

NPS-OR-01-011

# NAVAL POSTGRADUATE SCHOOL Monterey, California



## **Stochastic Models for Promoting and Testing System Reliability Evolution**

by

Donald P. Gaver  
Patricia A. Jacobs  
Ernest A. Seglie

September 2001

Approved for public release; distribution is unlimited.

Prepared for: Naval Postgraduate School  
Monterey, California 93943

20011016 074

NAVAL POSTGRADUATE SCHOOL  
MONTEREY, CA 93943-5000

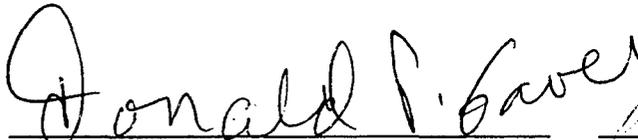
RADM David R. Ellison  
Superintendent

Richard Elster  
Provost

This report was prepared for and funded by the Director, Operational Test and Evaluation (DOT&E), The Pentagon (Room 3E318), Washington, DC 20301-1700. Research also supported by the Institute of Joint Warfare Analysis (IJWA) and the Modeling, Virtual Environments and Simulation (The MOVES) Institute at the Naval Postgraduate School.

Reproduction of all or part of this report is authorized.

This report was prepared by:



DONALD P. GAVER  
Distinguished Professor of  
Operations Research

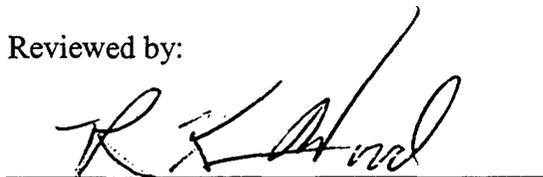


PATRICIA A. JACOBS  
Professor of Operations Research



ERNEST A. SEGIE  
Director, Operational Test and Evaluation

Reviewed by:



R. KEVIN WOOD  
Associate Chairman for Research  
Department of Operations Research

Released by:



JAMES N. EAGLE  
Chairman  
Department of Operations Research



DAVID W. NETZER  
Associate Provost and Dean of Research

# REPORT DOCUMENTATION PAGE

Form approved

OMB No 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE September 2001	3. REPORT TYPE AND DATES COVERED Technical Report
----------------------------------	----------------------------------	--

4. TITLE AND SUBTITLE Stochastic Models for Promoting and Testing System Reliability Evolution	5. FUNDING  MIPR NO. DVAM10001
---	--------------------------------------

6. AUTHOR(S) Donald P. Gaver, Patricia A Jacobs, Ernest A. Seglie	
--	--

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943	8. PERFORMING ORGANIZATION REPORT NUMBER NPS-OR-01-011
---	---

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Office of the Director, Operational Test and Evaluation (DOT&E) The Pentagon (Room 3E318) Washington, DC 20301-1700	10. SPONSORING/MONITORING AGENCY REPORT NUMBER
--	--

11. SUPPLEMENTARY NOTES
-------------------------

12a. DISTRIBUTION/AVAILABILITY STATEMENT	12b. DISTRIBUTION CODE
--	------------------------

<p>13. ABSTRACT (Maximum 200 words.)</p> <p>Many systems and systems-of-systems function in sequential-stage fashion, and are constantly <i>on</i> when operative, but are failure-susceptible. Communication systems, power generation and transmission, and vehicular transportation systems tend to fall into this category. We propose a reliability growth model for such systems that is based on design defect removal under a Test-Fix-Test (TFT) protocol: a system is assembled and put under test, for example for a fixed mission time, or multiple thereof. If the system fails during the test time its failure source in some stage is diagnosed, the stage is re-designed, and the new prototype system reassembled (system design is "fixed") and the system is re-tested. The test (TFT) process is repeated until a pre-determined test period elapses with no failures. This is analogous to the run-test criteria analyzed for one-shot devices [1]. In this model we also allow for occasional defective re-design: response to a test failure can actually (and realistically) <i>increase</i> the number of failure-generating design defects.</p> <p>Our model allows quick numerical assessment of TFT operating characteristics, given defining parameter values. It thus provides a planning tool for test designers.</p>
---

14. SUBJECT TERMS Operational test and evaluation; reliability growth; software testing; system with substages	15. NUMBER OF PAGES 15
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL
---	--	---	----------------------------------

