Directional Wavenumber Spectra of Ocean surface Waves

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DIRECTIONAL WAVENUMBER SPECTRA OF OCEAN SURFACE WAVES

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Abstract

It has been accepted as a truth that under a steady wind forcing condition, the wind generated waves travel in the direction of the wind vector. Based on field measurements, the directional beamwidth is narrowest near the spectral peak, and increases toward both higher and lower frequency components. Over the last several decades, such scenario is the basis for the design of the directional distribution function of any spectral model for engineering and scientific applications. We present results from a spectral analysis of 3D topography of random surface waves generated by a quasi-steady wind field. The directional spectra display clear bimodal pattern in the wavenumber region just above the spectral peak. The generation mechanism of the bimodal directional distribution in the short wave region is identified to be nonlinear wave-wave interaction. Quantitative comparisons of measured and simulated directional distribution functions are in very good agreement.

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Introduction

Presently, it is believed that weakly nonlinear processes largely control wind wave evolution. The nonlinear theory has been remarkably successful in predicting many aspects of wind wave behavior. Numerical computations show that application of the nonlinear theory results in a bimodal directional spreading (Banner and Young 1994). Measurements from directional wave buoys or wave gauge arrays, however, largely indicate that the wave field is unimodal. As a result, unimodal directional distribution has been assumed in all wave models for scientific and engineering applications. The directional resolution of the conventional wave measurements, however, is also known to be poor and significant variations are found in the analysis results using different processing methods. The contradiction between predicted bimodal and measured unimodal directional distributions has not been resolved even since the nonlinear wave-wave interaction theory was introduced in the early sixties, and the directional characteristics of random ocean waves have remained an unsettled issue over the past four decades. In this paper, we present results from analyzing the 3D ocean surface topography obtained by an airborne scanning lidar system. The analysis provides strong evidence of a bimodal feature in the directional distribution function. The measured bimodal properties are in very good agreement with prediction from nonlinear wave-wave interaction theory.

Directional Wavenumber Spectra

Airborne topographic mapper (ATM, an airborne scanning lidar system) acquires high-resolution spatial measurements of the 3D topography of ocean surface waves (Hwang et al. 2000a,b). From these spatial data, 2D wavenumber spectra can be directly calculated (Fig. 1). These 2D wavenumber spectra have excellent directional resolution, better than $10^{3} (k/k_{p})$ in the dataset analyzed here, where $k$ is wavenumber and subscript $p$ indicates the quantity at the spectral peak. The analysis of the resulting directional distribution shows that the spreading factor is narrowest near the spectral peak wavenumber ($k = 1.3 k_{p}$), and broadens toward both higher and lower wavenumbers from $k_{p}$. These results on the directional beamwidth are consistent with those derived from measurements using directional buoys and wave gauge arrays (Fig. 2), although the value of $k_{p}$ differs from earlier reports: $k_{p} = 0.9 k_{p}$ in Donelan et al. (1985), 1.0 $k_{p}$ in Mitsuyasu et al. (1975), and 1.1 $k_{p}$ in Hasselmann et al. (1980).

The development of bimodal distribution is clearly shown in the 2D wavenumber spectrum obtained from the 3D surface topography. The wavenumber dependence of the lobe angle and lobe ratio is established from the present dataset. Fourier decomposition of the directional distributions is performed. Coefficients of the third order polynomial fitting of the leading 9 Fourier coefficients are listed in Table 1. Compared with measured data, it is found that major features of the directional distributions such as the beam width (spreading factor), lobe angle, and lobe ratio can be sufficiently represented by 4 Fourier components of the distribution function.
Figure 1. An example of the 2D wavenumber spectrum calculated from 3D ocean surface topography. The result is plotted in (a) \([k_x, k_z]\), (b) \([k, \theta]\), and (c) \([\omega, \theta]\) coordinates.

Figure 2. Directional beamwidth calculated from the first moment of the directional distribution function. Measurements from ATM and analytical solutions from three spectral models (Donelan et al. 1985; Mitsuyasu et al. 1975; Håksmallmann et al. 1980) are presented.
Table 1. Polynomial fitting \( y = c_1x^3 + c_2x^2 + c_3x + c_4 \), where \( y \) is \( A_1, A_2, \ldots, A_9 \), and \( x \) is \( k/k_p \) of the Fourier coefficients of directional distributions.

