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<td></td>
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<td>Adelphi, MD 20783-1197</td>
<td></td>
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<tr>
<td>email: <a href="mailto:hauverma@arl.mil">hauverma@arl.mil</a></td>
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CONTENTS

Preface ................................................................. xi

SESSION I: SIMULATION/ANALYSIS

Visualization of Obscuration and Contrast Effects Using the Beams Models ........ 3

Donald W. Hoock and Patsy S. Hansen, U.S. Army Research Laboratory;
John C. Giever and Sean G. O'Brien, New Mexico State University

A Portable System for Data Assimilation in a Limited Area Model ....................... 15

Keith D. Sashegyi and Rangarao V. Madala, Naval Research Laboratory;
Frank H. Ruggiero, Phillips Laboratory; Sethu Raman, North Carolina
State University

Effect of High Resolution Atmospheric Models on Wargame Simulations ............... 25

Scarlett D. Ayres, U.S. Army Research Laboratory

An Assessment of the Potential of the Meteorological Office Mesoscale Model
for Predicting Artillery Ballistic Messages ......................................................... 37

Jonathan D. Turton and Peter F. Davies, Defence Services Division, Meteorological
Office, United Kingdom; Maj Tim G. Wilson, Projects Wings, Royal School
of Artillery, United Kingdom

Results of the Long-Range Overwater Diffusion (LROD) Experiment ..................... 47

James F. Bowers; U.S. Army Dugway Proving Ground; Roger G. Carter and
Thomas B. Watson, NOAA Air Resources Laboratory

Modeled Ceiling and Visibility ............................................................................. 57

Capt Robert J. Falvey, U.S. Air Force Environmental Technical Applications Center

A New PCFLOS Tool ............................................................................................. 65

K. E. Eis, Science and Technology Corporation

The Influence of Scattering Volume on Acoustic Scattering by Atmospheric
Turbulence .............................................................................................................. 75

Harry J. Auermann, U.S. Army Research Laboratory; George H. Goedecke
and Michael DeAntonio, New Mexico State University

Relationship Between Aerosol Characteristics and Meteorology of the Western
Mojave .................................................................................................................... 85

L. A. Mathews, and J. Finlinson, Naval Air Warfare Center; P. L. Walker, Naval
Postgraduate School
THE INFLUENCE OF SCATTERING VOLUME ON ACOUSTIC SCATTERING

BY ATMOSPHERIC TURBULENCE

Harry J. Auvermann
Army Research Laboratory, Battlefield Environment Directorate
White Sands Missile Range, New Mexico

George H. Goedcke and Michael DeAntonio
Dept. of Physics, New Mexico State University
Las Cruces, New Mexico

ABSTRACT

From a complete set of fluid equations, a complete set of coupled linear differential equations for the acoustic pressure, temperature, mass density, and velocity in the presence of stationary turbulence may be derived. To first order in the turbulent temperature variation and flow velocity, these coupled acoustic equations yield an acoustic wave equation given in the literature. Further reduction of this wave equation results in a second equation given in the literature which is good for turbulent length scales \( a \) much greater than the acoustic wavelength \( \lambda \). The length scale \( a_s \) of the scattering volume is found to be just as important as \( a \) and \( \lambda \) in predicting the general behavior of acoustic scattering by turbulence. In particular, if \( a < a_s \), then the first Born temperature and velocity scattering amplitudes for any ratio \( a/\lambda \) are the usual ones predicted by the first equation, and both the forward and backward velocity scattering are essentially zero for solenoidal turbulent flow velocity. The latter is not true if \( a > a_s \). If \( a \geq a_s > \lambda \), then the first Born scattering amplitudes are those predicted by the second equation. If \( \lambda \geq a \geq a_s \), other forms result for the scattering amplitudes. Implications of these findings for predicting results of acoustical scattering experiments where the scattering volume is often ill defined are discussed.

