GLOBAL POSITIONING SYSTEM (GPS)
ERROR SOURCE PREDICTION

THESIS

Marcus G. Ferguson, Second Lieutenant, USAF

AFIT/GOR/ENS/00M-12

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

20000613 099

REPORT DOCUMENTATION PAGE	Form Approved OMB No. 074-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of the collection of information, including suggestions for reducing this burden to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE March 2000	3. REPORT TYPE AND DATES COVERED Master's Thesis

4. TITLE AND SUBTITLE	5. FUNDING NUMBERS
GLOBAL POSITIONING SYSTEM (GPS) ERROR SOURCE PRECICTION	

6. AUTHOR(S)

Marcus G. Ferguson, Second Lieutenant, USAF

7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(S)	8. PERFORMING ORGANIZATION REPORT NUMBER
Air Force Institute of Technology Graduate School of Engineering and Management (AFIT/EN) 2950 P Street, Building 640 WPAFB OH 45433-7765	AFIT/GOR/ENS/00M-12

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING / MONITORING AGENCY REPORT NUMBER
HQ SWC/AE Attn: Suzanne Beers, Lt Col, USAF 730 Irwin Avenue, Suite 83 Schriever AFB, CO 80912-7383 DSN: 560-9286 e-mail: Suzanne.Beers@swc.schriever.af.mil	

11. SUPPLEMENTARY NOTES

Advisor: Jeffrey W. Lanning, Major, USAF
Comm: (937) 255-3636, ext. 4324   DSN: 785-3636, ext. 4324
e-mail: Jeffrey.Lanning@afit.af.mil

12a. DISTRIBUTION / AVAILABILITY STATEMENT	12b. DISTRIBUTION CODE
APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.	

ABSTRACT (Maximum 200 Words)

With the initiation of the navigation accuracy prediction algorithm used to estimate the amount of GPS solution (location and time) error for receivers, the capability to accurately predict solution errors due to the major GPS error sources is growing. Although some sources of error within the GPS solution have been previously analyzed, modeled, and/or accounted for within various modeling efforts, a formal evaluation of the seven major error sources that distort GPS activity has not been officially conducted up until this point. This research offers a logical assessment of all the major GPS error sources and their definitive impact on the end user.

This research describes the major error sources in the GPS solution, which includes error sources from the spacecraft, propagation of the signal through space, and receiver errors for a representative family of receivers. Once we define these error sources, we prioritize these sources with respect to benefit-to-cost ratios. We base the benefit-to-cost ratio on an error's accountability to the modeling effort required.

This research recommends a prioritized order of future enhancements for error source implementation and improvements in future GPS accuracy prediction models, with a complete explanation of the tradeoffs associated with each improvement.

14. SUBJECT TERMS	15. NUMBER OF PAGES 64
GPS, Global Positioning System, Error Sources, Error Budget, Ionosphere, Selective Availability, Satellite Clock, Ephemeris, Receiver, Multipath, Orbit, Measurement Noise, Error Prediction Accuracy, OMEGA, Troposphere, SPIDAR, Estimation Model, Validation, Satellite Selection, CEP, DOP	
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

GLOBAL POSITIONING SYSTEM (GPS)

ERROR SOURCE PREDICTION

THESIS

Presented to the Faculty

Department of Operational Sciences

Graduate School of Engineering and Management

Air Force Institute of Technology

Air University

Air Education and Training Command

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Operations Research

Marcus G. Ferguson, B.S.

