This final progress report summarizes the accomplishments of the three tasks conducted during the contract period. Task 1 dealt with mechanisms of damage under static and fatigue loading of composite laminates and laid the physical basis for the modeling efforts that followed in Task 2 and Task 3. Task 2 focused on the formulation of stiffness-damage relationships and provided a procedure for assessment of material coefficients experimentally as well as by computational simulation. Finally, Task 3 treated evolution of damage. A model was developed that predicted the increase of transverse crack density with cycles of a constant amplitude. The overall outcome of the project was a methodology for durability assessment of composite materials, in accordance with the goals and objectives set in the proposal.
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A SYNERGISTIC DAMAGE MECHANICS APPROACH TO DURABILITY OF COMPOSITE STRUCTURES

1. Goals and Objectives of Research

The overall technical goal of the research under this contract has been to develop a methodology for assessment of durability of composite structures operating under extreme environments typical of Army vehicles. The scientific goal has been to combine the conventional micromechanics and continuum damage mechanics approaches into a synergistic approach that maximizes advantages of the two approaches while minimizing their inherent deficiencies.

The specific objectives set for the three-year program have been to apply the concept of synergistic damage mechanics to composites of different configurations in order to validate the concept as well as to help formulate the structure of the methodology for life assessment.

The proposed research was planned to have three specific tests: Task 1 dealing with mechanisms of damage that will provide the physical basis of the modeling effort; Task 2 focusing on formulation of stiffness-damage relationships; and Task 3 treating the evolution of damage. These tasks have essentially been completed and the objectives set have been satisfactorily met.

2. Summary of the Most Important Results

2.1 Accomplishments of Task 1.

The first year's research in the program focused on conducting Task 1 dealing with damage mechanisms.

The first laminate configuration taken as an object of study was glass/epoxy [±θ/902]. This laminate configuration was also the object of a pilot study that was used as a motivation for the proposal. The preliminary results obtained with this class of laminate were described in the proposal. Further results were obtained during the course of this project. Those results were put together in a paper, "An Experimental Study of Crack Opening Displacement in Composite Laminates with Varying Constraints", published in International Journal of Damage Mechanics.
The laminate configuration described above was instrumental in validating the concept of utilizing the crack opening displacement (COD) as a micromechanics parameter in the second order tensor for characterization of damage, which then enters as the internal state variable in the continuum damage formulation of material response. The damage occurring in the [±θ/90₂], laminates consists of transverse cracks in the 90-plies while the ±θ-plies provide constraint to the opening of those cracks. However, the off-axis plies can also crack, providing another mode of damage. That damage can be isolated and systematically investigated by choosing a laminate configuration such as [0, ±θ₄, 0₁/₂]. This was done and the results were reported in the paper, "Damage in Composite Laminates with Off-axis Plies", published in Composites Science and Technology.

A summary of the results reported in the two papers mentioned above follows.

1. "An Experimental Study of Crack Opening Displacement in Composite Laminates with Varying Constraints"

This paper reports an experimental study of the opening displacement behavior of transverse cracks in a class of composite laminates where the constraint to cracking is varied systematically. The study is motivated by a synergistic approach to damage mechanics [1] where the crack opening displacement forms a key micromechanically determined parameter in a continuum damage formulation of the composite response. It is found that although the opening displacement shows scatter from point to point along a crack length, its average over the crack length forms a robust parameter. This parameter, when introduced as the damage influence coefficient in a continuum description of damage, allows satisfactory predictions of the elastic moduli changes due to transverse cracks in the considered class of laminates with varying constraints.

**Highlights of Results:**

Figure 1 shows the normalized average value of COD of transverse cracks plotted against θ in the [±θ/90₂], laminates. It is apparent that the constraint to COD is highest at θ = 0, i.e., for cross ply laminates, and it diminishes with increasing θ, vanishing at θ = 90. This COD provides a link between
stiffness changes due to transverse cracking in $[\pm \theta/90_2]_s$ laminates at different values of $\theta$. Defining a constraint parameter in the CDM formulation of stiffness-damage relationships, and setting it in proportion to the normalized COD, allows prediction of stiffness changes due to transverse cracking in $[\pm \theta/90_2]_s$ laminates when the stiffness changes in the cross ply laminate are known. The predictions by this procedure are compared with experimental data in Figs. 2-4.