# Stochastic Models for Promoting and Testing System Reliability Evolution

Donald P. Gaver  
Operations Research Department  
Naval Postgraduate School  
Monterey, CA 93943  
Email: dgaver@nps.navy.mil

Patricia A. Jacobs  
Operations Research Department  
Naval Postgraduate School  
Monterey, CA 93943  
Email: pajacobs@nps.navy.mil

Ernest A. Seglie  
Director, Operational Test and Evaluation  
The Pentagon  
Washington, DC 20301  
eseglie@dote.osd.mil

## ABSTRACT

Many systems and systems-of-systems function in sequential-stage fashion, and are constantly *on* when operative, but are failure-susceptible. Communication systems, power generation and transmission, and vehicular transportation systems tend to fall into this category. We propose a reliability growth model for such systems that is based on design defect removal under a Test-Fix-Test (TFT) protocol: a system is assembled and put under test, for example for a fixed mission time, or multiple thereof. If the system fails during the test time its failure source in some stage is diagnosed, the stage is re-designed, and the new prototype system reassembled (system design is "fixed") and the system is re-tested. The test (TFT) process is repeated until a pre-determined test period elapses with no failures. This is analogous to the run-test criteria analyzed for one-shot devices [1]. In this model we also allow for occasional defective re-design: response to a test failure can actually (and realistically) *increase* the number the number of failure-generating design defects.

Our model allows quick numerical understanding of TFT operating characteristics, given defining parameter values. It thus provides a planning tool for test designers.

## 1. Introduction and Model Formulation

Mathematical models are formulated for the *reliability evolution* (desirably growth [2], [3], [4], [5], but also occasional realistic decay) of a continuously operating ("always on") system that is tested, fixed (partially re-designed) if it fails, re-tested, etc, until a specified stopping condition is achieved. The stopping rule utilized here is analogous to a run test [1], [6]; here the entire system must survive without any failure for a time  $\tau$  in order to pass the test, have its design frozen, and be eligible for operational testing and eventual usage in the field.

Two test measures of effectiveness (MOEs) are analytically evaluated:

- (a) the probability that the system survives in the field, i.e., after the end-to-end testing period of specified duration  $\tau$  is survived without failure, and the design is frozen; and
- (b) the expected duration of such a test.

It is also possible to analytically evaluate other such measures by our backward equation technique: the variance of test duration, the probability distribution of remaining design defects or faults, and so forth. All of these measures are evaluated in terms of basic parameters, such as the initial number of design-fault-susceptible modules per stage ( $d_i$  for stage  $i$ ) the maximum number per stage ( $m_i$ ), the rate of design fault activation, hence failure per design fault module ( $\lambda_i$ ), the number of sequential stages ( $S$ ), the duration of the fault-free test interval that must be survived in order to pass the test ( $\tau$ ) (specified in advance by the planner/analyst), the probability of effective re-design/fault removal ( $\rho_i$ ), and the probability of ineffective re-design/fault addition ( $\alpha_i$ ). In the present model study the analyst must furnish values for these basic "what if" parameters, and the model then evaluates the MOEs

(a) and (b). The model is also extended to account for test-to-test environmental variability both random and systematic.

Our model can also form the basis for statistical inference concerning stage-wise fault population parameters. Given observations on failures at various stages, a likelihood function can be written down and analyzed, possibly making use of Bayesian methodology. This will be a topic for future research.

## 2. Generic Situation: Staged Systems in Continuous Time Under

### Test-Fix-Test

Consider a system,  $\mathcal{S}$ , that is made up of  $S$  stages,  $\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_i, \dots, \mathcal{S}_S$ , the  $i^{\text{th}}$ , ( $i=1, \dots, S$ ), of which has a maximum number of modules,  $m_i$ , all of which must operate for the  $i^{\text{th}}$  stage  $\mathcal{S}_i$  to be operative. However, stage  $i$  initially has  $d_i$ , ( $1 \leq d_i \leq m_i$ ), design defects, i.e., improperly designed failure prone modules. These are presumed to activate independently and randomly as exposure (test, or field operation) time elapses. Initially we presume the time to (activation/failure) of each design defect in stage  $i$  to be exponentially distributed, with rate  $\lambda_i$ . The  $m_i - d_i$  modules without defects at  $\mathcal{S}_i$  are assumed (for now) not to be failure-susceptible.

