A PROPOSED STATIONARY OR MOBILE OIL SHALE RETORTING SYSTEM

Elmer White
November 30, 1967

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Abstract

An investigation of the minimum power required for retorting oil shale in situ using the thermal heat from a nuclear reactor revealed the necessity of having a 340 MW reactor. In order to map temperature profiles in a nuclear chimney, a new equation was derived which computes the ratio of the solid-to-fluid temperature. Heat exchangers and pumping power requirements were established for the model under consideration. The cost of the nuclear retorting system is negligible when prorated over the lifetime of the reactor.

Introduction

Earlier studies at the Lawrence Radiation Laboratory have shown that it is feasible to extract oil from the broken oil shale rock in a chimney created by an underground nuclear explosion.\textsuperscript{1} One method suggests that a significant amount of oil from large chunks of oil shale can be removed by igniting the broken shale and maintaining combustion with air supplied to the chimney through drill holes.\textsuperscript{2} By this process, called retorting, the heat from the combustion process would cause the solid hydrocarbon in the shale to melt, producing oil which flows by gravity to the base of the chimney where it will be drained and pumped to surface storage facilities. Other techniques for retorting oil shale have been reported\textsuperscript{3} where hot gases, steam, and electrical heaters were employed.

Instead of utilizing heat from a combustion-driven front to enhance the retorting process or steam which may cause undue chemical reactions, oil from the field may be utilized in a heat exchange process with a nuclear reactor to liberate encapsulated oil in oil shale. An arbitrary retorting temperature of 700°F was chosen after an examination of experimental data reviewed by Williamson.\textsuperscript{4} Some investigators found that no oil was formed from kerogen when shales were heated while mixed with carnitite:
(1) for 28 days at 392 to 644°F
(2) for 26 days at 176°F
(3) for 30 hr at 500°F with various catalysts
(4) for one week at 509°F.

The following description of the general reaction of kerogen to temperature increases was reported:

617°F Kerogen softens, swells, becomes black, has general appearance of soft bitumen. Solid nitrogen compounds are converted to soluble compounds.

572—761°F The "water producing" period—water-of-combination and ammonia are released. Easily volatilized organic nitrogen compounds and hydrogen sulfide are formed.

734°F Heavy oil vapors and gas begin to be formed and distilled.

932—1112°F Production of condensable oil vapors ceases.

+ 1112°F Noncondensable hydrocarbons and ammonia continue to be produced.

It was also reported that for the rapid retorting of Colorado shales, oil started to be expelled from the shale at 689°F and ceased at 977°F. For slow retorting of shale from the Colorado fields, oil began to be expelled at 680°F and ceased at 860°F. No specific times were reported which distinguished slow or fast retorting but retorting times vary as the square of particle diameter.

This report proposes a system whereby readily available oil may be passed through a heat exchanger coupled to a nuclear reactor, cycled through a nuclear explosion-derived oil shale chimney to liberate oil from the shale.

The merit of this system is in the availability of most of its component parts as off-the-shelf items. None of the items indicated in the schematic (Fig. 1) require an extensive research and development phase; only an engineering design phase is required. The nuclear reactor and heat exchangers are items similar to those developed by the AEC where liquid sodium was the primary reactor heat exchange medium and graphite the moderator material.

There are three heat exchange passes in this system. The primary cycle of liquid sodium which removes heat from the reactor may become radioactive, the secondary cycle of sodium-potassium (Na-K) eliminates the possibility of radioactive contamination and hazard, and the third cycle or oil heating cycle transfers the reactor energy to the oil shale.

The nuclear reactor and heat exchanger units should be as compact as possible and fully shielded. A single reactor retorting unit may be sufficient for the total oil shale field or for isolated oil shale formations that show an economic promise.
Method of Operation

The retorting medium would be oil (hydrocarbon) readily derived from on-the-site natural oil operations or externally supplied. Residual heat from the effects of a nuclear device may cause partial retorting. Oil accumulation from adjacent wells may be sufficient to start the controlled retorting process. Otherwise, oil would be stored in a reservoir of sufficient quantity (as shown in Fig. 2) to perform the required heat transporting function of raising the temperature of the fragmented shale to cause the contained kerogen to become fluid. The heating of the oil would be performed by the heat exchanger which derives its energy from the secondary loop of the nuclear circuit. A pump would draw the oil from the oil storage reservoir through the heat exchanger where its temperature would be raised to a desirable level and would inject the hot oil into the shale pile in a random or spray manner. The hot oil would flow through the void spaces of the nuclear shattered oil shale, raising the temperature of the shale deposits as it flows. The oil initially injected would suffer a temperature drop in giving up its heat to the oil shale and the surrounding walls. A collector pump would be located such that oil which had passed through the flooded shale bed would be returned to the oil reservoir. The oil reheat cycle would begin again. The continuous injection of hot oil would soon raise the temperature of the total oil shale bed to the point where sufficient fluidity would be exhibited by the oil in the shale mass. Excess oil or oil derived from the shale mass would be pumped as overflow to a processing plant or new storage bins. After the retorted oil has been depleted from the nuclear chimney, processing may begin immediately on a new chimney. For optimum operation the retorting system should be strategically placed within an array of nuclear chimneys. The nuclear reactor would operate continuously over the lifetime of the system.
Fig. 2. Oil shale retorting by means of a nuclear reactor.
Chimney Model

The size and shape of nuclear-explosive-derived cavities are inferred from data reported at the Third Plowshare Symposium. The equation for scaling cavity radii is reported to be

\[ R = C \frac{W^{1/3}}{\left(\rho h\right)^{1/4}}, \tag{1} \]

where

\[ R = \text{cavity radius (m)} \]
\[ W = \text{yield (kt)} \]
\[ \rho = \text{average of overburden density (g/cm}^3) \]
\[ h = \text{depth of burial of nuclear device (m)} \]
\[ C = \text{a constant to be empirically derived.} \]

The chimney appears to be of cylindrical shape with a radius approximately equal to the initial nuclear-explosive-derived cavity. The chimney height is approximated by

\[ H = KR, \]

where K is empirically derived. If the volume of the cylinder is taken to be

\[ V = \pi R^2 H; \]

then

\[ V = \frac{K\pi C^3 W}{\left(\rho h\right)^{3/4}}. \tag{2} \]

If \( \phi \) is the percent porosity of the nuclear chimney, then the average void volume is

\[ V_v = \phi V. \]

A 100-kt nuclear device would create a nuclear chimney with the approximate dimensions of

\[ R = 43 \text{ m (140 ft)} \]
\[ H = 221 \text{ m (725 ft)}. \]
This represents a volume of $4.4 \times 10^7$ ft$^3$ of oil shale. If the chimney has an average porosity of 27%, then the approximate oil shale weight to be retorted is $5 \times 10^9$ lb.

In the calculations being presented, it is assumed that the oil yield of the shale is 28 (gpt).