![Graph showing normalized COD vs angle](image)

**Fig. 1.** Variation of the normalized COD with $\theta$ in $[\pm \theta/90_2]_s$ laminates.
Fig. 2. Predictions using experimental COD values of (a) longitudinal Young's modulus and (b) Poisson's ratio compared with experimental data for $[\pm 15/90_2]_s$ laminates.
Fig. 3. Predictions using experimental COD values of (a) longitudinal Young's modulus and (b) Poisson's ratio compared with experimental data for $[\pm 30/90_2]_s$ laminates.
Fig. 4. Predictions using experimental COD values of (a) longitudinal Young's modulus and (b) Poisson's ratio compared with experimental data for $[\pm 40/90_2]$_s laminates.
Conclusions: The applicability of continuum damage mechanics to a wide class of laminate configurations and crack geometries, as against micromechanics, which is laminate and crack geometry specific, is at the cost of the necessity to determine material constants experimentally. This situation can be improved by a synergistic damage mechanics approach where judiciously selected micromechanics, performed in a limited measure, provides input into a continuum damage mechanics formulation.

For the class of laminates given by [±θ/90₂], we have demonstrated here that the average crack opening displacement of 90-ply cracks provides the micromechanics parameter which relates well with the material constant which varies with the constraining ply angle θ. Thus, use of this micromechanics parameter will avoid the necessity to experimentally determine that material constant, which usually is a demanding task involving testing of a large number of specimens for each value of θ.

The crack opening displacements of 90-ply cracks were determined here by actual physical measurements. This was done to gain insight into the phenomenon of constraint effects on crack opening displacement, in addition to validating it as a micromechanics parameter for the purpose stated above. It is obviously not always feasible to determine this parameter by physical measurements. As we have suggested previously, this parameter should instead be computed by an approximate numerical scheme such as the finite element method. This was accomplished in a later work described below.

Finally, it is noted that the treatment presented here has been for the case of a linear dependence of moduli changes with crack density. For the cases where nonlinearity is displayed, as crack densities approach their saturation values, higher order terms must be included in the stiffness-damage relationships [2].

2. "Damage in Composite Laminates with Off-Axis Plies"

Damage in off-axis plies of composite laminates is studied by examining the configuration [0, ±θ₄, 0₁/₂], with θ = 25, 40, 55, 70 and 90 subjected to tensile loading in the axial direction. It is found that for the values of θ where the stress in the off-axis plies normal to the fibers is tensile, ply cracks
lying along fibers initiate and increase in number, while for other $\theta$ values the plies do not undergo this damage, as expected. However, the overall laminate elastic moduli are found to change also for the $\theta$ values where no ply cracks exist. It is postulated that a shear-induced degradation of the off-axis plies is responsible for the observed laminate moduli changes. The prediction of changes in these moduli by using the ply shear modulus measured on $[\pm \theta_4]$, appears to support this postulate. For the case of moduli changes caused by ply cracks the recently proposed synergistic damage mechanics approach [1] is applied. The implications of the findings of this work on a class of continuum damage mechanics formulations proposed in the literature are discussed.

**Highlights of Results:**

The average CODs of $[0, \pm \theta_4, 0_{1/2}]$, laminates for $\theta$ decreasing from 90 are shown in Fig. 5. The interesting result is that the COD vanishes at $\theta < 40$. For these cases the stress normal to fibers in the $\theta$-plies is compressive. However, the laminate stiffness properties are found to change whether the off-axis plies crack or not. When cracking occurs, the moduli changes are predicted by the synergistic damage mechanics procedure described in the previous paper. Predictions thus made are compared with experimental data in Fig. 6-7 for $[0, \pm 70_4, 0_{1/2}]$, laminate. For $[0, \pm 55_4, 0_{1/2}]$, laminate it is found that the ply cracks are only partly responsible for the laminate stiffness changes. Figures 8-9 shows how the stiffness predictions due to cracking, marked as CDM, do not agree with the experimental values. The other effect, responsible for the remaining stiffness changes, is due to the shear stress induced damage. Predictions due to that effect as well as the total effect by addition of the two are also shown in the figure. Finally, Figs. 10-11 compare the stiffness changes due to the shear stress induced degradation, when no ply cracks are present, for $[0, \pm 40_4, 0_{1/2}]$, laminate.