It is here assumed that if the system  $\mathcal{S}$  is put *on test* at  $t=0$  it operates successfully until the first design-defective module in any stage activates/fails; when that module fails,  $\mathcal{S}$  fails (no redundancy). Occurrence of such activation is an opportunity for re-design (permanent or temporary repair) of the failed module. If this step is (i) *positively effective* the module is no longer activation/failure-prone, i.e.,  $d_i$  is decreased by one; if this step is (ii) *negatively effective*, the re-design is not only ineffective, it adds a defective module, so the net number of defects is increased by

one; otherwise the re-design is (iii) *ineffective*, meaning that there is no change in the number of defective modules. Note that the above defect-removal/re-design option is only available at the testing (developmental, early operational) stage.

## 2.1 Test Protocol

We analyze properties of a no-mission time failure test protocol: specify a test time,  $\tau$ , and test the system for that time. Each such test event is called a subtest. If a failure occurs during that subtest, perform re-design and test again, continuing until the system survives for time  $\tau$  without failure. At this moment the test is complete and the design is frozen. This is clearly analogous for the *run of r* criteria analyzed [1].

There are two simple versions of this protocol.

- (A) The subtests all last for the basic test time  $\tau$ , even if a failure occurs during a subtest and the subtest has failed at that point. For the present we consider just one failure to be possible during a subtest. Generalizations will be furnished later.
- (B) The subtests each last until the time to first failure or time  $\tau$ , whichever occurs first.

This requires that the system be constantly monitored in real time to discover failure occurrence; if this is feasible it is undoubtedly more time efficient. But operational circumstances may compel the use of (A). It is the version we analyze first.

### 3. Test Protocol Modeling and Measures of Test Effectiveness Under Protocol (A)

The system model and test protocol yield expressions that allow numerical evaluation of measures of system test-fix, *etc*, effectiveness.

#### 3.1 Probability of Fielded (Design-Frozen) Success

Let  $p_\tau(d_1, \dots, d_i, \dots, d_S) =$  Probability that the tested and accepted (design-frozen) system survives without failure for time  $\tau_F$ .

Then by probability arguments that proceed from the first subtest (backward equation approach) we obtain

$$\begin{aligned}
 p_\tau(d_1, \dots, d_i, \dots, d_S) &= \underbrace{\left( e^{-\sum_{i=1}^S \lambda_i d_i \tau} \right)}_{\text{probability no failures, so no re-designs}} \underbrace{\left( e^{-\sum_{i=1}^S \lambda_{iF} d_i \tau_F} \right)}_{\text{probability no field failures}} + \\
 &+ \left( 1 - e^{-\sum_{i=1}^S \lambda_i d_i \tau} \right) \sum_{i=1}^S \frac{\lambda_i d_i}{\sum_{k=1}^S \lambda_k d_k} \left[ \underbrace{\rho_i(d_i) p_\tau(d_1, \dots, d_i - 1, \dots, d_S)}_{\text{defect in module } i \text{ removed}} \right. \\
 &+ \underbrace{\alpha_i(d_i) p_\tau(d_1, \dots, d_i + 1, \dots, d_S)}_{\text{new defect introduced by "redesign"}} + \\
 &\left. + \underbrace{(1 - \rho_i(d_i) - \alpha_i(d_i)) p_\tau(d_1, \dots, d_i, \dots, d_S)}_{\text{no change in number of defects}} \right] \quad (3.1)
 \end{aligned}$$

where  $\lambda_{iF}$  is the failure rate in the field of a remaining design defect in stage  $i$ .

The conditional probability of defect removal ( $\rho_i$ ) and addition ( $\alpha_i$ ) are assumed to be

$$\begin{aligned}
 \rho_i(d_i) &= \rho_i \quad \text{for } 1 \leq d_i \leq m_i \\
 &= 0 \quad \text{otherwise}
 \end{aligned} \quad (3.2,a)$$

$$\begin{aligned}\alpha_i(d_i) &= \alpha_i \quad \text{for } 1 \leq d_i \leq m_i - 1 \\ &= 0 \quad \text{otherwise}\end{aligned}\tag{3.2,b}$$

In the present model  $d_i \leq m_i$  for all stages, where  $m_i$  is the specified maximum number of defects in stage  $i$ . The above expression may be recursively solved, starting with  $p_\tau(0, 0, \dots, 0) = 1$ ; (3.2,b) prevents the number of defects from exceeding  $m_i$  in stage  $i$ .

