066	values of σ_v (kts)
20	33.0350	16.8570	8.3903	5.5775	standard deviation of velocity
50	84.2602	41.8125	21.0084	13.8999	estimate at t = 6 sec.
100	166.0383	83.3876	41.8152	27.8585	

Table 2. Monte Carlo Simulation Results for Least Square Fit Through Point Estimates of Velocity

	$\Delta\tau$ (sec)				
	0.5	**1.0**	**2.0**	**3.0**	
0	28.5216	27.2815	24.8014	22.3212	
10	28.4943	27.2972	24.8283	22.3620	values of μ_v (kts)
20	28.7232	27.3198	24.8558	22.3395	mean of velocity estimate
50	28.9005	27.2265	24.9550	22.2349	at t = 6 sec.
100	28.7893	28.1490	24.7464	22.0183	
σ_s (ft)					
0	0.0000	0.0000	0.0000	0.0000	
10	4.0757	3.7925	3.2304	2.8037	values of σ_v (kts)
20	8.2101	7.4663	6.5974	5.6261	standard deviation of velocity
50	20.4778	18.9782	16.5180	14.1353	estimate at t = 6 sec.
100	40.9821	37.7932	32.9881	28.0920	

Table 3. Monte Carlo Simulation Results for Least Square Parabolic Fit

	$\Delta\tau$ (sec)				
	0.5	**1.0**	**2.0**	**3.0**	
0	29.7616	29.7616	29.7616	29.7616	
10	29.7625	29.7634	29.7424	29.7402	μ_v (kts)
20	29.7851	29.7543	29.7669	29.7871	mean of velocity estimate
50	29.7741	29.6256	29.8761	29.7685	at t = 6 sec.
100	29.9171	29.9326	29.8169	30.3640	
σ_s (ft)					
0	0.0000	0.0000	0.0000	0.0000	
10	3.2951	4.0426	4.6289	5.0373	σ_v (kts)
20	6.5792	8.1009	9.2927	10.0810	standard deviation of velocity
50	16.4909	20.1938	23.0744	25.2568	estimate at t = 6 sec.
100	33.0101	40.4130	46.2655	50.4623	

The relationship between σ_s, $\Delta\tau$, and σ_v can be derived analytically for the three cases described above. For the case where the velocity estimate is computed as the derivative $E(v(t_i)) = 2\beta_2 t_i + \beta_1$ of a second order least squares fit through N position estimates $E(s(t)) = \beta_2 t^2 + \beta_1 t + \beta_0$. The derivation is given in Appendix B.

The results are shown in Table 4 in the relationship between σ_s, $\Delta\tau$, and σ_v for the case where the velocity estimate is computed as the derivative $E(v(t_i)) = 2\beta_2 t_i + \beta_1$ of a second order least squares fit through N position estimates $E(s(t)) = \beta_2 t^2 + \beta_1 t + \beta_0$.

Table 4. Relationship Between σ_s, $\Delta\tau$, and σ_v

N	$\Delta\tau^2\sigma_v^2/\sigma_s^2$
3	13/2
4	49/20
5	87/70
6	407/560
7	13/28
8	53/168
9	1037/4620
10	437/2640
11	49/390
12	391/4004
13	155/2002
14	909/14560
15	3161/61880
16	403/9520
17	275/7752
18	49/1632
19	1739/67830
20	1937/87780

Computing σ_v from above, a comparison can be made with the Monte Carlo simulation trials. After accounting for units, and noting that the ratio between σ_v and σ_s is linear for a given $\Delta\tau$ and N, we compare the Monte Carlo cases run for σ_s= 10 feet and σ_s=100 feet shown in Table 3 with the analytically derived values and see excellent agreement as shown in Table 5.

Table 5. Comparison of Analytically Derived Values and Monte Carlo Measured Values for σ_v at Values for σ_s of 10 and 100 Feet

σ_s = 10 feet

	$\Delta\tau$ 0.5	1.0	2.0	3.0
Monte Carlo	3.2951	4.0426	4.6289	5.0373
Analytic	3.29716	4.03710	4.63692	5.03514

σ_s = 100 feet

	$\Delta\tau$ 0.5	1.0	2.0	3.0
Monte Carlo	33.0101	40.4130	46.2655	50.4623
Analytic	32.9716	40.3710	46.3692	50.3514

Figure 45 shows the relationship between σ_v , σ_s and $\Delta\tau$ taken from Table 4 based on an N representing six seconds of position estimates.

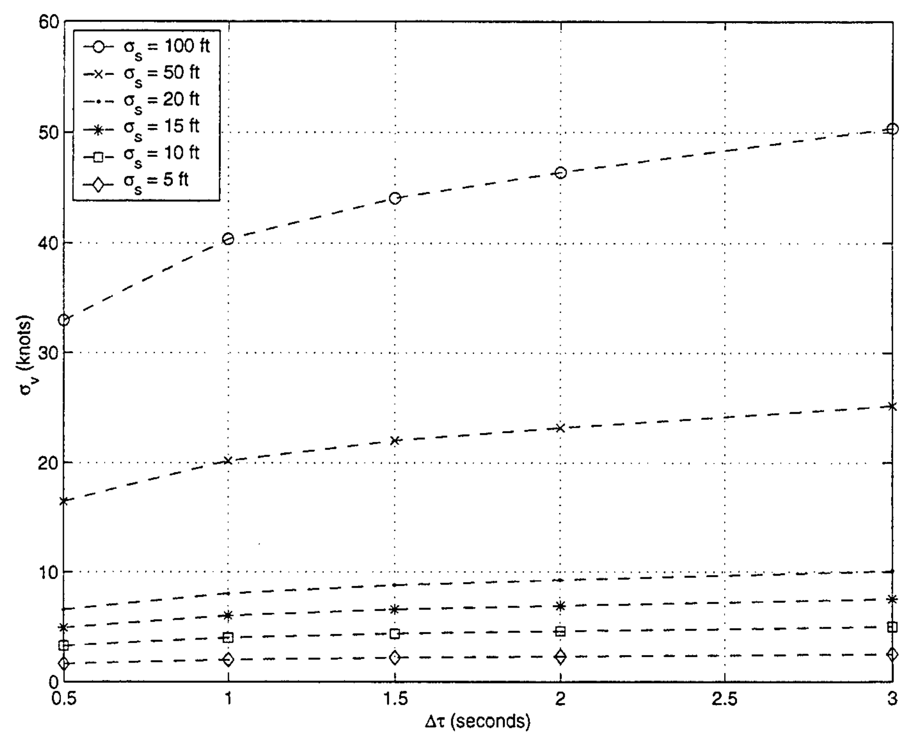


Figure 45. σ_v as a function of σ_s and $\Delta\tau$.

Since μ_v is unbiased for the case where the velocity is estimated by the second degree fit to the position estimates, the error in predicting velocity is due only to the σ_v and the alerting system will have to take in to account the value for σ_v in order to prevent false alerts. For the purposes of this analysis it is assumed that the system will need to wait until the estimated velocity is $2\sigma_v$ above the threshold (30 knots in this case) before alerting. Also note that the alert will not be made until the time of the next position estimate after the takeoff aircraft has accelerated $2\sigma_v$ above the threshold velocity. For instance, if the update interval $\Delta\tau$ is 2 seconds and the takeoff aircraft accelerates to a velocity $2\sigma_v$ above the threshold velocity shortly after a position estimate, the system will have to wait until the next estimate nearly 2 seconds later. The surveillance detection time can be computed as a function of σ_s and $\Delta\tau$ using an N equal to the number of position estimates made during a six second period and the analytically computed σ_v.

$$t_{\text{det}} = \frac{v_{thold} + 2\sigma_v(\sigma_s, \Delta\tau)}{a_{to}} + \Delta\tau$$

As an example, take $\sigma_s = 10$ feet and $\Delta\tau = 2$ sec. For an update interval of 2 seconds there will be four position estimates at t = 0,2,4,6 seconds so N = 4. From Table 4:

$$\frac{\Delta\tau^2\sigma_v^2}{\sigma_s^2} = \frac{49}{20}$$

or

$$\sigma_v = \sqrt{\left(\frac{49}{20}\right) \times \left(\frac{\sigma_s^2}{\Delta\tau^2}\right)} = 7.8262(feet / sec) = 4.6369(knots)$$

An aircraft accelerating at 0.26 g's will reach 30 knots in approximately 6.05 seconds. Now increase the velocity by $2\sigma_v$ to 39.27 knots. It will take the aircraft approximately 7.92 seconds to reach that velocity. Adding $\Delta\tau$ of 2 seconds, the maximum time to detect will be 9.92 seconds. That is the time that is given in Table 6 for $\sigma_s = 10$ feet and $\Delta\tau = 2$ seconds. The same procedure is used to compute the takeoff detection time results for all values of σ_s and $\Delta\tau$ given in Table 6 and Figure 46.

Table 6. Time in Seconds for the Surveillance and Alerting System to Detect a .26g Takeoff with a 30 knot Threshold Velocity as a Function of Surveillance Update Interval $\Delta\tau$ and Position Error σ_s

σ_s (feet)	$\Delta\tau$ (seconds) 0.5	1.0	1.5	2.0	3.0
0	6.6	7.0	7.6	8.0	9.0
5	7.2	7.9	8.4	9.0	10.1
10	7.9	8.7	9.3	9.9	11.1
15	8.5	9.5	10.2	10.9	12.1
20	9.2	10.3	11.1	11.8	13.1
50	13.2	15.2	16.4	17.4	19.2
100	19.8	23.3	25.3	26.7	29.4

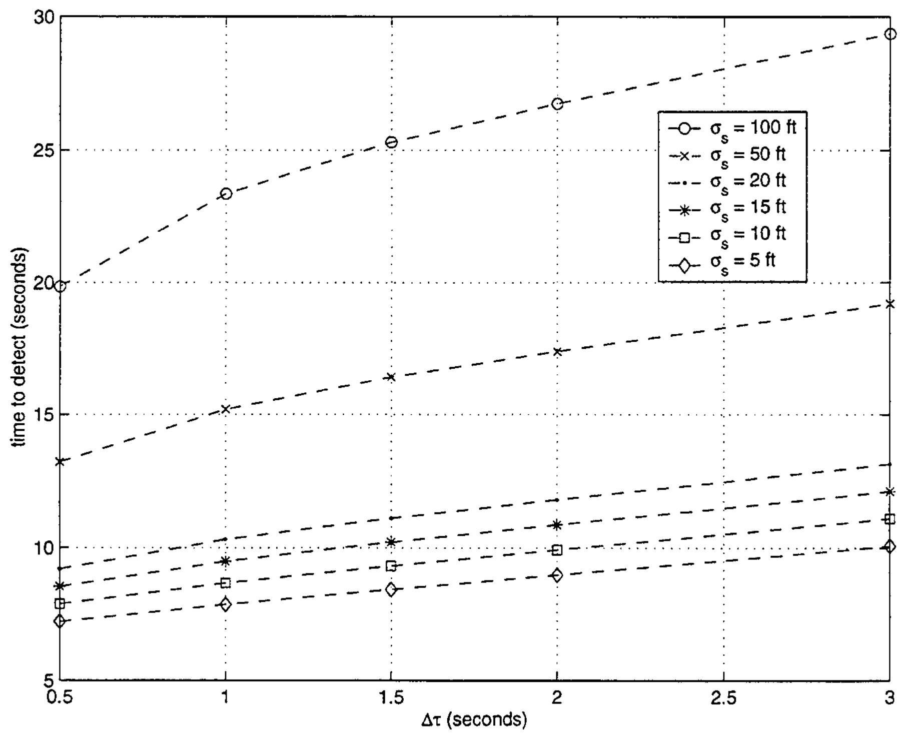


Figure 46. Time for the surveillance and alerting system to detect a 0.26g takeoff with a 30 knot absolute threshold velocity as a function of surveillance position error σ_s and update interval $\Delta\tau$.

If two seconds are added to the times in Table 6, one second for the system reaction time to turn on the status lights and one second for the pilot reaction time, we have the times, after the start of the takeoff roll of the departing aircraft, that the taxi aircraft will begin braking. These data are presented in Table 7.

Table 7. Time Required After the Start of a 0.26g Takeoff Roll for the Taxi Aircraft to Begin Braking as a Function of Surveillance Update Interval $\Delta\tau$ and Position Error σ_s

	$\Delta\tau$ (seconds)				
σ_s (feet)	**0.5**	**1.0**	**1.5**	**2.0**	**3.0**
0	8.6	9.1	9.6	10.1	11.0
5	9.2	9.9	10.4	11.0	12.1
10	9.9	10.7	11.3	11.9	13.1
15	10.5	11.5	12.2	12.9	14.1
20	11.2	12.3	13.1	13.8	15.1
50	15.2	17.2	18.4	19.4	21.2
100	21.8	25.3	27.3	28.7	31.4

To complete the timing analysis for the runway status light system we need to consider the case where the runway takeoff-hold lights are activated by the taxi aircraft. In this case the assumption is that the surveillance system will activate when the taxi aircraft is $2\sigma_s$ past the taxi hold line. If the taxi aircraft accelerates at 0.09 g's to a taxi velocity of 15 knots, then we can compute the time it takes for the taxi aircraft to taxi a distance $2\sigma_s$ past the taxi hold line. The update interval $\Delta\tau$ is taken in consideration to determine the latest time detection can take place. As an example, take σ_s of 5 feet and $\Delta\tau$ of 2 seconds. The time it takes the taxi aircraft, accelerating at 0.09 gs, to taxi $2\sigma_s$, or 10 feet, is 2.63 seconds. The maximum time it can take to detect that the aircraft has taxied the required distance is the time to taxi that distance plus $\Delta\tau$, in this case 4.63 seconds. The times as a function of σ_s and $\Delta\tau$ are presented in Table 8.

Table 8. Time in Seconds for the Surveillance and Alerting System to Detect a Taxi Aircraft Past the Taxi Hold Position as a Function of Surveillance Update Interval $\Delta\tau$ and Position Error σ_s

	$\Delta\tau$ (seconds)				
σ_s (feet)	**0.5**	**1.0**	**1.5**	**2.0**	**3.0**
0	0.5	1.0	1.5	2.0	3.0
5	3.1	3.6	4.1	4.6	5.6
10	4.2	4.7	5.2	5.7	6.7
15	5.1	5.6	6.1	6.6	7.6
20	5.8	6.3	6.8	7.3	8.3
50	8.8	9.3	9.8	10.3	11.3
100	12.8	13.3	13.8	14.3	15.3

If two seconds are added to the times in Table 8, one second for the system reaction time to turn on the status lights and one second for the pilot reaction time, we have the times, after the start of the taxi aircraft motion before the takeoff aircraft will begin braking. These data are presented in Table 9.

Table 9. Time Required After the Start of the Case 1 Taxi Aircraft Movement for the Takeoff Aircraft to Begin Braking as a Function of Surveillance Update Interval $\Delta\tau$ and Position Error σ_s

	$\Delta\tau$ (seconds)				
σ_s (feet)	**0.5**	**1.0**	**1.5**	**2.0**	**3.0**
0	2.5	3.0	3.5	4.0	5.0
5	5.1	5.6	6.1	6.6	7.6
10	6.2	6.7	7.2	7.7	8.7
15	7.1	7.6	8.1	8.6	9.6
20	7.8	8.3	8.8	9.3	10.3
50	10.8	11.3	11.8	12.3	13.3
100	14.8	15.3	15.8	16.3	17.3

Table 7 is basis for analyzing the timing for prevention of this category of incursion with the runway entrance lights. Table 9 will be used for analyzing the case where the takeoff aircraft must stop in Section 4.

3.4.3 Case 1

Case 1 is a representative nominal case. It involves a Boeing 727 on takeoff and a Shorts 360 taxiing across the runway at an intersection. The initial conditions and computed times and distances are shown in Table 10. Refer to Figure 27 for an illustration of the definition of the distances. Plots of the motion of the takeoff and taxi aircraft are given in Figures 29-34. A summary description is provided here.

The distance from the taxi hold position to the center of the runway is assumed to be 280 feet with the runway entrance lights located 180 feet ahead of the taxi hold position and offset 70 feet from the taxiway centerline. If across pavement lights are available, they are assumed to be in line with the off pavement lights. The maximum effectiveness angle of the off pavement lights is taken to be 60 degrees. The takeoff aircraft is assumed to accelerate at 0.26 g's and clear (fly over) the Shorts in 3500 feet. The threshold velocity of the safety system to detect the takeoff aircraft is 30 knots plus the increase due to the standard deviation of the surveillance position uncertainty σ_s and update interval $\Delta\tau$.

The taxi aircraft is assumed to accelerate at 0.09 g's and to brake at 0.20 g's. The taxi velocity is assumed to be 15 knots. Without braking, the taxi aircraft will accelerate to 15 knots passing the effectiveness of the off pavement status lights in 9.89 seconds 140 feet from the taxi hold position. The taxi aircraft will enter the danger zone in 13.3 seconds and exit in 20.4 seconds after the start of taxi. In order to stop short of the danger zone the aircraft would have to start braking at 11.33 seconds 176 feet from the taxi hold position. This is still short of an across pavement status light system.

Table 10. Case 1 Parameters, Boeing 727 Takeoff and Shorts 360 Taxi Aircraft

Case 1
B-727/Shorts
Nominal Taxi Profile, Nominal Takeoff Profile
Nominal Lights Placement, Nominal Threshold Velocity

Initial Conditions:

$span_{to}$	108.00 Feet
$length_{to}$	153.17 Feet
$span_{taxi}$	74.75 Feet
$length_{taxi}$	70.75 Feet
d_{taxi}	280.00 Feet
d_{lights}	180.00 Feet
d_{offset}	70.00 Feet
α_{max}	60.00 Degrees
a_{to}	.26 Gravity, 8.37 ft/sec^2
$d_{takeoff}$	3500.00 Feet
a_{taxi}	.09 Gravity, 2.90 ft/sec^2
b_{taxi}	.20 Gravity, 6.43 ft/sec^2
v_{taxi}	15.00 Knots, 25.32 ft/sec

Computed Values:

$d_{effective}$	139.59 Feet
d_{in}	226.00 Feet
d_{out}	404.75 Feet
v_{max}	25.32 ft/sec

v_{max} is equal to v_{taxi}

The following times and distances assume braking - starting at t_b = 11.3311 seconds to stop short of the danger zone:

t_{accel}	8.74 Seconds
t_{attaxi}	2.59 Seconds
$t_{braking}$	3.93 Seconds
t_{total}	15.27 Seconds
d_{accel}	110.68 Feet
d_{attaxi}	65.52 Feet
$d_{braking}$	49.80 Feet
d_{total}	226.00 Feet

The following times are without braking:

$t_{effective}$	9.89 Seconds
t_{in}	13.30 Seconds
t_{out}	20.36 Seconds

The time after which the off pavement status lights are effective, 9.89 seconds, and the time braking must begin to avoid the danger zone (while the across pavement lights are still effective), 11.33 seconds, are used in conjunction with Table 7 to assess the surveillance requirements. Table 7 gives the times after start of the takeoff roll that the surveillance and alerting system will turn on the runway entrance lights and taxi braking will begin as a function of the surveillance parameters σ_s and $\Delta\tau$. Referring to the collision region diagram for this case, Figure 35, the assumption is that the scenario where the takeoff roll and the start of taxi are simultaneous (delta taxi time equals zero) represents the most challenging case. The assumption is that if the takeoff roll starts first, it will be the role of the runway entrance lights to stop the taxi aircraft. If the taxi aircraft starts first, it will be the role of the takeoff-hold lights to stop the takeoff aircraft. This is the subject of incursion Category 2.

Comparing the required braking times (9.89 sec and 11.33 sec) to Table 7, contour plots for the two times are generated. This is presented in Figure 47. This represents the combinations of σ_s and $\Delta\tau$ that will detect the takeoff and turn on the runway entrance lights in time to prevent the collision. This allows for a one second delay to turn on the lights and a one second pilot reaction time to begin braking. Referring to Figure 47, we see that if across pavement entrance lights are available, then a surveillance system with a σ_s = 10 feet and $\Delta\tau$ = 1.5 seconds is sufficient, as is a system with σ_s = 20 feet and $\Delta\tau$ = 0.5 seconds. A system with only off pavement lights placed as described in Table 10 and shown in Figure 27 will require a surveillance system with σ_s = 5 feet and $\Delta\tau$ = 1.0 seconds or with σ_s = 10 feet and $\Delta\tau$ = 0.5 seconds to prevent the collision for the case described in Table 1. This is for simultaneous starts of both aircraft. Any delay in the motion of the taxi aircraft relative to the takeoff aircraft will relax the requirements.