**Heat Transfer Calculations**

The parameters in Table I are assumed in the oil and oil shale heat transfer calculations that follow. A list of symbols for the entire report appears in Appendix A.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Colorado shale oil$^6$</th>
<th>Mid-Continental oil$^7$</th>
<th>Colorado oil shale$^8,9,10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boiling point$^a$</td>
<td>°F</td>
<td>734</td>
<td>900</td>
<td>—</td>
</tr>
<tr>
<td>Characterization factor</td>
<td></td>
<td>11.87</td>
<td>12.2</td>
<td>—</td>
</tr>
<tr>
<td>Density</td>
<td>lb/ft$^3$</td>
<td>55.687</td>
<td>54.938</td>
<td>144</td>
</tr>
<tr>
<td>Enthalpy</td>
<td>Btu/lb</td>
<td>—</td>
<td>—</td>
<td>232</td>
</tr>
<tr>
<td>Gravity</td>
<td>°API</td>
<td>27</td>
<td>29</td>
<td>—</td>
</tr>
<tr>
<td>Group</td>
<td>III$^b$</td>
<td>II</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Melting point</td>
<td>°F</td>
<td>680</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Molecular weight</td>
<td></td>
<td>304</td>
<td>310</td>
<td>—</td>
</tr>
<tr>
<td>Specific heat</td>
<td>Btu/lb °F</td>
<td>0.765/0.675$^c$</td>
<td>0.775/0.675$^c$</td>
<td>0.313 (700°F)</td>
</tr>
<tr>
<td>Temperature</td>
<td>°F</td>
<td>700</td>
<td>700</td>
<td>80</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>Btu/ft hr °F</td>
<td>0.0794</td>
<td>0.0633</td>
<td>1.23</td>
</tr>
<tr>
<td>Vapor pressure</td>
<td>atm</td>
<td>0.7</td>
<td>0.1</td>
<td>—</td>
</tr>
<tr>
<td>Viscosity</td>
<td>lb/hr ft</td>
<td>0.6776</td>
<td>0.4114</td>
<td>—</td>
</tr>
</tbody>
</table>

$^a$Data are based on gas oils and heavy crude oils which indicate boiling points in excess of 500°F.

$^b$Has the same characteristics as Group III.

$^c$Liquid/vapor.

The objective of this heat transfer analysis of hot oil passing through a porous oil shale bed is to establish the minimum power requirements for a nuclear reactor to supply heat energy for retorting the oil shale.

By using techniques developed by Zabrodsky$^{11}$ fluid flow and heat transfer characteristics will be determined. Although Zabrodsky's approach is based on the
theory of fluidized beds, his formulations assume a static balance between the heating medium and the bed particles; i.e., the fluid temperature leaving the bed on its initial pass is assumed equal to the temperature of the material in the bed.

For the shale bed previously defined with 27% porosity, the void volume is approximately $1.2 \times 10^7$ ft$^3$. The total free flow area is

$$A = \frac{V_v}{H} = 1.66 \times 10^4 \text{ ft}^2.$$ 

The heat required to liberate all oil from the nuclear chimney may be determined by

$$q = hw,$$  

where $h$ is the enthalpy and $w$ the weight of oil shale, or

$$q = 1.206 \times 10^{12} \text{ Btu.}$$

Parameters of use in determining overall heat transfer characteristics are listed. The effective heat transfer coefficient $h_e$ is defined as

$$h_e = \frac{Nu\lambda}{d_e}.$$  

The effective diameter $d_e$ is assumed to be 2 ft.

The average temperature difference, $\Delta T$, between the heat transporting fluid and the bed solids is defined as

$$\Delta T = \frac{T_f - T_s - 37^\circ F}{\ln[(T_f - T_s)/3]}.$$  

It is assumed that no significant heat transfer occurs for temperature differences less than 37°F.

The heating surface, $S_v$, per foot of bed area parallel to the direction of flow within bed height, $H$, is defined as

$$S_v = \frac{[6(1 - \phi)H]}{d_e}.$$  

By utilizing these parameters, the heat transfer is defined as

$$Q = h_e S_v \Delta T.$$
Also,

\[ Q = w \rho_f C_f (T_f - T_s) \]  \hspace{1cm} (8)

The average temperature difference \( \Delta T \) does not appear because its value is insignificant compared to \( T_f - T_s \). It is obvious that the fluid velocity may be computed from Eq. (8). Based on Eqs. (4) through (8), the calculations for 1 through 6 mo of retorting times are made.

**One-Month (720 hr) Retort Time**

The required heat transfer

\[ Q = q/(tA) \]  \hspace{1cm} (9)

\[ = 1.206 \times 10^{12}/(7.2 \times 10^2 \times 1.66 \times 10^4) \]

\[ = 1.01 \times 10^5 \text{ Btu/hr ft}^2 \]  

Therefore, the flow velocity

\[ w = Q/\rho_f C_f (T_f - T_s) \]

\[ = 1.01 \times 10^5/[54.9 \times 0.775 (700 - 77)] \]

\[ = 3.81 \text{ ft/hr} \]

The effective heat transfer coefficient

\[ h_e = Q/S \sqrt{\Delta T} \]

\[ = 1.01 \times 10^5/1.585 \times 10^3 \times 1.1 \times 10^2 \]

\[ = 0.58 \text{ Btu/ft}^2 \text{ hr} \text{ °F} \]

Since

\[ Nu = h_e (d_e/\lambda_g) \]

\[ = 0.58 \times 2/6.33 \times 10^{-2} \]

\[ = 18.45, \]
and

\[ \text{Nu} = 1.25 \times 10^{-3} \ Re^{1.46} ; \]

then the Reynolds number

\[ \text{Re} = 708 . \]

By definition

\[ \text{Re} = \frac{\frac{d_e G}{\mu}} . \]

Then,

\[ G = \text{Re} \ \mu/d_e \]

\[ = 145.5 \ \text{lb/ft}^2 \ \text{hr}, \]

and

\[ W_f = GA \]

\[ = 2,420,000 \ \text{lb/hr} \]

\[ = 330,000 \ \text{gph}. \]

To compute the heat transfer coefficient, \( h_{c,12} \) between the oil and the shale

\[ \frac{h_{c,\text{d}}}{k_s} = 2.0 + 0.6 \ \text{Pr}^{1/3} \ \text{Re}^{1/2} , \] (10)

where the Prandtl number is defined as:

\[ \text{Pr} = \frac{C_f \mu_f}{k_f} ; \]

then

\[ h_c = 18.1 \ \text{Btu/hr ft}^2 \ ^\circ\text{F} . \]
If the prior data are used to compute temperature transient times in infinite slabs, the following characteristics are noted. According to McAdams, heat transfer problems on infinite slabs may be reduced to dimensionless parameters $X$ and $Y$:

$$X = \frac{k_s t}{\rho_s C_s r_m^2}$$

$$Y = \frac{(T_f - T)}{(T_f - T_s)}.$$

The symbols $r$ and $r_m$ are related according to Fig. 3. If $r_m$ is the half thickness of the infinite oil shale slab and $r$ is the distance from the midplane, then the parameters $m$ and $n$ are defined such that

