We have performed simulations of one and two dimensional Josephson-junction arrays to study the effects of disorder on power emitted from the arrays. To model a major source of disorder in real arrays, we simulated arrays with a Gaussian distribution of critical currents, as well as ideal arrays with no disorder.
Final Report
for
High-Frequency Properties of Two-Dimensional Josephson-Junction Arrays
AFOSR Grant Number F4962092J0041
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Objective
This research is designed to evaluate the potential of two-dimensional Josephson-junction arrays as high-Q, tunable, EHF sources of electromagnetic radiation. To achieve usable amounts of power, the oscillations of many junctions must be in phase with each other. To assure coherent oscillations, we must understand how the oscillations of single Josephson junctions are affected by the electromagnetic environment of the other junctions in the array, as well as the circuitry used to couple radiation out of the array. Thus, we are studying the high-frequency dynamics (the ac currents and voltages) of Josephson oscillations in arrays, and the mechanisms for emission and absorption of high-frequency radiation in these arrays.

Progress to date

Phase Coherence and Disorder in Josephson-Junction Arrays.\textsuperscript{1} We have performed simulations of one- and two-dimensional Josephson-junction arrays to study the effects of disorder on power emitted from the arrays. To model a major source of disorder in real arrays, we simulated arrays with a Gaussian distribution of critical currents, as well as ideal arrays with no disorder. We found that one-dimensional (series) arrays were rendered useless by a 5\% standard deviation in the critical currents, while two-dimensional square arrays were much less strongly affected, as shown in the spectra of Fig. 1. This difference between series and square arrays is due to the higher interconnectedness of the two-dimensional array, which, in effect, provides many parallel paths for current to follow, diminishing the effects of disorder.
Friction and Inertia of a Vortex in an Underdamped Josephson Array. Arrays designed to be used as sources have non-negligible shunt capacitances associated with each junction; the capacitances are added to raise the Q-factor of the oscillating junctions. One important consequence of these capacitances is that vortices in the array acquire an effective mass which is proportional to the junction capacitance. This is in contrast to the case of superconductors or overdamped arrays, where the vortices are massless. While many of the properties of overdamped arrays could be understood by analogy to superconductors, the analogy no longer holds for underdamped arrays.

To study this further, we have performed numerical simulations of vortex motion in arrays with non-negligible capacitance. We find that, as the capacitance and mass increase, the effective viscous drag force on vortices becomes independent of the junction resistance. This is in contrast to the low-mass limit, where the viscous drag is inversely proportional to the resistance. As a consequence, ballistic vortex motion should be difficult to observe, a result which is in agreement with experiment.

Dynamical States of Underdamped Josephson Arrays in a Magnetic Field. The work discussed in reference (1) dealt with a single vortex in an array, and would apply to experiments where the magnetic field is sufficiently small that vortices in the array do not interact significantly with each other. At higher fields, the situation is much more complex. We simulated the case where the field provides one-half of a flux quantum per unit cell of the array, the case of maximum frustration. The current-voltage characteristics display two distinct regimes, as shown in Fig 2, in contrast to the simpler behavior of single junctions. In the low voltage regime, the entire vortex superlattice moves collectively. At higher voltages, the behavior is analogous to single-junction behavior. In the low-voltage regime, quite complex behavior can occur, including periodic, bifurcating, and chaotic motion. These motions lead to corresponding voltages, because the vortex superlattice velocity is proportional to the time-dependent voltage across the array.

Effect of Inductance in Externally Shunted Josephson Tunnel Junctions. We measure current-voltage (I-V) characteristics of externally shunted Nb-Al$_2$O$_3$-Nb Josephson tunnel
junctions where the external shunts (Mo-Au normal metal film) have non-zero inductance. These junctions and shunts are of the type used in our Westinghouse arrays. At low temperatures ($T < 6.6\,K$), we observe standard I-V curves, as shown in Fig. 3 for $T = 4.2\,K$. When we increase the temperature above $T = 6.6\,K$ we observe an anomaly in the I-V curves, similar to an anomaly seen in the arrays. The anomalous I-V curve measured at temperature $T = 7.6\,K$ is shown in Fig. 4. We determined that this anomaly is caused by the non-zero inductance in the shunt loop. Numerical simulations which properly take into account the effect of inductance show that the dynamics in the anomalous region of the I-V curve are dominated by subharmonic relaxation oscillations. We used a load-line analysis to study the transition between the relaxation oscillations and the Josephson oscillations. The analysis is in good agreement with our data. Our analysis further predicts, and our data confirms, that in order for the relaxation oscillations to occur it is necessary to have

$$\beta_c = \frac{2\pi I_c R_s^2 C}{\Phi_0} < \left(\frac{4}{\pi}\right)^2$$

regardless of the value of the inductance. In Eq. 1, $I_c$ is the critical current of the junction, $R_s$ is the resistance of the shunt, $C$ is the capacitance of the junction, and $\Phi_0$ is the flux quantum.