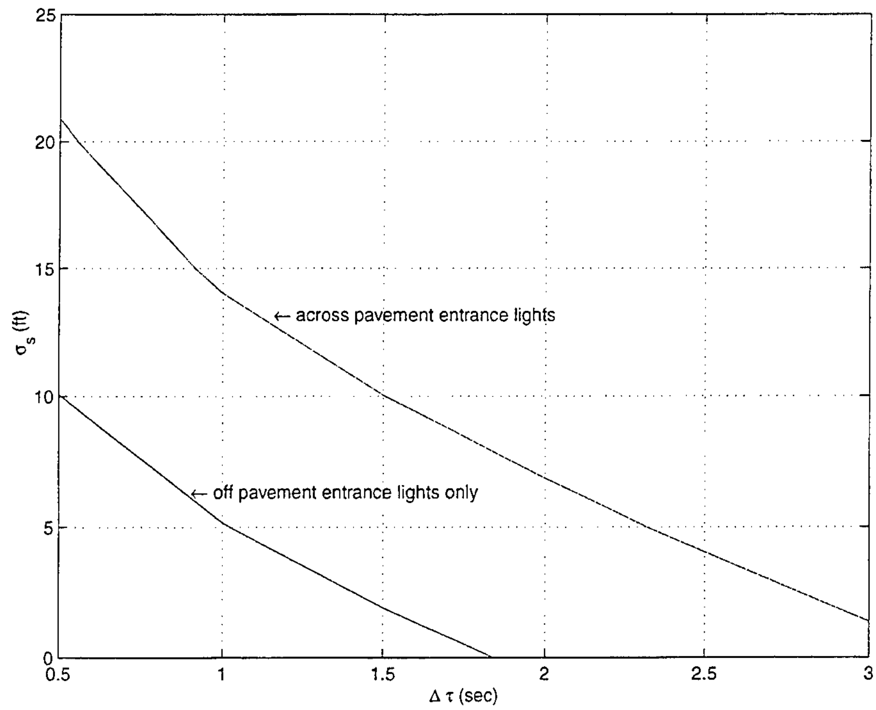


Figure 47. Contours of effective surveillance parameters to prevent an accident in the Case 1 incursion.

3.4.4 Case 2

The second case is a "worst-case" scenario. The takeoff aircraft is a Boeing 747, which has a much larger wingspan and therefore danger zone. The takeoff acceleration is decreased to 0.20 g's, which increases the time it takes to reach the threshold velocity. The takeoff distance is computed to be 7500 feet. The taxi aircraft is a Boeing 727 and the taxi acceleration is increased to 0.12 g's, the braking decreased to 0.12 g's, and the taxi velocity increased to 25 knots. The runway entrance lights are 25 feet closer to the taxi hold position and positioned further from the taxiway centerline. The effective angle of the lights is reduced to 45 degrees. The input parameters are listed in Table 11. Plots of the motion of the two aircraft are shown in Figure 48 and the potential collision region is shown in Figure 49.

Because of the large wingspan of the Boeing 747 and the fast taxi profile assumed for the Boeing 727, the taxi aircraft must begin braking only 6.87 seconds after start of taxi in order to stop short of the danger zone. The off pavement entrance lights are still effective at this point, even with their less desirable placement and effective viewing angle. However, at 6.87 seconds the takeoff aircraft has not yet attained 30 knots, so even with perfect surveillance it is not possible to warn the pilot to start braking in time to prevent entering the danger zone.

The accelerations of both the takeoff aircraft and taxi aircraft are different from Case 1 so the times to detect the relative motions are different and result in new times that correspond to the times in Tables 7 and 9 for Case 1. The corresponding times for Case 2 are computed the same way and presented as Tables 12 and 13.

Table 11. Case 2 Input

Case 2
B-747/B-727
Fast Taxi Profile, Slow, Long Takeoff Profile
Worst Case Lights Placement, Nominal Threshold Velocity

Initial Conditions:

$span_{to}$	195.67 Feet
$length_{to}$	231.00 Feet
$span_{taxi}$	108.00 Feet
$length_{taxi}$	153.17 Feet
d_{taxi}	280.00 Feet
d_{lights}	205.00 Feet
d_{offset}	100.00 Feet
α_{max}	45.00 Degrees
a_{to}	.20 Gravity, 6.43 ft/sec^2
$d_{takeoff}$	7500.00 Feet
$v_{tothreshold}$	30.00 Knots
a_{taxi}	.12 Gravity, 3.86 ft/sec^2
b_{taxi}	.12 Gravity, 3.86 ft/sec^2
v_{taxi}	25.00 Knots, 42.20 ft/sec

Computed Values:

$d_{effective}$	105.00 Feet
d_{in}	182.17 Feet
d_{out}	531.00 Feet
v_{max}	26.52 ft/sec

v_{max} is less than v_{taxi}

The following times and distances assume braking - starting at t_b = 6.86893 seconds to stop short of the danger zone:

t_{accel}	6.87 Seconds
t_{attaxi}	0.00 Seconds
$t_{braking}$	6.87 Seconds
t_{total}	13.74 Seconds
d_{accel}	91.08 Feet
d_{attaxi}	0.00 Feet
$d_{braking}$	91.08 Feet
d_{total}	182.17 Feet

The following times are without braking:

$t_{effective}$	7.38 Seconds
t_{in}	9.71 Seconds
t_{out}	18.05 Seconds

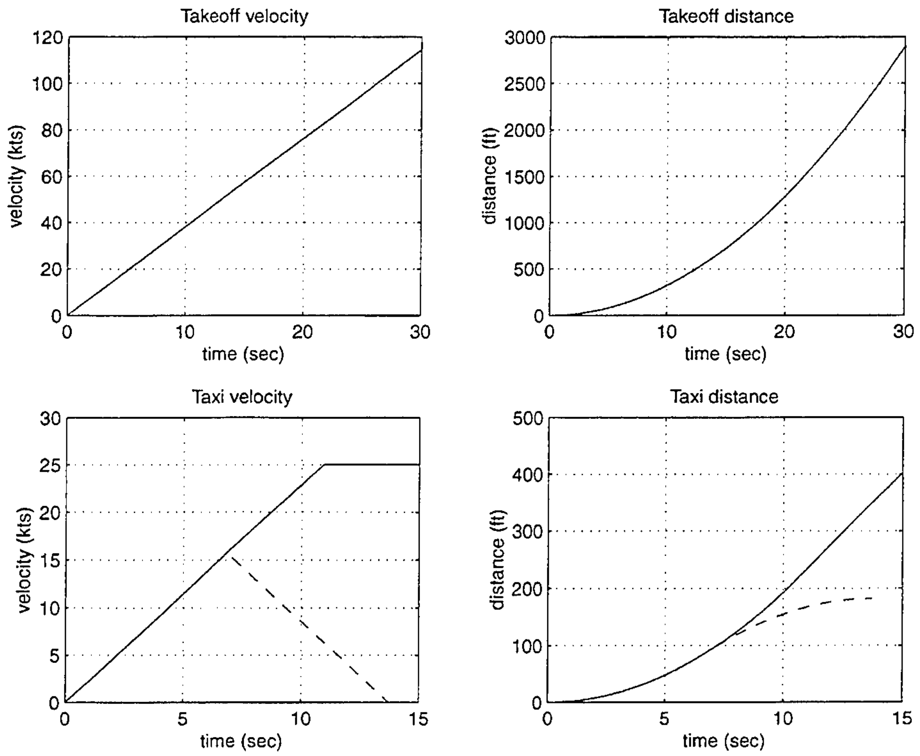


Figure 48. Aircraft motion profiles for Case 2 incursion.

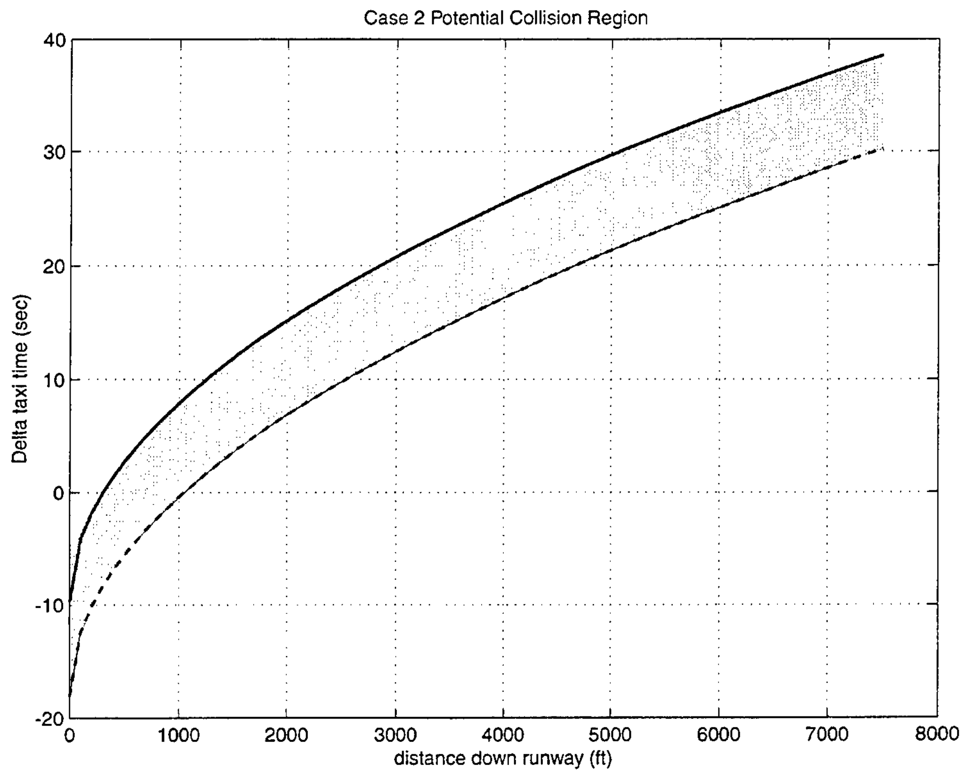


Figure 49. Potential collision region for Case 2 incursion.

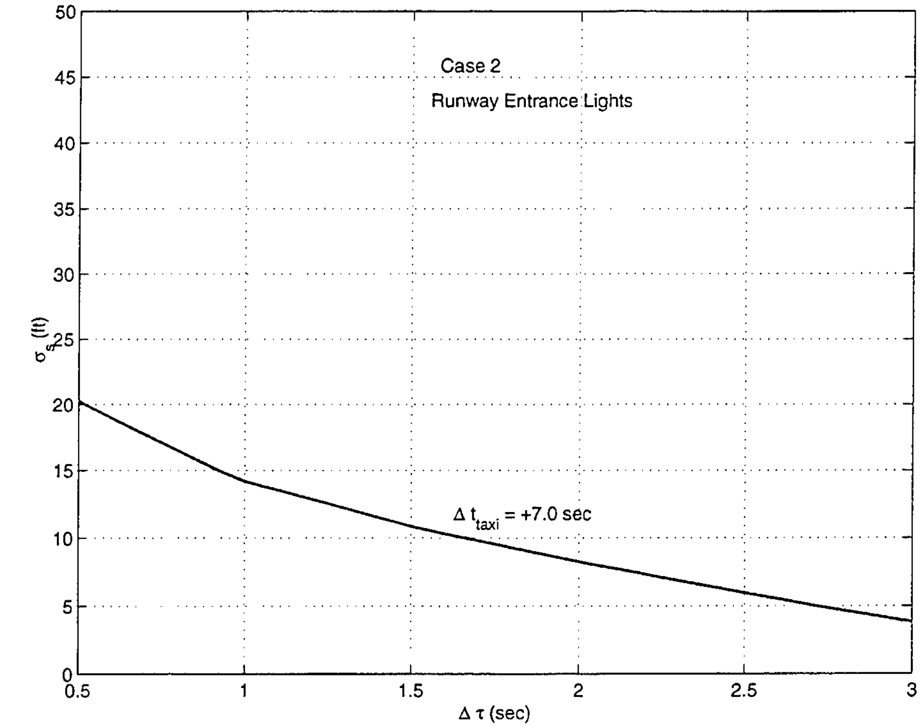
Table 12. Time Required After the Start of a 0.20 g Takeoff Roll for the Taxi Aircraft to Begin Braking as a Function of Surveillance Update Interval $\Delta\tau$ and Position Error σ_s.

	$\Delta\tau$ (seconds)				
σ_s (feet)	**0.5**	**1.0**	**1.5**	**2.0**	**3.0**
0	10.4	10.9	11.4	11.9	12.9
5	11.2	11.9	12.5	13.1	14.2
10	12.1	13.0	13.7	14.3	15.5
15	13.0	14.0	14.8	15.5	16.8
20	13.8	15.1	16.0	16.7	18.1
50	19.0	21.4	22.9	24.0	26.1
100	27.6	32.0	34.4	36.2	39.3

Table 13. Time Required After the Start of the Case 2 Taxi Aircraft Movement for the Takeoff Aircraft to Begin Braking as a Function of Surveillance Update Interval $\Delta\tau$ and Position Error σ_s.

	$\Delta\tau$ (seconds)				
σ_s (feet)	**0.5**	**1.0**	**1.5**	**2.0**	**3.0**
0	2.5	3.0	3.5	4.0	5.0
5	4.8	5.3	5.8	6.3	7.3
10	5.7	6.2	6.7	7.2	8.2
15	6.4	6.9	7.4	7.9	8.9
20	7.1	7.6	8.1	8.6	9.6
50	9.7	10.2	10.7	11.2	12.2
100	12.7	13.2	13.7	14.2	15.2

The time after the simultaneous start of motion of the two aircraft that the taxi aircraft must begin braking is very short, 6.87 seconds. The times shown in Table 12 are relatively longer because of the slow acceleration of the takeoff aircraft. In this case the runway entrance lights will not be effective regardless of the surveillance. If we look at the portion of the collision zone covered by a delta taxi time of 7 seconds or more, then we in effect remove 7 seconds from the times in Table 12, which gives times comparable to those in Table 7. Thus the portion of the collision region from a delta taxi time of zero to 7 seconds is not protected for this case as shown in Figure 50. The case for negative delta taxi times in which the takeoff aircraft rejects the takeoff is covered in incursion Category 2.



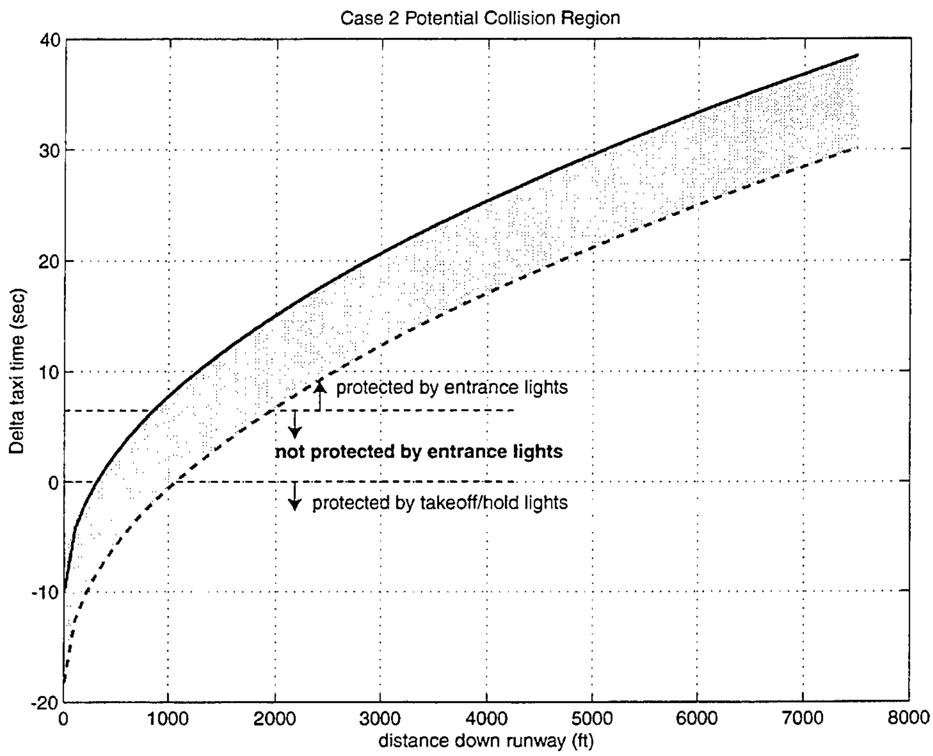


Figure 50. Effective surveillance parameters and protected portion of the Case 2 potential collision region.

3.4.5 Case 3

Case 3 is included as a less challenging case. An ATR-42 is the takeoff aircraft. It has a shorter wingspan, greater takeoff acceleration, and shorter takeoff distance than the aircraft in Case 1 and 2. The taxi aircraft is a DC-10 with a lower taxi acceleration and velocity. This plus the longer fuselage length has the effect of widening the collision region. The data for Case 3 are shown in Table 14. The distance and velocity plots for the takeoff and taxi aircraft are shown in Figure 51. The collision region is depicted in Figure 52.

The taxi motion profile in Figure 51 shows that the DC-10 will reach the taxi velocity of 12 knots well before braking is required. The distance from the taxi hold position to the runway centerline is increased to 330 feet as would be the case at an airport with a field elevation of 5000 feet [4]. This, of course, allows more time for the taxi aircraft to react. The offset of the runway entrance lights was reduced to 60 feet and the effective angle increased to 75 degrees. This plus the increased distance of the taxi hold position from the runway have the effect of increasing $d_{effective}$. This distance increased from 139.6 feet in Case 1 to 213.9 feet.

The time that the taxi aircraft must begin braking in order to stop short of the danger zone is 16.9 seconds after the start of taxi for the taxi profile given in Table 14 and illustrated in Figure 51. The entrance lights placement, as described in Table 14 and illustrated in Figure 27, are effective for 14.7 seconds after the taxi aircraft begins to taxi.

Table 14. Case 3 Input

Case 3
ATR-42/DC-10
Slow Taxi Profile, Fast Takeoff Profile
Good Lights Placement, Lower Threshold Velocity

Initial Conditions:

$span_{to}$	80.63 Feet
$length_{to}$	74.37 Feet
$span_{taxi}$	165.38 Feet
$length_{taxi}$	182.08 Feet
d_{taxi}	330.00 Feet
d_{lights}	230.00 Feet
d_{offset}	60.00 Feet
α_{max}	75.00 Degrees
a_{to}	.28 Gravity, 9.01 ft/sec^2
$d_{takeoff}$	3000.00 Feet
$v_{tothreshold}$	20.00 Knots
a_{taxi}	.08 Gravity, 2.41 ft/sec^2
b_{taxi}	.20 Gravity, 6.43 ft/sec^2
v_{taxi}	25.00 Knot, 20.25 ft/sec

Computed Values:

$d_{effective}$	213.92 Feet
d_{in}	289.69 Feet
d_{out}	552.40 Feet
v_{max}	20.25 ft/sec

v_{max} is equal to v_{taxi}

The following times and distances assume braking - starting at t_b = 16.9259 seconds to stop short of the danger zone:

t_{accel}	7.86 Seconds
t_{attaxi}	8.80 Seconds
$t_{braking}$	3.15 Seconds
t_{total}	19.81 Seconds
d_{accel}	85.00 Seconds
d_{attaxi}	172.81 Feet
$d_{braking}$	31.87 Feet
d_{total}	289.69 Feet

The following times are without braking:

$t_{effective}$	14.76 Seconds
t_{in}	18.50 Seconds
t_{out}	31.47 Seconds

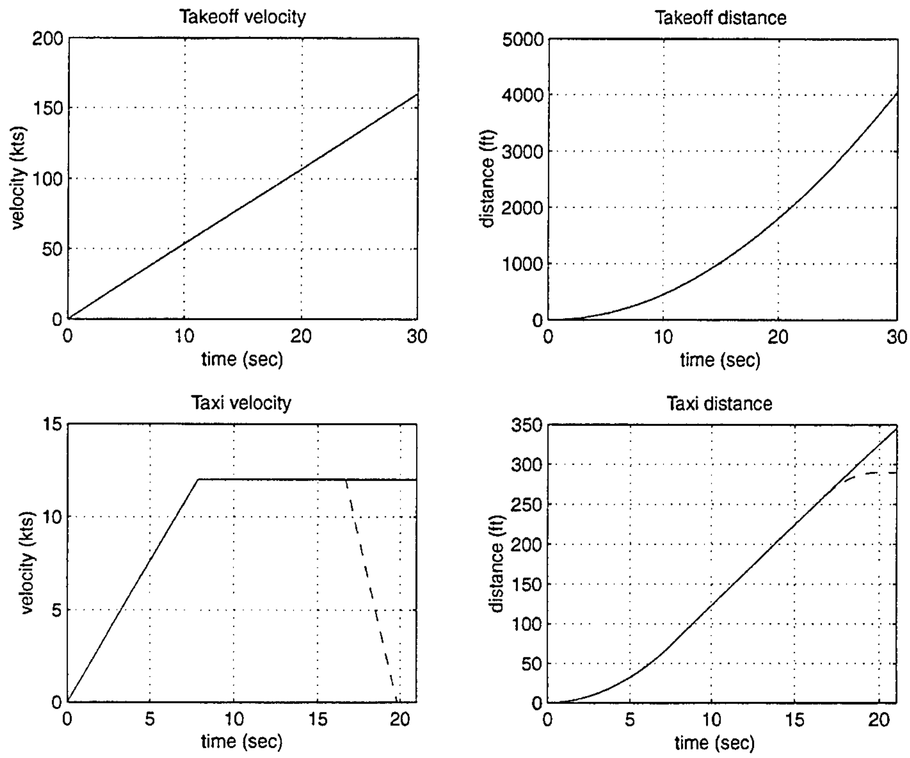


Figure 51. Aircraft motion profiles for Case 3 incursion.

