IMPLEMENTATION OF A TWO-PROBE TIP-TIMING TECHNIQUE TO DETERMINE COMRESSOR BLADE VIBRATIONS

by

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June 2000

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**ABSTRACT (maximum 200 words)**

This study involved the implementation and validation of a blade-tip time of arrival (TOA) measurement technique, and the development of a computer program to analyze TOA data using a recently published approach. The program was used to analyze experimental compressor data taken in-house using two laser light probes, data generated computationally, and data obtained by others in a compressor test. The in-house compressor data was compared successfully to amplitudes obtained by strobed digital photography. A resonance was successfully detected in the supplied compressor data set.
IMPLEMENTATION OF A TWO PROBE TIP-TIMING TECHNIQUE TO DETERMINE COMPRESSOR BLADE VIBRATIONS

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Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
June 2000

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I. INTRODUCTION

A. BACKGROUND

The overall aim of current research related to engine high cycle fatigue (HCF), is to develop the capability to predict, and consequently avoid, HCF-related failures of turbine and compressor blades. New testing procedures are required that will involve both excitation of vibrational modes in the blading of spinning rotors, and the measurement of the blades’ response. The present work deals with the problem of measuring the blade response, using a technique that does not require instrumentation to be in contact with the blades.

B. CONCEPT

Non-contact measurements of rotating turbomachinery are well documented, some dating to the early 1970’s (Zablotsky, 1970). The current experimental system employed two laser probes peripherally separated but at the same axial station over the blade tips. By determining the exact time required for each blade to pass from a reference position to the sensors, the deflection of the blade from its mean position could be determined. From such time of arrival (TOA) data, the harmonic oscillation of each compressor blade, at varying shaft speeds, may be determined.

The present study involved the implementation and validation of the measurement technique and the development of a computer program to analyze TOA data using a recently published approach (Heath, 1999). The program was used to analyze experimental compressor data taken from optical sensors, data generated computationally, and data obtained by contractors in a compressor test in another
laboratory. The in-house compressor data was also compared to amplitudes obtained by digital photography, in order to validate the TOA measurements.

C. BENEFITS

The thesis project will benefit the Navy, and other organizations and companies interested in affordable high cycle fatigue testing. However, the project’s immediate purpose is to provide non-contact HCF-related blade vibration measurements in the Naval Postgraduate School’s rotor spin pit. The technology developed will be transitioned to the Naval Air Warfare Center Aircraft Division, to be used in HCF-related rotor spin testing.

* Hood Technologies Corporation supplied the data from tests conducted at Wright Patterson Air Force Base. This cooperation is acknowledged and is appreciated.
II. TEST APPARATUS AND DATA ACQUISITION

A. COMPRESSOR AND BLADING

The test facility was a 36-inch diameter, two-stage compressor mounted horizontally in the laboratory. As is illustrated in figure 2.1, air was drawn from outside the building (through an inlet bellmouth mounted outside the wall) and exhausted into the room. The compressor was driven by a 150 HP synchronous motor with a nominal rotational speed of 1610 RPM. Due to the nature of the electric motor, speed variation could only be achieved by cutting the power to the compressor.

![Figure 2.1 Compressor and wall mounted inlet.](image)

A section through the compressor and the wall survey assembly are shown in Figures 2.2 and 2.3. To accommodate sensors over the tip of the second rotor, a matrix of 25 plug locations was machined in the case wall of the compressor (Moyle, 1991).^*

This allowed the angular separations between the two laser probes to be varied in increments of 2.8125 degrees, and the axial positions to be varied over the length of the

---

^* The wall survey assembly was designed to allow a single Kulite pressure sensor to be used for rotor surveys. The light probes in the present study were mounted in plugs that were identical to that used by the Kulite.
Figure 2.2 Compressor diagram showing the locations of the wall survey assembly and Plexiglas window in relation to the rotor stages. From Moyle (1991)

Figure 2.3 Detailed view of the wall survey assembly, window, and rotor blade with cross. From Moyle (1991)
rotor blades. Upstream of the wall survey plug assembly, a Plexiglas window, also visible in Figures 2.2 and 2.3, was located to visually observe the rotation of the first rotor stage. A red cross was drawn on one of the compressor blades of the first stage to serve as an angular reference for the experiment. Further technical information for the compressor is available in Moyle (1991).

