Adaptation of Flux-Corrected Transport Algorithms for Modelling Blast Waves


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Explosions  Adaptive gridding

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Flux-corrected transport represents an accurate and flexible class of methods for solving nonsteady compressible flow problems. In models which treat all the physical effects required for blast wave simulation, truncation errors inherent in the underlying finite-difference scheme are exacerbated by nonlinear coupling between the fluid equations and by the greater complexity of the phenomena being simulated. In order to improve the properties of the basic difference scheme, we propose a new algorithm for integrating generalized continuity equations over a timestep $\delta t$. 

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ADAPTATION OF FLUX-CORRECTED TRANSPORT ALGORITHMS
FOR MODELLING BLAST WAVES

Blast wave phenomena include reactive and two-phase flows associated with the motion of chemical explosion products; the propagation of shocks, rarefaction waves, and contact discontinuities through a nonideal medium (real air, possibly thermally stratified and containing dust and water vapor); and the interaction of the blast waves (including boundary layer effects) with structural surfaces. Flux-Corrected Transport (FCT) represents an accurate and flexible class of methods for solving such nonsteady compressible flow problems (Boris and Book, 1976). Coupled with a nondiffusive adaptive gridding scheme (Book et al., 1980; Fry et al., 1981), it enables complex time-dependent shocks to be efficiently "captured."

In models which treat all the physical effects required for blast wave simulation, truncation errors inherent in the underlying finite-difference scheme are exacerbated by nonlinear coupling between the fluid equations and by the greater complexity of the phenomena being simulated. Typical of these errors are the "terraces" which develop under some circumstances on the flanks of sloping profiles when the growth of ripples due to phase errors at short length scales is terminated by the action of the flux limiter. Two approaches are possible toward eliminating them: improving the short-wavelength phase and amplitude properties of the underlying algorithm, and switching on additional diffusion locally. The latter approach folds information about the shape of the profile and the nature of the physical process taking place (e.g., rarefaction) into the switch criterion, thus changing the FCT technique from a "convective equation solver" to a "fluid system solver." In doing this, care must be taken to avoid losing the accuracy, robustness and problem-independence which constitute valuable attributes of FCT algorithms (Book et al., 1981).

Tests carried out on scalar advection of simple density profiles by a uniform flow field show that terracing does not require either diverging velocities or discontinuities in the profile, but appears typically (for \( v > 0 \)) where the first and second derivatives of density have the same sign (Fig. 1). In order to improve the properties of the basic difference scheme, we propose a new algorithm for integrating generalized continuity equations over a timestep \( \Delta t \). Consider the following three-point transport scheme:

\[
\hat{\rho}_j = \rho_j^0 - \eta (\rho_{j+1}^0 - \rho_{j-1}^0) + \kappa (\rho_{j+1}^0 - 2\rho_j^0 + \rho_{j-1}^0);
\]

\[
\bar{\rho}_j = \hat{\rho}_j - \theta (\rho_{j+1}^0 - \rho_{j-1}^0) + \lambda (\rho_{j+1}^0 - 2\rho_j^0 + \rho_{j-1}^0);
\]

\[
\rho_j^n = \bar{\rho}_j - \nu (\phi_{j+1/2} - \phi_{j-1/2}),
\]

where

\[
\phi_{j+1/2} = \hat{\rho}_{j+1} - \hat{\rho}_j.
\]

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The arrays \( \{ \rho_j^O \} \) and \( \{ \rho_j^N \} \) are the old and new densities, \( \hat{\rho}_j \) and \( \tilde{\rho}_j \) are temporary intermediate densities, and \( \eta \), \( \theta \), \( k \), \( \lambda \), and \( \mu \) are velocity-dependent coefficients. Here \( \kappa \) and \( \lambda \) are diffusion coefficients, and \( \mu \) is the antidiffusion coefficient. In the actual algorithm, \( \phi_j^{c+1/2} \) is corrected (hence the name FCT) to a value \( \phi_j^{c+1/2} \) chosen so no extrema in \( \rho_j \) can be enhanced or new ones introduced in \( \rho_j \). Previous FCT algorithms had \( \theta = 0 \); the widely used EBFCT and related algorithms (Boris, 1976) have in addition \( \kappa = 0 \). If we define \( \rho_j \) to be sinusoidal with wave number \( k \) on a mesh with uniform spacing \( \delta x \), so that \( \rho_j^O = \exp (i j \beta) \) where \( \beta = k \delta x \), then the new density array satisfies
\[
\frac{\rho_j^N}{\rho_j^O} = A = 1 - 2i(\eta + \theta) \sin \beta + 2(\kappa + \lambda)(\cos \beta - 1) - 2\mu(\cos \beta - 1)[1 - 2i\eta \sin \beta + 2\kappa(\cos \beta - 1)].
\]