$$m = \frac{k_s}{r_m h_c}$$

and

$$n = r/r_m.$$

The time it would take the temperature of the slab to be raised to 690°F at its center plane if the slab is heated from both sides is now determined. If $r = 0$ and $r_m = 1$ ft, then $n = 0$ and $m = 0.07$. In the computation

$$Y = \frac{(700 - 690)}{(700 - 77)} = 0.016.$$

Observing the charts on pp. 32 to 35 of McAdams for $m = 0.07$ and $Y = 0.016$, it is seen that

$$X = 1.8.$$

Solving for the time, $t$, it can be seen that

$$t = 66 \text{ hr.}$$
When the slab is heated from one side such that

\[ r_m = 2 \text{ ft} \]

\[ m = 0.0358, \]

assume

\[ n = 0.8 \ (r = 1.6 \text{ ft}) \]

\[ Y = 0.016 ; \]

therefore, it is observed that

\[ X = 1.3 \]

and the time it takes the temperature to reach 690°F at a distance of 1.6 ft from the heated edge is

\[ t = 191.5 \text{ hr.} \]

\[ \text{Three-Month (2160 hr) Retort Time} \]

The following are the required and computed parameters:

- \[ Q = 3.37 \times 10^4 \text{ Btu/hr ft}^2 \]
- \[ w_o = 1.27 \text{ ft/hr (oil velocity)} \]
- \[ h_e = 0.1935 \text{ Btu/ft}^2 \text{ hr °F} \]
- \[ \text{Nu} = 6.11 \]
- \[ \text{Re} = 338.9 \]
- \[ G = 69.6 \text{ lb/ft}^2 \text{ hr} \]
- \[ W_o = 1,148,700 \text{ lb/hr (156,500 gph)} \]
- \[ h_c = 12.9 \text{ Btu/hr ft}^2 \text{ °F}. \]

Heating an infinite oil shale slab from both sides, \( t = 77.8 \text{ hr} \) to raise the midplane to 690°F. Heating from one side, \( t = 287 \text{ hr} \) to raise the temperature to 690°F at 1.6 ft from the heated edge.
Six-Month (4320 hr) Retort Time

The following are the required and computed parameters:

\[ Q = 1.68 \times 10^4 \text{ Btu/hr ft}^2 \]
\[ w_o = 0.636 \text{ ft/hr (oil velocity)} \]
\[ h_e = 0.09638 \text{ Btu/ft}^2 \text{ hr } ^\circ \text{F} \]
\[ Nu = 3.05 \]
\[ Re = 208.9 \]
\[ G = 43 \text{ lb/ft}^2 \text{ hr} \]
\[ W_o = 712,700 \text{ lb/hr (97,100 gph)} \]
\[ h_c = 10.4 \text{ Btu/hr ft}^2 \text{ } ^\circ \text{F}. \]

In heating an infinite oil shale slab from both sides, the time \( t = 80.2 \text{ hr} \) to raise the midplane to 690°F. Heating from one side, \( t = 294 \text{ hr} \) to raise the temperature to 690°F at 1.6 ft from the heated edge.

In observing the flow velocity and the heating transients of the infinite slab problems, it is apparent that retorting times greater than three months are excessive. Economics should dictate shorter retort times in the range comparable to twice the number of hours to heat an infinite slab from both sides. It may be necessary to fill the void spaces with oil at elevated temperatures and to maintain a flow predictable by Darcy Flow type equations. 14

Oil Flow Patterns

The Darcy Flow equation is of the form

\[ U = -\frac{AC}{\mu} \left[ \frac{\Delta P}{H} + g \rho \sin \theta \right] \frac{\text{ft}^3}{\text{sec}}, \]  \hspace{1cm} (11)

where

\[ C = \phi^3 \left[ \frac{5S_p^2}{V} (1 - \phi)^2 \right]. \]

The angle \( \theta \) is taken with respect to the horizontal direction. For flow downward, \( \theta = 90^\circ \). It can then be seen that the \( \sin \theta = -1 \). If no flow takes place with respect to the referenced vertical, the fluid is in static equilibrium and obviously
\[
\frac{\Delta P}{H} - g\rho = 0.
\]

If the bottom of a porous medium is vented and the rate of flow is influenced only by gravity, then,

\[
\frac{\Delta P}{H} - g\rho < 0,
\]
due to a small \(\Delta P\). The maximum rate of flow due to gravity takes place when \(\Delta P \to 0\). By inducing a slight vacuum at the bottom of a porous medium with respect to the atmosphere, \(\Delta P\) can be influenced to the extent that the flow rate will increase. The Darcy Flow equation is applicable with systems of variable porosity; i.e., elements of height may be considered if the porosity within a defined element is assumed constant.

**Temperature Profiles in Shale Bed**

The solution to the problem of the temperature profile in a porous medium due to a hot flowing liquid was approached from the formulations established by Schumann. A right circular cylinder consisting of crushed material at some uniform temperature was assumed. A fluid at some higher uniform temperature passes through the crushed material at a uniform rate of flow. The sides of the cylinder are assumed impervious to the fluid. The distribution of temperature in the crushed material and the fluid for all times is found based on the assumption that

1. The lumps of material are so small or have such high thermal diffusivity that any given lump may be considered at a uniform temperature at any instant.
2. Compared to the transfer of heat from fluid to solid, the transfer of heat by conduction in the fluid itself or in the solid itself is small and may be neglected.
3. The rate of heat transfer from fluid to solid at any point is proportional to the average difference in temperature between fluid and solid at that point.
4. Change in volume of fluid and solid due to change in temperature may be neglected.
5. The thermal constants are independent of the temperature.

In the application of Schumann's efforts to the oil shale problem, it became apparent that the curves generated by him did not extend far enough to give solutions for the ratio of the fluid to the solid temperatures. It was necessary to investigate the work of Furnas. These curves also were inadequate; therefore, it was necessary to derive new equations the solution of which involved Bessel Functions with large arguments.
According to Schumann, the equation expressing the temperature ratio between a solid and a fluid may be written as

\[
\frac{T_S}{T_o} = 1 - e^{-Y-Z} \sum_{n=0}^{\infty} Y^nM_n(YZ),
\]  

(12)

where the arguments \(Y\) and \(Z\) are defined as

\[
Y = h_c x/(C_f v_f \phi)
\]

and

\[
Z = \frac{h_c (t - X)}{C_s (1 - \phi)}
\]

The symbol \(x\) is the distance traveled by the fluid front into the porous medium.