The compressor blades were constructed by casting aluminum-filled epoxy resin in room temperature vulcanization silicone rubber molds. The blade profiles were based upon a circular arc camber line, with a thickness distribution that was flattened at the leading edge. Each blade was cast around a steel shank and threaded bolt, with glass cloth and carbon fiber to reinforce the root. Complete design information is available in Vavra (1970), while construction is detailed in Moyle (1981).

B. LASER LIGHT PROBE SYSTEM

A schematic of the instrumentation and the data acquisition system is shown in Figure 2.4. The data acquisition station for the light probe measurements is shown in Figures 2.5 and 2.6. A strobe (not shown in the figure), and the ‘Delayed Pulse’ feature of a Wavetek Model 145 function generator were used to position a rotor blade, with a red cross at its tip, in the center of the Plexiglas window and concurrently initiate Time of Arrival (TOA) measurements with a one-per-rev signal. A four-channel Hewlett Packard Infinium oscilloscope was used to set-up and monitor the acquisition process. The laser probe system used for the project was manufactured by Integrated Fiber Optic Systems Incorporated, of Stony Brook, New York (Dhadwal, 1999).

\footnote{The four channel system was provided on loan by the instrumentation group of Wright Pattersen Air Force Base}
Figure 2.4 Schematic of the Instrumentation and Data Acquisition System.
Figure 2.5 Test apparatus showing data acquisition equipment.

Figure 2.6 Laser Light Probe System and Blade Vibration Sensor Interface Board.
Each channel of the four channel system was composed of three elements; a transmitter module, the fiber optic blade tip sensor, and a receiving module. The transmitter module operated as a continuous laser light source for the sensor. Through fiber optic filaments, the laser light was delivered to the tip sensors where it struck each compressor blade, of the second rotor stage, as it passed the wall survey assembly pictured in Figure 2.3. At the passage of each blade, the light reflected from the blade tip was received by collecting fibers, and transmitted to the receiving module. The receiving module produced either analog voltages or square pulse signals that were input to the Blade Vibration Sensor Interface. Figure 2.6 depicts the transmitter and receiver modules rack-mounted with power supplies manufactured in house.

C. BLADE VIBRATION SENSOR INTERFACE (BVSI) AND DATA ACQUISITION

The Blade Vibration Sensor Interface was designed by the Hood Technology Corporation of Hood River, Oregon (Figure 2.6). A picture and details of the board are given in Appendix A. The purpose of the interface was to receive a one-per-rev and two tip-timing analog pulse trains, and to output corresponding time accurate TTL pulses to a National Instruments PCI counter board in the PC. The board was expressly designed to allow the conversion to digital pulses to be controlled (with adjustments to arming and trigger levels on each channel), and to allow the selection of which signals to interrogate on two oscilloscope channels. It must be noted that to avoid identifying extra blade pulses, the arming and triggering levels must be set at their lowest possible voltages to prevent triggering on blade tip imperfections.
The Blade Vibration Monitoring (BVM) Labview software for the PC (provided by Hood Technology Corporation) allowed the user to monitor the performance of the sensor systems in real time. The control window is shown in Figure 2.7. Once the start button was clicked, the software began recording data and continued until the stop button was pressed. The software then prompted the user to name the files and designate a location to save them. The automatic reduction of this TOA data by the software then converted the data for each blade to measurements of thousandths of an inch, and stored this information in the designated files. The software required the input of the rotor diameter, the angular positions of the two probes, and the angular position of the first rotor blade. The derivation of these precise angular measurements is described in Appendix B1. A sample raw data file is present in Appendix B2.
III. DATA ANALYSIS