### 3.2 Expected Test Duration, Protocol (A) (Each Subtest Requires Time $\tau$ )

Let  $w_\tau(d_1, d_2, \dots, d_i, \dots, d_s) =$  Expected/mean time to complete a test that terminates with system first failure-free survival of time  $\tau$ .

Then again by arguing from the first subtest

$$\begin{aligned}w_\tau(d_1, d_2, \dots, d_i, \dots, d_s) &= \tau + \\ &+ \left(1 - e^{-\sum_{i=1}^s \lambda_i d_i \tau}\right) \sum_{i=1}^s \frac{\lambda_i d_i}{\sum_{k=1}^s \lambda_k d_k} [\rho_i(d_i) w_\tau(d_1, \dots, d_i - 1, \dots, d_s) + \\ &+ \alpha_i(d_i) w_\tau(d_1, \dots, d_i + 1, \dots, d_s) + \\ &+ (1 - \rho_i(d_i) - \alpha_i(d_i)) w_\tau(d_1, \dots, d_i, \dots, d_s)]\end{aligned}\tag{3.3}$$

Here the initial/boundary condition is  $w_\tau(0, \dots, 0, \dots, 0) = \tau$ .

### 3.3 Generalization for Between-Test Variability

It is possible to explicitly account for an additional likely source of variability: subtest environmental variation, represented by a sequence of positive independent identically distributed random variables,  $\{\theta_t; t = 1, 2, \dots\}$  where  $t$  denotes the subtest number. Illustrate by generalizing (4.1). Conditional on the values  $\theta_1, \theta_2, \dots$ , and deconditioning subtest by subtest,

$$\begin{aligned}
p_\tau(d_1, d_2, \dots, d_S) &= E[p_\tau(d_1, \dots, d_S, \theta_1, \dots, \theta_i, \dots)] = \\
&= E \left[ e^{-\left(\sum_{i=1}^S \lambda_i d_i \tau\right) \theta} \right] E \left[ e^{-\left(\sum_{i=1}^S \lambda_i d_i \tau_F\right) \theta_F} \right] \\
&\left( 1 - E \left[ e^{-\left(\sum_{i=1}^S \lambda_i d_i \tau\right) \theta} \right] \right) \sum_{i=1}^S \frac{\lambda_i d_i}{\sum_{k=1}^S \lambda_k d_k} [\rho_i(d_i) p_\tau(d_1, \dots, d_i - 1, \dots, d_S) + \\
&+ \alpha_i(d_i) p_\tau(d_1, \dots, d_i + 1, \dots, d_S) + \\
&+ (1 - \rho_i(d_i) - \alpha_i(d_i)) p_\tau(d_1, \dots, d_i, \dots, d_S)] \quad (4.4)
\end{aligned}$$

#### 4. Test Protocol Modeling Under Protocol (B) (Each Subtest Requires the Time to Failure or $\tau$ , Whichever Occurs First)

In this protocol it is possible to correctly detect a failure in Stage  $i$  when it occurs, without waiting until the end of the test.

##### 4.1 Probability of Fielded (Design Frozen) Success

If  $p_\tau(d_1, d_2, \dots, d_i, \dots, d_S)$  is defined as in Section 4.1, then the backward equation for this function is the same as in (4.1). Furthermore, the expression (4.4) that incorporates independent between-test variability holds for this situation also.