Second Lieutenant, USAF

March 2000

GLOBAL POSITIONING SYSTEM (GPS)

ERROR SOURCE PREDICTION

Marcus G. Ferguson, B.S.
Second Lieutenant, USAF

Approved:
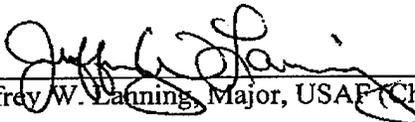
_____
Jeffrey W. Lanning, Major, USAF (Chairman)

13 Mar 00
date
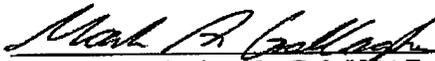
_____
Mark A. Gallagher, Lt Col, USAF (Member)

13 Mar 00
date

Table of Contents

List of Figures

List of Tables

Abstract

With the initiation of the navigation accuracy prediction algorithm used to estimate the amount of GPS solution (location and time) error for military receivers, the capability to accurately predict solution errors due to the major GPS error sources is growing. Although some sources of error within the GPS solution have been previously analyzed, modeled, and/or accounted for within various modeling efforts, a formal evaluation of the seven major error sources that distort GPS activity has not been officially conducted up until this point. This research offers a logical assessment of all the major GPS error sources and their definitive impact on the end user.

This research describes the major error sources in the GPS solution, which includes error sources from the spacecraft, propagation of the signal through space, and receiver errors for a representative family of military receivers. Once we define these error sources, we prioritize these sources with respect to benefit-to-cost ratios. We base the benefit-to-cost ratio on an error's accountability to the modeling effort required.

This research recommends a prioritized order of future enhancements for error source implementation and improvements in future GPS accuracy prediction models, with a complete explanation of the tradeoffs associated with each improvement.

I. Introduction

The Global Positioning System (GPS) is a navigational system that consists of
ground control stations, satellites in orbit, and receiver units. The ground control stations
upload navigation data to all GPS satellites. The user identifies which satellites are
within its view and selects the four satellites that provide the best solution (location and
time of receiver). The satellites then download the GPS signal to the receivers. From
this simple operation, a receiver computes a solution. Although this solution is accurate
to within a few meters of the actual location, the receiver does not compute an exact
solution. This solution error is due to several error sources.

Accurate predictions of GPS error sources inform users of the magnitude of
solution error to expect for a given place and time in the future. In order to accomplish
an accurate estimation of the solution error, error prediction models were generated.
Current error prediction models estimate the magnitudes of position errors that a user can
expect to incur at a given place and time in the future. Although error prediction models
address some of the major sources of error, modelers can still improve these models.
Possible improvements include refining the estimates of the error sources currently
modeled and consideration of additional error sources that have not yet been modeled.
Some error sources do not warrant modeling consideration because they do not affect
military receivers; due to their negligible effects on GPS, we only briefly discuss these
controlled sources and corrected sources. (A controlled error source is a source
manipulated by the Department of Defense and a corrected error source is a source of
error that no longer affects GPS activity.) This allows us to restrict the number of error
sources we consider in detail. We primarily focus on the remaining major error sources

(both modeled and unpredicted) in this research. We examine all of the error sources and determine the error sources that warrant further research based on their error prediction potential. We offer recommendations on how to implement the remaining error sources in future models. Although other sources of error can cause disruption in the GPS, we focus strictly on the major sources.

Enhancement of these prediction models is necessary because a small amount of error from a GPS error source can have a large effect on the solution error. A better prediction of the amount of position error for military GPS receivers is vital for the precise planning of missions that depend on GPS. The ability to predict GPS errors accurately should result in the accurate planning and execution of more effective missions. Improved predictions develop by properly modeling the error sources. A better understanding of these sources leads to better error modeling.

Error sources corrupt position accuracy for every type of GPS receiver. We center our discussion on the use of state-of-the-art military receivers in standard GPS situations and not differential GPS (DGPS). The major difference between standard GPS and DGPS is that standard GPS frequently utilizes only a single kinetic receiver, whereas DGPS frequently utilizes two or more stationary receivers (usually for reference checking purposes).

In general, military receivers utilize dual-frequency (P-code) capabilities whereas civilian receivers use only single-frequency (C/A-code) capabilities. Civilian receivers normally offer a significantly degraded performance when compared to military receivers. We limit our study to the use of mobile, military GPS receivers because too

many compromises may generate errors that are greater than those offered in this document.