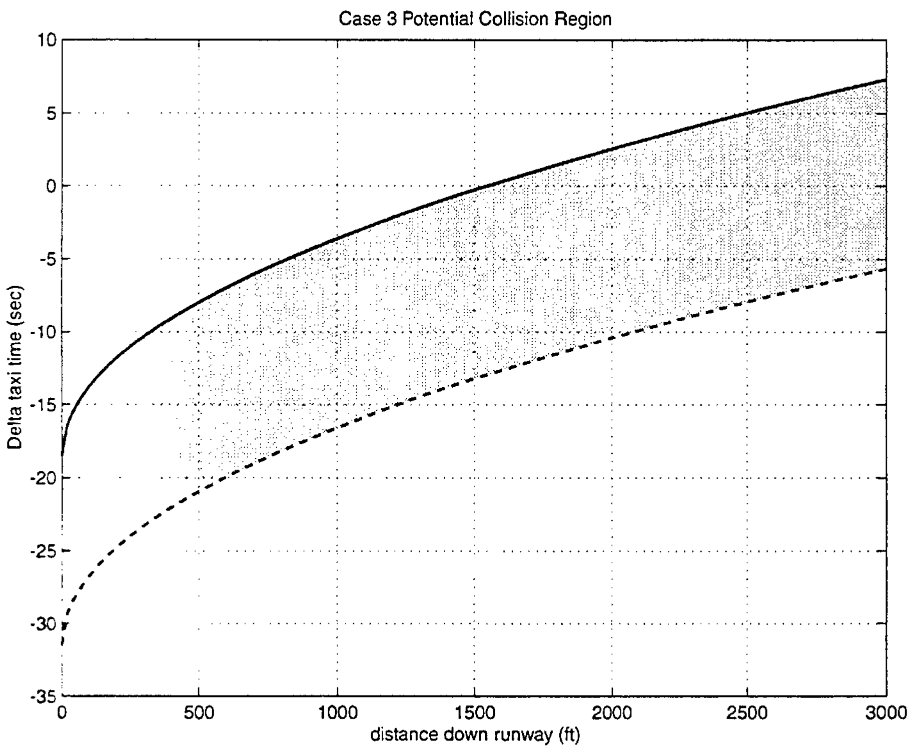


Figure 52. Potential collision region for Case 3 incursion.

The faster takeoff acceleration requires less time to detect the takeoff and turn on the runway entrance lights as shown in Table 15. This table compares to Table 7 for Case 1 and Table 12 for Case 2 and includes the time to detect using a 30 knot absolute threshold and includes a one second delay for the system to turn on the lights and a one second delay for the pilot reaction time. Table 16 showing the time to detect that the taxi aircraft has crossed the taxi hold position (by 2 σ_s) and turn on the aircraft takeoff-hold lights and for the pilot of the takeoff aircraft to begin braking is included for completeness and compares to Table 9 for Case 1 and Table 13 for Case 2.

Table 15. Time Required After the Start of a 0.28 g Takeoff Roll for the Taxi Aircraft to Begin Braking as a Function of Surveillance Update Interval $\Delta\tau$ and Position Error σ_s

	$\Delta\tau$ (seconds)				
σ_s (feet)	**0.5**	**1.0**	**1.5**	**2.0**	**3.0**
0	8.1	8.6	9.1	9.6	10.6
5	8.7	9.4	9.9	10.5	11.6
10	9.4	10.1	10.8	11.4	12.5
15	10.0	10.9	11.6	12.2	13.4
20	10.6	11.6	12.4	13.1	14.4
50	14.3	16.2	17.4	18.3	20.0
100	20.5	23.7	25.6	27.0	29.5

Table 16. Time Required After the Start of the Case 3 Taxi Aircraft Movement for the Takeoff Aircraft to Begin Braking as a Function of Surveillance Update Interval $\Delta\tau$ and Position Error σ_s

	$\Delta\tau$ (seconds)				
σ_s (feet)	**0.5**	**1.0**	**1.5**	**2.0**	**3.0**
0	2.5	3.0	3.5	4.0	5.0
5	5.3	5.8	6.3	6.8	7.8
10	6.4	6.9	7.4	7.9	8.9
15	7.3	7.8	8.3	8.8	9.8
20	8.1	8.6	9.1	9.6	10.6
50	12.4	11.9	12.4	12.9	13.9
100	16.3	16.8	17.3	17.8	18.8

Figure 53 shows two time contours through the data presented in Table 15. The two times correspond to the time the taxi aircraft passes the point where the runway entrance lights are not longer effective (14.76 seconds) and when it must start braking (16.9 seconds) to avoid the danger zone. The surveillance requirements are far more relaxed due to the large amount of time available to detect the takeoff and turn on the entrance lights.

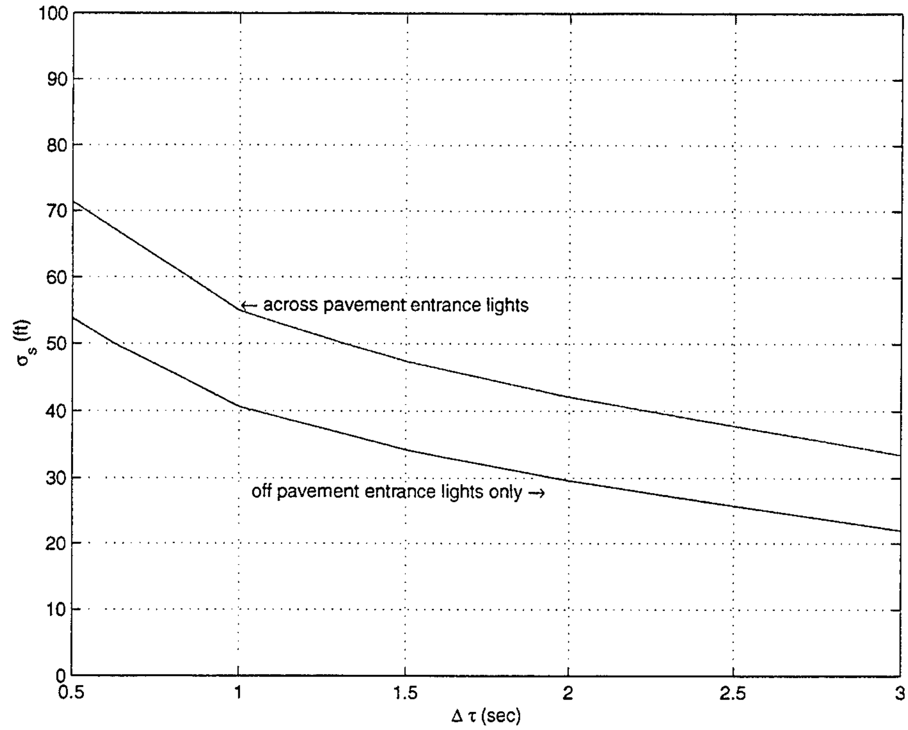


Figure 53. Contours of effective surveillance parameters to prevent an accident in the Case 3 incursion.

3.4.6 Results for Runway Entrance Lights

The timing analysis and study of three individual cases above provide some insight into the effectiveness of runway entrance lights to prevent this category of incursion and for the surveillance requirements. First, runway entrance lights are totally effective if the lights are designed and placed properly for cases where the runway is hot with an arriving or departing aircraft. Cases 1,2,3 are examples where the runway was not yet determined to be hot. They examined relative timings for the start of the motion of the two aircraft to determine a potential collision zone. The presumption is that the runway entrance lights must protect the runway if the takeoff aircraft starts its motion simultaneous with or before the start of motion of the taxi aircraft. The most challenging case is a simultaneous start of motion. For these cases the effectiveness of the system depends greatly on the positioning and effectiveness of the runway entrance lights and the motion profiles of the two aircraft. Cases where the taxi aircraft starts first will be considered in the next category of incursion.

Results for the cases presented above assumed that the safety system would determine that an aircraft was a departing takeoff when it reliably determined that the velocity was above 30 knots. This threshold velocity was increased by an amount determined by the uncertainty in velocity prediction to avoid nuisance alerts. The time and accuracy of detection is also a function of update interval. If the threshold velocity required to determine that an aircraft is taking off can be reduced even slightly, the effectiveness of the system will increase.

Because of the time sensitivity of this type of incursion with simultaneous initial motion of both aircraft, it is reasonable to require that the takeoff aircraft be detected as having crossed the threshold velocity within two seconds of the event. This conclusion is derived from the observations of Cases 1,2,3 above. From the timing analysis in Section 3.4.2, Table 6 shows the times the surveillance and alerting system can determine that an aircraft accelerating at 0.26g's will be detected. The aircraft actually achieves 30 knots at 6.05 seconds. If we draw a contour line at 8.05 seconds through the data, this represents a reasonable requirement on the surveillance system needed to support the runway entrance light system in the most trying circumstances. This is presented in Figure 54.

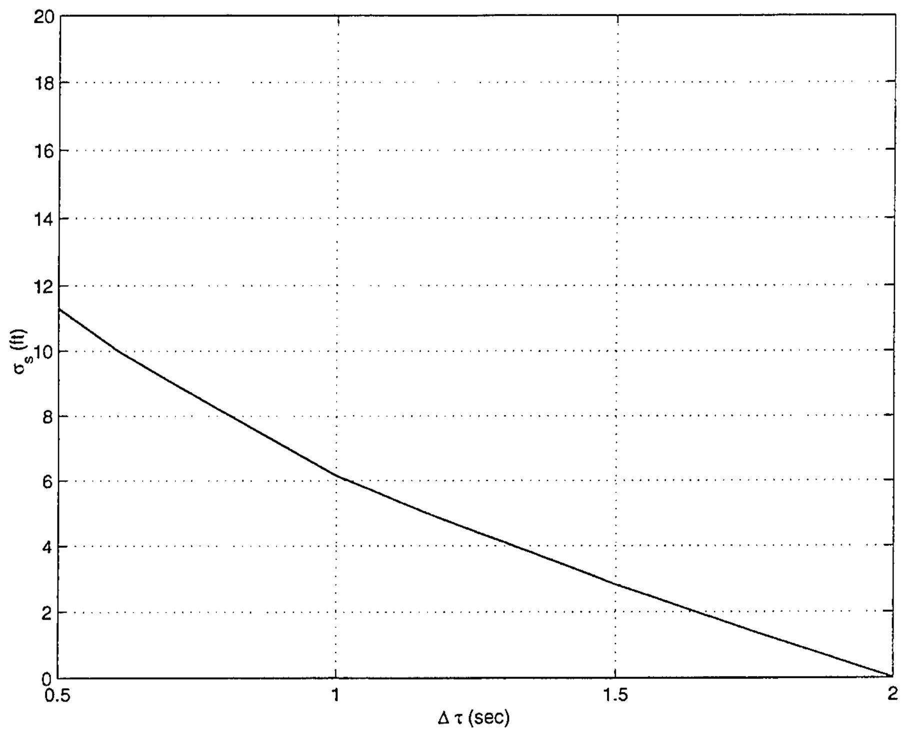


Figure 54. Contour of effective surveillance parameters to detect takeoff within 2 seconds of achieving a 30 knot absolute threshold velocity.

3.5 RESULTS FOR CATEGORY 1 INCURSION

The length of time it takes a taxi aircraft, whether stopped or not, to taxi across the taxi hold position and into the danger zone is short enough that it taxes any alerting system. This category of incursion may not be totally protected using an alerting system even with perfect surveillance. Results from the cases analyzed in Section 3 indicate that the effectiveness of either a tower cab alerting system or a direct to the cockpit alerting system drops off after the surveillance system positional error standard deviation σ_s exceeds 5 feet ($2\sigma_s$ = 10 feet) and the update interval $\Delta\tau$ exceeds 2 seconds.

The runway entrance lights are effective at indicating a hot runway and thus preventing the incursion by the taxi aircraft and the need to alert. Only when the taxi and takeoff aircraft begin motion at nearly the same time will an incursion occur. In this case, the entrance lights are effective at preventing the collision for cases where the takeoff aircraft starts simultaneous with or before the taxi aircraft in all but the most challenging motion profiles. A surveillance system with a positional error standard deviation σ_s of 6 feet ($2\sigma_s$ = 12 feet) and the update interval $\Delta\tau$ of 1 second will reliably detect the takeoff within 2 seconds of the aircraft achieving a threshold velocity of 30 knots without nuisance lighting of the entrance lights. This is sufficient to alert the taxi aircraft to brake before entering the danger zone for nominal taxi velocity profiles.

4. CATEGORY 2 INCURSION: DEPARTURE-BLOCKED RUNWAY

4.1 APPROACH

This scenario, depicted in Figure 55, assumes a normal departure from the end of the runway with the runway obstructed. The runway may be blocked by a previous arrival, an aircraft that had a rejected takeoff, a crossing or back taxiing aircraft, or an aircraft cleared into position at an intersection. This analysis investigates the requirements for rejecting the takeoff of the departure aircraft to prevent a collision. It is assumed that the only evasive action available is the braking of the departure aircraft. Whether or not a departure aircraft will, of its own accord, notice that the runway is blocked depends on the visibility; how far down the runway the obstruction is located; and pilot vigilance outside the cockpit.

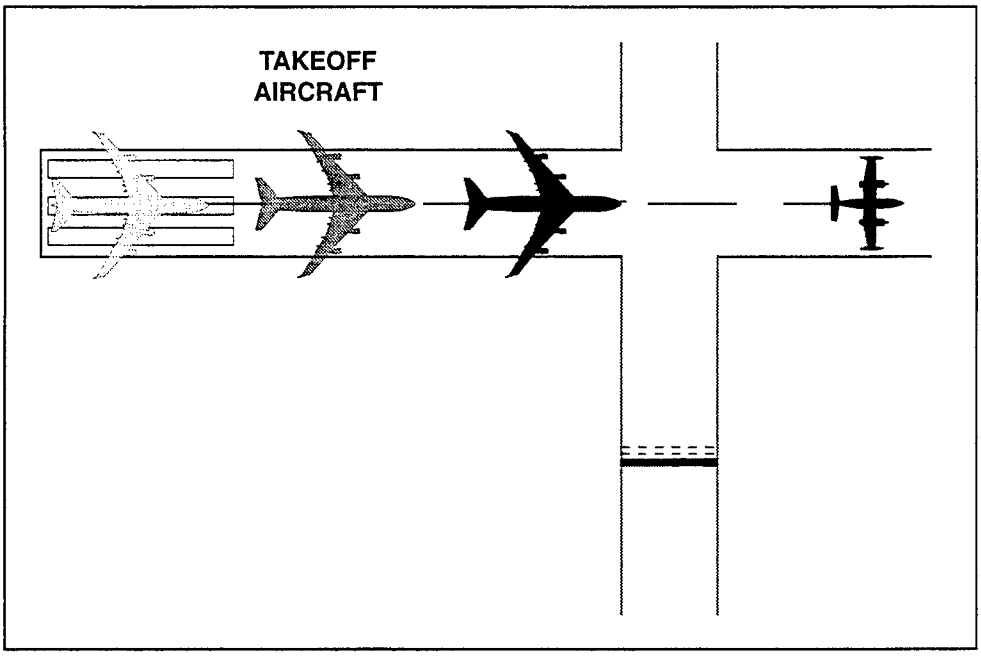


Figure 55. Category 2 incursion.

There are several considerations that affect the analysis. First, a system that simply detects a blocked runway can convey that information to the controller in a number of ways. For example, the system could place a bar at the takeoff[4] end of the runway on a surface radar display. However, this is not a true alert because the runway will be blocked on a regular basis, often with an aircraft in position at the takeoff end of the runway. An alerting system that detects a departure with a blocked runway must delay an alert until the aircraft is determined to be a departure in order to reduce nuisance alarms. This will, depending on the safety algorithm,

[4] Conventional air traffic terminology refers to the end of the runway where an arrival first crosses the threshold as the "arrival end" of the runway. A takeoff aircraft climbs out over the "departure end" of the runway. Technically, a departing aircraft begins its takeoff at the "arrival end" of the runway. To avoid confusion, throughout this report, the "takeoff end" of the runway refers to the end of the runway where the takeoff roll begins.

require that the departure aircraft accelerate to some velocity while the runway is blocked. How long it takes the safety system to correctly declare that the aircraft is departing depends on this threshold velocity and the surveillance parameters. In order to remain consistent with the previous analysis, an absolute threshold velocity requirement of 30 knots will be assumed that will, as in the previous analysis, be increased to eliminate uncertainties introduced by the positional uncertainty σ_s and update interval $\Delta\tau$ of the surveillance system. This is the same approach used in Section 3.

In the case of a tower alerting system, the controller reaction time, the VHF channel availability, and the pilot reaction time after the start of the message must be taken into account before evasive braking will begin. In the case of direct cockpit alerts, only the pilot reaction time need be considered.

For the case of the takeoff-hold lights, the lights are on if the runway is obstructed. It is not necessary to determine that the aircraft is departing before declaring an alert. The lights will be on if the runway is obstructed so that no takeoff would begin. The case of interest is, as in Section 3, the near simultaneous motion of a taxi aircraft that will block the runway and the beginning of the takeoff. The three cases for the intersection taxi-takeoff conflict in Section 3 are revisited using the rejected takeoff option for the evasive action. This then investigates that portion of the collision region introduced in Section 3 that can be protected by braking the departure aircraft.

4.2 REJECTED TAKEOFF MOTION PROFILE

Observations taken at Boston's Logan airport suggest, for a nominal profile, assume 0.26g takeoff acceleration and a 0.33g maximum effort braking. In addition, there is assumed to be a two-second interval at zero thrust between takeoff acceleration and the start of braking. For example, if evasive action were to begin at t_{evade} = 10 seconds, the rejected takeoff profile would include 10 seconds of acceleration at 0.26g, 2 seconds of coasting at the maximum attained velocity, followed by braking at 0.33g to a stop. This is illustrated in Figure 56 for t_{evade} = 10 seconds. Figure 57 is a plot of the total runway used to come to a stop versus t_{evade}; the time evasive action begins.

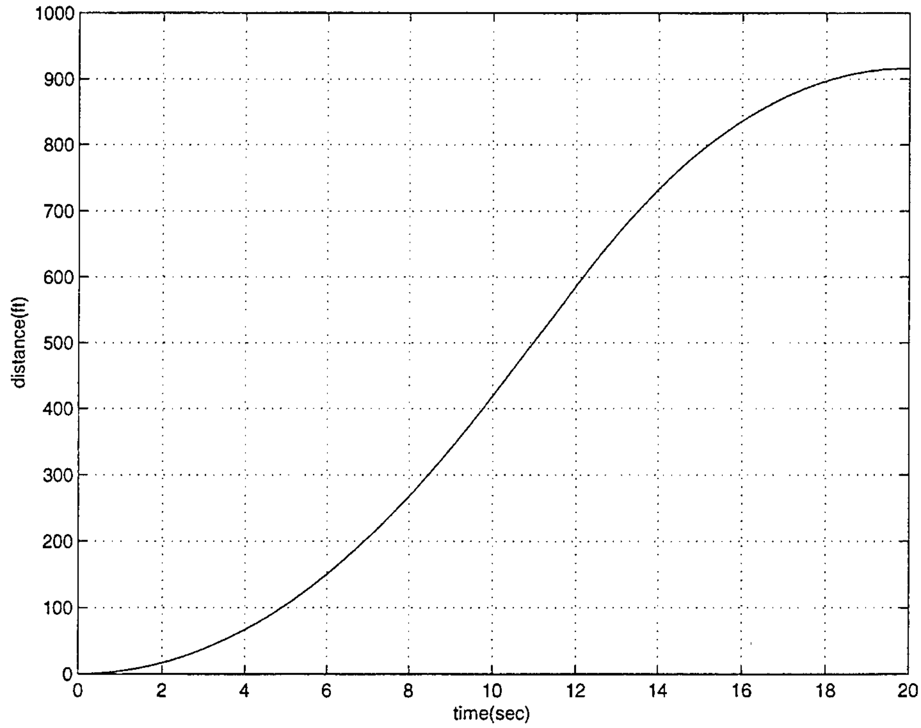


Figure 56. Distance vs. time for nominal takeoff rejected at t_{evade} *=10 seconds.*

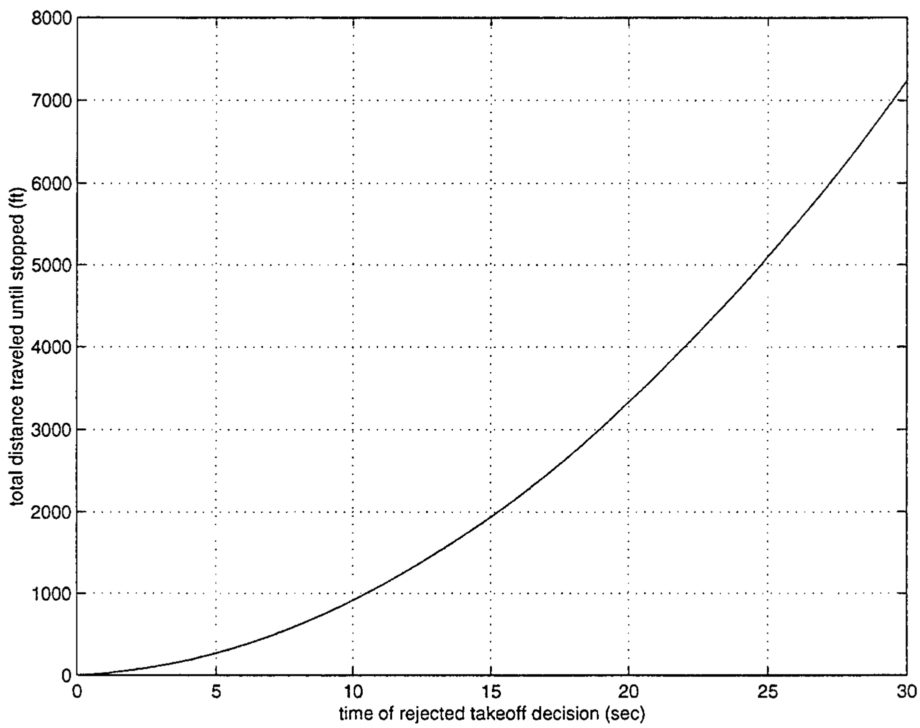


Figure 57. Total runway used versus t_{evade}*, time evasive action began for the nominal takeoff profile.*

4.3 ANALYSIS FOR TOWER CAB ALERTS

4.3.1 Timing Analysis

The reaction times are taken to be the same as those used in Section 3. These include the controller reaction time, VHF channel availability, and the reaction time of the pilot from the start of the controller message. These probability density functions (pdfs) are reproduced in Figure 58 along with the convolution. Figure 59 is the cumulative distribution represented by their convolution of the three pdfs, which indicates that the time required with 95% confidence to begin evasive action after the system alerts is 17.4 seconds. This is 17.4 seconds after the surveillance system detects that the aircraft has accelerated to 30 knots. Accelerating to 30 knots at 0.26g requires 6.05 seconds. With perfect surveillance, the t_{evade} = 23.45 seconds for 95% confidence. From Figure 57, with a t_{evade} = 23.45 seconds, the total distance traveled down the runway is 4500 feet. If the surveillance system added only 2 seconds so that t_{evade} = 25.45 seconds, the total distance covered increases to 5270 feet.