If the following transformations are made:

\[
Y + Z = \phi
\]

\[
YZ = \xi
\]

then Eq. (12) may be rewritten as

\[
\frac{T_S}{T_o} = 1 - e^{-\phi} \sum_{n=0}^{\infty} Y^nM_n(\xi).
\]  

(13)

By definition\(^{17}\)

\[
M_n(\xi) = \frac{\partial^n M_0(\xi)}{\partial \xi^n},
\]  

(14)

where

\[
M_0(\xi) = 1 + \xi + \frac{\xi^2}{(2!)^2} + \frac{\xi^3}{(3!)^2} + \ldots
\]

\[
\xi = \frac{Y}{Z}
\]
In expanding Eq. (13),

\[
\frac{T_S}{T_0} = 1 - e^{-\Psi} \left[ M_0(\xi) + YM_1(\xi) + Y^2M_2(\xi) + \ldots \right],
\]

(15)

but utilizing Eq. (14)

\[
\frac{T_S}{T_0} = 1 - e^{-\Psi} \left[ M_0(\xi) + Y \frac{\partial M_0(\xi)}{\partial \xi} + Y^2 \frac{\partial^2 M_0}{\partial \xi^2} + \ldots \right].
\]

(16)

The function \(M_0(\xi)\) is usually defined

\[
M_0(\xi) \equiv I_0(2\sqrt{\xi}) = I_0(\sigma),
\]

where

\[
\sigma = 2\sqrt{\xi}
\]

and \(I_0(\sigma)\) is a modified Bessel Function. Keeping in mind that

\[
\frac{\partial^2 I_0(\sigma)}{\partial \sigma^2} = I_1(\sigma),
\]

Eq. (16) may be rewritten

\[
\frac{T_S}{T_0} = 1 - e^{-\Psi} \left[ I_0(\sigma) + Y \frac{\partial I_0(\sigma)}{\partial \xi} + Y^2 \left( \left( \frac{\partial I_0(\sigma)}{\partial \xi} \right)^2 - \frac{\partial^2 I_0(\sigma)}{\partial \sigma^2} \right) + \ldots \right].
\]

(17)

Also,

\[
\frac{\partial \sigma}{\partial \xi} = \frac{2}{\sigma}.
\]
If the modified Bessel Functions are expanded into infinite series and all higher
order terms neglected for \( \sigma \gg 1 \), then

\[
\frac{T_s}{T_0} \approx 1 - \frac{e^{-\phi+\sigma}}{\sqrt{2\pi \sigma}} \left[ 1 + \frac{2Y}{\sigma} + Y^2 \left( \frac{2}{\sigma} \right)^2 - \frac{1}{\sigma} \left( \frac{2Y}{\sigma} \right)^2 \right] + \ldots
\]

or

\[
\frac{T_s}{T_0} \approx 1 - \frac{e^{-\phi+\sigma}}{\sqrt{2\pi \sigma}} \left[ 1 + \frac{2Y}{\sigma} + \left( \frac{2Y}{\sigma} \right)^2 \left( 1 - \frac{1}{\sigma} \right) + \ldots \right]
\]

\[
= 1 - \frac{e^{-\phi+\sigma}}{\sqrt{2\pi \sigma}} \left[ 1 + \frac{2Y}{\sigma} + \left( \frac{2Y}{\sigma} \right)^2 + \ldots \right]. \tag{18}
\]

If \( \left( \frac{2Y}{\sigma} \right)^2 < 1 \) such that \( Y < Z \), then this solution of the temperature ratio is good
only for time

\[
t > \frac{X}{V_f} \left[ 1 + \frac{C_x}{C_f} \left( 1 - \frac{\phi}{\sigma} \right) \right]; \tag{19}
\]

therefore,

\[
\frac{T_s}{T_0} = 1 - \frac{e^{-\phi+\sigma}}{\sqrt{\pi \sigma}} \frac{1}{\left( 1 - \frac{2Y}{\sigma} \right)}, \tag{20}
\]

and

\[
\phi = Y + Z
\]

\[
\sigma = 2\sqrt{YZ}.
\]

**Design of Heat Exchangers**

Heat exchanger design techniques are similar to those presented for sodium (Na)
and sodium-potassium (Na-K) liquid metal systems by Fraas and Ozisik. In the
proposed triple cycle heat exchanger, oil is the third cycle coolant, but the underground oil shale represents the heat sink. In each heat exchanger the fluid flows in a parallel manner.

On the initial pass it is assumed that the oil enters the heat exchanger at a temperature equal to the initial oil shale temperature (77°F). The oil will be raised to a temperature of 700°F before being injected into the cool oil shale chimney. From the equation

\[ Q = W_o C_o \Delta T, \]

the heat flow to the oil for a 30-day retorting system is

\[ Q = 1.17 \times 10^9 \text{ Btu/hr}. \]

The parameters governing the Na-K secondary heat exchange cycle are listed below.

**Sodium-Potassium Secondary Heat Exchange Cycle**

The following are the required and computed parameters:

- \( T_1 = 795°F \)
- \( T_2 = 945°F \)
- \( C_{NK} = 0.2503 \text{ Btu/lb°F} \)
- \( K_{NK} = 16 \text{ Btu/lb ft °F} \)
- \( \mu = 0.494 \text{ lb/hr ft} \)
- \( \rho_{NK} = 51 \text{ lb/ft}^3 \)
- \( Pr = 0.00842 \)
- \( Pr^{0.4} = 0.14775 \)
- i.d. = 0.305 in.
- o.d. = 0.375 in.

The subscript NK means sodium-potassium (Na-K).

For the oil heating cycle, the Na-K flows inside small tubes, while the oil flows against the outer surfaces of the tubes. An expression for the required number of tubes is

\[
 n = \frac{W_{NK}}{6.31} \left[ \frac{fL}{\rho_{NK} D^5 \Delta P_{NK}} \right]^{1/2}.
\] (21)
If the heat load between the oil and Na-K is the same, then for the oil:

\[ Q_o = Q_{NK} \]

\[ W_{NK} = \frac{Q_{NK}}{C_{NK} \Delta T_{NK}} = 8.61 \times 10^3 \text{ lb/sec}. \]

Assume \( f = 0.03 \), \( L = 10 \text{ ft} \), \( \rho_{NK} = 51 \text{ lb/ft}^3 \) and \( \Delta P = 1440 \text{ lb/ft}^2 \) then

\( n = 32,200 \) tubes.

The flow rate per tube is then

\[ W'_{NK} = W_{NK} / n \]

\[ = 0.2674 \text{ lb/sec}. \]

The tube inside diameter (i. d.) is assumed to be \( (D = 0.305) \text{ in.} \); therefore, its cross sectional area

\[ A = 0.0731 \text{ in}^2. \]

The mass flow rate per tube is

\[ G'_{NK} = \frac{W'_{NK}}{A} = 526 \text{ lb/sec ft}^2, \]

and the Reynolds number

\[ \text{Re} = \frac{G'_{NK} D}{\mu} = 97,500. \]

The heat transfer coefficient for Na-K is computed from the expression

\[ h_{NK} = 0.02 \left( K_{NK} / D \right) \text{Pr}^{0.4} \text{Re}^{0.8}. \tag{22} \]

From the known parameters

\[ h_{NK} = 55,200 \text{ Btu/hr ft}^2 \circ\text{F}. \]
On the side of the oil, \( Q = 1.17 \times 10^9 \text{ Btu/hr} \). The total oil flow rate through the exchanger is

\[
W_o = 673 \text{ lb/sec}.
\]