**Conclusions**

Based on a systematic investigation of damage and the associated deformational response of laminates of configuration $[0, \pm \theta_4, 0_{1/2}]$, with $\theta = 25, 40, 55, 70$ and 90 loaded in axial tension it has been found that two different damage modes exist depending on the angle $\theta$. For the case where the ply stress component normal to the fibers is tensile, the familiar mechanism of multiple matrix cracks along fibers exists and the laminate
elastic moduli change as a function of the density of these cracks. A procedure for predicting moduli changes based on the synergistic damage mechanics proposed earlier [1] is found to be successful for this case. When the stress component normal to fibers is compressive the multiple matrix cracking does not occur, but the elastic moduli still undergo changes. It has been demonstrated that the underlying cause of these moduli changes is damage induced by shear stresses in off-axis plies. A procedure for predicting these moduli changes has been proposed and demonstrated to work well.

The nature of the shear stress induced ply damage is a subject of further research. Its existence raises questions concerning applicability of certain continuum damage models that define damage in terms of the moduli changes.

![Graph](image)

Fig. 5. The average COD plotted against the orientation angle $\theta$ in $[0/\pm\theta_4/0_{1/2}]_s$ laminates at an applied axial strain of 0.5%.
Fig. 6. The longitudinal Young’s modulus normalized by its virgin state value plotted against the crack density in $[0/\pm704/0_{1/2}]_s$ laminate. The solid line is the prediction of the CDM model.

Fig. 7. The Poisson’s ratio normalized by its virgin state value plotted against the crack density in $[0/\pm704/0_{1/2}]_s$ laminate. The solid line is the prediction of the CDM model.
Fig. 8. The longitudinal Young's modulus normalized by its virgin state value plotted against the crack density in \([0/\pm 55/0/1/2]\), laminate. The dotted line is the prediction of the CDM model.

Fig. 9. The Poisson's ratio normalized by its virgin state value plotted against the crack density in \([0/\pm 55/4/0/1/2]\), laminate. The dotted line is the prediction of the CDM model.
Fig. 10. The longitudinal Young’s modulus normalized by its virgin state value plotted against the applied axial strain in [0/±40\(\frac{4}{4}\)/0\(\frac{1}{4}\)], laminate.

Fig. 11. The Poisson’s ratio normalized by its virgin state value plotted against the applied axial strain in [0/±40\(\frac{4}{4}\)/0\(\frac{1}{4}\)], laminate.
2.2 Accomplishments of Task 2

The research during the first year (6/1/97 - 5/31/98) was focused on Task 1 dealing with damage mechanisms caused by mechanical loading of composite materials. Two configurations of composite laminates were considered, namely, \([\pm \theta/90\]_m and \([0/\pm \theta/0\]_m, with \(\theta = 25, 40, 55, 70\) and 90 degrees. These configurations were selected to reveal and to characterize the constraint effects on crack surface displacements and the nature of damage in off-axis plies. These objectives were accomplished during the first year's work. In order to continue on to Task 2 aimed at formulating stiffness-damage relationships in a synergistic damage mechanics approach we needed to develop micromechanics analyses of crack surface displacements as well as evaluate the shear stress induced off-axis ply damage that was discovered during the first year's research. This was accomplished as described below.