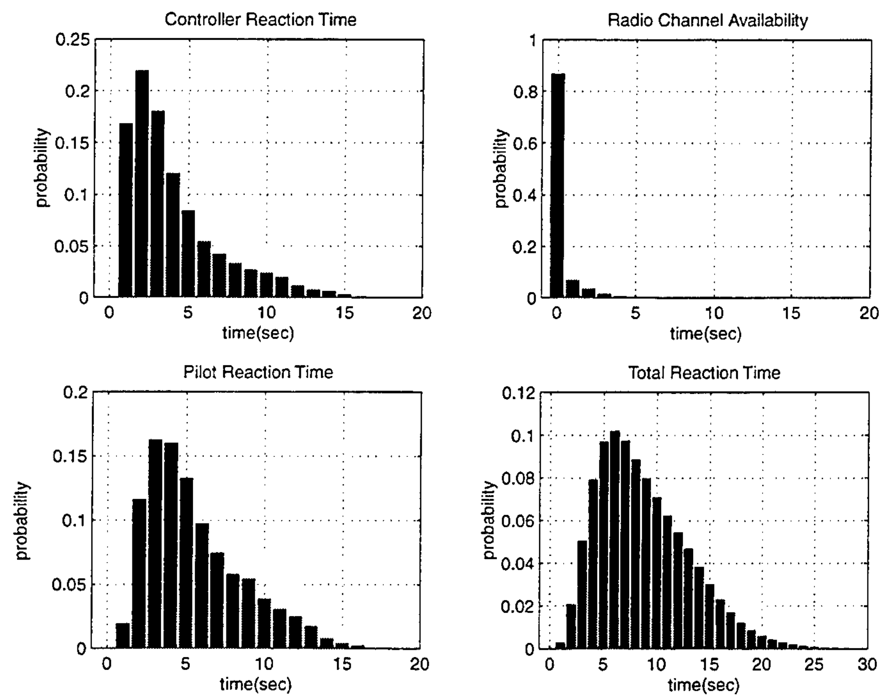


Figure 58. Probability density functions for reaction times for tower cab alerting systems.

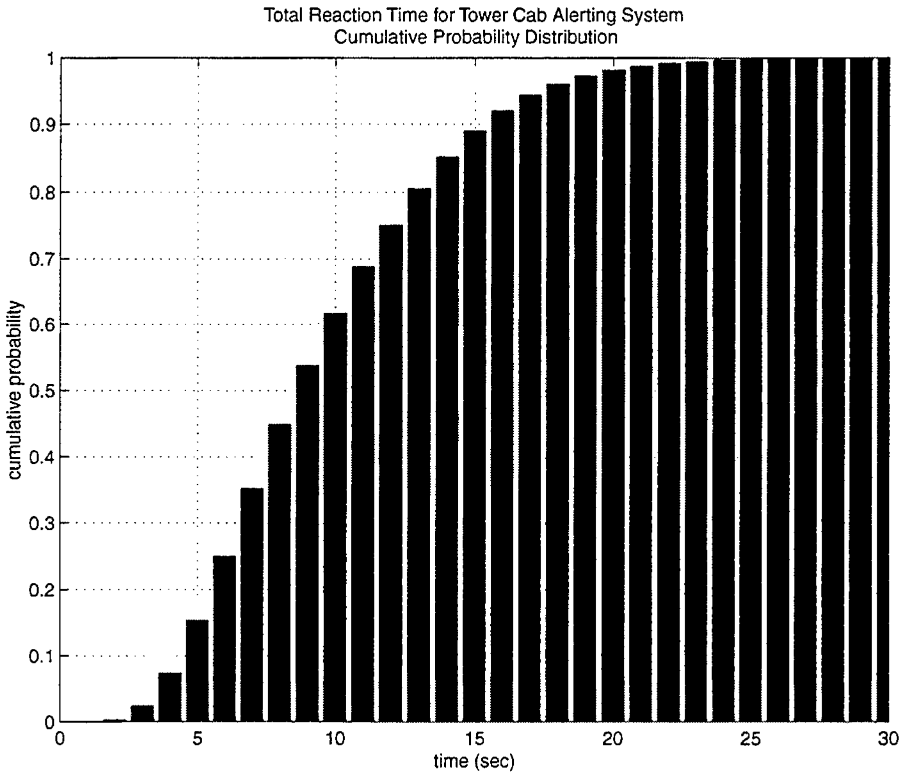


Figure 59. Cumulative distribution function for total reaction time for alerting systems.

4.3.2 Results for Tower Cab Alerts

The conclusion from the timing analysis presented above is that a tower cab alerting system cannot reliably protect a reasonable portion of the runway regardless of the surveillance parameters. The reason is that it is normal operations to have an aircraft, in motion at the takeoff end of the runway, while the runway is obstructed with another aircraft. The only way to avoid nuisance alarms is to wait until the departure aircraft has begun takeoff acceleration and by that time it is too late to protect a large portion of the runway.

4.4 ANALYSIS FOR DIRECT COCKPIT ALERTS

4.4.1 Timing Analysis

For the purposes of this analysis, it is assumed that a properly designed direct cockpit alerting system that incorporates human factors, coupled with proper pilot training can provide a pilot reaction time for a rejected takeoff of 3 seconds or less. The calculation of t_{evade} will be the sum of the time it takes the surveillance system to determine that the aircraft has reached 30 knots plus the pilot reaction time of 3 seconds. The time for the surveillance and alerting system to detect a 0.26g takeoff is presented as a function of surveillance update interval and position error in Table 6. Adding 3 seconds for the pilot reaction time to the results in Table 6 results in Table 17, the time of evasive action t_{evade} as a function of the surveillance system parameters.

Table 17. Time of the Start of Evasive Action t_{evade} as a Function of Surveillance Update Interval $\Delta\tau$ and Position Error σ_s for a .26g Takeoff with a 30 knot Threshold Velocity for Detecting a Takeoff.

	$\Delta\tau$ (seconds)				
σ_s (feet)	**0.5**	**1.0**	**1.5**	**2.0**	**3.0**
0	9.6	10.0	10.6	11.1	12.0
5	10.2	10.9	11.4	12.0	13.1
10	10.9	11.7	12.3	12.9	14.1
15	11.5	12.5	13.2	13.9	15.1
20	12.2	13.3	14.1	14.8	16.1
50	16.2	18.2	19.4	20.4	22.2
100	22.8	26.3	28.3	29.7	32.4

4.4.2 Analysis for Direct Cockpit Alerts

Using the time of the start of evasive action given in Table 17, the total runway used is computed as described above and shown in Figure 57. The results are presented in Table 18, showing the total runway used as a function of the surveillance parameters.

Table 18. Total Runway Used in Feet for a Rejected Takeoff as a of Surveillance Update Interval $\Delta\tau$ and Position Error σ_s for a .26g Takeoff with a 30 knot Threshold Velocity for Detecting a Takeoff.

	$\Delta\tau$ (seconds)				
σ_s (feet)	**0.5**	**1.0**	**1.5**	**2.0**	**3.0**
0	842	924	1009	1099	1288
5	952	1065	1170	1275	1496
10	1068	1216	1343	1465	1719
15	1190	1377	1528	1668	1958
20	1320	1547	1724	1884	2212
50	2234	2780	3149	3455	4060
100	4287	5628	6469	7119	8374

Contour lines representing the total runway used as a function of the surveillance parameters are presented in Figure 60 using the data in Table 18.

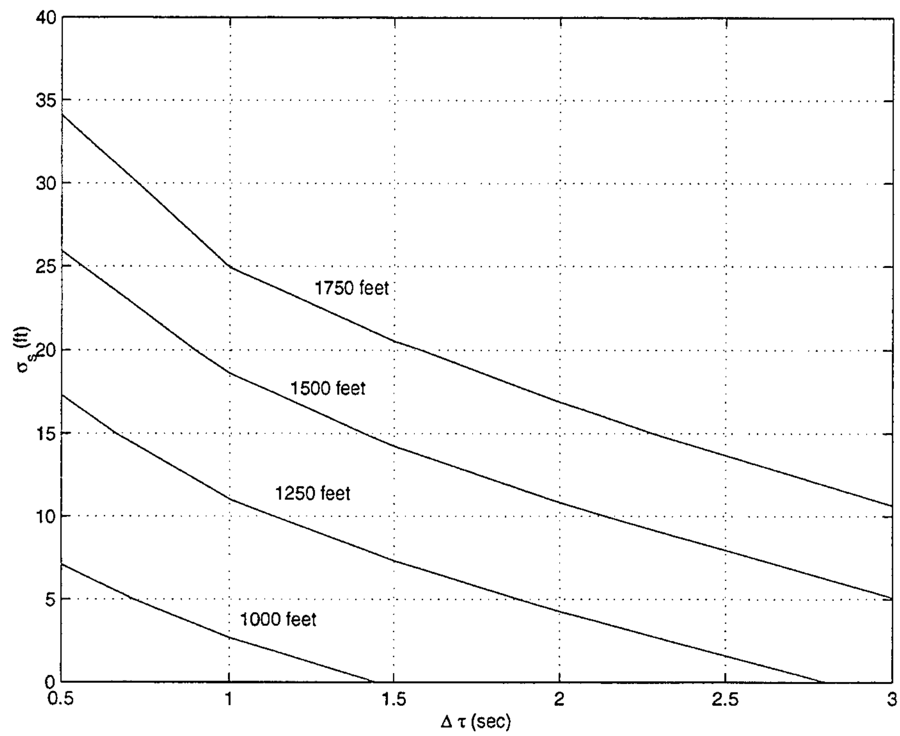


Figure 60. Contour lines for total runway used in a rejected takeoff as a function of surveillance update interval and position error for a .26g takeoff with a 30 knot threshold velocity for detecting a takeoff.

4.4.3 Results for Direct Cockpit Alerts

Direct cockpit alerts are effective in protecting the runway beyond 1000 feet to 1500 feet depending on the surveillance update interval $\Delta\tau$ and positional uncertainty σ_s. A positional uncertainty of 10 feet and an update interval of 1 second will protect the runway past approximately 1200 feet. The portion of the runway protected is not very sensitive to the surveillance system as seen in Figure 60. An aircraft will require 375 feet to accelerate to 30 knots, begin evasive action, and come to a stop. If we introduce only an additional pilot reaction time of 3 seconds then the runway required is 765 feet. This can be considered a system with instantaneous perfect surveillance. Depending on the visibility at takeoff, an obstruction within the first 1000 feet or so, especially an aircraft, should be visible to the takeoff aircraft.

4.5 ANALYSIS FOR RUNWAY STATUS LIGHT SYSTEM

4.5.1 Approach

The runway status light system utilizes takeoff-hold lights to show that a runway is obstructed and not safe for takeoff. These lights are visible to an aircraft at the takeoff end of the runway in position to start a takeoff roll. While the design is not final, it is envisioned that there will be two sets of takeoff-hold lights, one set near the end of the runway and one set approximately 700 feet down the runway. For the purposes of this analysis, it is assumed that the takeoff-hold lights are no longer effective after a departure aircraft has traveled 450 feet. This may be somewhat conservative. An aircraft that accelerates at .26g's will travel 450 feet in approximately 10.4 seconds and will have attained a velocity of a little over 50 knots.

A blocked runway will activate the takeoff-hold lights and the departure aircraft will know that it is not safe to depart. Unlike the case of tower cab alerts, it is not necessary to determine that the aircraft is departing because there will be no nuisance alerts. The lights simply indicate that the runway is obstructed.

As in Section 3, the analysis will investigate the case where a taxi aircraft taxis onto the runway at or near the same time the takeoff aircraft begins its takeoff roll. The same three cases used in Section 3 will be examined but the analysis will investigate that portion of the collision region where the departure aircraft must reject the takeoff.

4.5.2 Collision Region

Refer to the collision region for Case 1 shown in Figure 61. The takeoff aircraft must reject the takeoff and come to a stop before it enters the danger zone. The data for the takeoff and taxi aircraft for Case 1 are given in Table 10 in Section 3.4.3. The taxi aircraft in Case 1 takes 13.3 seconds to enter the danger zone after it begins to taxi across the taxi-hold position. If both the taxi aircraft and the takeoff aircraft begin at the same time (delta taxi time = 0) then they both simultaneously enter the danger zone at 13.3 seconds at a point 740.46 feet down the runway as shown in Figure 61. If the taxi aircraft starts first (delta taxi time negative) then the aircraft would meet along the solid line in Figure 61 at a point closer than 740.46 feet. It will still take the taxi aircraft 13.3 seconds to reach the danger area but because the taxi aircraft starts first, the takeoff aircraft will travel a shorter distance. If the takeoff aircraft starts first (delta taxi time positive) then the two aircraft will meet in the danger zone further down the runway.

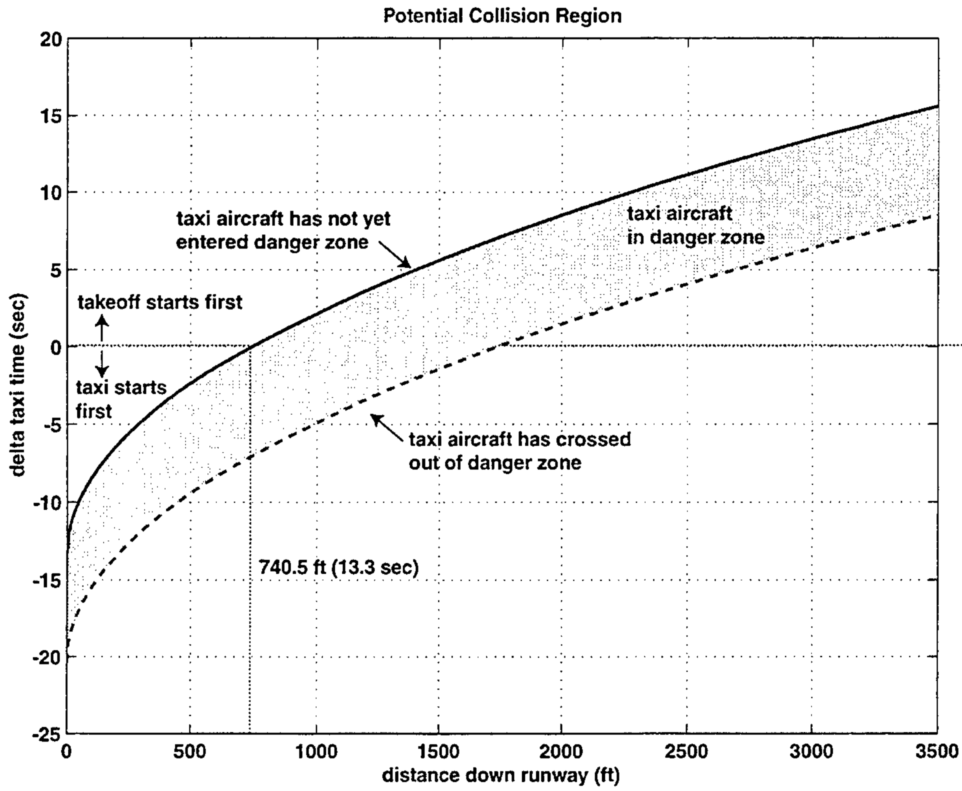


Figure 61. Potential Collision Region for Case 1.

4.5.3 Timing Analysis

The surveillance system must detect that the taxi aircraft has taxied past the taxi-hold position and turn on the takeoff-hold lights in time to prevent the collision. This must be before the takeoff aircraft has past the lights and in time for the aircraft to reject the takeoff and come to a stop before the intersection.

The times to detect that the taxi aircraft passes the taxi-hold position were computed in Section 3. Recall that the surveillance detection times were computed as the time required for the taxi aircraft, according to its taxi profile, to taxi a distance $2\sigma_s$ past the taxi hold line. The update interval $\Delta\tau$ was added to this time. An additional second was added for the system to turn on the lights and another second added for pilot reaction time. This resulted in Tables 9, 13, and 16 for Cases 1,2, and 3, respectively. These represent the times after the taxi aircraft begins to taxi before the rejected takeoff can be initiated. This corresponds to the time evasive action begins (t_{evade}) for the rejected takeoff motion profile described in Section 4.2 above.

Figure 57 in Section 4.2 above shows a plot of the total distance traveled as a function of the time evasive action begins for the rejected takeoff motion profile. The rejected takeoff motion profile assumes the aircraft accelerates at $a\left(ft / sec^2\right)$ until t_{evade}, the time evasive action begins. This is followed by two seconds at the maximum velocity attained followed by a braking deceleration of $b(ft / sec^2)$. Assuming the motion profile described above, the time evasive action must begin (t_{evade}) in order to stop short a distance s down the runway can be computed as:

$$t_{evade} = \frac{-2a + \sqrt{(2a)^2 + 4s\left(\frac{a(a+b)}{2ab}\right)}}{\frac{a(a+b)}{2ab}}$$

The time evasive action must begin in order to stop short of s can be compared with the time evasive action can begin as calculated above. This will be a function of the surveillance parameters. The surveillance parameters required to meet the time evasive action must begin are then determined.

For example, the time evasive action can begin relative to the start of motion of the taxi aircraft is given in Table 9 as a function of the surveillance parameters for Case 1. If the taxi aircraft and takeoff aircraft start simultaneously, then they will arrive simultaneously at the danger zone at a point 740.46 feet down the runway. As listed in Case 1 in Table 10, the acceleration is at 0.26g's or 8.37 ft/sec^2. The braking in all cases is assumed to be at .33 g's or 10.63 ft/sec^2. Using these values, the time evasive action must begin can be computed according to the equation above as t_{evade}= 8.89 seconds. This can be compared to the times in Table 9 to draw a contour line of acceptable surveillance parameters that will result in the takeoff aircraft stopping before the collision.

If the taxi aircraft were to start before the takeoff aircraft by 5 seconds (Δt_{taxi} = -5.0 sec.), then s would be 288.37 feet, which is the distance the takeoff aircraft would travel in 8.3 seconds. This is the time it takes for the taxi aircraft to reach the danger zone, 13.3 seconds in Case 1, less the five second lead time of the taxi aircraft. The time evasive action must begin is computed as t_{evade}= 5.19 seconds for a distance of 288.37 feet. This is compared to the times listed in Table 9, but with 5 seconds added to the times. The 5 seconds are added because t_{evade} is computed relative to the start of the takeoff aircraft and the times in Table 9 are relative to the start of the taxi aircraft. A contour line of acceptable surveillance parameters can then be drawn. Similarly contour lines can be drawn for positive delta taxi times where the taxi aircraft starts after the takeoff aircraft. Note that if a t_{evade} greater than 10.37 seconds is required, the system is not effective because in the Case 1 takeoff profile the takeoff aircraft will have passed the 450 foot distance considered the limit of effectiveness for the takeoff-hold lights. This puts a limit on the area of the collision region that can be protected by the takeoff-hold lights.