1. INTRODUCTION

This is the third paper given at Battlefield Atmospherics Conferences that deals with acoustic scattering by atmospheric turbulence. In the first paper (Auvermann, Goedcke, DeAntonio 1992), experimental evidence was presented showing that atmospheric turbulence near the ground was neither homogeneous nor isotropic, two conditions
required for the usual statistical model of turbulence to be valid. An alternate model consisting of a collection of isolated vortices of different sizes was proposed. This model was termed a structural model previously. The more descriptive name of Turbulence Ensemble Model (TEM) will now be adopted. A turbule is an isolated inhomogeneity of either fluid temperature or fluid velocity. The first paper (Auvermann, Goedecke, DeAntonio 1992) presented a general formulation of the method by which acoustic signal levels in shadow zones may be estimated using TEM. The second paper (Auvermann, Goedecke, DeAntonio 1993) showed how TEM may be used to explain theoretically the extreme variability of acoustic shadow zone signals that has been documented experimentally. In TEM, the scattering pattern of the various individual turbules is assumed known. The analysis proceeds by assuming a distribution function for the sizes, and then locating the turbules of each size randomly within the atmospheric region of interest. The shadow zone signal is then the summation of the contributions from each turbule. To carry out the summation in the first paper (Auvermann, Goedecke, DeAntonio 1992), a uniform concentration of turbules (number of turbules per unit volume) was assumed accompanied by a reasonable estimation of the volume from which the detector could receive signals. The summation in the second paper (Auvermann, Goedecke, DeAntonio 1993) was carried out directly because a relatively small number of turbules was postulated, the position and size of each being chosen randomly within appropriate limits.

In this paper, the problem of determining the volume from which significant scattering can occur is addressed in a more rigorous manner. Acoustical signals of interest to the Army are in general low frequency. The import of this is that the wavelengths of interest are large compared to the dimensions of either source or detector. Therefore, both source and detector are nearly omni-directional and thus cannot serve to define a scattering volume. This is a complication not usually experienced in optical scattering scenarios. For example, optical scattering by atmospheric aerosol is usually modeled with a narrow beam from a laser source and a narrow field-of-view detector system, the overlap or union of the two defining a small scattering volume. This geometric construction is the usual way scattering volume is defined. In this paper (except in section 4), scattering volume will denote the volume from which significant scattering can occur. An additional simplification that can be taken advantage of in aerosol scattering is the fact that the largest aerosol particle dimension is small compared to the dimension of the scattering volume. Even though acoustic wavelengths are large, turbule sizes can be larger, and may approach the scattering volume dimension. In section 2, the scattering pattern of individual turbules is used to define a scattering volume as a function of turbule size. Then, in section 3, scattering cross-section modified by number concentration is used to determine the relative contributions from the various size classes. Section 4 contains general results that may be applied to determine if surface integrals from scattering theory may be ignored, as is usually done in optics. Conclusions that may be drawn from this work are discussed in section 5.

The symbols for the some of the variables, parameters and mathematical operations are summarized in the following list. Others are defined in the text. Bold quantities in the
list (for example r) are three-vectors.

**NOTATION**

- \( a \)  = turbule characteristic size, m
- \( c_\infty \)  = asymptotic acoustic wave speed = 344 m·s\(^{-1}\)
- \( \partial_i \)  = \( \partial / \partial x_i \); \( x_1 = x; x_2 = y; x_3 = z \)
- \( \exp(-j\omega t) \)  = time dependence of acoustic wave
- \( f \)  = acoustic wave frequency = 500 hz
- \( k \)  = propagation vector, m\(^{-1}\)
- \( k \)  = magnitude of \( k = \omega / c_\infty \)
- \( \lambda \)  = wavelength, m
- \( r \)  = position vector, m
- \( r \)  = magnitude of \( r \)
- \( r_1 \)  = integration variable position vector
- \( t \)  = time variable, s
- \( \tau(r) \)  = turbulence field temperature difference ratio
- \( v_0(r) \)  = turbulent flow velocity in the absence of the acoustic wave
- \( v_{0i} \)  = i-th component of \( v_0 \)
- \( \kappa \)  = scattering volume limit angle for velocity turbule ensemble
- \( \xi \)  = velocity turbule ensemble relative scattering cross-section
- \( \chi \)  = size parameter = \( ka \)
- \( \psi \)  = scattering angle between \( k \) and \( r \)
- \( \omega \)  = wave angular frequency = \( 2\pi f \)