In the following chapters, we fully address and analyze the major error sources that corrupt GPS operation. In the background chapter, we discuss the basics of GPS, an introduction to the major error sources, a key component in determining GPS accuracy called Dilution of Precision (DOP), and current error prediction models. This elementary foundation in GPS paves the way for a thorough explanation of each error source in the third chapter. In the third chapter, we will organize the error sources according to sources that do not affect military receivers, sources that have already been predicted in the latest error prediction model, and sources that possess good potential for model consideration. For the error sources that we recommend for consideration in future prediction models, we present a full investigation. We examine the following properties of the potential error sources: the source's causes, the modeler's ability to accurately predict the source's magnitude, how researchers explain and model the source, and modeling capabilities. The analysis chapter describes the effort required for implementing different error sources in future prediction models, and the benefits that we expect to result from these additions. Finally, in the conclusion chapter, we state a suggested order for implementing and reworking all the major error sources as well as provide recommendations for further research.

II. Background

This chapter provides a short discussion and/or refresher to the reader who is unfamiliar with GPS's inception, progression, activity, and sources of error. Also covered are the error prediction models currently used to estimate GPS solution error.

GPS Basics

This section contains a brief history and development of GPS and how GPS functions. Recognizing the advances in GPS technology should provide an appreciation for the developments to date. A basic understanding of how GPS works is essential in order to effectively analyze the error sources.

Condensed History and Development of GPS

Several United States government organizations, particularly the military, showed interest in developing satellite systems for position determination in the early 1960s. Kaplan (1996) points out that the optimum system was to provide global coverage, continuous all weather operation, the ability to serve high-dynamic platforms, and high accuracy.

Kaplan also notes that the first space-based navigational systems received wide acceptance for use only on low-dynamic platforms. These systems offered a high-accuracy positioning service for only two-dimensions. The frequency of obtaining a position fix varies with time; as the latitude increases, the time to obtain a position fix decreases. Each position fix needs an estimate of the user's position requiring approximately 15 minutes of receiver processing. These features are appropriate for

shipboard navigation, but are not suitable for aircraft and other high-dynamic users. These shortcomings for high-dynamic systems led to the creation of the GPS in the early 1970s. Kaplan points out that many developments took place to overcome earlier shortcomings and provide better accuracy. The insertion of highly-stable, atomic clocks in the satellite systems achieves precise time transmission and offers a satellite-to-user ranging capability for two-dimensional position determination (Parkinson, 1994). Ranging using pseudorandom noise (PRN) modulation with digital signals then provides three-dimensional coverage along with continuous worldwide service (Kaplan, 1996).

GPS is now completely operational and satisfies the criteria established in the 1960s for an optimal navigational system. The current system provides accurate, continuous, global, three-dimensional information to users with suitable receivers.

How GPS Works

GPS is a space-based navigational system, consisting of 24 active satellites and five ground support stations. The satellites are located approximately 20,200 kilometers above the earth (Dana, 1999). GPS provides users with accurate information about their position, velocity, and time anywhere in the world under all weather conditions. Figure 1 shows the constellation of 24 satellites in orbit around the earth providing users information regarding their position and movement. This network of satellites is positioned in six orbital planes with four satellites per plane. These planes as surrounding the earth like a box would surround a sphere.

GPS determines the user's position by calculating the difference between the time when the satellite transmits a signal and the time the receiver actually receives the signal.