4.5.4 Case 1

Following the procedures described above, effective contour lines for three delta taxi times were computed for Case 1 and are shown in Figure 62. For the motion profiles described in Case 1, taxi aircraft that start later than 1.99 seconds after the takeoff aircraft will not be in a position to turn on the takeoff-hold lights in time. This is computed as follows. The time for the takeoff aircraft to reach 450 feet is 10.37 seconds. After this point the takeoff-hold lights are assumed to be not effective. If evasive action were to begin at 10.37 seconds the aircraft would come to a stop 978.15 feet down the runway. For a normal takeoff without evasive action it will take the aircraft 15.29 seconds to reach a point 978.15 feet down the runway. The taxi aircraft requires 13.3 seconds to reach the danger zone so it would have to start its motion 1.99 seconds after the takeoff aircraft begins in order to reach the danger zone simultaneous with the takeoff aircraft at a point 978.15 feet down the runway. At any point less than 978.15 feet, the taxi aircraft would have to start its motion less than 1.99 seconds after the takeoff aircraft in order to reach the danger zone in time to meet the takeoff aircraft. For taxi aircraft that start later than 1.99 seconds after the start of the takeoff aircraft, the delta taxi time would be added to the times in Table 9, which represents the earliest time evasive action can take place. However, the time required for the takeoff aircraft to begin evasive action can never go higher than 10.37 seconds. This greatly reduces the effectiveness of the takeoff-hold lights for that portion of the collision region. As an example, if the taxi aircraft were to begin 11 seconds after the takeoff aircraft then there is a point down the runway where the two aircraft would meet as shown in the collision region in Figure 61. However, even with perfect surveillance, the takeoff aircraft would be past the region where the takeoff-hold lights were effective so no evasive action would be possible.

Compare the results in Figure 62 with the results for Case 1 for the runway entrance lights given in Figure 47 in Section 3.4.3. The surveillance requirements for the rejected takeoff for $\Delta t_{taxi} = 0$ seconds are less stringent than those for stopping the taxi aircraft shown in Figure 47, even for the across pavement lights. The difference between the two lines in Figure 47 represents the difference in time between when the off pavement lights are effective and when the across pavements lights are effective. According to Table 10 this is the difference between 13.3 seconds and 9.89 seconds or 3.41 seconds. This indicates that if the dividing line between the portion of the collision region protected by the runway entrance lights and takeoff-hold lights is shifted to $\Delta t_{taxi} = 2.0$ seconds, the surveillance requirements for both will be approximately the same, and correspond to the $\Delta t_{taxi} = +1.99$ second line in Figure 62.

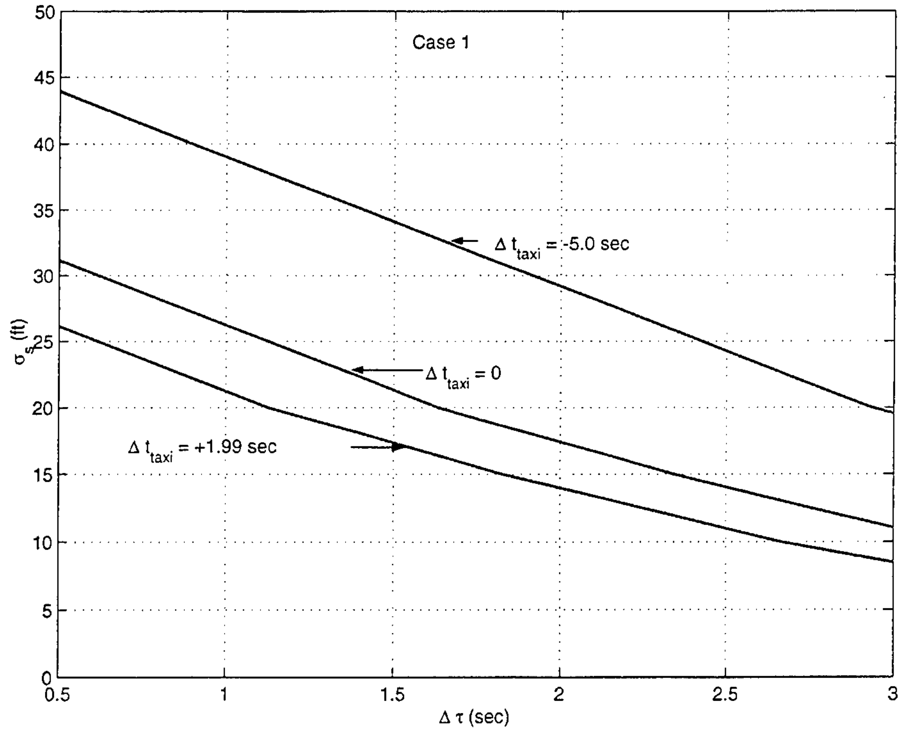


Figure 62. Surveillance requirements for rejected takeoff for Case 1.

4.5.5 Case 2

The data describing Case 2 were presented in Section 3.4.4 in Table 10. This was a worst-case scenario with a B747 as the takeoff aircraft and a fast taxi towards the runway. The results for the runway entrance lights required that the taxi aircraft start approximately seven seconds after the start of the takeoff aircraft (Δt_{taxi} = 7 sec.) in order for the runway entrance lights to be effective. This left a portion of the potential collision region uncovered as shown in Figure 50.

The takeoff-hold lights are analyzed in the same manner as in Case 1. The earliest time that the surveillance and alerting system can detect that the taxi aircraft has crossed the takeoff-hold position, turn on the takeoff-hold lights, and the pilot of the takeoff aircraft can begin evasive action is given in Table 12. This includes one second for the lights to turn on and one second for the pilot to react.

The takeoff aircraft acceleration is .20g's and the takeoff aircraft will reach the point 450 feet down the runway where the takeoff-hold lights are assumed not effective, in 11.82 seconds after takeoff. If the aircraft were to begin evasive action at this point it would come to a stop 875 feet down the runway. Without evasive action the takeoff aircraft would reach this point in 16.48 seconds. A taxi aircraft following the taxi profile described for Case 2 will enter the danger zone 9.71 seconds after it starts taxiing. Therefore the taxi aircraft would have to start 6.77 seconds after the takeoff aircraft (Δt_{taxi} = +6.77 sec.) in order to meet at that point down the runway. The point 875 feet down the runway is significant because for points further down the runway, the taxi aircraft can start even later and still enter the danger zone but the takeoff aircraft must see the takeoff-hold lights by 11.82 seconds after the start of takeoff. The contour for effective surveillance parameters for this point on the collision region is found by comparing 11.82 seconds with the times in Table 12 with 6.77 seconds added to the values. Adding the 6.77 seconds is required because the taxi aircraft starts 6.77 seconds after the takeoff aircraft. The 11.82 seconds represents the latest time after the start of takeoff that the takeoff-hold lights are effective. This contour for $\Delta t_{taxi} = +6.77$ sec. is shown in Figure 63.

Contour lines are shown for simultaneous motion of the two aircraft ($\Delta t_{taxi} = 0$ sec.) and for the case where the taxi aircraft start five seconds before the takeoff aircraft ($\Delta t_{taxi} = -5.0$ sec.).

The contour line for $\Delta t_{taxi} = +6.77$ sec. will effectively cover the portion of the collision region left unprotected by the runway entrance lights but the surveillance requirements for this contour appear to stringent. Most likely only cases where the taxi aircraft starts at the same time or earlier than the takeoff aircraft will the takeoff-hold lights be effective.

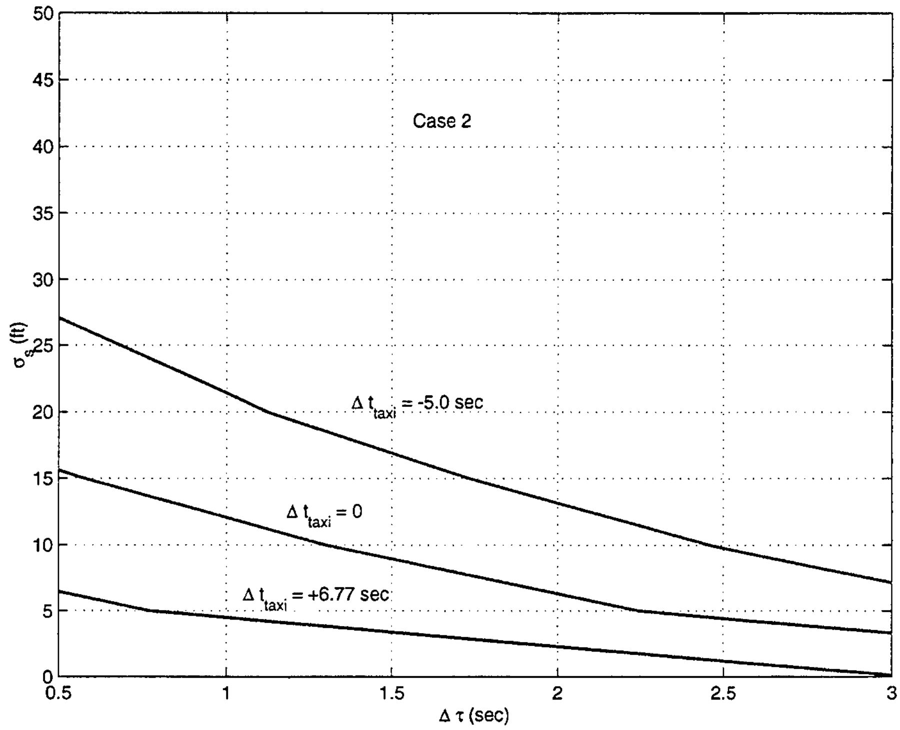


Figure 63. Surveillance requirements for rejected takeoff for Case 2.

4.5.6 Case 3

The data for Case 3 are presented in Table 13. This was a scenario with a relatively slower taxi profile and faster takeoff profile. The potential collision region is shown in Figure 52. Analysis presented in Section 3.4.5 show that the surveillance requirements for the runway entrance lights were not as tight as the other cases as shown in Figure 53.

Similarly, the analysis for the takeoff-hold lights, shown in Figure 64, indicate less stringent requirements for the surveillance. Note the scale change compared to the same graphs for Case 1 and Case 2. The contour line for Δt_{taxi} = -3.52 sec. represents that point on the collision region 1012 feet down the runway which is where the takeoff aircraft would stop if the lights were seen at the last possible point during the takeoff. In this case, the takeoff aircraft crosses the point where the takeoff-hold lights are no longer effective in 9.99 seconds.

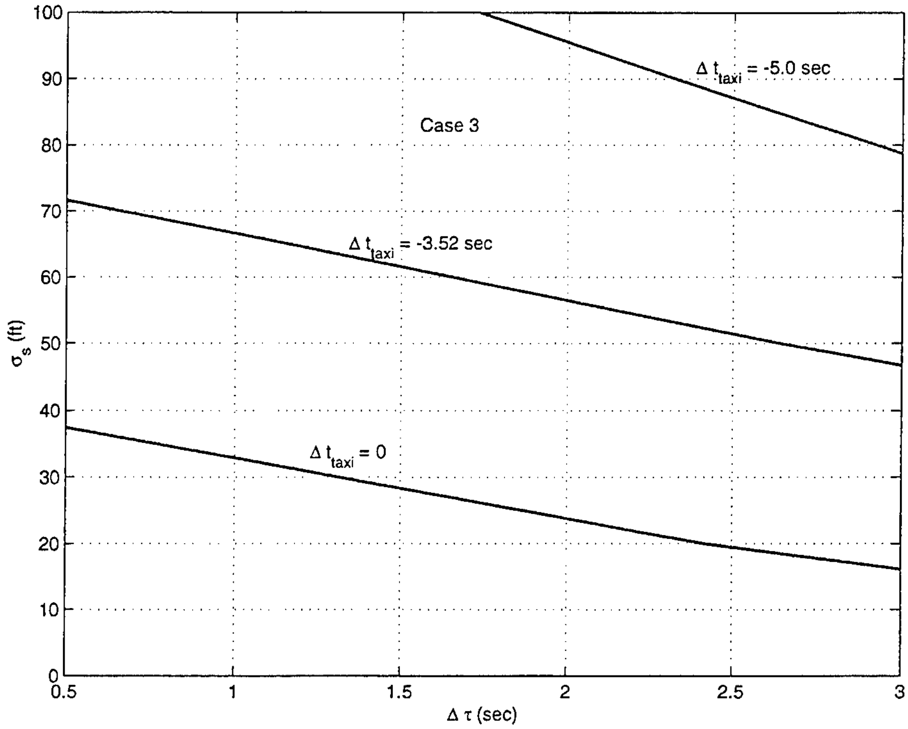


Figure 64. Surveillance requirements for rejected takeoff for Case 3.

4.5.7 Results for Takeoff-Hold Lights

The surveillance requirements for the takeoff-hold lights are similar in value to the requirements for the runway entrance lights. In Case 1, comparing Figure 62 to Figure 47, the surveillance requirements are less stringent for the takeoff-hold lights at Δt_{taxi} = 0 seconds than for the runway entrance lights even for runway entrance lights across the taxi pavement. For Case 1, a surveillance system with σ_s = 15 feet and update interval $\Delta\tau$ = 1 second will protect the entire collision region. The runway entrance lights will be effective for cases where the takeoff aircraft starts first and the take-off hold lights will be effective for cases where the taxi aircraft starts first. Both sets of lights will be effective for a simultaneous start of both aircraft. Case 2 was a much more challenging scenario. The runway entrance lights are effective for cases where the takeoff aircraft starts approximately 7 seconds before the taxi aircraft. The takeoff-hold lights are effective for cases where the taxi aircraft starts at the same time or earlier than the takeoff aircraft with σ_s = 12 feet and update interval $\Delta\tau$ = 1 second. These leaves a portion of the collision region shown in Figure 50 unprotected. Case 3 was less challenging for both the takeoff-hold lights and the runway entrance lights. The more stringent requirements were on the takeoff-hold lights and the data shown in Figure 64 indicate that σ_s = 32 feet and update interval $\Delta\tau$ = 1 second are sufficient.

4.6 RESULTS FOR CATEGORY 2 INCURSION

The analysis shows that a tower cab alerting system cannot reliably protect a takeoff from collision in the case of an obstructed runway. This is because the system must wait until the aircraft is determined to be a takeoff and then alert the tower controller who in turn must have a voice channel available and then alert the pilot. The pilot in turn must react and begin evasive action. The timing analysis shows that this takes too long. A direct cockpit alerting system can protect all but the first 1250 feet of the runway with surveillance system parameters shown in Figure 60, such as σ_s = 12 feet and update interval $\Delta\tau$ = 1 second. The takeoff-hold lights do not require that the safety system determine that the departing aircraft has begun a takeoff. The takeoff-hold lights indicate that the runway is obstructed without producing nuisance alarms. Three cases were studied where a taxi aircraft began to taxi towards the runway near the time the takeoff aircraft began its takeoff. These were the same three cases analyzed for the runway entrance light system for incursion B. In this case, the portion of the potential collision region that could be protected by the takeoff-hold lights was analyzed and compared with the portion of the collision region protected by the runway entrance lights. The surveillance requirements for the takeoff-hold lights were less stringent than those for the runway entrance lights.

A surveillance positional uncertainty of σ_s = 15 feet and update interval $\Delta\tau$ = 1 second will be sufficient.

5. CATEGORY 3 INCURSION: ARRIVAL TO A BLOCKED RUNWAY

5.1 APPROACH

This scenario, depicted in Figure 65, involves an arriving aircraft to a blocked runway. The runway may be obstructed by a previous arrival, an aircraft taxing across or on the runway, an aircraft in position to depart, or a ground vehicle. It is assumed that the evasive action will be a go-around by the arriving aircraft.

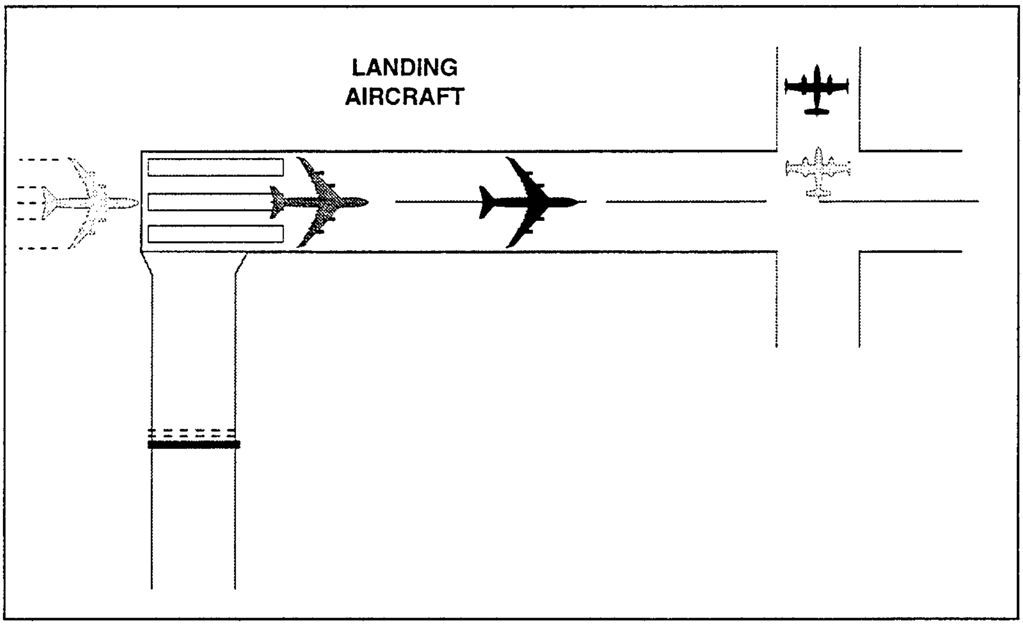


Figure 65. Category 3 incursion.

Detecting that a runway has an obstruction is not challenging to the surveillance system as long as the obstruction is of sufficient size and reflectivity. For a multilateration system, a transponder is required. The concern is whether or not the obstruction will clear the runway in time to prevent an incursion. The system must alert the arrival aircraft in time to execute a go-around in cases where continuing the arrival will cause an incursion but avoid the possibility of numerous nuisance alerts.

Two cases are considered in this section. In the first case the obstruction is an aircraft in position to depart and awaiting departure clearance. The challenge for the safety system is to determine whether the departing aircraft will begin its takeoff roll in time to prevent a loss of separation. If the aircraft does not depart, the arrival aircraft must be warned in time to execute a go-around. If the aircraft does depart in time to avoid a loss of separation, a nuisance alert must be avoided. In the second case, the obstruction is a previous arrival or a crossing taxi aircraft that has not yet cleared the runway. If the aircraft on the ground exiting the runway is past the taxi-hold position when the arrival crosses the runway threshold, then no incursion occurs. The challenge again is to warn the arrival aircraft in time to prevent an arrival to a blocked runway but not issue nuisance alerts.

5.2 CASE 1 ARRIVAL WITH DEPARTURE IN POSITION

5.2.1 Case 1 Timing Analysis

The runway separation requirement for Same Runway Separation (SRS) Category III aircraft (everything except light single and twin engine propeller aircraft) is that the departing aircraft must have crossed the runway departure end before the arriving aircraft crosses the landing threshold. There is an exception if the controller can determine distances by references to suitable landmarks, and the departing aircraft is airborne. In that case, the departing aircraft need not have crossed the other end of the runway as long as it is at least 6000 feet beyond the runway approach threshold when the arriving aircraft crosses the threshold. The assumption made for this analysis is that 6000 feet minimum separation is sufficient to avoid an incursion. If an alert occurs and this separation is subsequently lost, then that is not considered a nuisance alarm.

The timing analysis is illustrated in Figure 66. In this example it is assumed that the arrival aircraft has a ground speed of 150 knots maintained to the runway threshold. The departing aircraft is assumed to accelerate to 150 knots at 0.26g's and then maintain 150 knots. The horizontal axis in the upper plot of Figure 66 is time with zero being the time the takeoff aircraft begins its takeoff roll. The vertical axis is distance from the runway threshold with positive values for distance down the runway and negative values for distances before the threshold. The linear distance versus time plot for the arrival aircraft is adjusted to maintain a minimum separation of 6000 feet from the departing aircraft at the threshold. Because the two aircraft have the same speed in this example that separation is maintained. The lower plot in Figure 66 shows the arrival aircraft's altitude and distance from the threshold consistent with the time scale of the upper plot. This assumes a 3-degree glide slope with a 55 foot threshold crossing height. Touchdown point is 1049 feet down the runway. It is assumed that the final time a go-around can be executed is when the aircraft is at the standard Category I decision height of 200 feet above touchdown level.

