Implementation of a Kalman Filter for Single-Sensor Tracking

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Abstract/Background
This research investigates the use of a single passive acoustic sensor for vehicle tracking. The sensor is part of a network of acoustic sensor arrays which use bearings-only measurements for the tracking process. The motion estimates from these deployed arrays are then fused at a gateway where an overall estimate is formed. Although tracking is performed with a single sensor, the bearings-only tracking process is initially unobservable. Previous work [1,2] has shown that for sensors mounted on a moving platform the process becomes observable after a maneuver. In this work initialization is performed using two triangulated sensors to invoke observability. One to three bearing measurements per sensor are used for the initialization process depending on the number of filter states to be initialized. The second sensor is then released and tracking performed on the initialized filter. The residual is monitored for filter divergence and consequently the need for filter re-initialization.

Introduction
Passive bearings-only tracking provides a method of battlefield surveillance without the use of an active, or radiative source. Thus, except for the actual physical structure or array, it is virtually undetectable. In passive tracking, the bearing to the object-under-surveillance relative to a reference direction is the measurement parameter. The 'target' states to be estimated based on this measurement are usually the x and y positions, velocities, and depending of the application, accelerations. Bearings-only tracking has been used in the naval arena for some time, again, as a method to provide the tracking function without emitting a radiative signature. However, one problem with the method is that the process of is unobservable until the tracking vehicle performs a maneuver. For example, for the same bearing measurement, a source could be close to the sensor moving slowly, or at distance from the sensor moving at a proportionately higher velocity. This is presented mathematically in [1,2]. In the naval scenario, this unobservability condition can be eliminated by ownship performing a maneuver. It has been shown [2] that some maneuvers are more optimum than others, leading to more rapid convergence and stability of the tracking filters.

Thus, when trying to implement bearings-only tracking from a stationary array, a method must be used to obtain this observability. In this work, initial source location is obtained by two sensors triangulating to the source. Simulations are then run using a four-state and a six-state extended Kalman filter (EKF) to quantify the tracking performance of a single sensor given that the tracking filters were initialized close enough to the actual source location.
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Model Equations
The Kalman filter is model-based estimator. If the filter is properly designed and the behavior of the source is close enough to the model assumptions, the Kalman filter is an efficient estimator. In the current scenario, the measurement equation is non-linear, i.e.,

\[ z[n] = \tan^{-1} \left( \frac{x_t[n] - x_o}{y_t[n] - y_o} \right) \]

where, \( x_t[n], y_t[n] \) are the coordinates of the acoustic source at time 'n', and \( x_o, y_o \) is the location of the tracking sensor. This requires the use of an extended Kalman filter (EKF). The measurement matrix, \( H \), then consist of the partial derivatives of the measurement equation with respect to the various state variables.

The model assumption is a vehicle moving at a constant velocity. However, this is not the real-world condition, and mechanisms must be provided to handle vehicle maneuvers and random acceleration components. This can be done by one of several methods; the first is to increase the process variance, for example when implementing a four-state estimator. A maneuver indicates a change in acceleration in one, or both, of the coordinates, thus, a second method is to use an augmented model of increased dimensionality to model the acceleration explicitly (i.e., six-state estimator).

The general Kalman filter system and measurement equations are given by

\[
X[n+1] = \phi[n+1,n] X[n] + Bu[n] \\
\text{and,} \\
Z[n] = h[X[n]] + \eta[n]
\]

where,

- \( X[n] \) - State vector
- \( \phi[n+1,n] \) - State transition matrix
- \( Z[n] \) - Measurement
- \( Bu[n] \) - Random acceleration inputs
- \( \eta[n] \) - Random measurement noise

Filter Initialization
Although it is desired to use a single-sensor to perform the tracking task, initialization is performed by using an additional cooperating sensor or sensor array. This eliminates unobservability of the single-sensor-only geometry. Over the first three measurement time steps these sensors use triangulation to provide an accurate estimate of the source location to within the measurement error of the two sensors. In this work the additional problem of track assignment will not be covered. It is assumed that by other means, the two sensors are tracking the same acoustic source. If only position estimates are required, only a single measurement is necessary. If velocity is required, an additional time step is required, i.e., \( v_s = (x_2 - x_1)/T \), where \( x_2 \) is the source location at time 2, \( x_1 \) is the source location at time 1 and \( T \) is the sampling interval. If acceleration is required, an additional time step is required to obtain the second velocity. Once these quantities are obtained, the states of the tracking sensor are initialized. The second cooperating sensor is then 'released'. The general geometry is illustrated in Fig. 1.
Simulations
In operation, the tracking processor receives only lines of bearing to the source. However, to quantify the performance of the various models, a target-track data generator was developed. The output of the EKF thus consisted of both target state truth and target state estimates. The generator can be configured to produce any number of 'legs' to the track. For each leg the source dynamics can be specified including acceleration, duration, and where applicable, initial velocities. The initial inputs to the generator are the sampling interval, \( T \), and the x and y locations, velocities, and accelerations. The generator then produces an output file containing x-y position, velocity, and acceleration data pairs. This file is then used by the EKF to form the bearing measurement. In the simulations, the system sampling/update rate is 1 sample-per-second. The tracking sensor is arbitrarily located at \( x=0, y=0 \). The initial target dynamics are:

\[
\begin{align*}
x_0 &= 2000.0 \text{ meters} \\
y_0 &= 10,000.0 \text{ meters} \\
v_{x0} &= 0.0 \text{ meters/second} \\
v_{y0} &= -15.0 \text{ meters/second} \\
a_{x0} &= 0.0 \text{ meters/second} \\
a_{y0} &= 0.0 \text{ m/second}^2
\end{align*}
\]

These dynamics are held from \( t=0 \) to \( t=400 \) seconds. On leg 2, from \( t=401 \) to \( t=600 \) seconds, an x and y acceleration value of 0.075 m/s\(^2\) is applied. On leg 3, from \( t=660 \) to \( t=1000 \) seconds, the x and y accelerations are again set to zero. The target trajectory is shown in Fig. 2.
Two EKFs were developed to investigate the concept. The models included a four-state constant-velocity model where deviations from the constant velocity assumption, i.e., accelerations, were handled by a variance component proportional to the expected deviations. The second model is a six-state model which models the acceleration components as part of the state.

Results
The tracking performance of the four-state EKF is shown in Fig. 3. This plot was obtained by averaging the results over several trials. In this case, the model is initialized to the correct vehicle position. Several other simulations were performed with the filter initialized to 0.5, 1.0, 2.0, and 5.0 degree initial measurement error. The results were similar for 0.5, 1.0, and 2.0 degrees but degraded when the starting location error was 5.0 degrees. Similar results are shown for the six-state model in Fig. 4.

Conclusions
It was shown that an EKF could be successfully implemented to perform the task of ‘single’-sensor bearings-only tracking. The models shown here have not had their process and measurement variances tuned to the dynamics of the actual vehicles, so the maneuver tracking performance is expected to improve, i.e., in the simulations, the region from approximately 400 to 490 seconds. By tuning the measurement variance, better performance is anticipated while initializing the EKF.
Future Work
There are currently two more Cartesian-coordinate based models under development. The first is a four-state model which uses psuedo-linear measurements. The second is an interacting multiple model (IMM) [3,4,5] implementation which will initially use two models; a low noise variance model for constant velocity vehicle dynamics, and a high noise variance model for constant acceleration dynamics. Additional candidate models are a stop model and a coordinated turn model. A polar coordinate implementation is also possible. The polar coordinate model has the benefit that all of the model states are observable.

References