Outside the tube, the flow rate per tube

\[
W'_o = 0.0209 \text{ lb/sec},
\]

and the mass flow rate \( G'_o = 37.8 \text{ lb/sec ft}^2 \) at a Reynolds number

\[
Re = 10,370.
\]

The Prandtl number computes to be \( (Pr = 5.04) \); therefore, the heat transfer coefficient

\[
h_o = 177 \text{ Btu/hr ft}^2 \ ^\circ \text{F}.
\]

A columbium tube wall is assumed between the two media:

\[
k_w = 34 \text{ Btu/hr ft} \ ^\circ \text{F}.
\]

The wall conductance (heat transfer coefficient)

\[
h_w = \frac{k_w}{thk}, \ thk = 0.035 \text{ in.}
\]

\[
= 11,650 \text{ Btu/hr ft}^2 \ ^\circ \text{F}.
\]

The overall heat transfer coefficient \( h_u \) is derived:

\[
\frac{1}{h_u} = \frac{L}{h_{NK}} + \frac{1}{h_w} + \frac{1}{h_o(i.d./o.d.)}
\]

\[
\text{(23)}
\]

The tube outside diameter is 0.375 in. and the inside diameter is 0.305 in.

\[
h_u = 138.5 \text{ Btu/hr ft}^2 \ ^\circ \text{F}.
\]

The total tube external surface required is calculated by equating the power from the Na-K to the oil:

\[
Q = Ah_u\Delta T
\]

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where
\[ \Delta T = \frac{T_{NK} - T_o - 37}{\ln \left[ \left( \frac{T_{NK} - T_o}{3} \right) \right]} \]  
(24)

Remembering that \( T_{NK} = 945^\circ F \), \( T_o = 77^\circ F \); then
\[ \Delta T = 147^\circ F. \]

Since
\[ Q = 1.17 \times 10^9 \text{ Btu/hr}, \]
\[ A = 5.74 \times 10^4 \text{ ft}^2. \]

The external surface per tube is
\[ a = A/n = 1.78 \text{ ft}^2/\text{tube}. \]

The tube length
\[ L = A/C, \]

where \( C \) is the circumference of the outside diameter, or
\[ L = 18.15 \text{ ft}. \]

For the Na-K pressure drop
\[ \Delta P = f_{NK} \frac{G_{NK}^{1/2} L}{(2\rho D)}. \]  
(25)

Assume \( f_{NK} = 0.03 \); therefore, \( \Delta P = 1.885 \times 10^3 \text{ lb/ft}^2 \) (13 psi) which is fairly close; therefore, \( 3.22 \times 10^4 \) tubes are a good approximation. For the oil, if \( f_o = 0.035 \),
\[ \Delta P = 8.23 \text{ lb/ft}^2. \]
Sodium Primary Heat Exchange Cycle

The following are the required and computed parameters:

\[ T_1 = 950^\circ F \]
\[ T_2 = 1050^\circ F \]
\[ C_N = 0.302 \text{ Btu/lb } ^\circ F \]
\[ k_N = 37.2 \text{ Btu/lb ft } ^\circ F \]
\[ \mu = 0.56 \text{ lb/hr ft} \]
\[ \rho_N = 51.2 \text{ lb/ft}^3 \]
\[ Pr = 0.00455 \]
\[ Pr^{0.4} = 0.1155 \]
\[ \Delta P = 1440 \text{ lb/ft}^2 \text{ (allowed)} \]
\[ i.d. = 0.305 \text{ in.} \]
\[ Q = 1.17 \times 10^9 \text{ Btu/hr} \]

The subscript N refers to sodium (Na).

Employing the same expressions used for the secondary cycle, the following are the parametric requirements computed:

\[ W_N = 10,750 \text{ lb/sec} \]
\[ n = 33,600 \text{ tubes} \]
\[ W_N = 0.3212 \text{ lb/sec tube (inside)} \]
\[ W_{NK} = 0.257 \text{ lb/sec tube (outside)} \]

The mass flow rates are:

\[ G_N = 633 \text{ lb/sec ft}^2 \text{ (inside)} \]
\[ G_{NK} = 465 \text{ lb/sec ft}^2 \text{ (outside)} \]
\[ Re_N = 99,000 \]
\[ Re_{NK} = 1.07 \times 10^5 \]

The heat transfer coefficient values are:

\[ h_N' = 33,700 \text{ Btu/hr ft}^2 \cdot ^\circ F \]
\[ h_{NK}' = 30,200 \text{ Btu/hr ft}^2 \cdot ^\circ F. \]

Between the Na and the Na-K, the average

\[ \Delta T = 48^\circ F. \]
Assume a columbium tube wall at 1000°F. Then

\[ \frac{k_w}{\rho} = 0.067 \]
\[ k_w = 35.8 \text{ Btu/hr ft °F} \]
\[ \text{thk} = 0.035 \text{ in.} \]
\[ h_w = 12,300 \text{ Btu/hr ft}^2 \text{ °F}. \]

The overall heat transfer coefficient

\[ h_u = 2420 \text{ Btu/hr ft}^2 \text{ °F}. \]

The total required external surface

\[ A = 10,000 \text{ ft}^2. \]

The tube length

\[ L = 4.82 \text{ ft}. \]

The pressure drop on both sides of the tube becomes

\[ \Delta P_N = 761 \text{ lb/ft}^2 \text{ (inside)} \]
\[ \Delta P_{NK} = 824 \text{ lb/ft}^2 \text{ (outside)} \]

Therefore, the number of tubes is reasonable since the pressure drops are less than those assumed.

For all designs, the tube spacing should be square and at least 1.25 times the tube outside diameter or larger. Heat exchanger designs should be comparable to those developed by the AEC for liquid metals and molten salts for best performance.

**Nuclear Reactor Heat Transfer**

In the computation of fuel element temperatures, it is assumed that the lengthwise heat distribution is generated in a cosine pattern. The maximum fuel element temperature in Eq. (26) depends upon three basic reactor characteristics; i.e., core coolant inlet temperature \( T_1 \), coolant temperature rise \( \Delta T \), and the average coolant film temperature drop \( \theta_a \). \[ ^{19} \]
The expression for the maximum fuel element surface temperature

\[ T_m = T_1 + F_{\Delta T} \frac{\Delta T}{2} + 1/2 \sqrt{F_{\Delta T}^2 \Delta T^2 + F_{\theta}^2 \theta^2 \theta_a^2} \]  \hspace{1cm} (26)

where the average temperature rise in the core

\[ \Delta T = 0.008 \left[ \frac{F_1}{\rho_p} \frac{F_2}{\rho_r} \frac{L_0}{D} \frac{1}{H} \right]^{0.357} \mu \frac{0.0714}{D_e^{0.429}} \frac{Q_r}{C_p \beta}^{0.643} \frac{n}{D^{0.929}} \]  \hspace{1cm} (27)

and the average coolant film temperature drop in the core

\[ \theta_a = \frac{58.2 Q_r^{0.2} \Delta T^{0.8} D e^{1.2} C_p^{0.4} \mu^{0.4}}{D^{1.4} (L_0 / D)^{0.6} \beta^{0.2} n^{0.8}} \]  \hspace{1cm} (28)

The hot channel factor accounting for nonuniform radial heat generation and average dimensional deviations is chosen to be \( F_{\Delta T} \approx 1.5 \). For a coolant film temperature drop small compared to the average temperature rise in the core when \( \Delta T = 100^\circ F \),

the maximum fuel element surface temperature is approximately

\[ T_m \approx 1200^\circ F. \]

**Sodium Graphite Nuclear Reactor**

The following are the required parameters:

- \( T_1 = 950^\circ F \)
- \( T_2 = 1050^\circ F \)
- \( T_w = 1200^\circ F \)
- \( \rho = 51.2 \text{ lb/ft}^3 \)

The subscript \( w \) refers to the reactor coolant flow walls.