**2. SCATTERING PATTERN INFLUENCE ON SCATTERING VOLUME**

The theory of acoustical scattering from turbules is too lengthy to be covered in this paper. It is covered elsewhere (Goedecke, DeAntonio, Auvermann 1994a). The following is a brief synopsis of how scattering patterns are determined theoretically. Beginning with a complete set of fluid equations in density, pressure, temperature, and velocity, each variable is assumed to be made up of a time independent part representing the inhomogeneous medium plus a small time dependent part representing the acoustic wave. Expressions representing the above are substituted in the fluid equations and terms second order or higher in the ratios \( (v_0 / c_\infty, \tau) \) are discarded. Assuming harmonic time dependence, the resulting wave equation is

\[ (\nabla^2 + k^2) \eta(\boldsymbol{r}) = \partial_i \tau \partial_i \eta(\boldsymbol{r}) + 2j\omega^{-1} \partial_i (v_{0i} \partial_i \tau) \eta(\boldsymbol{r}) = -4\pi S(\boldsymbol{r}) \eta(\boldsymbol{r}) \]  

(1)

where the summation convention is used for repeated subscripts and

\[ S(\boldsymbol{r}) = S_T(\boldsymbol{r}) + S_\chi(\boldsymbol{r}) = -(4\pi)^{-1} \partial_i (\tau \partial_i) - (j/2\pi \omega) \partial_i (v_{0i} \partial_i \tau). \]  

(2)
\[ [S_T(\vec{r}), S_T(\vec{r})] \] define the scattering operators for (temperature, velocity) inhomogeneities respectively. \( \eta(\vec{r}) \) is the acoustic wave field quantity, the ratio of the acoustic pressure to the total fluid pressure.

The Green's function solution for an incident plane wave is written down, and a Born approximation is made in the scattering integral. The Born approximation involves replacing the field in the scattering integral by the incident field. The result is where

\[
f_B(\vec{r}) = \int d^3 r_{1} \exp(-jkr_1) S(\vec{r}_1) \exp(j\vec{k} \cdot \vec{r}_1)
\]

Equation (3) has been used (Goedcke, DeAntonio, Auvermann 1994a) to derive the scattering cross-section for temperature and velocity inhomogeneities. The scattering cross-section is equal to the scattering amplitude multiplied by its complex conjugate, and has units of length squared. Only velocity inhomogeneities will be considered further in this paper.

It has been shown (Goedcke, DeAntonio, Auvermann 1994b) that an isotropic ensemble of turbulences having a given scale length a but arbitrary velocity morphology except for \( \nabla \cdot \vec{v}_0 = 0 \) can be replaced by an ensemble with velocity \( \vec{v}_0 \) given by \( \vec{v}_0 = \hat{\Omega} \times \vec{r} f(\vec{r}) \), where \( f(\vec{r}) \) is a function of the distance \( r \) from turbulence "center", and \( \hat{\Omega} \) is a randomly oriented angular velocity. A Gaussian form of \( f(\vec{r}) \) has proved convenient (Goedcke 1992), so that

\[
\vec{v}_0(\vec{r}) = \hat{\Omega} \times \vec{r} \exp(-r^2/a^2)
\]

The scattering cross-section obtained from the theory outlined above was given in the report (Goedcke 1992). To simplify the presentation in this paper, the cross-section averaged over orientation angles will be used. This cross-section is (Goedcke 1992)

\[
\sigma_v(\vec{k}, \vec{r}) = \left( \frac{\pi}{3} \right) \left( \frac{\Omega a \chi^4}{4k_c} \right)^2 [\sin(\psi) \cos(\psi)]^2 \exp\left\{-\chi^2[1 - \cos(\psi)]\right\}
\]

Two normalized expressions are defined in eq. (6) below for the purpose of illustration of

\[
\sigma_{vN} = 4[\sin(\psi) \cos(\psi)]^2 \exp\{-\chi^2[1 - \cos(\psi)]\}
\]
\[
\sigma'_{vN} = 4\left( \frac{\chi^4}{8^4} \right)^2 [\sin(\psi) \cos(\psi)]^2 \exp\{-\chi^2[1 - \cos(\psi)]\}
\]

the behavior of eq. (5). The first contains all of the angular dependence and the second includes the multiplicative size parameter dependence. These two functions are plotted in
the next figures to show the nature of the velocity cross section in the \((\chi, \psi)\) domain. Figure 1 shows the angle dependent part, the axis label "Sigma" being \(\sigma'_{vN}\). Hereafter, the axis label "Chi" is \(\chi\) and the axis label "Psi" is \(\psi\). Figure 2 shows the influence of the size parameter on the cross-section, the axis label "Sigma" being \(\sigma^{'}_{vN}\).