A. METHOD OF ANALYSIS

After initial examination of the amplitude of tip motion, the method of analysis developed by Heath was programmed to derive frequencies and amplitudes of resonant behavior from TOA measurements (Heath, 1999). The procedure requires the acquisition of two sets of tip-timing measurements from optical probes mounted at the same axial plane in the case wall. By plotting the tip-deflection data for a particular blade, taken when the compressor is transitioning a resonance, for two probes, one against the other, both the resonance order and the maximum amplitude of the resonance can be determined. Figure 3.1 illustrates this method. The plotted data for the blade deflections forms a near ellipse. Through the method of least squares, a smooth curve is fitted to the data and the lengths of the major and minor axes are measured. The ratio of the minor axis to the major axes is a measure of the engine order of the resonance.

![Graph showing tip-deflection data for a particular blade with a near-elliptical plot representing Heath's Method.](image)

**Figure 3.1** Heath's Method.
B. PROGRAM DESCRIPTION

Matlab was used to implement the method of analysis (Appendix C1) described above, and to program a test data generator (Appendix C2). The necessity for the latter will be described in greater detail in the following chapter. The Matlab language was chosen because of its compatibility with the data generated by the Labview software, the ease with which its programs can be modified, and its superb graphical analysis tools. Both programs are fully documented in the Appendix C.
IV. EXPERIMENTAL PROGRAM AND RESULTS

A. OVERVIEW

Following the integration of the sensor system with the compressor test rig, and the completion of the analysis software, the entire system was tested to ensure the accuracy of the blade displacement measurements output by the analysis program. A digital camera was fixed above the Plexiglas window, and digital movies were taken of blade 1 as it was flashed by a strobe light. The strobe was triggered by the delayed one-per-rev pulse. The digital movies were then analyzed, using a PC to display each strobe-illuminated frame. The blade’s displacement, with reference to the gradations etched in the window, was measured, and the results compared to those given by the analysis program.

Once the displacement measurements were validated, a spare rotor blade was bowed to identify its resonant frequencies. A Campbell diagram, generated earlier using a similar rotor blade was used to verify the results of the bowing. Transient tests were then conducted by cutting power to the compressor while recording TOA data. The results given by the analysis program, were then compared to those expected from resonant crossings on the Campbell diagram. Indications of resonance were found, but the deceleration rate was too rapid to allow a clear definition of frequency and amplitude.

Since the deceleration rate could not be controlled, in order to test the validity of the analysis program, an artificial data generator was created in Matlab. This program, listed in Appendix C2, was used to simulate the data that would result from the behavior of a resonating rotor blade. Using an error function, the amplitude of the blade’s vibration about its mean position could be set to maximize at a selected frequency. By
successfully analyzing the artificially generated TOA data file, the accuracy and utility of the analysis program were verified.

Finally, a series of TOA data files were obtained from the Hood Technology Corporation. The files contained data that was taken during compressor tests conducted at Wright Patterson Airforce Base. These files were analyzed by the analysis program as a last check.

**B. SYSTEM VALIDATION WITH DIGITAL PHOTOGRAPHY**

The goal was to compare blade vibration amplitudes derived from TOA measurements by the analysis program, and amplitudes obtained by direct measurement, through the use of a digital camera. The camera was fixed to a rigid bracket that was attached to the compressor case just below the Plexiglas window. The bracket enabled the camera to be positioned perpendicular to the window, and to be held firmly in position during the compressor’s operation. A series of digital movies in an ‘mpeg’ format were then taken while the compressor was running at constant speed, and converted to still ‘jpeg’ images using a PC and ‘Paint Shop Pro’ software. Figures 4.1 and 4.2 show images taken on successive revolutions, as converted jpeg images. The motion of the compressor blade is clearly evident between the two pictures. (The gradations etched into the window have been computer enhanced). In order to define a mean position for the blade during the compressor’s operation, the red cross drawn on the tip was aligned with the bottom gradation, while the tip of the leading edge was aligned with the center gradation. Adjusting the time delay of the pulse controlled the visible position of the blade. The deviation of the blade away from this mean position shown in the second picture represents the maximum for this run.
Figure 4.1 Mean blade position during compressor operation.