##### 4.2 Expected Test Duration, Protocol (B)

Define  $w_\tau(d_1, d_2, \dots, d_i, \dots, d_S)$  to be the mean time to test termination (after the system survives time  $\tau$ ). Then in this situation the backward equation becomes

$$\begin{aligned}
w_\tau(d_1, d_2, \dots, d_i, \dots, d_S) &= \tau e^{-\sum_{i=1}^S \lambda_i d_i \tau} + \\
&+ \sum_{i=1}^S \int_0^\tau e^{-\sum_{i=1}^S \lambda_i d_i x} \lambda_i d_i dx [x + \rho_i(d_i) w_\tau(d_1, \dots, d_i - 1, \dots, d_S) + \\
&+ \alpha_i(d_i) w_\tau(d_1, \dots, d_i + 1, \dots, d_S) + \\
&+ (1 - \rho_i(d_i) - \alpha_i(d_i)) w_\tau(d_1, \dots, d_i, \dots, d_S)].
\end{aligned}$$

This simplifies to

$$\begin{aligned}
w_\tau(d_1, d_2, \dots, d_i, \dots, d_s) &= \frac{1}{\sum_{i=1}^s \lambda_i d_i} \left[ 1 - e^{-\sum_{i=1}^s \lambda_i d_i \tau} \right] + \\
&+ \left[ 1 - e^{-\sum_{i=1}^s \lambda_i d_i \tau} \right] \sum_{i=1}^s \frac{\lambda_i d_i}{\sum_{k=1}^s \lambda_k d_k} [\rho_i(d_i) w_\tau(d_1, \dots, d_i - 1, \dots, d_s) + \\
&+ \alpha_i(d_i) w_\tau(d_1, \dots, d_i + 1, \dots, d_s) + \\
&+ (1 - \rho_i(d_i) - \alpha_i(d_i)) w_\tau(d_1, \dots, d_i, \dots, d_s)]. \quad (5.1)
\end{aligned}$$

To generalize (5.1) to account for between-test variability it is only necessary to

replace the first term  $\frac{1}{\sum_{i=1}^s \lambda_i d_i} \left[ 1 - e^{-\sum_{i=1}^s \lambda_i d_i \tau} \right]$ , by  $E \left[ \frac{1 - e^{-\left(\sum_{i=1}^s \lambda_i d_i \tau\right) \theta}}{\theta \sum_{i=1}^s \lambda_i d_i} \right]$  and, in the second term,  $e^{-\sum_{i=1}^s \lambda_i d_i \tau}$  by  $E \left[ \left( e^{-\left(\sum_{i=1}^s \lambda_i d_i \tau\right) \theta} \right) \right]$ , where the expectation is on  $\theta$ .

Any distribution having positive support and with an explicit Laplace-Stieltjes transform provides tractable closed form models for Protocol (A). To obtain a closed-form expression for Protocol (B) it must be possible to *integrate* the Laplace transform of  $\theta$  from zero to a finite limit  $\sum_{i=1}^s \lambda_i d_i \tau$ .

## 5. Illustrative Numerical Example

The backward equations may be solved iteratively to provide numerical insights into system performance under the TFT testing protocol. Here is a brief, isolated, but suggestive example.

## 5.1 A Test-Stage Situation

The parameters used are the following:

$$\lambda_1 = 0.01, \lambda_2 = 0.05 \quad (\text{Test defect activation rate (hours)}^{-1})$$

$$\rho_1 = \rho_2 = 0.75 \quad (\text{Defect rectification/correction probability})$$

$$\alpha_1 = 0.20, \alpha_2 = 0.10 \quad (\text{Defect mis-identification/addition of one defect probability})$$

$$m_1 = m_2 = 4 \quad (\text{Maximum number of defects in each stage})$$

$$\tau_F = 100 \quad (\text{The field mission time (hours)})$$

$$\lambda_{1F} = 0.05, \lambda_{2F} = 0.05 \quad (\text{Field defect activation rate (hours)}^{-1})$$

(A) Examine the effect of the basic sub-test time,  $\tau$ , on the probability of surviving a field operation without failure. The numbers in the small table below indicate the surprisingly systematic effect of test duration on probability of successful field operation.

**Table 1: Probability of Surviving  $\tau_F$  (Field Operation)**

Initial Defects ( $d_i$ )		Test Time ( $\tau$ )			
$d_1$	$d_2$	50	100	200	300
0	1	0.89 (119) [70]	0.99 (252) [127]	1.00 (508) [229]	1.00 (761) [329]
2	2	0.25 (276) [106]	0.52 (651) [204]	0.83 (1462) [375]	0.94 (2264) [510]
2	4	0.29 (426) [120]	0.55 (945) [215]	0.84 (2035) [380]	0.94 (3119) [513]
4	4	0.16 (519) [139]	0.45 (1201) [261]	0.81 (2625) [453]	0.93 (4021) [593]

( )=Expected Test Time, Protocol (A)

[ ]=Expected Test Time, Protocol (B)

However, the required number of tests tends to increase substantially particularly under Protocol (A). If the test can be stopped as soon as a failure occurs, considerable time can be saved. The moral is that only by considerable testing and fixing (in an

error-prone “fix” environment) can we eventually hope to have a highly reliable (small, two-stage) system.