![Figure 1. Normalized velocity turbule cross-section (angle part only)](image1)

![Figure 2. Velocity turbule scattering cross-section](image2)

The range of the independent variables \(\chi\) and \(\psi\) were chosen in figure 2 to illustrate interesting features of the function \(\sigma^{'}_{vN}\). When \(\chi\) is 13.4 and \(\psi\) is 0.1, \(\sigma^{'}_{vN}\) peaks at very nearly unity. For \(\psi\) not near zero, the exponential drives \(\sigma^{'}_{vN}\) to zero. As \(\chi\) increases beyond the values in the figure, the peak of \(\sigma^{'}_{vN}\) continues to increase. However, the width of the peak decreases and the value of \(\psi\) at the peak decreases. These curves for the cross-section are not the entire story because other factors influence the scattered signal at a detector. Consideration of these other factors is undertaken in the next section.

3. NUMBER CONCENTRATION INFLUENCE ON SCATTERING VOLUME

In the previous section, the dependence of the cross-section upon size parameter was shown to emphasize the importance of large turbules which have narrow scattering patterns. To do an incoherent summation for the signal scattered from an ensemble of turbules, the first expedient to employ is to simply add the effects of the many by multiplying by a number concentration. Since the scattering volume as a function of turbule size is the desired quantity in this paper, the number concentration as a function of size is necessary. An estimate of this has been made (Goedecke, DeAntonio, Auvermann 1994c). Furthermore, an estimate of the characteristic velocities of the turbules is necessary. The following power law scaling functions are first assumed:
\[
\frac{N_\alpha}{N_1} = \left(\frac{a_\alpha}{a_1}\right)^\beta, \quad \frac{v_\alpha}{v_1} = \left(\frac{a_\alpha}{a_1}\right)^\nu = \Omega_\alpha a_\alpha
\] (7)

The meaning of eq. (7) is as follows. The largest turbules are identified by the subscript 1. The largest turbules have the concentration \( N_1 \). Their characteristic size is \( a_1 \). The largest velocity turbules have characteristic velocities \( v_1 \). Other sized turbules are identified by the index \( \alpha \). The exponents (\( \beta, \nu \)) are chosen so that a homogeneous isotropic ensemble of turbules matches the Kolmogorov spectrum. The results are (Goedecke, DeAntonio, Auvermann 1994c) \( \beta = 3, \quad \nu = 1/3 \). An interesting feature of this result is the concentration exponent being 3. This means that the packing fraction is the same for all size classes. The velocity scaling exponent is 1/3 as may be derived from a simple energy cascade calculation.

It is now possible to write the size parameter dependence of the cross-section for each size of turbules. Multiplying eq. (5) by \( N_\alpha \) and using eq. (7) yields for the cross-section per unit volume for turbule size \( a_\alpha \) and size parameter \( \chi_\alpha \)

\[
\sigma_v(\mathbf{k}, \mathbf{r}) = \pi a_\alpha^2 \left(\frac{v_1^2 N_1}{48 c_\infty^2}\right) (k a_\alpha)^{1/3} \chi_\alpha^{17/3} [\sin(\psi) \cos(\psi)]^2 \exp \{-\chi_\alpha^2 [1 - \cos(\psi)]\}
\] (8)