The sodium flow rate through the reactor core is the same as through the primary heat exchanger:

\[ W_N = 10,750 \text{ lb/sec.} \]
Assuming a coolant passage inside diameter of 1 in., an allowable pressure drop of 10 psi, and a 10-ft core length, the following are the resulting parameters computed:

\[ n = 5440 \text{ tubes} \]
\[ G_N = 1.98 \text{ lb/sec} \]
\[ G_{f} = 363 \text{ lb/sec ft}^2 \]
\[ Re_N = 195,000 \]
\[ h_N = 1.74 \times 10^4 \text{ Btu/hr ft}^2 \text{°F} \]

Assuming a wall temperature of 1200°F and the coolant inlet temperature of 950°F, the average temperature difference between the walls and the bulk fluid becomes

\[ \Delta T = 57^\circ F \]
\[ h_u = 1520 \text{ Btu/hr ft}^2 \text{°F}. \]

The coolant total free flow area \( A = 1.35 \times 10^4 \text{ ft}^2 \). Therefore,

\[ a = 2.48 \text{ ft}^2/\text{tube}. \]

The computed \( L = 9.5 \text{ ft} \), and the \( \Delta P \) for 9.5 ft is 144 psf, but the \( \Delta P \) for 15 ft is 216 psf. For a square core where the diameter and the length equal 9.5 ft, the volume fraction of sodium in the core causes a high fuel inventory. It is necessary to assume a core diameter and length of 15 ft, each. For a 1-in. diam flow passage the total sodium frontal area is computed to be

\[ A_N = 29.648 \text{ ft}^2 \]

The reactor frontal area

\[ A_r = 176.7 \text{ ft}^2 \]

The reactor core volume

\[ V_r = 2650.5 \text{ ft}^3 = 7.506 \times 10^8 \text{ cm}^3 \]

The sodium void fraction equals 16.77%.
For a 100°F temperature rise in the core,

\[ Q = \frac{10,750 \times 3600 \times 0.302 \times 100}{5440 \times 3413} \]

= 63 kw/channel

Therefore, the total reactor power is

\[ Q_r = 340 \text{ MW}. \]

**Reactor Criticality Calculations**

The volume fraction of the core materials are as indicated in Table II.

**Table II. Reactor composition**

<table>
<thead>
<tr>
<th>% Vol</th>
<th>Element</th>
<th>Volume (cm³)</th>
<th>Weight^a (g)</th>
<th>Core density (g/cm³)</th>
<th>( \frac{\text{barns}}{\sigma_a \sigma_s} )</th>
<th>Atoms/cm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1677</td>
<td>Na</td>
<td>1.26 \times 10^8</td>
<td>1.17 \times 10^8</td>
<td>0.1555</td>
<td>0.4 4</td>
<td>4.07 \times 10^{21}</td>
</tr>
<tr>
<td>0.0310</td>
<td>SS</td>
<td>2.33 \times 10^7</td>
<td>1.86 \times 10^8</td>
<td>0.2480</td>
<td>2.4 11</td>
<td>2.67 \times 10^{21}</td>
</tr>
<tr>
<td>0.0500</td>
<td>Nat. U</td>
<td>3.75 \times 10^7</td>
<td>7.13 \times 10^8</td>
<td>0.9500</td>
<td>2.40 \times 10^{21}</td>
<td></td>
</tr>
<tr>
<td>0.7513</td>
<td>U235</td>
<td></td>
<td></td>
<td>687 9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7513</td>
<td>U238</td>
<td></td>
<td></td>
<td>2.75 8.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0000</td>
<td>Graphite</td>
<td>5.64 \times 10^8</td>
<td>1.00 \times 10^9</td>
<td>1.3345</td>
<td>0.0045 4.8</td>
<td>6.70 \times 10^{22}</td>
</tr>
</tbody>
</table>

^aTotal weight without shield (4.445 \times 10^6 \text{ lb}).

For natural uranium 1.111% enriched with U235, the ratio of the atomic densities

\[ N_{238}/N_{235} = 90. \]

By solving the expression

\[ \sigma_{au} = \frac{\sigma_{235} + (N_{238}/N_{235}) \sigma_{238}}{1 + (N_{238}/N_{235})} \]

the thermal absorption cross section for the enriched natural uranium is computed to be \( \sigma_{au} = 10.27 \) barns. The macroscopic absorption cross section \( \Sigma_{au} = 0.025 \text{ cm}^{-1} \) for the enriched uranium where \( \Sigma = N_0 \). The scattering cross section is computed with a .
similar expression such that the macroscopic scattering cross section $\Sigma_{su} = 0.0199 \text{ cm}^{-1}$. In summing the macroscopic cross sections for the absorption and the scattering process separately, the total absorption and scattering macroscopic cross sections are $\Sigma_a = 0.0339$ and $\Sigma_s = 0.39716$, respectively.

From the parameters known, a thermal utilization factor $f$ may be computed, where

$$f = \frac{\Sigma_{su}}{\Sigma_a} = 0.725$$

represents the percent of neutrons utilized to carry on fissions.

The number of fast fission neutrons produced per thermal neutron absorbed in uranium ($\eta$) is computed from the expression\textsuperscript{20}

$$\eta = \frac{2.5}{1.184 + 0.00474 \left( \frac{N_{238}}{N_{235}} \right)} .$$

For this case, $\eta = 1.552$. The number of second generation fissions in U235 per fission of U235 from the first generation in a reactor of infinite extent is computed as the multiplication factor $K_\infty$, defined by

$$K_\infty = \eta f$$

$$K_\infty = 1.125 .$$

The diffusion coefficient $D$ may be computed from the expression

$$D = \frac{1}{3\Sigma_s (1 - \bar{\mu}_0)} ,$$

where $\bar{\mu}_0 = 2/\bar{\alpha}$. Since this is a homogenous reactor, $\bar{\alpha}$ represents the weighted average of the atomic weights in the core:

$$\bar{A} = 21.5091$$
$$\bar{\mu}_0 = 0.030994$$
$$D = 0.86612 \text{ cm} .$$

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The square of the diffusion length $L$ is computed from

$$L^2 = \frac{D}{\Sigma_a}$$

$$L^2 = 25.4988 \text{ cm}^2.$$ 

For a pure graphite moderator, the slowing-down length $L_s = 18.7$ cm. If only the moderator influences a neutron lifetime in the core materials, then the modified slowing-down length

$$L_s' = 18.7 \frac{\text{density of the graphite}}{\text{density of graphite per unit volume of core}} = 22.406 \text{ cm}.$$ 

By definition,

$$L_s'^2 = \tau,$$

the Fermi age.