The signal includes information about the locations of the satellites within the receiver's view and corrections necessary for accurate positioning. The receiver uses the time offset between the time that the signal is received and the time that the satellite broadcasts the signal to calculate the distance from the receiver to the satellite. In doing so, the receiver must account for propagation delays of the signal caused by the atmosphere (Kruczynski, 1998).
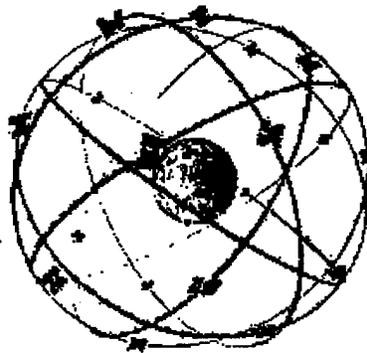
Figure 1: 24 Satellites Orbit around the Earth in GPS (copied from Kruczynski, 1998).

In order to compute a receiver's solution (location and time), the receiver algorithm selects four satellites from all of the satellites in the receiver's view. In mathematical terms, the user's receiver solves a system of equations with four equations and four unknowns; the four equations represent the four satellites selected by the receiver to compute a solution and the four unknowns represent the receiver's latitude, longitude, altitude, and time (Trimble, 1999).

GPS requires three segments to accurately process a user's position: control, space, and user. Figure 2 shows the control segment that consists of the master control

station (MCS, located in Colorado) and four monitor stations (strategically located on different sectors of Earth).
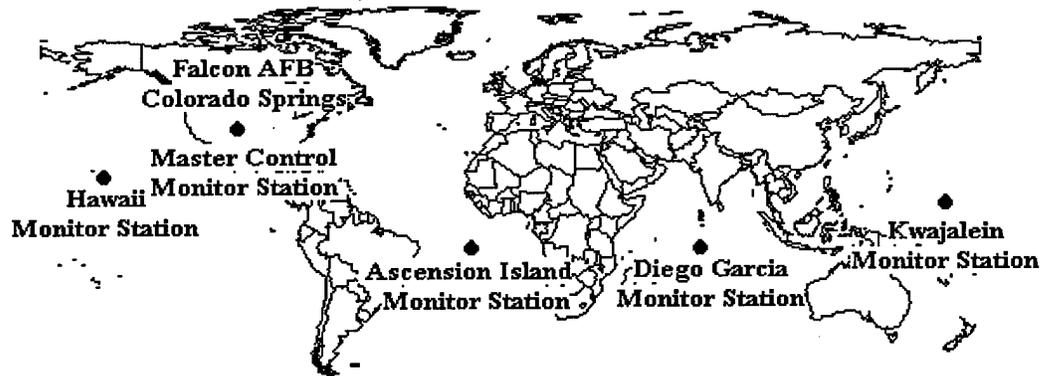
Global Positioning System (GPS) Master Control and Monitor Station Network

Figure 2: Five Ground Stations that Monitor GPS Activity (copied from Dana, 1999).

Together, these stations monitor the health and status of the satellites. The control segment uploads navigation information and other data to the satellites of the space segment. The satellites then download calculated data to the receivers. Figure 3 shows how the segments work together to upload and download data. This figure also identifies the different kinds of data that are uploaded and downloaded. From this figure, we can see the many places where errors can develop in GPS.

We normally discuss GPS accuracy in terms of average position measurements, but GPS actually provides instantaneous position measurements. The instantaneous accuracy is driven by several factors, specifically the seven major GPS error sources that impact a receiver's solution. The error estimation of these sources is critical to predicting accurate GPS solutions. Some of the major error sources do not apply to military

receivers, some are currently modeled in error prediction models, and other error sources