Software that can be activated to exercise programs to evaluate various situations (and parameter variations) appears at <http://www.nps.navy.mil/opnsrsch/testeval/>.

## REFERENCES

- [1] Gaver, D. P., Glazebrook, K. D., Jacobs, P. A., and Seglie, E. A., "Probability models for sequential-stage system reliability growth via failure mode removal," Naval Postgraduate School Technical Report NPS-OR-00-07, (September 2000).
- [2] Balaban, H. S., "Reliability Growth Models," *Journal of Environmental Sciences*, **21**, 11-18 (January/February 1978).
- [3] Fries, A., "Discrete Reliability-Growth Models Based on a Learning-Curve Property," *IEEE Transactions on Reliability*, **42**, 303-306 (June 1993).
- [4] Gaver, D. P. and Jacobs, P. A., "Testing or Fault-Finding for Reliability Growth: A Missile Destructive-Test Example," *Naval Research Logistics*, **44**, 623-637 (1997).
- [5] Crow, L. H. (1998), "Using reliability growth models in the management and structure of a failure review board," *Reliability Growth Modeling: Objectives, Expectations, and Approaches*, ed. by K. J. Farquhar and A. Mosleh. The Center for Reliability Engineering, University of Maryland, College Park, MD 20742.
- [6] Seglie, E. A. (1998), "Reliability growth programs from the operational test and evaluation perspective," *Reliability Growth Modeling: Objectives, Expectations, and Approaches*, ed. by K. J. Farquhar and A. Mosleh. The Center for Reliability Engineering, University of Maryland, College Park, MD 20742.

## BIBLIOGRAPHY

- Ascher, H. and Feingold, H., *Repairable Systems Reliability: Modeling, Inference, Misconceptions and Their Causes, Lecture Notes in Statistics*, 7, Marcel Dekker, New York, 1984.
- Barlow, R. E. and Scheuer, E. M., "Reliability Growth During a Development Testing Program," *Technometrics*, 8, 53-60 (February 1966).
- Barr, D. R., "A Class of General Reliability Growth Models," *Operations Research*, 18, 52-65 (1970).
- Benkherouf, L. and Bather, J. A., "Oil exploration: sequential decisions in the face of uncertainty," *Journal of Applied Probability*, 25, 529-543 (1988).
- Benkherouf, L., Glazebrook, K. D., and Owen, R. W., "Gittins indices and oil exploration," *Journal of the Royal Statistical Society*, B54, 229-241 (1992).
- Bhattacharyya, G. K., Fries, A., and Johnson, R. A., "Properties of Continuous Analog Estimators for a Discrete Reliability-Growth Model," *IEEE Transactions on Reliability*, 38, 373-378 (August 1989).
- Block, Henry W. and Savits, Thomas H., "Burn-In," *Statistical Science*, 12/1, 1-13 (1997).
- Bogdonavicius, V. and Nikulin, M. S., "Estimation in Degradation Models with Explanatory Variables. Report, Centre des Consultations Opti-Stats, Responsable de L'Equipe d'Accueil" Statistique mathématique et ses Applications. Univ. Victor Segalen Bordeaux, France (2000).
- Calabria, R., Guida, M., and Pulcini, G., "A Bayes Procedure for Estimation of Current System Reliability," *IEEE Transactions on Reliability*, 41, 616-621 (December 1992).
- Chernoff, H. and Ray, S. N., "A Bayes Sequential Sampling Inspection Plan," *Annals of Mathematical Statistics*, 36/5, (October 1965).
- Chernoff, H., "Sequential Models for Clinical Trials," *Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability*, University of California Press, Berkeley, CA, 1966.
- Easterling, Robert G., Mazumdar, Mainak, Spencer, Floyd W., and Diegert, Kathleen V., "System-based component-test plans and operating characteristics: binomial data," *TECHNOMETRICS*, 33/3, 287-298 (August 1991).
- Feller, W., *An Introduction to Probability Theory and Its Applications*, Volume II. John Wiley & Sons, Inc., New York, 1966.
- Fries, A., "Discrete Reliability-Growth Models Based on a Learning-Curve Property," *IEEE Transactions on Reliability*, 42, 303-306 (June 1993).
- Gaver, D. P. and Jacobs, P. A., "Methodology for an operationally-based test length decision," *IIE Transactions*, 30, 1129-1134 (1998).
- Gaver, D. P. and Jacobs, P. A., "WEBSIGHT: An internet tool for uncertainty analysis in operational testing," American Statistical Association Joint Annual Meetings, Baltimore, MD, August 8-12, 1999.
- Gittins, J. C., *Multi-armed Bandit Allocation Indices*, New York: Wiley, 1989.
- Glazebrook, K. D., "On a family of prior distributions for a class of Bayesian search models," *Advances in Applied Probability*, 25, 714-716 (1993).
- Gross, A. J. and Kamins, M., "Reliability Assessment in the Presence of Reliability Growth," *Annals of Assurance Sciences: 1968 Symposium on Reliability*, 406-416 (1968).