Emission of Radiation from Josephson-Junction Arrays. A Josephson junction with a voltage across it has a time-dependent supercurrent given by

$$I = I_c \sin\left(\frac{2e}{h} \int V dt\right)$$

so that a junction biased with a constant voltage $V_{DC}$ will have an oscillating current at frequency
\[ f = 2\pi \frac{2e}{\hbar} V_{DC} = 483 \frac{\text{GHz}}{\text{mV}} V_{DC} \] (3)

This oscillating current will in turn create electromagnetic radiation at the same frequency.

A number of problems limit the usefulness of this radiation. First, the radiation from a single junction is too weak, because the characteristic current \( I_C \) is small (0.1 mA is typical) and the impedance match between the device (typically 1 \( \Omega \)) and external circuitry (typically 100 \( \Omega \)) is poor.

To raise the impedance, series arrays have been used, with external circuitry to guarantee that the series junctions oscillate in phase.\(^7\) As discussed in reference (1), series arrays are strongly affected by disorder, and an alternative worth exploring is arrays in the form of square lattices of Josephson junctions. These arrays are much less affected by disorder, and allow much flexibility in design. One can, for example, raise the impedance by making an array with \( N \) junctions in the direction of current flow, and \( M \) junctions in the perpendicular direction, making the impedance \( N/M \) times the impedance of a single junction.

The simplest way to detect emitted radiation is to look for Shapiro steps in a detector junction placed nearby. Our efforts to do this with the detector junction on another chip have so far been unsuccessful. With the detector junction on the same chip, as shown schematically in Fig. 5, radiation has been detected from the NIST arrays from Benz, but not from the Hypres arrays. We are currently uncertain about the cause of this difference, but suspect that the initial Hypres arrays were less uniform than the NIST arrays.

A typical current-voltage (I-V) curve for the detector junction when the array is emitting radiation is shown in Fig. 6. The array is current biased, with an average voltage drop across each junction of \( (1.4 \text{ mV})/6=0.23 \text{ mV} \) and \( (2.0 \text{ mV})/6=0.33 \text{ mV} \) for the two curves shown. The curves show Shapiro steps at integer multiples of these voltages, as expected from theory, with the first step of the 2.0 mV curve being suppressed. These steps indicate radiation is being emitted by the array at 110 GHz and 160 GHz; more detailed analysis of other data shows detected power of order 1/2 \( \mu \text{W} \) at frequencies between 75 GHz and 160 GHz.
One interesting aspect of the emitted power is that it turns on and off suddenly as the voltage across the array is swept; one might have expected a broad resonance with the signal gradually being washed out. A clue to what is going on is offered by the IV curves of the array and detector, each taken with no current through the other, in Fig. 7. The detector, Fig. 7(a), shows a strange two-part IV characteristic, with a critical current around 0.9 mA followed by a slowly-rising voltage regime between 0.2 mV and 0.28 mV, and a second jump in voltage up to an ohmic line. The array characteristic, Fig. 7(b), is qualitatively similar, with the voltage plateau extending over a somewhat greater range. Interestingly, the broad voltage plateau, which falls between (1mV/6) and (1.9mV/6) per junction, is approximately the range of voltages over which radiation is emitted. The midpoint corresponds roughly with the LC resonance frequency estimated for the structure which couples the radiation from the array to the detector.

This data suggests that the array and detector circuits are being strongly affected by the coupling circuit. This raises the possibility that the right external impedance is essential for phase-locking to occur in the array, a situation analogous to that seen in one-dimensional arrays.\textsuperscript{8} This may explain why emitted power is not seen from the Hypres and Westinghouse arrays.

This is clearly a matter which must be explored. If the external circuitry is playing a key part in square array oscillators, understanding the connection may allow more power to be obtained at higher frequencies. By measuring junctions with different parallel impedances and different coupling schemes, we will determine how to optimize performance of these devices.

