4. **TITLE AND SUBTITLE**
High-Order Algorithms for 3D Plasma Simulations on Unstructured and Hybrid Meshes

6. **AUTHOR(S)**
George Karniadakis

7. **PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
Division of Applied Mathematics
Brown University
Providence, RI 02912

9. **SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
AFOSR
801 North Randolph Street, Room 732
Arlington, VA 22203-1977

13. **ABSTRACT** *(Maximum 200 words)*
The work completed under this project builds on our previous work sponsored by AFOSR on developing a universal spectral basis for formulating algorithms for high-order accurate solutions of arbitrary nonlinear systes of PDEs in complex-geometry domains. To this end, several classes of classical numerical methods (e.g., finite elements and finite volumes) as well as modern methods (e.g., spectral and spectral elements) are simply subcases of the developed new numerical methodology. One of the unique features of the new methods is that its accuracy is insensitive to the mesh quality, which allows simulation in moving and very distorted domains (without re-gridding) at extremely high-efficiencies, not possible with any other method.

17. **SECURITY CLASSIFICATION OF REPORT**
18. **SECURITY CLASSIFICATION OF THIS PAGE**
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20. **LIMITATION OF ABSTRACT**
FINAL REPORT
High-Order Algorithms for 3D Plasma Simulations on Unstructured and Hybrid Meshes
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PI: George Em Karniadakis
Center for Fluid Mechanics
Division of Applied Mathematics
Brown University, Providence, R.I. 02912
Tel: 401-863-1217
email: gk@cfm.brown.edu
Objectives

- Develop high-order discretizations on unstructured and hybrid meshes for the viscous unsteady MHD equations in two- and three-dimensions. No such capability exists in the AFOSR labs or elsewhere in the world.
- Advance the state-of-the-art in multi-resolution algorithms and parallel software.
- Apply the new capability to parallel simulations of isothermal and MHD turbulence in complex-geometry domains.

Accomplishments/New Findings

The work completed under this project builds on our previous work sponsored by AFOSR on developing a universal spectral basis for formulating algorithms for high-order accurate solutions of arbitrary nonlinear syste of PDEs in complex-geometry domains. To this end, several classes of classical numerical methods (e.g. finite elements and finite volumes) as well as modern methods (e.g. spectral and spectral elements) are simply subcases of the developed new numerical methodology. One of the unique features of the new method is that its accuracy is insensitive to the mesh quality, which allows simulation in moving and very distorted domains (without re-gridding) at extremely high-efficiencies, not possible with any other method.

The specific tasks completed include:
- Development of Discontinuous Galerkin projections. This is a breakthrough in variational numerical methods, which will have great impact in many AFOSR applications. It allows general high-order differential operators to be discretized with finite jumps ($L^2$ continuity) across subdomains. This, in turn, implies that “sliding” domains (of great interest to weapons simulation technology) can now be discretized with no extra computational effort. In addition, intelligent trial bases can be used in different subdomains to deal with singularities, discontinuities and sharp gradients while optimizing their efficiency locally without global inter-dependence. The reduced continuity requirements make the method inherently parallel and naturally suited to modern parallel platforms at AFOSR labs.
- Development of Discontinuous Galerkin methods for the viscous MHD equations for compressible and incompressible turbulence, and demonstration of the high-order accuracy of the method. The parallel code we developed is the only MHD code with high-order capability in complex-geometry domains. The new formulation is extension
to high-order and unsteady flows of the work of Peterkin et al. (Kirkland AFB),
Shumlak (University of Washington) and Powell et al. (University of Michigan),
who are the others PIs in the AFOSR Computational Math program. In fact, their
algorithms are a subset of the new formulation in the degenerate case of setting
the spectral order equal to zero, which is the finite volume limit of the discontinuous
Galerkin formulation. Therefore, this makes for an easy porting of our numerical
algorithms to the current MACH2 and MACH3 (AFOSR codes), which are only
limited to second-order accuracy at best and only cartesian grids. Our algorithm is
suitable for cartesian grids, unstructured grids, as well as hybrid grids consisting of
polymorphic domains.

Personnel

• Faculty: G.E. Karniadakis (US), Professor of Applied Mathematics
• PhD Students: T.C. Warburton and I. Lomtev

Publications

1. T.C. Warburton, “Spectral/hp Methods on Polymorphic Multi-Domains: Algo-
rithms and Applications”, PhD thesis, Brown University, 1998 (supervised by
the PI).


Galerkin method for the compressible Navier-Stokes equations on hybrid grids”,
Proc. Tenth International Conference on Finite Elements in Fluids, January 5-8,


Interactions/Transitions

The PI will organize the first International conference on Discontinuous Galerkin Methods on May 24-26, in Newport, RI USA. Participants will include several academic and national lab researchers from US.

The PI was invited in the past year to present the AFOSR-sponsored research at:
- Cornell Workshop on POD-Galerkin Models for the Dynamics and Control of Complex Flows
- University of Michigan, Department of Aerospace Engineering
- AFOSR/Princeton Workshop on Plasma-Assisted Drag Reduction
- DARPA/NUWC Workshop on Electromagnetic Turbulence Control
- DOE/Oakridge Workshop on Discontinuous Galerkin Methods for Materials
- NSF Workshop on New Computational Challenges
- AIAA Fluid Dynamics Conference on LES
- SIAM Annual Meeting/Symposium on MHD
- ICOSAHOM'98 Symposium on Corner Singularities
- Japanese Society of Fluid Mechanics 30th Anniversary Symposium
- University of Tokyo, Department of Mechanical Engineering
- Turkey Workshop on Industrial and Environmental Applications of DNS/LES

Software Distribution: The code NEKTAR has been distributed to more than one dozen Universities and Laboratories. Some of them include MIT, Clatech, Cornell University, Penn State University, University of Wisconsin, Imperial College, North Carolina University, Florida State University, OAK Ridge Labs, Nielsen, Inc., Boeing, etc. There is limited documentation of the code, which has made this distribution somewhat difficult. However, certain researchers, e.g. Hussaini & Erlebacher at FSU have used NEKTAR extensively in LES of compressible turbulence and have developed new extensions for their applications. The porting of the new developments to the AFOSR codes MACH2 and MACH3 has not been done yet and it will require additional resources.