The size parameter subscript will be dropped hereafter. The largest turbule size has been assumed to be 10 m, which is appropriate for a velocity turbule centered ten meters above the ground. The size parameter for this size turbule is 91.325. A velocity turbule is produced by wind shear with the wind velocity zero at ground level. Further assuming that the velocity ratio (\( v_1/c_\infty \)) is 0.01, this size turbule would have a characteristic velocity of 3.44 m s\(^{-1}\). This velocity would be produced by a wind gradient with this velocity at turbule center at 10 m height and twice this velocity at turbule upper edge at 20 m height. The scenario including the ground is more complex than is possible to treat in this paper. Rather, the assumption will be made that both source and detector are in free space and that the atmospheric turbulence is homogeneous and isotropic with characteristics of that at 10 m height. Considering the line joining source and detector a reference line from which to measure scattering angles and that the source is a long distance away, the scattering angle is the offset angle from the detector to a differential scattering volume. The total signal received by the detector will be a volume integral of the differential scattering volume times the expression in eq. (8). A further simplification will be taken in that no further specification of scenario parameters will be given. Thus, no range dependent effects will be considered. The volume integration will be confined to a shell around the detector of radius \( R_\psi \) and thickness \( \Delta R_\psi \). Figure 3 depicts this geometry on a plane section through the scattering volume under consideration. The scattering angle is \( \psi \) and the view angle is \( \psi' \). These two angles are equal for this choice of source wave incident direction. The differential scattering volume will then be the ring around the axis.
whose cross-section is shown in the figure and is \([2\pi R_s^2 \Delta R_s \sin(\psi') d\psi']\). Equation (9) below derived from eq. (8) gives the specific mathematical form for the velocity turbule ensemble scattering cross-section.

\[
\xi_v(\chi, \xi) = \left( \frac{\pi^2 a_r^2 v_1^2 (ka_r)^{1/3} R_s^2 \Delta R_s N_1}{24 c^2} \right)^{1/3} 
\int_0^\xi \psi' \sin^3(\psi') \cos^2(\psi') 
\exp[-\chi^2(1 - \cos(\psi'))] 
\]

The expression involving the size parameter \(\chi\) and the scattering volume limit angle \(\xi\) in eq. (8) will be plotted in figure 4 for the case of \(\xi = \pi\). This is the case in which the cross-section for an entire spherical shell has been determined and which contains the maximum of the expression. The maximum occurs at \(\chi = 91.325\) and is 3701.82. That which has been plotted in figure 4 is the ratio of the scattering cross-section to this value, so the maximum in figure 4 is unity. The curves for which the relative scattering cross-section is 0.01 and 0.95 are plotted in figure 5. A few of the calculated points are listed below:

\[
\begin{align*}
\xi_{v,rel}(88.5592, \pi) &= 0.95 & \xi_{v,rel}(91.3250, 0.033716) &= 0.95 \\
\xi_{v,rel}(60.2799, \pi) &= 0.50 & \xi_{v,rel}(91.3250, 0.020059) &= 0.50 \\
\xi_{v,rel}(6.2246, \pi) &= 0.01 & \xi_{v,rel}(91.3250, 0.005969) &= 0.01
\end{align*}
\]

These curves are interpreted as an aid to forming a turbule distribution as follows. As an example limit angle, take 0.05 radian on figure 5 at the largest size parameter. This is larger than the size for the 0.95 contour. Extend a cone of this central angle out to 200 m and it will contain an entire 10 m radius turbule. Even if turbules were placed in a hexagonal close-pack configuration, the six turbules in the next circle can at most contribute 0.05 to the total scattering. Thus, our distribution need contain only one turbule of the largest size, or at the most seven. Assume that the shell thickness \(\Delta R_s\) is 20.0 m, large enough to accommodate a collection of the largest turbules. At the other end of the distribution, consider those whose size parameter is 6.2246. The radius of these would be 0.682 m. Some 5,047,000 of these could be packed into the entire spherical shell at a range of 200 m. However, this entire ensemble would scatter no more than 0.01 times that scattered by the single turbule of the largest size. Consider now turbules with size parameter 88.5592 or radius 9.697 m. The complete shell would need to be filled with turbules of this size to scatter 0.95 times the scatter of the largest turbule. The number of these would be in the neighborhood of 1,755. For turbules of size
parameter 60.2799 or radius 6.600 m, the number in the shell would be 5,566. These

would contribute only 0.50 times the scatter of the largest turbule. Although integration
with respect to size parameter has not been attempted, and therefore it cannot be said
definitely, it does seem likely that the scatter from turbules less than 0.682 m radius will
total a great deal less than 0.01 times the scatter from those of larger radius. This means
that an upper bound has been established above on the number of turbules that need to be
placed in a representative distribution for the scenario considered. It is clear that the
larger turbules dominate the scattering to be considered.