\textbf{References} \hspace{1em} (*indicates work acknowledging support from this grant)


7. Siyuan Han, Baokang Bi, Wenxing Zhang, and J. E. Lukens, Appl. Phys. Lett. 64, 1424 (1994), and references cited therein.


Conference presentations acknowledging support from this grant


Fig. 1. (a) Power spectral density for the voltage across a 64-junction series array as a function of normalized frequency $\Omega = \hbar \omega / 2eV_{DC}$ for $I_{DC}=1.8 I_c$. Dashed curve is for perfect array, showing sharp spikes at $\Omega=1$ and harmonics; solid curve is for disordered array with 5% standard deviation in critical currents.
Fig. 1. (b) Power spectral density for the voltage across an 8 by 8 square array as a function of normalized frequency $\Omega = \hbar \omega / 2eV_{DC}$ for $I_{DC} = 1.8 I_C$. Dashed curve is for perfect array, showing sharp spikes at $\Omega = 1$ and harmonics; solid curve is for disordered array with 5% standard deviation in critical currents. Note that disordered case is much sharper than disordered case in series array, Fig. 1 (a).
Fig. 2. Normalized dc voltage $V_{DC}/I_c R$ as a function of normalized bias current $I/I_c$ for $\beta_c=5$ (solid line). Dashed line shows a single-junction characteristic for a junction with the same parameters as the junction in the array.
Figure 3: Current-voltage (I-V) characteristic of one of our samples measured at temperature $T = 4.2K$. This I-V curve is in good agreement with the RSJ model.

Figure 4: I-V characteristic of the same sample as in Fig. 3 measured at temperature $T = 7.6K$. The anomaly referred to in the text is the structure above and near the critical current, which is unexpected in the RSJ model.
Fig. 5. Schematic diagram of array and detector circuit. Radiation is coupled out of the array capacitively to the detector junction, denoted d.j. A ground plane (not shown) covers both the array and the coupling capacitor.
Fig. 6. Detector junction current-voltage characteristics for arrays biased at 1.4 mV (solid curve) and 2.0 mV (dashed curve).
Fig. 7. (a) Current-voltage characteristic for detector junction when array is unbiased. Note plateau between 0.09 and 0.15 mA. (b) Current-voltage characteristics for 6 by 6 array when detector junction is not biased.
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OBJECTIVES

- To understand the high-frequency dynamics (the ac currents and voltages) of two-dimensional Josephson junction arrays.
- To understand the mechanisms for emission and absorption of high-frequency radiation from two-dimensional Josephson junction arrays.
- To evaluate the potential and limitations of Josephson junction arrays as tunable high-frequency radiation sources.
- To make spatially-resolved low and high-frequency measurements using a scanning Superconducting QUantum Interference Device (SQUID).

STATUS OF EFFORT

When a Josephson junction is biased at a DC voltage $V_{DC}$, it emits radiation at a frequency $f$ given by

$$ f = \frac{2e}{h} V_{DC} = 483.6 \frac{GHz}{mV} V_{DC} $$

Nb junctions can, for example, oscillate at frequencies up to 1 THz. Unfortunately, single junctions can only produce power on the order of 1 nW to a load of order 1 Ω, making them useless as practical sources.

Our work has been aimed toward building multiple-junction circuits, especially square lattices, to increase the power output. It is necessary for junctions to operate at the same frequency and in phase with each other to achieve maximum power output; we have made great progress here.

- In overdamped arrays, arrays in which the effective Q is low, we have identified the
coupling mechanism which causes synchronization, showing that earlier work in this area was incorrect. This has lead to higher powers and frequencies of operation.

- In *underdamped arrays*, arrays of unshunted junctions in which the internal losses are very low, and the effective Q is high, we have observed amplification via stimulated emission. While this has been known to be a theoretical possibility since the 1970s, this is the first reported observation.

- Our *scanning SQUID microscope* has been successfully used to scan arrays. Remarkably, we find that when a sample is cooled in a field, the response is often paramagnetic, in contrast to the diamagnetic response expected in superconductor. The microscope has also been used to image high-$T_c$ tapes from ORNL, as well as thin films of YBCO.

These topics are discussed in more detail below.

**ACCOMPLISHMENTS/NEW FINDINGS**

*Overdamped Arrays:* Most theories describing the synchronization and phase locking of Josephson-junction arrays use a lumped approach. A major result of these theories is that feedback through an external load is required for phase locking [1,2]. Our experiments indicated the contrary: the load had very little effect on array performance, while details of the coupling to the ground plane over or under the array did.