Figure 4.2 Maximum deviation away from the mean.
The (enhanced) gradations were 0.5 inches apart on the Plexiglas surface, and the width of the red cross was approximately 0.05 inches. Using these measurements to scale the displacements visible on the PC image (using lengths in numbers of pixels), the maximum movement of the blade for this particular run was determined to be approximately 0.03 inches. The corresponding displacements recorded by the two sensors and output by the analysis program for this same run, are shown plotted in Figure 4.3. It can be seen that the maximum amplitude derived from the photographs, and that deduced from probe TOA measurements, were in close agreement. Although the pictures (taken through the Plexiglas window), and the sensors record separate stages of the compressor, the behavior of the blades for each stage is similar. A plot depicting amplitude versus time, at constant speed, for both sensors, showing blade number one is presented in Appendix D1, while a plot showing data for all thirty blades per sensor is illustrated in Appendix D2.

![Graph](image)

**Figure 4.3** Constant Compressor Speed Results for the Displacement of Blade 1. (Scale is in 0.001 inches).

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C. IDENTIFICATION OF BLADE RESONANCES

Potential blade resonance was determined by bowing a compressor blade to establish modal response, and constructing the Campbell diagram for the blade. The Campbell diagram for the present rotor blade is shown in Figure 4.4. Typical results of bowing the blade are shown in Figure 4.5. The figure shows the FFT of the amplified sound produced by bowing.

![Diagram of Campbell diagram](image)

**Figure 4.4** Rotor Blade Campbell Diagram.

* The diagram has been replotted using data obtained in an earlier graduate class laboratory experiment.
Figure 4.5 Frequencies excited by bowing a rotor blade.

D. ANALYSIS OF RESULTS WITH DECELERATION

Compounding the problem of being unable to control the deceleration (the electric drive motor was synchronous and could not be adjusted), the damping caused by the air flowing through the compressor restricted the excitation of the blades as they transitioned the resonances. Figure 4.6 shows the results of a deceleration.
In Figure 4.6, the x and y axes have been rotated -45 degrees so that the 45 degree slope line lies along the x axis. This was done to enable the major and minor axes of any resulting ellipses to be more easily measured (Heath, 1999). The string of data points represents the blade unbending as the compressor slowed. The y-axis depicts the deviation away from the mean bending line. The points clustered to the far right were taken at the constant operating speed of approximately 1610 RPM. Data points for speeds below approximately 400 RPM could not be plotted because of hardware limitations. (The data points were plotted in groups of ten, with neighboring groups possessing different colors.) Zero on the graph is the approximate unbent position of the blade. Due to the uncontrollable deceleration and dampening, the regions where the compressor transitions blade resonances appear only as areas of unpronounced excitation.
These groupings of data points occurred between 1400-1200 RPM (23.33-20 Hz), and 900-600 RPM (15-10 Hz). When compared to the Campbell diagram these speeds correspond to the fourth and sixth engine order crossings of the first bending mode. The torsional crossings apparent in the Campbell diagram were not detected due to their smaller amplitudes and the effects of damping in the compressor.

E. ANALYSIS OF TEST DATA

To ensure that the analysis program was functioning properly, an artificial data set was generated to simulate a resonating compressor blade. The complete program is listed in Appendix C.2. Using an error function, the program generated a data file that, when plotted, created a figure that spiraled out to form an ellipse when a specific frequency was transitioned. The test data was designed to simulate the first bending mode at the 900 RPM crossing of the rotor blade. Figure 4.7 shows the results of analyzing the test data. As is illustrated by the ellipse fit to the test data, visible in blue, despite the deliberate addition of noise to the program, the original parameters were deduced.