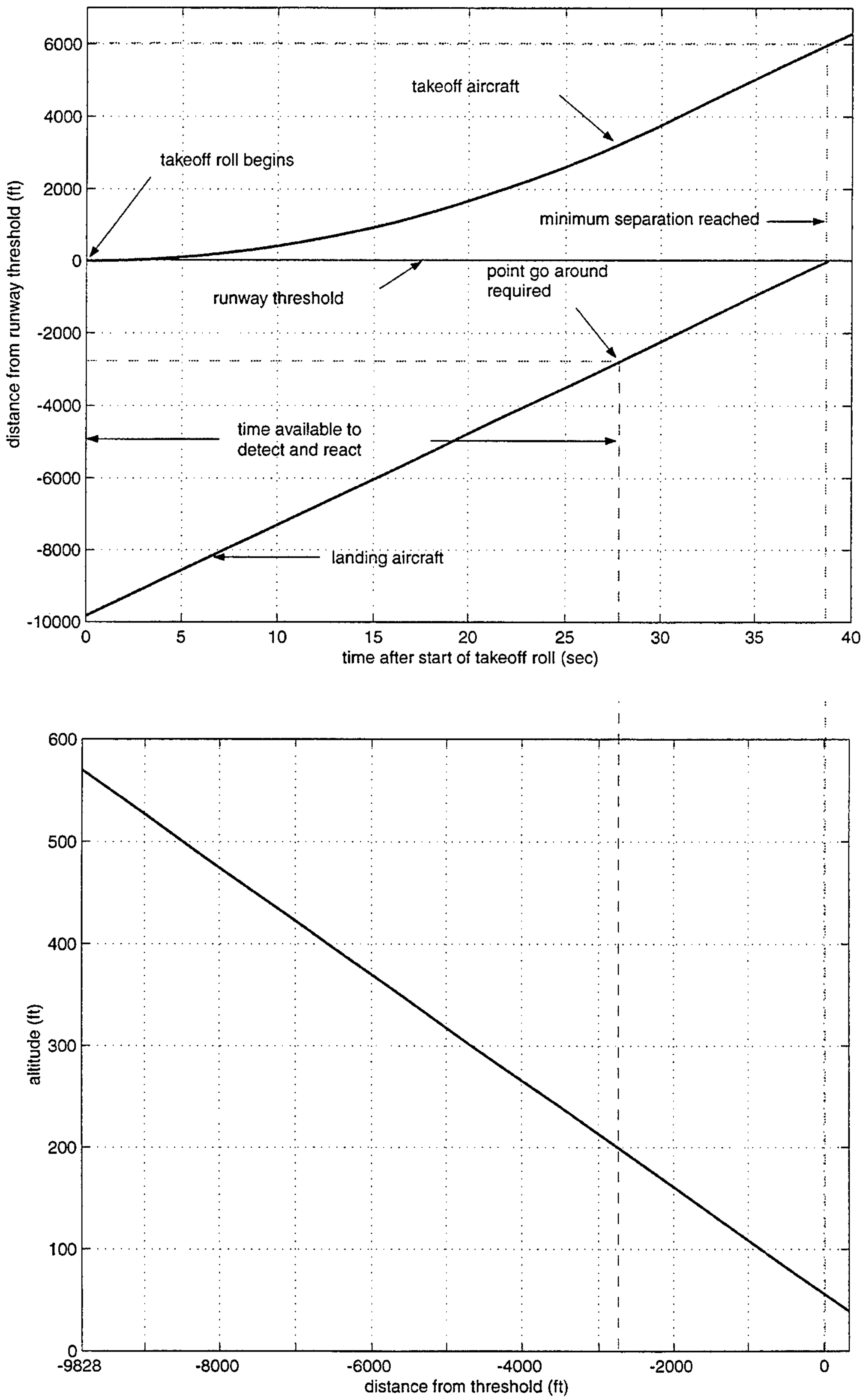
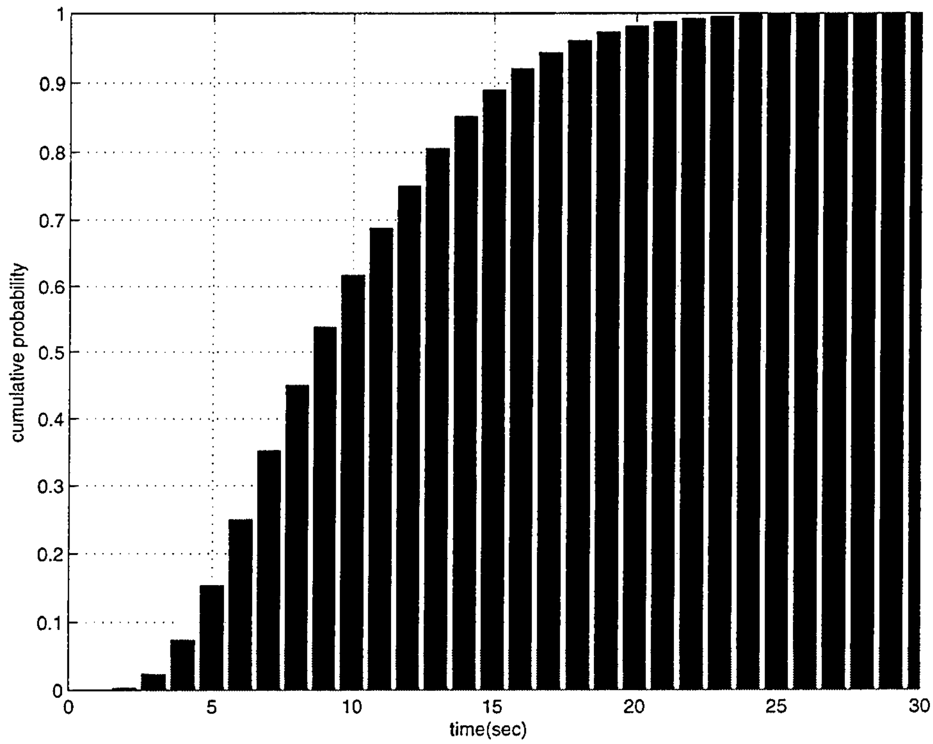


Figure 66. Timing analysis for a 150 knot arrival and a 0.26g acceleration to 150 knot departure.

5.2.2 Results for 150-knot Arrival and 0.26g Departure

In this example, it takes the takeoff aircraft 38.8 seconds to reach 6000 feet down the runway. Therefore the takeoff aircraft must begin its takeoff roll when the arriving aircraft is 38.8 seconds and 9828 feet away from crossing the threshold in order to meet the 6000-foot separation requirement. At this point the arrival aircraft is at an altitude of 570 feet. The go-around must be executed at 27.9 seconds after the last time the takeoff aircraft must have started its takeoff roll. The arrival aircraft will be at an altitude of 200 feet and 2767 feet and 10.9 seconds from the threshold. Therefore, there is 27.9 seconds available to detect that the aircraft in position to depart is not a takeoff and to react. The reaction times are the same as those used in the earlier incursions. The cumulative distribution function for the controller reaction time, channel availability, and pilot reaction time is duplicated in Figure 67.



Confidence Level	time
50%	8.575
80%	12.9077
95%	17.379
99%	21.5063

Figure 67. Total reaction times and confidence levels for controller reaction to the alert, VHF channel availability, and pilot reaction time.

This distribution can be interpolated to provide confidence levels for total reaction times. The total reaction time for the 50%, 80%, 90%, and 99% confidence levels are given in the table below the graph. These times, subtracted from the total time available of 27.9 seconds, leave the time available to the surveillance system to detect that the takeoff aircraft is departing. The time required for the surveillance and alerting system to detect a 0.26g takeoff as a function of surveillance update rate and position error was developed in Section 3.4.2 and given in Table 6. The table is reproduced below as Table 19. Matching the time available against the surveillance time requirements listed in Table 19 derive contours of surveillance requirements for the different confidence levels. The results are shown in Figure 68 for the case described above; a 150 knot arrival and a 0.26g acceleration to 150 knot takeoff.

Table 19. Time Required for the Surveillance and Alerting System to Detect a .26g Takeoff Using a 30 knot Threshold Velocity as a Function of Surveillance Update Interval $\Delta\tau$ and Position Error σ_s

	$\Delta\tau$ (seconds)				
σ_s (feet)	**0.5**	**1.0**	**1.5**	**2.0**	**3.0**
0	6.6	7.0	7.6	8.0	9.0
5	7.2	7.9	8.4	9.0	10.1
10	7.9	8.7	9.3	9.9	11.1
15	8.5	9.5	10.2	10.9	12.1
20	9.2	10.3	11.1	11.8	13.1
50	13.2	15.2	16.4	17.4	19.2
100	19.8	23.3	25.3	26.7	29.4

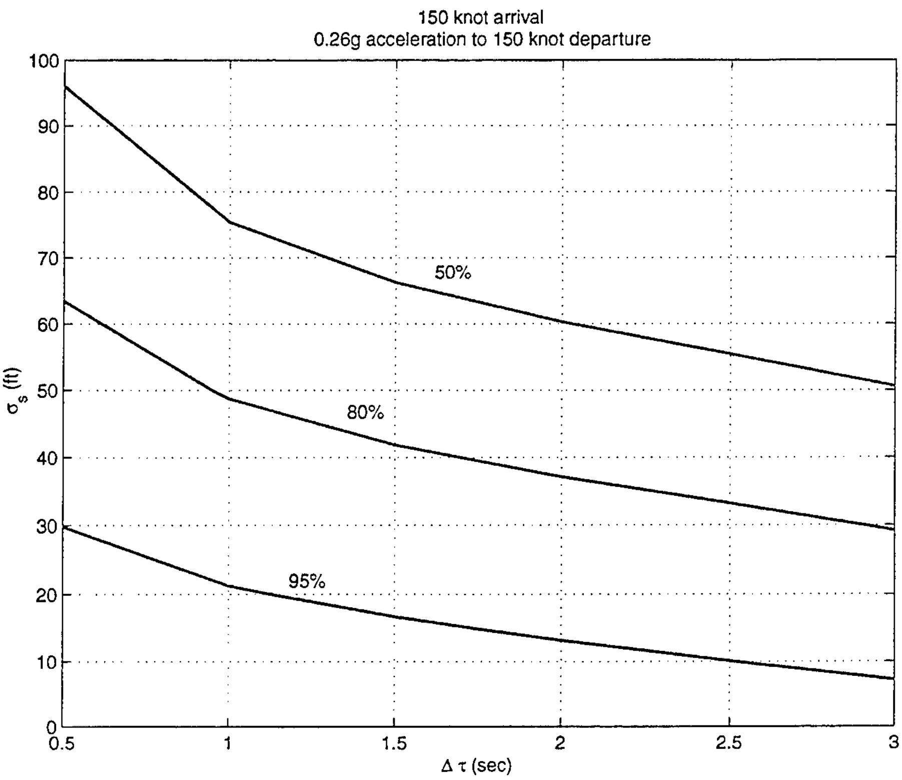


Figure 68. Surveillance requirements to detect a non-takeoff and react for a go-around to a blocked runway for various confidence levels of reaction time. The arrival aircraft has a 150 knot ground speed. The departure aircraft is a 0.26g acceleration to a 150 knot maximum velocity.

5.2.3 Results for 135 knot Arrival and 0.26g Departure

For comparison, Figure 69 shows the results for a 135 knot arrival and the same 0.26g acceleration to 150 knot departure. A slower arrival has the effect of rotating the landing aircraft time versus distance plot about the runway threshold crossing point. It still requires the 38.8 seconds for the takeoff aircraft to reach 6000 feet down the runway. However, a slower arrival aircraft is not as far from the threshold when the takeoff aircraft must start so that it reaches the threshold 38.8 seconds later. The 135 knot aircraft will take 12.14 seconds to travel from the go-around point to the threshold whereas the 150 knot aircraft will take 10.93 seconds. Both aircraft are 38.8 seconds from the threshold when the takeoff aircraft must start so that it is 6000 feet down the runway when the arrival aircraft crosses the threshold. This means the time available to detect and react for the 135 knot arrival is 26.68 seconds whereas the time available for the 150 knot arrival is 27.89 seconds. This is the consequence of having a fixed point on the approach where the arrival must go-around but adjusting the arrival timing so that the arrival aircraft will cross the threshold just as the departing aircraft reaches 6000 feet down the runway. The system must allow for different arrival speeds that provide legal spacing without giving a nuisance alarm.

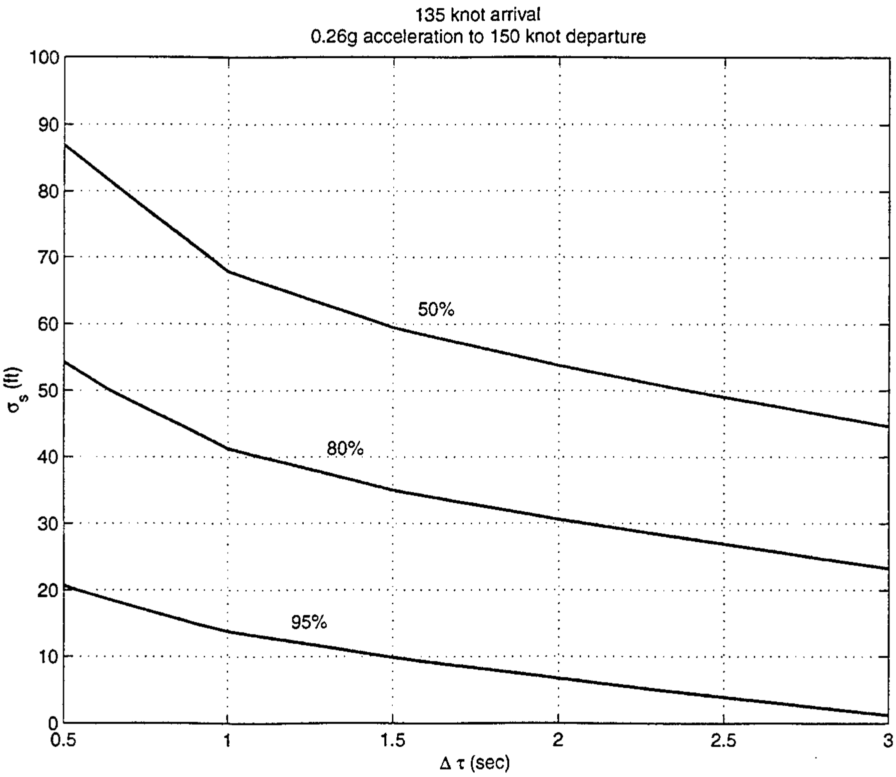


Figure 69. Surveillance requirements to detect and react for a go-around to a blocked runway for various confidence levels of reaction time. The arrival aircraft has a 135 knot ground speed. The departure aircraft is a 0.26g acceleration to a 150 knot maximum velocity.

5.2.4 Results for 150 knot Arrival and 0.20g Departure

A slower acceleration takeoff will take longer to detect and will take longer to reach a point 6000 feet down the runway. Assume a 0.20g acceleration to 150 knots for the takeoff aircraft and the same 150 knot arrival aircraft. The time to detect the takeoff as a function of surveillance parameters is given in Table 20. This was computed as described in Section 3.4.2.

Table 20. Time Required for the Surveillance and Alerting System to Detect a .20g Takeoff Using a 30 knot Threshold Velocity as a Function of Surveillance Update Interval $\Delta\tau$ and Position Error σ_s

	$\Delta\tau$ (seconds)				
σ_s (feet)	**0.5**	**1.0**	**1.5**	**2.0**	**3.0**
0	8.4	8.9	9.4	9.9	10.9
5	9.2	9.9	10.5	11.1	12.2
10	10.1	11.0	11.7	12.3	13.5
15	11.0	12.0	12.8	13.5	14.8
20	11.8	13.1	14.0	14.7	16.1
50	17.0	19.4	20.9	22.0	24.1
100	25.6	30.0	32.4	34.2	37.3

It also takes longer for the takeoff aircraft to reach 6000 feet. In this case the aircraft will reach 6000 feet in 43.4 seconds. It takes 10.9 seconds for the arrival to travel from the go-around point to the threshold. This leaves 32.4 seconds for detection and reaction before the arrival must go around. In the same manner as before, the reaction times for several confidence levels are subtracted from the 32.4 seconds and the remaining time compared to Table 20 and contour lines for acceptable surveillance parameters is derived. The results for this case are shown in Figure 70. Even though it takes longer to detect the takeoff (compare Table 20 to Table 19), it takes longer (43.4 sec vs. 38.8 sec) for the 0.20g acceleration aircraft to reach 6000 feet. The takeoff must begin 43.3 seconds before the arrival aircraft reaches the threshold. The go-around point remains the same, 10.9 seconds from the threshold. This gives more time in the case of the 0.20g acceleration aircraft to detect and react. This is why the surveillance requirements are less stringent in Figure 70 than for the .26g takeoff shown in Figure 68.

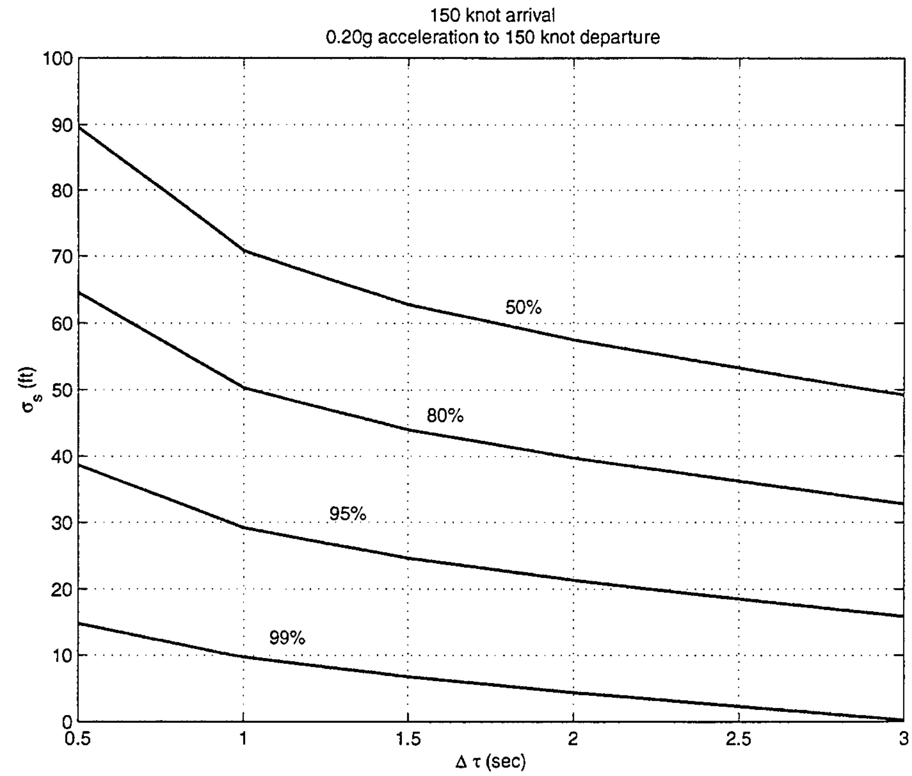


Figure 70. Surveillance requirements to detect and react for a go-around to a blocked runway for various confidence levels of reaction time. The arrival aircraft has a 150 knot ground speed. The departure aircraft is a 0.20g acceleration to a 150 knot maximum velocity.

5.3 CASE 2 ARRIVAL WITH PREVIOUS ARRIVAL EXITING OR TAXI AIRCRAFT CROSSING

An aircraft on the runway, whether it is a previous arrival or an aircraft taxiing along or across the runway, will be in one of two states. It will be in a state where it is able to exit the runway before the arrival crosses the threshold or it will be in a state where it cannot exit the runway before the arrival crosses the threshold. If the aircraft is in a state where it cannot exit the runway before the arrival crosses the threshold, then the arrival aircraft should be alerted and executes a go-around.

In order to determine the state of the aircraft on the runway the surveillance and alerting system must know its position relative to runway exits, velocity, and achievable maneuvering (acceleration, braking, and turning.) An aircraft can go from the state where it is able to exit the runway before the arrival aircraft crosses the threshold to the state where it cannot, very quickly, i.e., missing a high-speed exit.

If the aircraft on the runway is in a state where it can exit the runway before the arrival crosses the threshold, then an alert cannot be issued because it would be considered a nuisance alert. Separation standards are met as long as the aircraft on the runway exits the runway before the arrival aircraft crosses the threshold. Exiting the runway means crossing past the taxi-hold position.

Average runway occupancy times at major airports are approximately 50 to 60 seconds [5]. A three-mile separation distance (some airports allow 2.5 miles) is covered by an aircraft traveling at 135 knots in 80 seconds. During normal operations there is only 20 to 30 seconds between the exit of a landing aircraft and the arrival at the threshold of the next aircraft. Given the total reaction time cumulative probability distribution function, this is not enough time to alert the aircrew of the arriving aircraft.

An aircraft that is apparently exiting the runway can suddenly stop short with little or no warning moments before the arrival aircraft crosses the runway threshold. There is no system that can prevent the incursion in this case, although the accident may be prevented if there is enough distance to the blocked taxiway.

Figure 45 in Section 3.4.2 shows the values of σ_v as a function of σ_s and $\Delta\tau$. A surveillance system that can predict the state of the taxi aircraft will have to track velocity and position with a high degree of precision. This is because the taxi or exiting aircraft will be slowing down to speeds between 10 and 60 knots. High-speed exits are designed to handle exiting aircraft traveling at speeds up to 60 knots. Taxi aircraft are typically traveling at much slower speeds. In order to predict an aircraft's ability to exit the runway and it exit time, the velocity estimate must be accurate. This means that σ_s must be on the order of 5 to 10 feet and $\Delta\tau$ 1 second. Even with this surveillance, there is no assurance that an exiting aircraft will not stop suddenly or will miss an exit option.

Given these considerations, it seems that it will be very difficult to design an effective alerting system that will prevent these categories of incursions except in cases where it is obvious that an aircraft cannot exit the runway for a period of time.

In practice, controllers must monitor arrivals and crossing taxi aircraft and anticipate that an exit from the runway may not occur in order to alert the succeeding arrival.

An alternative approach would be to provide situational awareness to the arriving aircraft that provides an indication that a runway is obstructed and the position of the obstruction. This will allow the pilots to anticipate situations where the runway may be obstructed as they approach the threshold.