These results led us to a distributed model for an array. This model recognizes that there is a short section of transmission line between each junction. Numerical simulations based on the transmission line model are in very good agreement with experiment. The correct model can be used to design larger more powerful arrays which work at higher frequencies. In addition to applications for Josephson-junction arrays, the idea of distributed coupling is a new paradigm for the coupling of non-linear oscillators in general. This work has been published in Physical Review B [3].

*Underdamped Arrays:* In last years annual report we showed that arrays of underdamped (unshunted) junctions emit coherently in a novel synchronized state, analogous to the state of a laser: The junctions in the array are strongly coupled to a resonance, single junctions are analogous to single two-level atoms [4], and there is a threshold for radiation output in the array. This work has been published in Physical Review Letters [5] and led to 13 invited talks, with 7 of them at...
conferences.

This work has led to some controversy, with claims being made that the performance can be explained completely within a classical model [6]. While this is always a possibility, the growing weight of evidence favors our original interpretation. While not definitive, recent work from Ohio State [7] has exploited the laser model, and may explain our data.

**Scanning SQUID Microscope:** We have used our low-temperature scanning-SQUID microscope to image arrays cooled in non-zero field [8]. Surprisingly, the net magnetization is sometimes positive and sometimes negative, depending sensitively on the value of the field. (See Figs. 1 and 2.) This is not expected in a superconductor, which is expected to be diamagnetic.

This effect has been seen in granular high and low-Tc superconductors. In the high-Tc case, it has been attributed to the d-wave symmetry of the order parameter. Our results clearly show that d-wave symmetry is not necessary. The effect is caused by flux trapping.

More recently we have used the microscope to study YBCO films on rolled annealed biaxially textured structure (RABiTS) tapes provided by Oak Ridge National Labs. Samples were cooled in nominally zero field and imaged. The field was then increased, and then decreased to zero, and the sample was imaged again. A line cut through the sample, showing a line segment of two such images, is shown in Fig. 3.

The first striking aspect of the figure is that there is a complex magnetic structure even in a zero-field cooled sample. This is the result of the fact that the RABiTS substrate is magnetic, being made largely of Ni. At first site, this seems inconsistent with the low losses of these tapes. It seems, however, that flux does not easily move--the remanence image (taken after the application and removal of a 1.15 Oe field) is quite close to the original image. The places where there are differences (indicated by arrows) are relatively few, and are correlated with micrometer-size holes in the YBCO. Improved processing to removed these holes would probably lead to lower losses.

A simpler example which demonstrates the power of the scanning SQUID is shown in Fig. 4. Here, a line scan was done across a good quality YBCO film on STO. The sample was zero-field cooled, and then a field of 224 mOe was applied and not removed. Experimentally, this field is below \( H_{C1} \), in that flux motion is completely reversible--that is, if the external field were removed,
there would be field trapped in the sample. Fig. 4 is thus a scan of a film in the Meissner state. Analysis indicates good agreement with theory. To our knowledge, this is the first scan of a thin-film sample in the Meissner state. Manuscripts are in preparation for these results.

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PERSONNEL

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PUBLICATIONS


A. P. Nielsen, A. B. Cawthorne, P. Barbara, F. C. Wellstood, C. J. Lobb, R. S. Newrock, and


**INVITED TALKS** (* indicates invited talks at conferences)


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Fig. 1 Representative magnetization images for (a) paramagnetic $\Phi_{\text{external}} = 4.8\Phi_0$ and (b) diamagnetic $\Phi_{\text{external}} = 1.2\Phi_0$. The color scales runs from $-0.6\Phi_0$ (white) to $+0.6\Phi_0$ (black). In (a) the black rectangle delineates the region analyzed to compute the mean magnetization.
Fig. 2. Histograms of the local magnetization for (a) paramagnetic image Fig. 1(a), with an external cooling field of $\Phi_{\text{external}} = 4.8\Phi_0$ and (b) diamagnetic image Fig. 1(b), with an external cooling field of $\Phi_{\text{external}} = 1.2\Phi_0$. Note that the means are positive in (a), indicating paramagnetism, and negative in (b). The histograms are generated using the rectangular region marked off in Fig. 1(a).
Fig. 3. Line scans of magnetic field vs. position of YBCO on RABiTS sample. Open circles indicate zero-field cooled scan; solid squares are scan after an external field of 1.15 Oe has been applied and removed.
Fig. 4. Line scan of YBCO film on STO substrate in the Meissner state. External field is 224 mOe.