**Figure 4.7** Analysis of Test Data (0.001 in. per division)
F. HOOD TECHNOLOGIES DATA

The data files provided by Hood Technologies from vibration tests at Wright Patterson Airforce Base enabled the analysis program to be further tested. The data files were generated with better controlled test conditions than was possible with the compressor at the Postgraduate School. However, some processing of the raw data was required before the application of Heath’s method was successful. After a process of trial and error it was found that an eighth order polynomial to smooth the TOA data for each blade, and averaging the rotor speed over the operating time, provided reasonably noise free results. Figure 4.8 depicts the results of plotting the data files for blade one against one another after implementing the smoothing process. The outline of an ellipse is clearly visible. By narrowing the field of data input to the analysis program, the ellipse was found to be generated between 12960 and 13000 RPM. Figure 4.9 shows the results of restricting the window, and Figure 4.10 shows the ellipse fit by the analysis program, in applying Heath’s method.

![Figure 4.8 Analysis of Hood Technologies data file from tests conducted at Wright Patterson AFB](image)
Figure 4.9 Ellipse at resonant rotor speed (12960-13000 RPM)

Figure 4.10 Ellipse fit for Heath’s Method
Applying Heath’s method to the axis ratio, the vibration was correctly identified as a fourth engine order resonance at approximately 13000 RPM, and the amplitude may be easily read from the displacement scale. It is noted that the peak amplitude of the blade vibration was only 0.002-0.003 inches, which is an order of magnitude smaller than was measured in the NPS low speed compressor. Thus the method is capable of identifying blade resonances in the NPS transonic compressor, which has a rotor diameter of eleven inches, and in similar, and all larger, rotors which are to be tested in the HCF/Spin Test research program.

The ellipse of data points in Figure 4.10 is canted slightly down on the left side because the sensors were not placed on the same axial plane. It is noted that Heath’s method requires that the sensors be on the same axial plane. The reason that the resonance was found successfully with the present data is that the resonance was a bending mode. Indeed, the attempts to identify reported non-bending modes in the data set, were not successful.
V. CONCLUSIONS AND RECOMMENDATIONS

A two-channel laser light probe system was implemented successfully to obtain blade tip timing, ‘time of arrival’ (TOA) data on a low-speed compressor. The amplitudes of tip deflection measured by the system were successfully verified using strobed photography. No significant problem was encountered in using light probes with the BVM interface board and acquisition software, which were developed and used by Hood Technologies with other types of sensors. The completed system constitutes the ‘front-end’ of a ‘non-contact stress measurement system’ (NSMS) (Jones, 1996), which can now be used in the HCF/Rotor Spin Research Facility.

The method of Heath, to detect and identify blade resonance from TOA data, was programmed and used successfully to identify a blade resonance in compressor test data acquired from the Hood Technologies Corporation. It was found that data smoothing procedures were required before the method gave definitive results on the supplied data set. However, the amplitudes involved were very small, (0.002-0.003 inches), and smoothing may not always be necessary. Attempts to identify blade resonance from data taken in the NPS Low Speed Multi-Stage Compressor were not successful because the deceleration of the compressor could only be effected by cutting power, generating too-rapid a deceleration. Confidence in the programmed analysis method was established, however, by programming the calculation of a virtual data set for a compressor decelerating through a blade resonance, and then identifying the programmed resonance using the analysis. The programmed calculation of data can be used to investigate the robustness of the analysis method.

With the knowledge gained thus far, recommendations are to:
-Implement the probe and acquisition system in the Rotor Spin Research Facility and obtain data as specific resonance conditions are deliberately excited in the blades.
- Solicit and analyze other acceleration or deceleration data sets (containing resonant behavior) which were acquired with probes located at the same axial plane.