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Numerical experiments (e.g., Komen et al. 1984; Banner and Young 1994) demonstrate that the directional distribution function is sensitive to different formulations of the sources terms. Accurate determination of the directional distribution is needed to enhance our understanding of the ocean wave dynamics, and to validate the performance of nonlinear ocean wave models. Fig. 3 shows a comparison of the measured and computed directional distributions by Banner and Young (1994).

Figure 3 Directional distributions of spectral components based on the numerical results given by Banner and Young (1994) [left two panels]. For comparison, the directional distributions derived from spectral analysis of 3D topography acquired by an airborne scanning lidar system are shown on the right two panels. The wind direction of field data is at \(-45^\circ\).

The directional bimodality is clearly illustrated in both datasets. The bimodal feature of the directional distribution function can be quantified by the lobe angle, defined as the separation angle of the side lobe from the wind direction, and the lobe ratio, defined as the amplitude of the side lobe of the directional distribution to the...
reference value of the directional distribution function at the wind direction. Fig. 4 plots the measured and simulated lobe angles and lobe ratios. The simulated results are in agreement with the measurements, especially in terms of the lobe angle. For the lobe ratio, the two datasets are in good agreement for \( k/k_p < \sim 4 \), at higher wavenumber, the measured data continue to increase, while the computed results appear to reach a saturation level.

![Diagram](image)

Figure 4. A comparison of (a) lobe angles, and (b) lobe ratios measured by the ATM and numerical calculation (Banner and Young 1994) shown in Fig. 3.

**Discussions**

In this paper, we present results of the directional distribution of wind-generated waves. The data source is the spatial measurement of ocean surface topography obtained by an airborne scanning lidar system. Directional spectral analysis of the 3D surface wave topography provides convincing evidence of bimodal features in the wavenumber region just above the spectral peak. For the wave components shorter than the dominant wavelength, the lobe angle and lobe ratio of the directional distribution increase monotonically as wavenumber increases. The bimodal directional distribution is clearly different from the conventional unimodal directional distribution functions presently adapted in ocean wave models.

Earlier analyses of temporal measurements by wave gauge arrays or directional buoys show unimodal distributions (e.g., Mitsuyasu et al. 1975; Hasselmann et al. 1980; Donelan et al. 1985). Bimodal features have been extracted from temporal measurements more recently using maximum likelihood method (MLM) or maximum entropy method (MEM) (Young et al. 1995; Ewans 1998). These results highlight the major difficulty in resolving directional distribution properties from a small number of sensor elements. Depending on the chosen method in the analysis procedure, significant quantitative differences occur. For example, Young (1994)
compares the directional resolutions of the Fourier expansion method (FEM) and the MLM. Considerable broadening of the bimodal feature using either method is illustrated (e.g., Young 1994, Fig. 4). Ewans (1998, Fig. 8) shows a comparison of the bimodal analysis using MLM and MEM. The directional resolution of MEM is much "sharper," and the method is known to produce false bimodal distribution in tests using synthetic data (Ewans 1998; Lyger and Krogstad 1986). Despite these shortcomings, significant progress has been made from Ewans' (1998) MEM analysis. For example, results of the lobe separation angle as a function of dimensionless wave frequency over a wide range of wave age conditions are established from one-year's data collected in an offshore station with well-defined fetch conditions and steady wind fields. He also shows that the simulation results on the lobe angle based on nonlinear wave model simulations are in excellent agreement with field data (Fig. 13 of Ewans 1998). Wang and Hwang extend the analysis of directional buoy measurements to the condition of transient development stage of wind-wave generation. They found that the bimodal characteristics (lobe angle and lobe ratio) under transient development stage are very similar to those under steady wind conditions; and the robust bimodal directional distribution exists throughout the evolution of wind-wave generation.