For a reactor of the given dimensions, the buckling term ($B^2$) is computed by the expression

$$B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$$

$$B^2 = 1.5456 \times 10^{-4}.$$ 

The effective multiplication factor $K_{\text{eff}}$ due to a finite geometry can now be computed:

$$K_{\text{eff}} = \frac{K_{\infty} e^{-B^2 \tau}}{1 + L_s'^2 B^2}$$

$$= 1.0369.$$ 

The excess multiplication, $K_{\text{ex}} = K_{\text{eff}} - 1 = 0.0369$, will determine the reactor lifetime. For a reactor where $K_{\text{eff}} = 1$, $N_{235} \approx 2.45738807 \times 10^{19}$ atoms/cm$^3$ ($7.2 \times 10^6$ g), the thermal neutron flux

$$\psi = \frac{1.2 \times 10^{13}}{W_{\sigma_t}} P_r.$$ 

*The excess reactivity which affects the reactor lifetime was an arbitrarily built-in factor.*
where W is the weight of U235 in grams and $P_r$, the reactor power in watts. For

$$\psi = 1.03 \times 10^{12} \text{ neutrons/cm}^2\text{-sec}$$

and

$$\Sigma_f = 0.0135 \text{ cm}^{-1},$$

the average number of fissions per second $\Psi$ is determined by

$$\Psi = V \Sigma_f \psi.$$  

For $K_{\text{eff}} = 1$, the fission rate $\Psi = 1.04 \times 10^{19}$ fissions/sec. The lifetime of this reactor with an arbitrarily selected 3.69% reactivity is calculated to be

$$t = N_{235} V / \Psi \text{ sec}$$

$$t = 1.8947 \times 10^9 \text{ sec}.$$  

Since there are $3.145 \times 10^7 \text{ sec/year}$, therefore, $t = 63 \text{ years}$.  

The reactor lifetime may be reduced to any desired number of years by reducing the fuel inventory. Also, for the sake of mobility, the reactor size may be reduced, governed by the allowed coolant pressure drop across the core. Any reductions cause favorable savings in the overall system weight.

**Theoretical Pumping Power**

Computations on pumping power make no allowance for pump efficiency. The following equation expresses pumping power:

$$P_p = (\Delta P) (A_T) (v_f)/(33,000) (60)$$

For the primary cycle, the following parameters

$$A_T = 17.06 \text{ ft}^2\]$$

$$\Delta P = 761 \text{ lb/ft}^3\]$$

$$v_f = 4.44 \times 10^4 \text{ ft/hr}$$

yield a pumping power of

$$P_p = 290.5 \text{ hp (minimum)}.$$
For the secondary cycle, the following parameters

\[ A_T = 16.35 \text{ ft}^2 \]
\[ \Delta P = 1885 \text{ lb/ft}^2 \]
\[ v_f = 3.72 \times 10^4 \text{ ft/hr} \]

yield a pumping power of

\[ P_p = 578 \text{ hp.} \]

For the oil cycle, the following parameters

\[ W_o = 2.4228 \times 10^6 \text{ lb/hr} \]
\[ H_p = 3750 \text{ ft (pumping height)} \]

yield a pumping power of

\[ P_p = \frac{W_o H_p}{(33,000)(60)} \]
\[ = 4590 \text{ hp.} \]

The pumping power requirement through the heat exchanger is negligible compared to the pumping requirement from the bottom of the chimney.

**Factors Affecting System Economics**

An economical appraisal of this system is beyond the scope of this report. Some of the factors\(^{21,22}\) affecting the economics of this system are commented upon. Consideration must be given to the reactor system, heat exchanger, pipe lines, land leasing, roads, interest and amortization rates, operational cost over specific time spans, moving of the reactor, nuclear explosive services and manpower.

The fact that 66 hr is required to retort a 2-ft thick oil shale slab at a background temperature of 700°F suggests that a minimum of 10 nuclear chimneys may be retorted in 4 wk. This is equivalent to 132 chimneys per year. For an oil shale field of a 28-gpt yield, the oil shale tonnage for the given chimney is \(2.5 \times 10^6\) tons. The oil yield would be \(7 \times 10^7\) gal or \(2.22 \times 10^6\) barrels of oil per chimney.

The lifetime of a nuclear reactor will affect the economics. A nuclear reactor can be designed for any lifetime. Coupled to the lifetime is the reactor power.
requirement of diminishing output over the retorting time of a single chimney. As the oil shale approaches retorting temperature, the reactor power requirement falls off.

Another factor affecting the economics is the residual heat in a nuclear chimney. The residual heat in the oil shale is a function of time after chimney formation. For example, the maximum observed temperature 11 mo after detonation in grandiorite was 88°C for the Hardhat experiment.

The nuclear reactor power requirement will be governed by the equilibrium temperature in a nuclear chimney. Nevertheless, the size (weight) of a nuclear reactor and its mobile unit will be governed by the required energy the reactor must supply to take the oil shale to retorting temperatures.

System Advantages and Disadvantages

System Advantages

This system has the following advantages:

1. The maximum temperature of retorting is controllable.
2. The retorting time is controllable.
3. When a hydrocarbon is employed, no chemical reaction is expected between the hydrocarbon and the oil shale.
4. Residue from the oil shale retorting process is left in a favorable condition for additional reclamation processes. No new compounds have been formed as a result of the retorting process.
5. The nuclear chimney walls give up oil to the retorting process. Because of this, oil yield can exceed the calculated yield of the broken oil shale within the chimney.
6. Oil losses are kept at a minimum by constant temperature medium retorting.
7. Oil retorted from residual heat in neighboring chimneys may be used to start the retorting process.
8. Only one externally supplied oil reserve is required to start the retorting process over the life of the nuclear reactor.
9. By centrally locating a mobile retorting system among several nuclear chimneys, the cycling of oil through the heat exchanger need not terminate with the emptying of a single nuclear chimney.
10. The residual heat in a nuclear chimney remaining from the retorting process may be used to generate electrical power by steam generators or MHD power conversion devices.
11. The rate of oil extraction is controllable.
12. The minimum time for oil shale to be brought up to 700°F is approximately 66 hr (2.75 days). Two chimneys should be retorted completely in 1 wk.

13. A diesel-driven mobile retorting unit with appropriate electrical generators could supply the necessary power for all pumps.

14. Fluids other than hydrocarbons may be used as heat transporting media.

15. An alternate to nuclear power is the direct application of heater coils for heating the heat transporting medium.

**System Disadvantages**

The following are disadvantages of the system:

1. The mobile oil shale retorting unit is heavy.

2. Building more than one mobile oil shale retorting device would require an outlay by the Federal Government or by a combination of several industrial firms.
References

9. W. H. Somerton, Thermal Conductivities of Oil Sands and Clearwater Shale, University of California, Berkeley, Rept. DMT-59-7C.