Currently there are no plans to use the runway status light system to provide information on runway obstructions to arriving aircraft. However, the technology does exist to provide cockpit display of traffic information including traffic on the ground as long as the traffic is equipped with a transponder.

6. PRACTICAL APPLICATION

The analysis presented in Sections 3 through 5 examine three types of runway incursion prevention systems and provide results for individual categories of incursions. In order to determine the overall surveillance requirements to support a safety system, it is worthwhile to apply the results of the analysis presented above in a practical application. Current FAA plans call for the deployment of a new surface surveillance system at selected airports. The Airport Surveillance Detection Equipment X band radar (ASDE-X) tracks targets using both primary radar and multilateration of the aircrafts' Modes S squitter signals. The multilateration portion of the ASDE-X has been demonstrated to provide a positional accuracy of $\sigma_s = 20$ feet in one dimension with an update interval of $\Delta\tau = 1$ second [9]. In addition, it may be possible to decrease the update interval to $\Delta\tau = 0.5$ second for selected targets.

The safety system considered here is one with direct alerts to the tower cab together with a runway status light system. Direct alerts to the aircraft are not considered because there are no systems currently available. The surveillance parameters of $\sigma_s = 20$ feet and $\Delta\tau = 1$ second are applied to the three cases of motion for the taxi and takeoff aircraft analyzed for the runway status light system in Sections 3 and 4. The portion of the potential collision region that is protected by the status lights can determined as well as the portion of the potential collision region that might be protected by alerts to the tower cab. In addition, the effectiveness of the tower cab alerts in preventing landings on a blocked runway is analyzed. This provides an assessment of the overall effectiveness of a safety system with a demonstrated surveillance system. In addition, the potential collision region is compared with a specific runway and taxiway configuration at the Dallas/Fort Worth International airport, which is a candidate for a demonstration runway safety system utilizing both tower cab alerts and runway status lights.

The approach is as follows. The taxi and takeoff motion profiles for Case 1, the "nominal" case, are given in Table 10. In order for the taxi aircraft to stop short of the takeoff aircraft, the taxi aircraft must begin braking at 11.3 seconds after it starts to taxi. From Table 7, the time required to detect the nominal takeoff and begin braking the taxi aircraft for a surveillance system with $\sigma_s = 20$ feet and $\Delta\tau = 1$ second is 12.3 seconds. This means that the portion of the collision region with a delta taxi time (time after the start of the takeoff that the taxi aircraft begins motion) of 1 second or more is protected by the runway entrance lights. Now note from Table 10 that it takes 13.3 seconds for the taxi aircraft to enter the danger zone. An aircraft with a takeoff acceleration of 0.26 gs will travel 740.5 feet in 13.3 seconds. In order to stop short of this distance the takeoff aircraft will have to begin evasive action, according to the equation in Section 4.5.3, at 8.9 seconds after starting the takeoff roll. According to Table 9, for Case 1, a surveillance system with $\sigma_s = 20$ feet and $\Delta\tau = 1$ second will require 8.3 seconds for the takeoff aircraft to begin breaking. This means the portion of the collision region with a delta taxi time of 0.6 seconds or less can be protected by the takeoff-hold lights. This leaves the region between a delta taxi time of 1.0 and 0.6 seconds unprotected, which is within the accuracy of the analysis. This is shown in Figure 71.

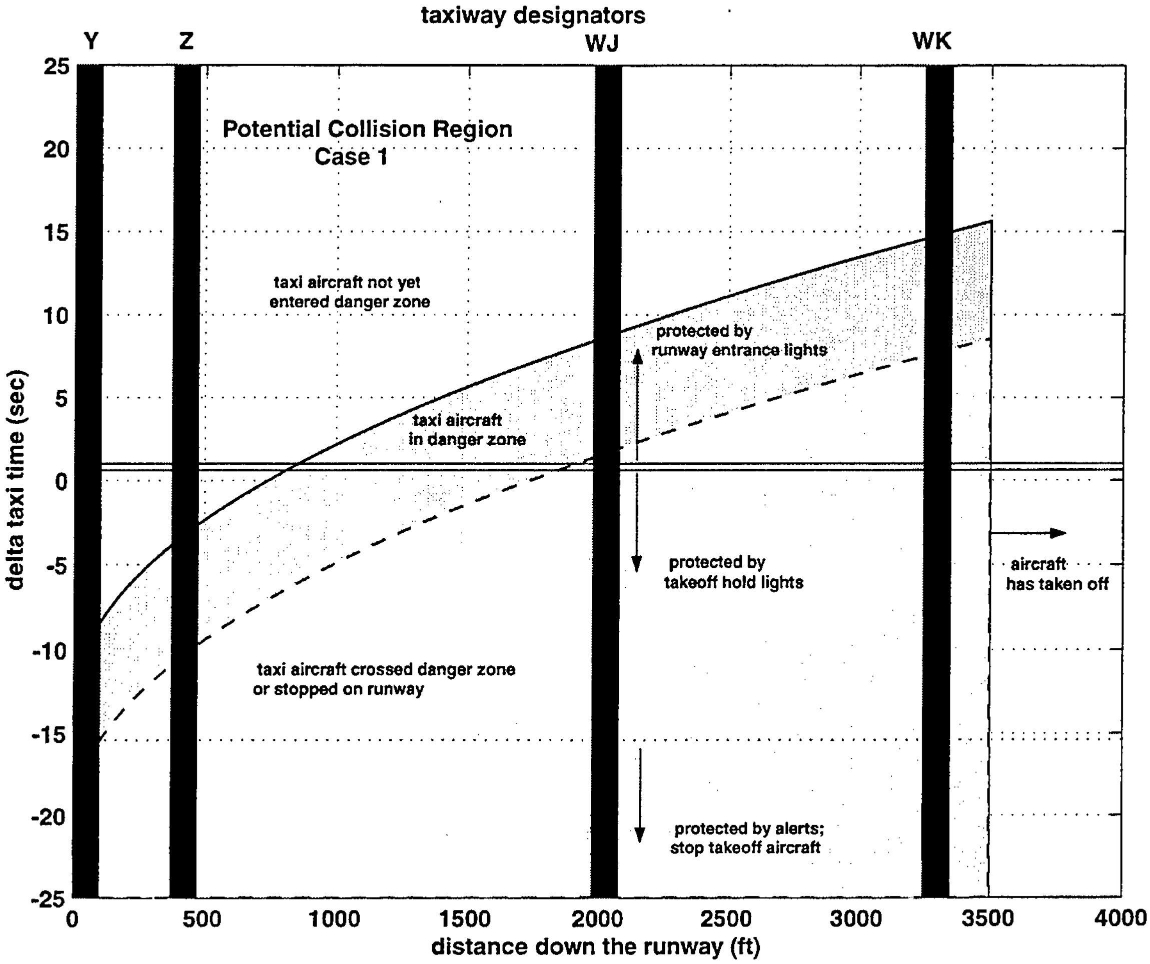


Figure 71. Potential collision region and protected area with σ_s= 20 feet and $\Delta\tau$ = 1.0 second for Case 1.

Now the effectiveness of the tower cab alerting system is analyzed for the same case. From Figure 6 we conclude that over 95% of the time evasive action will begin within 18 seconds after detection by the surveillance system. Because two seconds are included in the light system for reaction of the lights and pilot, the portion of the collision region protected by the alerting system can be depicted as the region protected by the status lights with 16 seconds added for the additional reaction time required by the tower cab alerting system. This is also illustrated in Figure 71 as the region protected by the alerting system. Thus for the nominal case we conclude that within the accuracy of the analysis, all of the potential collision region can be protected by status lights for cases where the both aircraft begin motion at nearly the same time. The alerting system will be effective in stopping the takeoff aircraft for cases where the taxi aircraft starts approximately 15 seconds or more before the takeoff aircraft. The alerting system will also be effective in stopping the taxi aircraft in cases where the taxi aircraft starts 17 seconds or more after the takeoff aircraft begins its takeoff. The location and identifiers for the applicable crossing taxiways for runway 18L at Dallas/Fort Worth International Airport (see Figure 72) are also depicted on Figure 71.

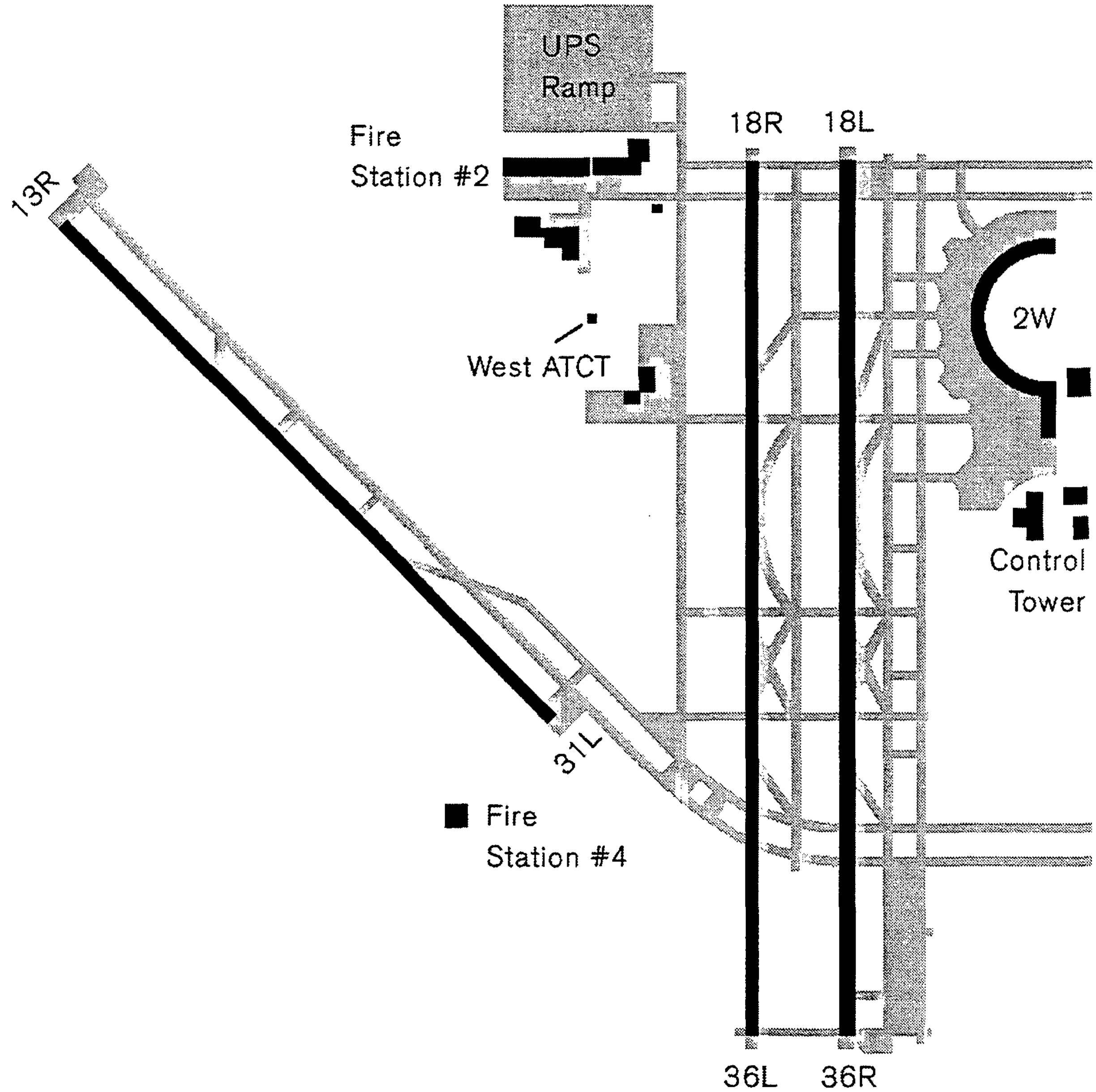


Figure 72. Dallas/Fort Worth International Airport (DFW) Runway 18L and crossing taxiways.

Case 2, a "worst-case" scenario and Case 3, a representative "easy-case" scenario are analyzed in the same manner. The specifics of the scenario are given in Tables 11 and 14, respectively. For Case 2, the times required for the taxi aircraft to begin braking are given in Table 12, and the time for the takeoff aircraft to begin braking in Table 13. The times for Case 3 are in Tables 15 and 16, respectively.

For Case 2, the worst-case scenario, all but a period of about 9 seconds is protected by the runway status lights as shown in Figure 73. This can be reduced somewhat if $\Delta\tau$ is reduced to 0.5 seconds. This case involved a large wingspan B747 with a slow takeoff acceleration and a fast taxiing B727 and included a "worst case" runway entrance light placement. Under those assumptions, if the takeoff and taxi aircraft started motion within a few seconds of each other, the status light system would not protect a portion of the collision region.

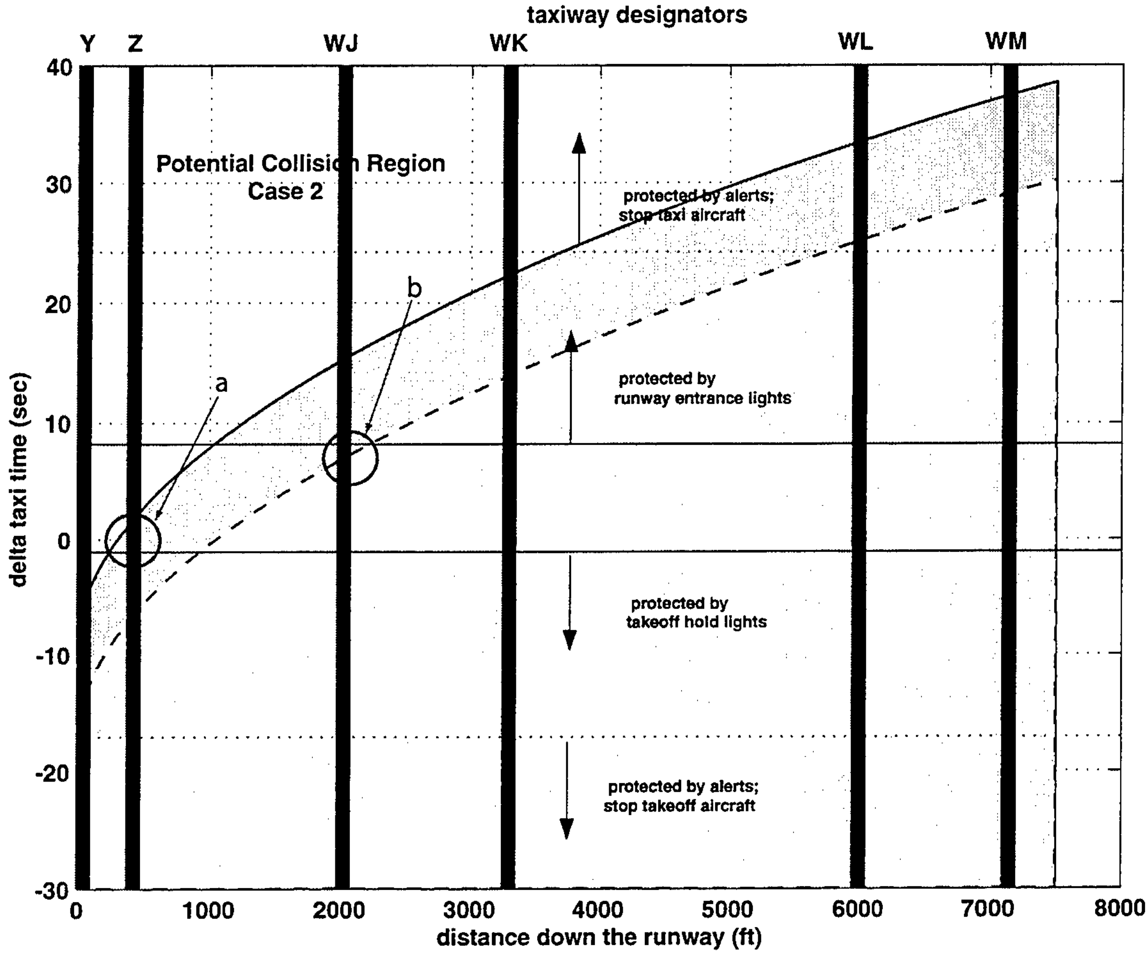


Figure 73. Potential collision region and protected area with σ_s = 20 feet and $\Delta\tau$ = 1.0 second for Case 2.

The interpretation of this figure is as follows. In this extreme worst-case scenario, for runway 18L at Dallas/Fort Worth International Airport, the runway status lights would offer protection except in a few incursion scenarios of very specific space-time combinations. Specifically, for an incursion at taxiway Z, there is a small, unprotected area, noted on the figure as area "a", where the taxi and takeoff aircraft start within approximately 1 second of each other. There is also a small unprotected area on taxiway WJ, denoted as "b" in the figure, where the taxi aircraft starts approximately seven or eight seconds after the takeoff aircraft. Note also that there are some unprotected areas on taxiways WJ, WK, WL, and WM further down the runway if the taxi aircraft were to stop and remain on the runway.

Case 3, the least challenging motion profile case has overlapping protection from the status lights as shown in Figure 74. In addition, the tower cab alerting system is effective over a portion of the collision region.

For the case of an arrival aircraft with a runway blocked by an aircraft in position to depart, only the tower cab alerting is available. In Section 5.2 there were three cases analyzed. A 150 knot arrival with a 0.26g departure, a 135 knot arrival with a 0.26g departure, and a 150 knot arrival with a 0.20g departure. As analyzed in section 5.2, the times available to detect whether the aircraft on the runway is a departure is 27.9 seconds for the first case, 26.7 seconds for the second case and 32.4 seconds for the third case. The time required to detect a 0.26g takeoff is given as 10.3 seconds in Table 19 for a surveillance system with $\sigma_s = 20$ feet and $\Delta\tau = 1$ second. Similarly the time required is given as 13.1 seconds for a 0.20g departure. This leaves 17.6 seconds for the first case, 16.4 seconds for the second case, and 19.3 seconds for the third case for the controller to react, find a clear voice channel, and for the pilot to begin a go-around before reaching the decision height. Comparing this to the cumulative distribution function presented in Figure 67, the probability for a successful go-around before reaching the decision height is 95.4%, 93.0%, and 97.6% for the three cases respectively. This means that the alerting system is effective for preventing the incursion by having the arrival aircraft go-around before reaching the decision height without issuing false go-arounds for cases where separation would not have been violated.

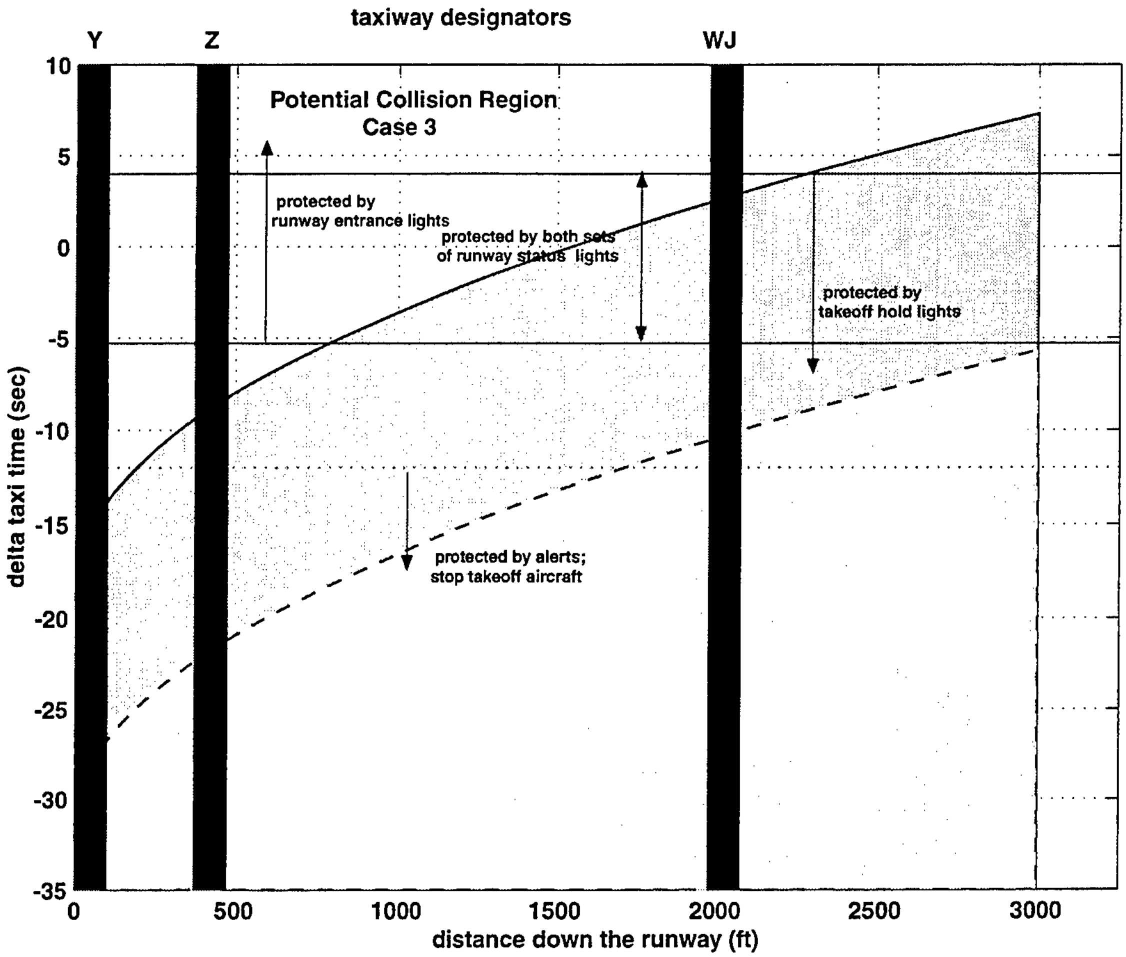


Figure 74. Potential collision region and protected area with σ_s= 20 feet and $\Delta\tau$ = 1.0 second for Case 3.