Bimodal directional distribution has been observed from spatial measurements using aerial stereo photographic technique (Phillips 1958; Cote et al. 1960, Holthuijsen 1983), airborne radar system (Jackson et al. 1985), land-based imaging radar (Wyatt 1995), and airborne scanning lidar system (Hwang et al. 2000a,b). In contrast to the analysis of temporal measurements from wave gauge arrays or directional buoys, standard 2D Fast Fourier Transformation (FFT) procedure is sufficient to bring out the multi-modal feature in the directional distribution from 3D spatial topographic images. Data quality of earlier stereophotography, however, was not very high. Holthuijsen (1983) estimates the dynamic range in their spectral results to be approximately 10 dB. Their data are also significantly affected by the presence of nontrivial swell. The dynamic range of the spectra presented in Cote et al. (1960) is much higher. Based on the contour plots such as those shown in their Fig. 10.12, it is judged that the dynamic range of that dataset is close to 20 dB. Technology has advanced significantly since those earlier wave mapping missions. Specifically, the aircraft motion can be determined more accurately due to the advent of the kinematic GPS (Global Positioning System) technology. As a result, the signal to noise ratio of the measurement also improved considerably. The dynamic range of the of the airborne scanning lidar system measurements presented in Hwang et al. (2000a-b) is approximately 30 dB, i.e., 10 to 100 times improved over the earlier topographic datasets.

The bimodal feature described above mainly addresses the short wave components with wavenumbers higher than the peak wavenumber. The bimodal feature is produced by nonlinear wave-wave interaction. In the early stage of wind wave generation, a different kind of bimodal directional distribution exists. For this situation, the phase speed of the dominant waves is much slower than the wind speed. In the absence of strong background swell, airborne measurements show that two dominant wave systems travel at oblique angles (Fig. 5).
Figure 5. A second kind of bimodal directional distribution produced by resonant propagation of the dominant wave systems. At the young stage of wave generation, resonant condition results in two systems of waves propagating at oblique angles with respect to the wind vector (from right to left), creating the crosshatched surface wave pattern shown in (a). (b) and (c) are the corresponding directional wavenumber spectrum presented in \((k, k_x)\) and \((k, \theta)\), coordinates, respectively. The wind direction is at 0° in the coordinates shown.

Theoretical analysis of wave dynamics suggests that at the young wave stage, resonant propagation between winds and waves results in bimodal or polymodal directional distributions (Phillips 1957). Because the phase velocities of young waves are slower than the wind speed, in order to maintain in resonance condition waves propagate at oblique angles satisfying \(C_p = U \cos \theta\), where \(C_p\) is the phase velocity of the wave component, \(U\) is the wind speed at a reference height proportional to the wavelength, and \(\theta\) is the direction of wave propagation with respect to the wind direction. More detailed discussions are presented in Hwang et al. (2000c).

Clarification of various physical mechanisms is a key step to the improvement of numerical models that provide forecast, nowcast and hindcast. The confirmation of directional bimodality of ocean waves is significant in clarifying the role of nonlinear wave-wave interaction mechanism governing the evolution of ocean waves and in enhancing our wave modeling capabilities. Accurate prescription of the directional properties is also crucial to realistic representation of the ocean surface, which is required input for many electro-optical and electromagnetic remote sensing applications. In other applications such as coastal and harbor engineering, beach protection, and offshore engineering projects, wave amplitude and wave direction are key parameters defining the wave force on the beach and coastal or offshore structures. These ocean engineering projects are costly. Accurate
formulation of the wave directional properties is also important to design optimization.

Summary

In this paper, we present results of the directional distribution of wind-generated waves. The data source is spatial measurements of ocean surface topography obtained by an airborne scanning lidar system. Directional spectral analysis of the 3D surface wave topography provides convincing evidence of bimodal features in the wavenumber region just above the spectral peak. The lobe angle and lobe ratio of the directional distribution increase monotonically as wavenumber increases. The bimodal directional distribution is clearly different from the conventional unimodal directional distribution functions presently adapted in ocean wave models. The generation bimodal directional distribution in the short wave region is generated by nonlinear wave-wave interaction. Quantitative comparisons of measured and numerically calculated directional distribution functions show quantitative agreement.

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References


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