A systematic computational study was conducted to study the effects of the constraining \(\pm \theta\) plies on crack opening displacement (COD) of 90-ply cracks in laminates. Replacing the \(\pm \theta\) plies by an equivalent sublamine \(S\) the COD behavior in \([S_n/90_m]\), laminates was studied by varying parameters such as the sublamine stiffness and thickness (i.e. numbers \(n\) and \(m\)) of \(S\) the 90-plys. The three cases of plane stress, plane strain and generalized plane strain were considered. It turned out that they all produced essentially the same COD. Figures 12-14 show the variation of COD with the relative stiffness (longitudinal Young's modulus) of the sublamine \(S\). The COD plotted in these figures is normalized by the longitudinal strain and the half thickness (d) of 90-plys.

![Diagram](image)

**Fig. 12.** The calculated variation of COD of 90-ply cracks in \([S_n/90_m]\) laminates with the relative stiffness of sublamine \(S\).
Fig. 13. The calculated variation of COD of 90-ply cracks in [S₄, 90₄]s laminates with the relative stiffness of sublamine S.

\[ u_a = 0.965 + 0.956 \left( \frac{E_x(S)}{E_x(90)} \right)^{-0.964} \]

Fig. 14. The calculated variation of COD of 90-ply cracks in [S₈, 90₄]s laminates with the relative stiffness of sublamine S.

\[ u_a = 0.96 + 0.76 \left( \frac{E_x(S)}{E_x(90)} \right)^{-0.95} \]
Comparison of these three figures provide several interesting results:

1. For highly anisotropic composites such as carbon-epoxy cross ply laminates, where the ratio of the 0-ply to 90-ply stiffness is large, the constraining ply thickness plays a minor role in affecting the COD of the 90-ply cracks normalized by the 90-ply thickness.

2. For constraining ply orientation θ deviating significantly from the axial direction, say by 20 degrees or more, and for moderately anisotropic laminates, e.g., glass-epoxy cross ply laminates, the constraining ply thickness has a significant effect on COD of 90-ply cracks.

3. For all cases studied the normalized COD tends asymptotically to 1.0 for large normalized sublaminates to 90-ply stiffness ratio.

It is to be noted that the results presented here are for non-interacting cracks. Previous studies by this author and his associates [3,4] have indicated that in most cases linearized stiffness-damage relationships suffice, in which instance CODs of non-interacting cracks will provide the results needed.

The expressions shown in Figs. 12-14 provide functional form for variation of COD that can be incorporated in this author's synergistic damage mechanics formulation [1]. Preliminary comparisons of stiffness changes by use of these expressions and by use of experimentally determined CODs [4] show better agreement with experimental data. A paper is under preparation to report this comparison.

2.3 Accomplishments of Task 3

The work in Task 2 had focused on determining the changes in the stiffness properties at fixed states of damage. The work in this task combines this with prediction of damage evolution. Thus the stiffness properties are described as functions of the applied loading. The specific case considered is that of transverse ply cracking constrained by sublaminates of varying stiffness. Figure 15 shows the laminate with cracks in the 90 degree layers sandwiched between sublaminates of ±θ plies. The angle θ is varied to have values 0, 15, 30 and 40 degrees. The previously described constraint parameter is calculated by a parametric study in which the ply thickness and ply properties are changed. A relationship is developed to describe the effects of relative ply stiffness and the ratio of crack size to the constraining ply thickness. A fracture mechanics based procedure is developed to predict the evolution of ply cracking. This procedure involves Monte-Carlo
simulation incorporating the statistical variation of the fracture toughness in the fiber direction.

Figures 16 and 17 illustrate the predictions of deformational response (in terms of the axial Young’s modulus and Poisson’s ratio) with applied strain for the cases of $\theta = 15$ and $\theta = 40$.

Fig. 15. A composite laminate loaded in tension axially, developing multiple cracks in the middle 90 degree layers with the outer sublamine S remaining uncracked.
Fig. 16. The variations of the axial Young's modulus and Poisson's ratio with the applied strain for $\theta = 15$. 
Fig. 17. The variations of the axial Young’s modulus and Poisson’s ratio with the applied strain for $\theta = 40$. 
REFERENCES


LIST OF PUBLICATIONS SUPPORTED BY THE CONTRACT

a) Papers published in peer-reviewed journals


b) Conference Papers Without Proceedings


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