4. APPLICABILITY OF STANDARD SCATTERING THEORY

Standard scattering theory for scattering from a single localized scatterer involves a plane
wave incident on the scatterer and a Green's function solution of the wave equation. The
quantities obtained are the scattering amplitude and the scattering cross-section in the far
field of the scatterer. In the analysis of the previous section, standard theory results were
used for the scattering from individual turbules, in the Born approximation. The
summation of cross-sections for the collection of turbules in each size range neglects the
coherence properties of the scattered signals which is appropriate for the ensemble average
scattering by a collection of scatterers having random locations except in the near forward
direction (Goedcke, DeAntonio, Auermann 1994c). Standard scattering theory for an
ensemble of scatterers is usually applied when the detector is in the far field of the
scattering volume occupied by the ensemble. For the scenario used in this paper, the
detector is assumed to be in the "near field" of the scattering volume, but the far field of
each turbule. This will not actually be true for the larger turbules in a more realistic
scenario. Also, in many scenarios, the incident wave is not plane. The importance of
deviations from standard far field scattering geometry has not been determined.

Some general conclusions may be drawn from the standard scattering theory, general in the sense that they do not depend on the form of the temperature and velocity distributions considered (Goedecke, DeAntonio, Auvermann 1994a). Using the formal result of eq. (3), it is recognized that the volume integral need only extend over a finite scattering volume. In this section, the expression scattering volume is thought of as the intersection of the illuminated region and the detector field-of-regard, perhaps made of limited extent by the use of a parabolic reflector. Although this volume may have a complicated shape, a single scale length \( a \), is ascribed to it for convenience. The effect of turbulence outside the scattering volume rapidly goes to zero. Integration of eq. (3) by parts yields two terms, the first being essentially the Fourier transform of the turbulence distribution (either temperature or velocity) and the other a surface integral over the scattering volume surface. In the commonly treated case where \( a << a_s \), there are many turbules in the scattering volume and the surface integral will be negligibly small. The result is the standard theory that has been used in the first parts of this paper. If, however, the turbule scale \( a \) is greater than \( a_s \), the surface integral is not negligible. The result is the same as our result above except the cosine factor, which makes scattering at right angles to the incident wave zero, is not present. In the third case, where the wavelength is greater than the turbule size which is in turn greater than \( a_s \), the surface integral is again not negligible. The scattering pattern can deviate appreciably from the standard results.

5. CONCLUSIONS

Battlefield scenarios in which acoustic propagation can have significant importance will often be such that standard scattering theory will not fully apply. This may occur when the wavelength, the length scale of the turbulence, and the length scales of some of the turbulent eddies, and the length scale of the scattering volume are of the same order, or when the detector is not in the far field of the scattering volume and/or the individual turbules. Thus, definition of the scattering volume is an important element in calculating turbulence scattering.

Using specific examples of turbule morphologies and a particular simplified scenario, it was shown that large scale turbulence dominates scattering and that the scattering pattern of these large scale entities tends to limit the effective scattering volume. The scenario involved a \( 4\pi \) detector assumed to be in the far field of each turbule in an ensemble of randomly located turbules of different sizes.

Future work will need to deal with more complex scenarios. However, considerable information may be derived with the use of standard scattering theory, such information giving an initial approximation to the true result. Also, to be included are the effects of ground reflections, shadow zones, and the \( 1/r^2 \) effects of source and scattered fields. All of these effects may further limit effective scattering volumes and/or modify the current results.
Concerning the enormous fluctuations of scattered signals measured in shadow zones (Auvermann, Goedcke, DeAntonio 1993), these must occur because of relative motion among a moderate number of large entities. This situation requires consideration of scattering amplitudes rather than cross-sections. Even if the signal from all turbules at 6.22 size parameter were in phase and the signal from all turbules at 91.325 size parameter were in phase separately, the former could produce only a plus or minus 20% variation in the overall summed signal. The experiment showed the variation was 100%, indicating that the relative amplitudes of the different significant contributors are nearly equal.

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