Appendix A
List of Symbols

A, a  area (ft²)
A̅  weighted average of atomic weights in core
A_N  total sodium frontal area
A_T  reaction frontal area
A_T  total flow area (ft²)
B²  buckling term (cm⁻²)
C  circumference of outside diameter
C  constant to be empirically derived
C_f  specific heat to fluid at constant pressure (Btu/lb °F)
C_N  specific heat of sodium at constant pressure
C_NK  specific heat of sodium-potassium at constant pressure
C_o  specific heat of oil at constant pressure
C_P  specific heat at constant pressure
C_s  specific heat of solid at constant pressure
D  diffusion coefficient
D  reactor core diameter (ft)
D_e, d_e  equivalent (effective) diameter (ft)
F_∆T  hot channel factor accounting for nonuniform radial heat generation and average dimensional deviations
f  thermal utilization factor
f  friction factor
f_N  sodium friction factor
f_NK  sodium-potassium friction factor
F_θ  hot channel factor applied to coolant film temperature drop for nonuniform heat generation with average and local off-design deviations
F_1  ratio of system pressure drop to core and reactor vessel pressure drop
F_2  ratio of core and reactor vessel pressure drop to core friction pressure drop
G  mass flow (lb/hr ft²)
G_N  mass flow for sodium
G_NK  mass flow for sodium-potassium
g  acceleration of gravity (ft/sec²)
H_p  pumping height (ft)
H  bed height
H  height of chimney (m)
h  enthalpy

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\( h \) depth of burial of nuclear device (m)
\( h_c \) heat transfer coefficient between oil and shale
\( h_e \) effective heat transfer coefficient
\( h_N \) heat transfer coefficient of sodium
\( h_{NK} \) heat transfer coefficient of sodium-potassium
\( h_o \) heat transfer coefficient of oil
\( h_u \) overall heat transfer coefficient
\( h_w \) wall heat transfer coefficient
\( I_0(\sigma) \) modified Bessel function
\( K \) empirically derived constant
\( k_f \) thermal conductivity of a fluid (Btu/ft hr °F)
\( k_N \) thermal conductivity of sodium (Btu/ft hr °F)
\( k_{NK} \) thermal conductivity of sodium-potassium (Btu/ft hr °F)
\( k_w \) wall thermal conductivity (Btu/ft hr °F)
\( k_s \) thermal conductivity of a solid (Btu/ft hr °F)
\( K_{eff} \) effective multiplication factor
\( K_{ex} \) excess multiplication factor
\( K_\infty \) multiplication factor
\( L \) length (ft)
\( L_s \) slowing down length (cm)
\( ln \) natural logarithm
\( N \) atomic density
\( Nu \) Nusselt number
\( m \) parameter
\( n \) number
\( n \) parameter
\( P \) pressure (psf)
\( P_r \) Prandl number
\( P_{r'} \) reactor power (W)
\( P_N \) pressure for sodium (psf)
\( P_{NK} \) pressure for sodium-potassium (psf)
\( P_p \) pumping power (hp)
\( Q \) heat transfer
\( Q_N \) heat transfer for sodium
\( Q_{NK} \) heat transfer for sodium-potassium
\( Q_r \) thermal output from reactor (kW)
\( q \) heat required to liberate all oil from nuclear chimney (Btu)
\( R \) cavity radius (m)
\( Re \) Reynolds number
\( Re_N \) Reynolds number for sodium
\( Re_{NK} \) Reynolds number for sodium-potassium
r distance from the midplane of an infinite slab (ft)
\( r_m \) half thickness of an infinite slab (ft)
\( S_v \) heating surface (ft\(^2\))
\( T \) temperature
\( T_m \) maximum fuel element surface temperature (°F)
\( T_w \) temperature of reactor coolant flow walls (°F)
\( T_{1} \) inlet temperature (°F)
\( T_{2} \) outlet temperature (°F)
\( T_f \) fluid temperature (°F)
\( T_o \) initial fluid temperature (°F)
\( T_s \) temperature of a solid (°F)
\( t \) time
\( V \) volumetric flow rate (ft\(^3\)/sec)
\( V \) volume of cylinder (ft\(^3\))
\( V_v \) void volume (ft\(^3\))
\( V_f \) flow velocity
\( V_r \) reactor core volume
\( W \) weight of U\(^{235}\) (g)
\( W \) yield (kt)
\( W_f \) fluid mass flow rate (lb/hr)
\( W_n \) mass flow rate of sodium (lb/hr)
\( W_{NK} \) mass flow rate of sodium-potassium (lb/hr)
\( W_o \) mass flow rate of oil (lb/hr)
\( w \) weight of oil shale (lb)
\( w_f \) flow velocity
\( w_o \) oil flow velocity
\( \beta \) coolant volume fraction
\( \Delta P \) pressure change (lb/ft\(^2\))
\( \Delta P_n \) pressure change for sodium
\( \Delta P_{NK} \) pressure change for sodium-potassium
\( \Delta T \) average temperature difference (°F)
\( \Delta T_n \) average temperature difference for sodium
\( \Delta T_{NK} \) average temperature difference for sodium-potassium
\( \eta \) number of fast fission neutrons produced per thermal neutron absorbed in uranium
\( \theta_a \) average coolant film temperature change (°F)
\( \lambda_g \) molecular specific heat (Btu/lb °F)
\( \mu \) viscosity (lb/hr ft)
\( \mu_f \) fluid viscosity
\( \rho \) average of overburden density (g/cm\(^3\))
\( \rho_f \) fluid density (lb/ft\(^3\))
\( \rho_N \)  sodium density (lb/ft\(^3\))
\( \rho_{NK} \) sodium-potassium density (lb/ft\(^3\))
\( \rho_p \)  coolant density at the pump (lb/ft\(^3\))
\( \rho_r \)  mean coolant density in the reactor core (lb/ft\(^3\))
\( \rho_s \)  density of a solid
\( \Sigma_f \)  macroscopic fission cross section (cm\(^{-1}\))
\( \Sigma_{au} \) macroscopic absorption cross section (cm\(^{-1}\))
\( \Sigma_{su} \) macroscopic scattering cross section (cm\(^{-1}\))
\( \Sigma_a \) total macroscopic absorption cross section (cm\(^{-1}\))
\( \Sigma_s \) total macroscopic scattering cross section (cm\(^{-1}\))
\( \sigma_a \)  microscopic absorption cross section (barns)
\( \sigma_{au} \) thermal absorption cross section for enriched natural uranium (barns)
\( \sigma_f \) microscopic fission cross section (cm\(^{-1}\))
\( \sigma_s \) microscopic scattering cross section (barns)
\( \tau \)  the Fermi age (cm\(^2\))
\( \phi \)  % porosity
\( \chi \)  distance traveled by fluid front into porous medium (ft)
\( \Psi \)  average number of fissions per second
\( \psi \)  thermal neutron flux
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