It is noted that the data acquired in spin-testing, in which the rotor is hung on a slender spindle, may require a different analysis than data from compressor tests, in which the rotor is spun between nearly rigid bearings. Thus the recommendations should be followed consonantly.
APPENDIX A. HOOD TECHNOLOGIES BLADE VIBRATION SENSOR INTERFACE BOARD

Electric Characteristics:

Power In:

<table>
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<tr>
<th>Ground</th>
<th>DIN Pin 1</th>
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<tr>
<td>+5VDC</td>
<td>≤500mA DIN Pin 3</td>
</tr>
<tr>
<td>-12VDC 300mA</td>
<td>DIN Pin 4</td>
</tr>
<tr>
<td>+12VDC</td>
<td>300mA DIN Pin 5</td>
</tr>
</tbody>
</table>

Power In: Use ELPAC WM113-TT

Power Out:

<table>
<thead>
<tr>
<th>Ground</th>
<th>Sub-D Pins 6,7,8,9</th>
</tr>
</thead>
<tbody>
<tr>
<td>+12VDC</td>
<td>Sub-D Pin 4</td>
</tr>
<tr>
<td>-12VDC</td>
<td>Sub-D Pin 5</td>
</tr>
</tbody>
</table>
APPENDIX B1. DETERMINATION OF ANGULAR POSITIONS FOR THE BVM LABVIEW SOFTWARE

The angular positions for each of the probes was measured using the machine drawings for the compressor to obtain key dimensions. The horizontal seam between the upper and lower sections of the compressor was used as the reference, with 0 degrees represented by the seam nearest the wall survey assembly. The probes were used in positions corresponding to 61.8750 degrees and 73.1250 degrees. (This was the greatest separation allowed by the wall survey assembly, and was used to provide the largest possible movement of the rotor blades between the sensors.)

The BVM Labview software required the input of the angle of the first rotor blade at the one-per-rev pulse. This angle was calculated by using the 'Delayed Pulse' function of a Wavetek signal generator. By inputting the one-per-rev pulse into the Wavetek, and delaying the pulse until the rotor blade with the red cross was visible in the center of the Plexiglas window, the timing difference between the one-per-rev pulse and the delayed pulse was measured to be 1.6727 ms (Figure B.1). (It is important to note that the Wavetek must output a negative pulse in order to operate the strobe.) Since the center of the window was known to be 55 degrees in reference to the seam, and the compressor operated at a mean speed of 1611.8 RPM, the angle of blade 1 at the one-per-rev. (the rotor with the red cross) was calculated to be 43.9 degrees.
Figure B1. Oscilloscope window for the determination of the timing delay.
### APPENDIX B2. SAMPLE DATA FILE
(Data for blades 2-30 are not shown)

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<thead>
<tr>
<th>TIME (SEC)</th>
<th>SPEED (RPM)</th>
<th>TOA (BLADE 1) (0.001 in.)</th>
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APPENDIX C1. MATLAB IMPLEMENTATION OF HEATH'S METHOD

clear all;
%All previous variables are cleared

sen = input('Input the number of sensors currently in use:');
%The user is asked to input the number
%of sensors used.

for i=1:sen
%The for loop is initiated once for each sensor in use.

    in = input('Enter the name of the desired ASCII input file with its extension:');
    s = load (in);
%The user is asked for a Labview ASCII file
%and it is read into Matlab.

    m = mean(s);
%The mean value of each column of the ASCII file
%is computed and saved as a row vector.

    [q,r] = size(s);
%The size of the ASCII file is computed: [rows,columns].

    for y = 3:r
        for z = 1:q
            dat(z,y-2) = s(z,y) - m(1,y);
        end
    end
%The tip-timing data for each blade is normalized about
%its mean position, calculated previously, by subtracting
%the mean for each blade from its respective column of
%values. This data is saved into the matrix dat.

    e = num2str(i);
    d = ['in_' e];
    eval([d, ' =in;']);
%Each ASCII file is saved under the variable in_(and the
%number of its corresponding sensor).

    a = 'dat_';
    b = num2str(i);
    c = [a b];
    eval([c, ' =dat;']);
% Each matrix of normalized tip-timing data is saved under
% the variable dat_(and the number of its corresponding
% sensor.)

end

blade=input('Enter the blade you wish to observe:');
figure(1)
x1=dat_1(:,blade);
y1=dat_2(:,blade);
plot(x1,y1,'b.')
grid on
% The tip-timing data of sensor 1 for the selected blade
% is plotted against sensor 2