From \( A \) we can determine the amplification \( \alpha = A \) and relative phase error \( R = (1/\epsilon^2\tan^{-1}(-\Im A/\Re A)) - 1 \), where \( \epsilon = \nu \delta t / \delta x \) is the Courant number. Expanding in powers of \( \beta \) we find
\[
\alpha = 1 + \alpha_2 \beta^2 + \alpha_4 \beta^4 + \alpha_6 \beta^6 + \ldots;
\]
\[
R = R_0 + R_2 \beta^2 + R_4 \beta^4 + R_6 \beta^6 + \ldots.
\]
First-order accuracy entails making \( R_0 \) vanish, which requires that \( \eta + \theta = \epsilon/2 \). Second-order accuracy (\( \alpha_2 = 0 \)) implies that \( \mu = \kappa + \lambda - \epsilon^2/2 \). Analogously, the "reduced-phase-error" property \( R_2 = 0 \) (Boris and Book, 1976) determines \( \mu = (1-\epsilon^2)/6 \), thus leaving two free parameters. One of these can be used to make \( R_4 \) vanish also. The resulting phase error \( R(\beta) \) is small not only as \( \beta \to 0 \), but also for larger values of \( \beta \), corresponding to the short wavelengths responsible for terraces (Fig. 2). The remaining parameter \( \eta \) can be chosen to relax the Courant number restriction needed to ensure positivity from \( \epsilon < 1/2 \) to \( \epsilon < 1 \). When coded, these changes necessitate a small increase in the operation count of EBFCT along with a small increase in overhead to precalculate the two new arrays of velocity-dependent transport coefficients. On advection tests, the new algorithm completely eliminated terraces (Fig. 3). When applied to the coupled systems of gas dynamic equations, it produced profiles which closely approximate the Riemann solution of the exploding diaphragm problem (Fig. 4).

The second approach uses a rarefaction flux limiter (RFL) to eliminate numerical ripples in strong rarefaction waves. This approach is physically motivated. Raw anti-diffusive fluxes \( \phi_j^{c+1/2} \) are limited so that the slope of local flow field profiles decays with time in a rarefaction wave. In effect, additional diffusion is left in the field to maintain monotonicity of local slopes. For multi-material calculations a "contact surface sensor" is needed to detect physical discontinuities and shut off the RFL locally.

In addition we found that some care was required when applying generalized continuity equation solvers to a system of equations. Truncation errors of the various equations can interact, causing undershoots or overshoots in nonconvective quantities such as pressure. We found that it was necessary to monotize derived quantities (pressure, velocity) before using them in minimal-diffusion transport algorithms.
The above methodology has been applied to a series of test problems initiated by a spherical high-explosive (HE) detonation in air. An ideal Chapman-Jouguet detonation was used to specify the initial conditions; afterburning was neglected. In the absence of reflecting surfaces, spherical symmetry is maintained and the calculation remains one-dimensional. A nonuniform radial grid was used with extremely fine zoning near the shock front. The grid was moved so that the shock remained approximately fixed with respect to the mesh. The original version of the FCT algorithm gave rise to pronounced terraces in the rarefaction region. This would have rendered any two-dimensional calculation involving shock diffraction or nonideal effects dubious. The techniques described here improved the blast wave results considerably. The decrease in phase error reduced terracing dramatically.

Next, a two-dimensional (2D) numerical calculation was performed to simulate one of Carpenter's (1974) height-of-burst experiments which used spherical 8-lb. charges of PBX 9404 at 51.6 cm. The previous fine-zoned 1D calculation was used to initialize the problem. It was mapped onto the 2D grid just prior to the onset of reflection. The solution was then advanced in time, with pressure being calculated from a real-air equation of state and a JW equation of state for the combustion products. The front of the blast wave was captured in a finely gridded region which moved outward horizontally. Special care was taken to ensure that the grid moved smoothly. The resulting solution, particularly the curve of peak overpressure vs. range, was consistent with Carpenter's experimental data (Fig. 8). Although this calculation represents a reasonable accurate simulation of the double-Mach-stem region, no doubt improvements can and will be made to numerically model such phenomena.

References


Fig. 1 — Rounded half circle used in passive scalar advection tests (a) initially, and (b) after propagation for 14 cycles using JPBFC. Note that terraces form even, as here, in the absence of corners in the profile. Tick marks indicate computational zones (N = 100).
Fig. 2 — Contour plot of $R(\beta, \epsilon)$ for new multicoefficient FCT algorithm. Note $R \approx 0$ except for $\beta \geq 3 \pi/2$. The relative phase error vanishes exactly for $\epsilon = 1/2$ and $\epsilon = 1$. 
Fig. 3 — (a) Blowup of Fig. 1(a) (dashed line) compared with (b) same profile as computed using new sixth order-phase-accurate FCT algorithm. Solid traces are exact solutions.
Fig. 4 — (a) Exact and (b) computed solution of exploding diaphragm problem (10-to-1 initial density jump, 100-to-1 initial pressure jump)
Fig. 5 — One-dimensional solution of expanding HE products and air calculated with the new algorithm using 500 equally spaced zones. Note contact surface separating HE products from air.
Fig. 6 — Adaptive grid for height-of-burst problem shown (a) initially and (b) at time when transition to Mach reflection occurs.
Fig. 7 — Pressure-time histories directly beneath burst site. Note second peak, associated with interaction between shock reflected from ground and following contact surface.
Fig. 8 — Computed peak overpressure vs distance along ground surface.
Broken curve represents Carpenter's (1974) data.
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