The conclusion is that a safety system employing both runway status lights and tower cab alerts can be an effective system with input from a surveillance system having a position error of approximately σ_s = 20 feet and an update interval of $\Delta\tau$ = 1 second or less.

7. SUMMARY AND CONCLUSIONS

The purpose of this analysis is to derive quantitative requirements for area surveillance systems to support runway incursion prevention safety systems. The approach taken is to determine surveillance accuracy and update interval required to support three types of safety systems for three specific instances of runway incursions. The effectiveness of a given safety system will necessarily start to fall off when the quality of the surveillance inputs degrades beyond a certain critical point. The goal in this analysis is to find that critical point for various safety systems and different categories of incursion.

Three categories of incursion were investigated; 1) the case of a taxi aircraft taxiing onto a "hot" runway, 2) the case of a takeoff aircraft with an obstruction on or about to enter the runway, and 3) the case of an arrival aircraft with an obstruction on or about to enter the runway. Three types of runway safety systems were considered; 1) a system that provides alerts to the tower cab that result in controller instructions to the appropriate aircraft, 2) a system that provides alerts directly to the aircraft cockpit, and 3) a system of runway status lights consisting of runway entrance lights at taxiway entrances and takeoff-hold lights at the takeoff end of the runway.

Representative motion profiles for taxi and takeoff aircraft were generated for each category of incursion and a timing analysis was used to determine the surveillance parameters that would be required to support each of the three safety systems. Reaction time probability density functions for controller reaction, VHF channel availability, and pilot reaction were used to determine the cumulative probability distribution for total reaction time for the case of the tower cab alerts. Pilot reaction time was used for the direct cockpit alerting system.

In Section 3, two taxi aircraft motion profiles were examined for the case of a taxi aircraft entering a hot runway with an alerting system. In one case an aircraft failed to stop at the taxi hold position and continued on to a hot runway with a Boeing 747 traveling down the runway. In the other case the taxi aircraft started from a stopped position at the taxi-hold position and taxied towards a runway with a Boeing 727. The results were disappointing for all but the slowest taxi profiles. Because of the reaction time requirements for a tower cab alerting system, the system is not effective in preventing this type of incursion even with very good surveillance. The system with a direct alert to the cockpit can be effective as long as the taxi speed is below 10 knots or the aircraft starts from a stopped position.

The runway entrance light system, assuming that the lights are effectively designed from a human factors standpoint, will indicate when a runway is hot and prevent the incursion. In order to determine surveillance requirements, three scenarios of taxi and takeoff motion profiles were examined that tested the runway entrance and takeoff-hold light system with near simultaneous motion of the two aircraft. The concept of a collision region was developed that defined the combinations of start times and distances down the runway that would result in a collision. Three cases were examined; a nominal case, a challenging case, and an easy case. In all but the challenging case all of the potential collision region would have been protected with as surveillance system with $\sigma_s = 20$ feet and $\Delta\tau = 1$ second. For the challenging case, the portion of the collision region where the taxi aircraft started seven seconds after the takeoff aircraft would have been protected. The major challenge to the surveillance system in this case is determining when the takeoff aircraft is actually a takeoff. For the purposes of this analysis it was assumed that the takeoff aircraft must have accelerated to an absolute threshold of 30 knots. This threshold was increased to accommodate the uncertainty introduced by the surveillance system. It may be worthwhile to consider decreasing the absolute threshold to 25 knots and considering algorithms that employ the position on the runway to determine a takeoff. The potential drawback of this approach is that it is more likely to produce false departure declarations when an aircraft taxies forward on the runway (for any number of reasons, including to vacate the runway).

Section 4 considers the case of the departure with a blocked runway. It was assumed that the takeoff aircraft must reject the takeoff and brake the aircraft to a stop. Timing analysis for the tower cab alerting system indicates that even with perfect surveillance, an unacceptable distance of the runway would be unprotected because of the reaction times and because it is necessary to wait until the aircraft is determined to be a takeoff. An absolute threshold of 30 knots was assumed. A direct cockpit alerting system could protect all but the first 1250 feet of the runway with a surveillance system of $\sigma_s = 10$ feet and $\Delta\tau = 1$. Analysis for the takeoff-hold lights used the same three cases of dual aircraft motion used to analyze the runway entrance lights. In all three cases, the portion of the collision region where the taxi aircraft started simultaneously with or before the takeoff aircraft could be protected with less stringent surveillance requirements than those for the runway entrance lights. This is because it was no longer necessary to wait until the takeoff aircraft reached a threshold velocity.

For the incursion involving an arrival to a blocked runway, two cases were considered in Section 5. The case where a departure was in position and holding awaiting departure and the case where a previous arrival or a taxi aircraft was not yet off the runway. In the case of an aircraft in position for takeoff, the challenge is to alert the arrival in time to execute a go-around without issuing nuisance alerts. Legal separation was assumed to be 6000 feet between the departing aircraft and the arrival as the arrival crossed the threshold. It was assumed that a go-around would have to be executed at a decision height of two hundred feet. Depending on the takeoff acceleration and arrival ground speed, a tower cab alerting system will be effective 93% to over 98% of the time with surveillance parameters of $\sigma_s = 20$ feet and $\Delta\tau = 1$ second. It was determined that no safety system could adequately warn an arrival if a previous arrival or a taxi aircraft failed to exit the runway by suddenly stopping shortly before the arrival aircraft crossed the threshold. A situational awareness system that provides information directly to the cockpit indicating the status of the runway may be effective in this case.

Finally, in Section 6, a practical application was investigated using a representative surveillance system with a positional accuracy of $\sigma_s = 20$ feet and an update interval of $\Delta\tau = 1$ second. All three categories of incursion were analyzed using a safety system that employed both a tower cab alerting system and a runway status light system.

The conclusion from this analysis is that a safety system incorporating runway status lights with tower cab alerting will be effective in preventing most surface accidents and runway incursions with a surveillance system providing σ_s = 20 feet ($2\sigma_s$ = 40 feet)[5] and $\Delta\tau$ = 1 second. The runway status lights are required to prevent accidents when a takeoff aircraft and a taxi aircraft both begin motion to the same runway within a few seconds of each other. In cases where one aircraft moves 10 to 15 seconds before the other aircraft, runway status lights will prevent the incursion and the tower cab alerting system will be able to prevent the accident. For cases where an arrival is landing on a runway that is occupied by an aircraft in position to takeoff, the tower cab alerting system will be effective in alerting in time to conduct a go-around without issuing nuisance alerts (i.e., alerts when minimum separation standards would have been maintained had the arrival continued and landed). Runway status information provided to the cockpit will be required for the case where a previous arrival or a taxi aircraft fails to exit the runway as anticipated shortly before the arrival crosses the threshold.

Track integrity must be very reliable for a safety system to be effective. The track integrity depends on the probability of detection and probability of false detection by the surveillance system as well as the tracker design. The probability of detection or false detection depends on the type of surveillance system implemented and the location on the airport. An operational system will need to assess the surveillance systems track performance for that individual installation.

Another conclusion of the analysis is that – assuming good but achievable surveillance performance – surveillance, *per se*, does not materially limit the performance of these systems. Other aspects of the runway incursion problem and the airport surveillance environment do, however, affect the degree of protection that these systems can provide. One of the fundamental challenges is the limited warning time in many conflict situations: time-critical incursions cannot be prevented with a tower cab alerting system, regardless of surveillance quality, because they develop too fast to allow time for effective controller intervention. Another major challenge is to achieve the required high track integrity in the difficult surveillance environment of the airport surface: without accurate representation and prediction of the airport traffic situation neither alerts nor lights will perform as required.

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[5] σ_s is the standard deviation of uncorrelated position reports. System performance is often specified in terms of 95% or $2\sigma_s$.

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APPENDIX A

The time available for the controller and pilot to react to an incursion from a taxiway can be computed as a function of σ_s and taxi velocity. For a taxi-hold position 280 feet from the runway centerline and a 150 feet wide runway, the distance from the taxi-hold position to the entrance to the runway is 205 feet. The braking acceleration is assumed to be 0.33 g's. It is also assumed that the surveillance system will be set to alert after the taxi aircraft has taxied a distance $2\sigma_s$ past the taxi-hold position. The time available for the surveillance update, controller reaction time, VHF channel availability, and pilot reaction time can be plotted as a function of σ_s and taxi velocity. This is presented in Figure A-1.

If controller reaction time were 6 seconds, pilot reaction time were 5 seconds, surveillance update interval 1 second and VHF channel availability instantaneous, it would take 12 seconds to react and begin braking. From Figure A-1, even with perfect surveillance (σ_s =0), there would not be enough time available to prevent the incursion for aircraft with taxi velocities over 10 knots.

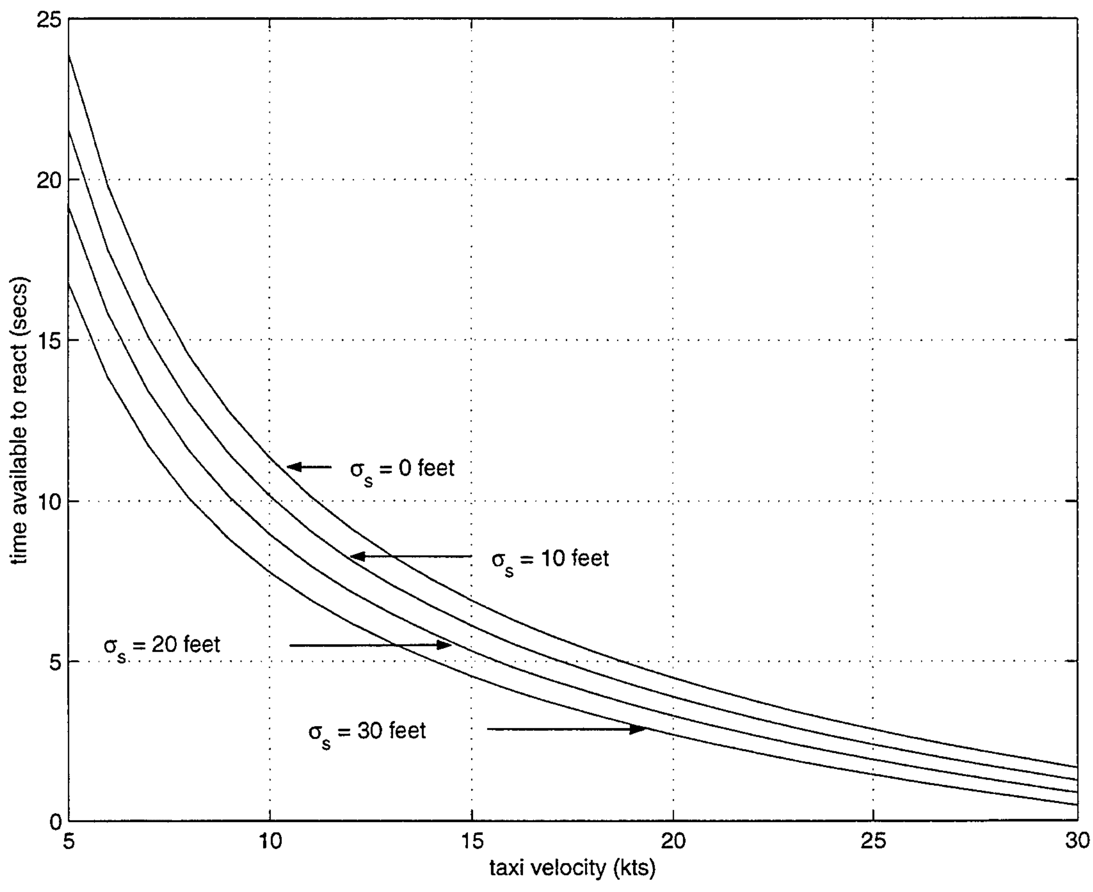

Figure A-1 Time available to react to a taxi aircraft incursion as a function of taxi velocity and surveillance position accuracy.

APPENDIX B

The least-squares algorithm can be used to fit a parabola to N points. A least-squares technique is required for N>3, but also works for N=3, and starts with the definition of chi-squared:

$$\chi^2 = \sum_{i=-N+1}^{0} w_i^2 \left(s_0 + v_0 t_i + \tfrac{1}{2} a_0 t_i^2 - x_i\right)^2, \tag{1}$$

weighted by the parameters w. We calculate the present estimated position s_0, velocity v_0, and acceleration a_0 parameters from the surveillance inputs x_k and times t_k in the normal fashion:

$$\frac{\partial \chi^2}{\partial s_0} = 0, \quad \frac{\partial \chi^2}{\partial v_0} = 0, \quad \frac{\partial \chi^2}{\partial a_0} = 0, \tag{2}$$

which leads to the matrix equation

$$\begin{pmatrix} s_0 \\ v_0 \\ a_0/2 \end{pmatrix} = \begin{pmatrix} T_0 & T_1 & T_2 \\ T_1 & T_2 & T_3 \\ T_2 & T_3 & T_4 \end{pmatrix}^{-1} \begin{pmatrix} w_0^2 & w_{-1}^2 & \cdots & w_{-N+1}^2 \\ w_0^2 t_0 & w_{-1}^2 t_{-1} & \cdots & w_{-N+1}^2 t_{-N+1} \\ w_0^2 t_0^2 & w_{-1}^2 t_{-1}^2 & \cdots & w_{-N+1}^2 t_{-N+1}^2 \end{pmatrix} \begin{pmatrix} x_0 \\ x_{-1} \\ \vdots \\ x_{-N+1} \end{pmatrix} = T^{-1} W X, \tag{3}$$

where

$$T_k = \sum_{i=-N+1}^{0} w_i^2 t_i^k . \tag{4}$$

Predicted values of future positions, velocities, and accelerations are provided by the assumption of constant future acceleration:

$$\begin{pmatrix} s(t) \\ v(t) \\ a(t)/2 \end{pmatrix} = \begin{pmatrix} 1 & t & t^2 \\ 0 & 1 & 2t \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s_0 \\ v_0 \\ a_0/2 \end{pmatrix} = P T^{-1} W X = A X. \tag{5}$$

The simplest representation is obtained, however, if time $t = 0$ is used for the current time.

To estimate the effect of surveillance noise on the accuracy of position predictions $s(t)$, we make the simplifying assumptions that the measurement uncertainties of each of the surveillance points are independent and equal. Thus the covariances $\sigma_{x_i x_j}^2$ between the ith and the jth measured positions are equal to a constant variance σ_x^2 if i and j are the same, and zero otherwise. This relation can be written using the Kronecker delta as

$$\sigma_{x_i x_j}^2 = \sigma_x^2 \delta_{i,j}, \tag{6}$$

where

$$\delta_{i,j} \equiv \begin{cases} 1, & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Then we may write the variances of the projected position, velocity, and acceleration as

$$\begin{pmatrix} \sigma^2_{s(t)} \\ \sigma^2_{v(t)} \\ \sigma^2_{a(t)} \end{pmatrix} = \text{diag}\left(AA^T\right)\sigma^2_x, \tag{8}$$

where diag() selects the diagonal elements of its argument. The variance in the predicted position is thus

$$\sigma^2_{s(t)} = \begin{pmatrix} 1 & t & t^2 \end{pmatrix} T^{-1} U T^{-1} \begin{pmatrix} 1 \\ t \\ t^2 \end{pmatrix} \sigma^2_x, \tag{9}$$

where

$$U = WW^T = \begin{pmatrix} U_0 & U_1 & U_2 \\ U_1 & U_2 & U_3 \\ U_2 & U_3 & U_4 \end{pmatrix}, \quad U_k = \sum_{i=-N+1}^{0} w_i^4 t_i^k. \tag{10}$$

If all the weights are equal to one (unweighted case), then T=U.

The variances of the current (i.e., $t = 0$) position, velocity, and acceleration estimates produced by the unweighted parabolic fit can be calculated from equation 8 as a function of the number of points used in the fit if all the weights are equal to one and the times are equally spaced with a spacing τ. As the number of points increases, the variance of the fit parameters decreases. The results are shown in Table B-1.

Table B-1. Fit Parameter Variance Ratios for an Unweighted Parabolic Fit to N Points.

N	σ_s^2/σ_x^2	$\tau^2\sigma_v^2/\sigma_x^2$	$\tau^4\sigma_a^2/\sigma_x^2$
3	1	13/2	3/2
4	19/20	49/20	1/4
5	31/35	87/70	1/14
6	23/28	407/560	3/112
7	16/21	13/28	1/84
8	17/24	53/168	1/168
9	109/165	1037/4620	1/308
10	34/55	437/2640	1/528
11	83/143	49/390	1/858`
12	199/364	391/4004	3/4004
13	47/91	155/2002	1/2002
14	137/280	909/14560	1/2912
15	79/170	3161/61880	3/12376
16	361/816	403/9520	1/5712
17	409/969	275/7752	1/7752
18	23/57	49/1632	1/10336
19	257/665	1739/67830	1/13566
20	571/1540	1937/87780	1/17556

If the parabolic least-squares fit procedure is used for parameter estimation in a tracker, then the question arises how well the tracker works as a function of N, τ, and σ_x^2. As N is increased, the results shown in Table B-1 show that the parameter variances decrease. This is not without cost, however. If measurements are made at a constant interval τ, for increasing N, the estimates will be based on an increasing time interval. Then if the true target motion temporarily deviates from simple acceleration, the effect on the estimated parameters will persist for a time N τ. Thus, a change in target behavior will result in a lag in the estimated parameter values.

It is useful to investigate the case of varying surveillance update rates but with constant parameter estimation lag times. To accomplish this, the number of measurement points must vary to keep the fit period N τ constant. In this case, the variance of the fit parameters also decreases with increasing N, but with a constant estimate lag. The variance as a function of the number of points in the case of a constant fit period is shown in Figure B-1. The upper scale is drawn for convenience to show that N and τ vary inversely. The value of τ_0 is unspecified and the product N τ is constant but unspecified because the latter's value is factored out of the quantities being plotted.

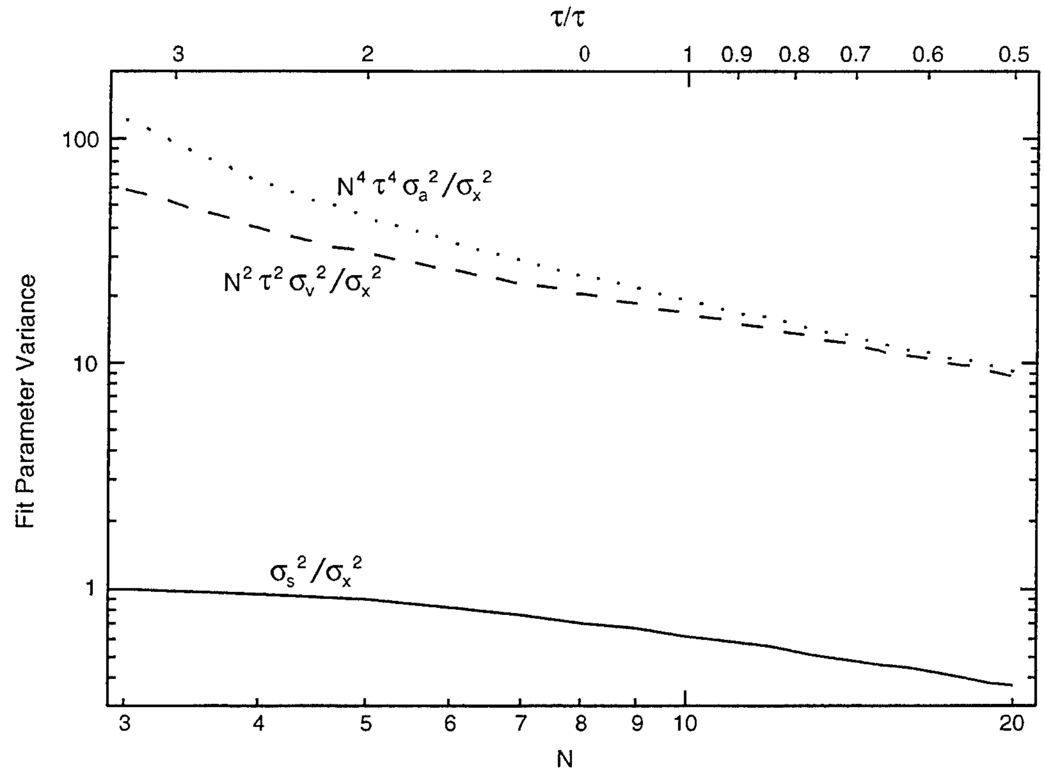

Figure B-1. Dependence of the variance of the fit parameters on the number of points used in the unweighted parabolic fit, with the fit period N τ *held constant.*