- Jayachadran, T. and Moore, L. R., "A Comparison of Reliability Growth Models," *IEEE Transactions on Reliability*, **R-25**, 49-51 (April 1976).
- Johnson, N. L., Katz, S., and Balakrishnan, N., *Continuous Univariate Distributions*, Vol. 1, Second Edition, New York: Wiley, 1994.
- Lynn, N. J. and Singpurwalla, N. D., "Comment: 'Burn-In' makes us feel good," *Statistical Science*, **12**, 13-19 (1997).
- Manor, G. and Kress, M., "Optimality of the greedy shooting strategy in the presence of incomplete damage information," *Naval Research Logistics*, **44**, 613-622 (1997).
- Mazzuchi, T.A. and Soyer, R., "A Bayes Method for Assessing Product-Reliability During Development Testing," *IEEE Transactions on Reliability*, **42**, 503-510 (September 1993).
- Olsen, D. E., "Estimating Reliability Growth," *IEEE Transactions on Reliability*, **R-26**, 50-53 (April 1977).
- Pollock, S. M., "A Bayesian Reliability Growth Model," *IEEE Transactions on Reliability*, **R-17**, 187-198 (December 1968).
- Sen, A., Basu, S. and Banerjee, M., "Statistical Analysis of Life-Data with Masked Cause-of-Failure," *Handbook of Statistics: Reliability* (C.R. Rao and N. Balakrishnan, eds.) (2000, to appear).
- Woods, W. M., "The Effect of Discounting Failures and Weighting Data on the Accuracy of Some Reliability Growth Models," *Proceedings of the Annual Reliability and Maintainability Symposium*, Los Angeles, CA, Institute of Electrical and Electronics Engineers, Inc., 200-204 (1990).
- Yang, M. C., Wackerly, D. D., and Rosalsky, A., "Optimal stopping rules in proof-reading," *J. Appl. Prob.* **19**, 723-729 (1982).

## Initial Distribution List

1. Defense Technical Information Center .....2  
8725 John J. Kingman Road, STE 0944  
Ft. Belvoir, VA 22060-6218
2. Research Office (Code 09).....1  
Naval Postgraduate School  
Monterey, CA 93943-5000
3. Dudley Knox Library (Code 013) .....2  
Naval Postgraduate School  
Monterey, CA 93943-5002
4. Richard Mastowski (Editorial Assistant) .....2  
Dept. of Operations Research  
Naval Postgraduate School  
Monterey, CA 93943-5000
5. Prof. Donald P. Gaver (Code OR/Gv) .....1  
Dept. of Operations Research  
Naval Postgraduate School  
Monterey, CA 93943-5000
6. Prof. Patricia A. Jacobs (Code OR/Jc).....1  
Dept. of Operations Research  
Naval Postgraduate School  
Monterey, CA 93943-5000
7. Prof. Ernest A. Seglie.....1  
Science Advisor; DOT&E  
1700 Defense Pentagon  
3E318  
Washington, DC 20301-1700