[TH,R] = CART2POL(x1,y1);
TH=TH-45*pi/180;
[X2,Y2] = POL2CART(TH,R);
figure(2)
hold on
plot(X2,Y2,'r. ')
grid on
% The grid points are rotated -45 degrees

ao=[10 10];
options=foptions;
options(1)=1;
c1=sum(X2)/length(X2);
c2=sum(Y2)/length(Y2);
[a,options]=leastsq('ellipsela', ao, options, [],
X2,Y2,c1,c2);
t=0:pi/100:2*pi;
plot(c1+a(1)*cos(t), c2+a(2)*sin(t), 'b')
% The matlab ellipse fitter is initiated

a_r = a(2)/a(1) % Ellipse axis ratio
ssr_1 = -7.2264*a_r^3 + -21.82787*a_r^2 + 119.08946*a_r + -0.16976
ssr_2 = -7.28363*a_r^3 + -21.82579*a_r^2 + 119.15622*a_r + 179.82559
% The two sensor separations, derived using Heath's method,
% are calculated in degrees of resonance.

hold off
APPENDIX C2. MATLAB IMPLEMENTATION OF TEST DATA GENERATOR

clear all;
%All previous variables are cleared.

in1 = input('Enter the name of the x ASCII input file with its extension:');
xo = load (in1);
in2 = input('Enter the name of the y ASCII input file with its extension:');
yo = load (in2);
%The time and rpm vectors are reused from existing run-down %files. The old files to be used are entered using the %above expression.

sz=size(xo);
L=900;
sig=200;
ta=0:.25:(sz(1)/4);
y=3.033*exp(-.5*((yo(:,2)-
L)/sig).^2).*sin(ta(1:sz(1)))+.5*rand(size(ta(1:sz(1))));
x=10*exp(-.5*((xo(:,2)-
L)/sig).^2).*cos(ta(1:sz(1)))+.5*rand(size(ta(1:sz(1))));
%This segment of code implements an error function centered %about 900 rpm to simulate the existence of the second %engine order resonance.

ao=[10 10];
options=foptions;
options(1)=1;
c1=sum(x)/length(x);
c2=sum(y)/length(y);
[a,options]=leastsq('ellipsela', ao, options, [],
x,y,c1,c2);
figure(1);
plot(x,y,'b.'), hold on, grid on
t=0:pi/100:2*pi;
plot(c1+a(1)*cos(t), c2+a(2)*sin(t), 'r')
hold off
%An ellipse is fitted to the artificial data to check its %validity.

[th,r] = CART2POL(x,y);
th=th+45*pi/180;
[x2,y2] = POL2CART(th,r);
figure(2);
plot(x2,y2,'r.')
grid on
%The data is rotated 45 degrees in order to mimic the %appearance of actual test data.

x3=x2+2069.1;
y3=y2+4706.6;
%Average blade positions are added to make the data appear %more realistic.

A1 = [xo(:,1) xo(:,2) x3(1:sz(1),:)];
A2 = [yo(:,1) yo(:,2) y3(1:sz(1),:)];
dlmwrite('dat1a.aal',A1,'\t')
dlmwrite('dat1a.aal2',A2,'\t')
%The artificial data is written to the files listed above.
APPENDIX D1. AMPLITUDE VERSUS TIME AT CONSTANT SPEED:
BLADE 1

Amplitude (0.001 in.)

Time (sec)

48
48.5
49
49.5
50
50.5
51
51.5
52
-30
-20
-10
0
10
20
30
APPENDIX D2. AMPLITUDE VERSUS TIME AT CONSTANT SPEED:
ALL BLADES (30)
LIST OF REFERENCES


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<td>3</td>
<td>Professor Raymond Shreeve</td>
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<td>Naval Air Warfare Center – Aircraft Division&lt;br&gt;Propulsion and Power Engineering&lt;br&gt;22195 Elmer RD, Unit 4&lt;br&gt;Patuxent River, Maryland 20670-1534</td>
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9. C. Gorton ................................................................. 1
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   Patuxent River, Maryland 20670-1534

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    Hood River, Oregon 97031