have not yet been implemented in error prediction models.
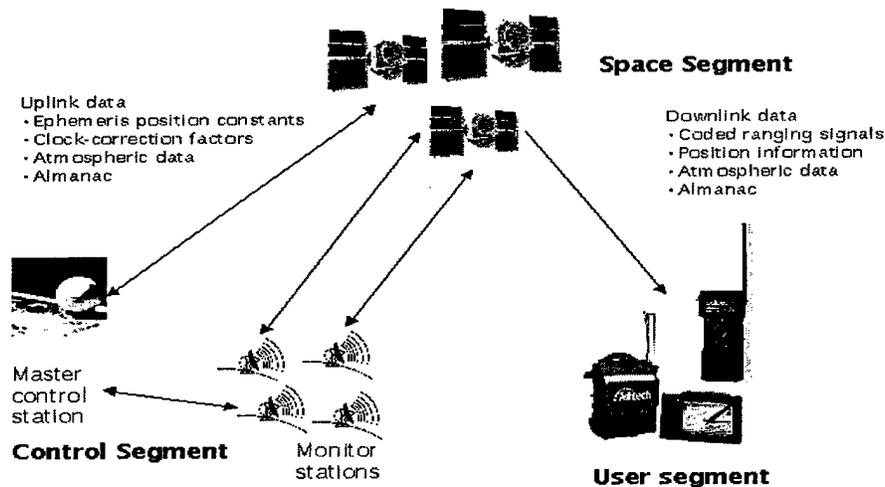
Figure 3: GPS Program Segment (copied from Bak, 1999).

Identification of the Error Sources

When GPS was first conceived, it was designed to be as accurate as possible.

However, several error sources still affect the performance of GPS. Kalafas (*et al.,* 1986)

notes the following seven major error sources impacting GPS accuracy:

> *Selective Availability (SA) errors* – artificial errors introduced at the satellites by
> the Department of Defense (DoD) for security reasons

> *Ionosphere delay errors* – signal propagation group delay errors caused by
> charged space particles in the upper atmosphere

> *Satellite clock errors* – differences between the actual satellite's clock time and
> the time predicted by the satellite data

> *Ephemeris (orbital) errors* – differences between the actual satellite location and
> the location predicted in the satellite orbital data

> *Receiver error* – error incurred due to receiver signal noise that can be caused by
> several different influences (i.e., inferior receiver design, algorithm
> problems)

Multipath error – error in satellite signal where the signal bounces off various obstructions in the environment before it gets to the receiver

Troposphere delay errors – signal propagation delay errors caused by weather conditions in the lower atmosphere

We tabulate the average error values of these error sources in Table 1 for both unauthorized standard positioning system (SPS) users and authorized precise positioning system (PPS) users. SPS generally consists of civilian users and PPS consists primarily of military users. (These values are within 1 standard deviation and measured in meters.)

Table 1: Average GPS Positioning Errors with SPS (with and without Selective Availability) and PPS Receivers Per Platform of 4 Satellites (copied from Parkinson, 1994 and Raquet, 1999).

Error Source \ Positioning System	PPS (military use)	SPS (civilian use)	
		With SA	Without SA
Ionosphere	0.01	4.0	4.0
Satellite Clock	2.1	20	2.1
Ephemeris Data	2.1	2.1	2.1
Receiver	1.0	1.0	1.0
Multipath	1.4	1.4	1.4
Troposphere	0.7	0.7	0.7

From this list of major error sources, we note the influential sources of GPS errors and the average values of these errors for military and civilian users. We show the table above only to demonstrate the differences between PPS and SPS error magnitudes and the impact of Selective Availability (SA) on civilian receivers. The errors for PPS are similar to those of SPS without SA. SA is the military's ability to inject errors into the GPS solution thereby hampering an enemy's ability to use the system. The authorized PPS users can access the artificially induced SA errors and eliminate them entirely. Although its use remains an option, SA is currently turned off and hence is not applicable

to either SPS or PPS at this time. The dominant sources appear to develop from the satellite ephemeris and clocks for PPS and the ionosphere for SPS. The biggest distinction between the PPS and the SPS sources is that the ionosphere error is significantly less for PPS than for SPS. This is due to the dual-frequency correction capability that only PPS receivers possess. For this research, we are only interested in the error contributions incurred by military receivers.

Bak (1999) allocates the GPS error sources into three physical regions, the spacecraft, space, and the receiver. He shows this graphically in a figure reproduced as Figure 4.
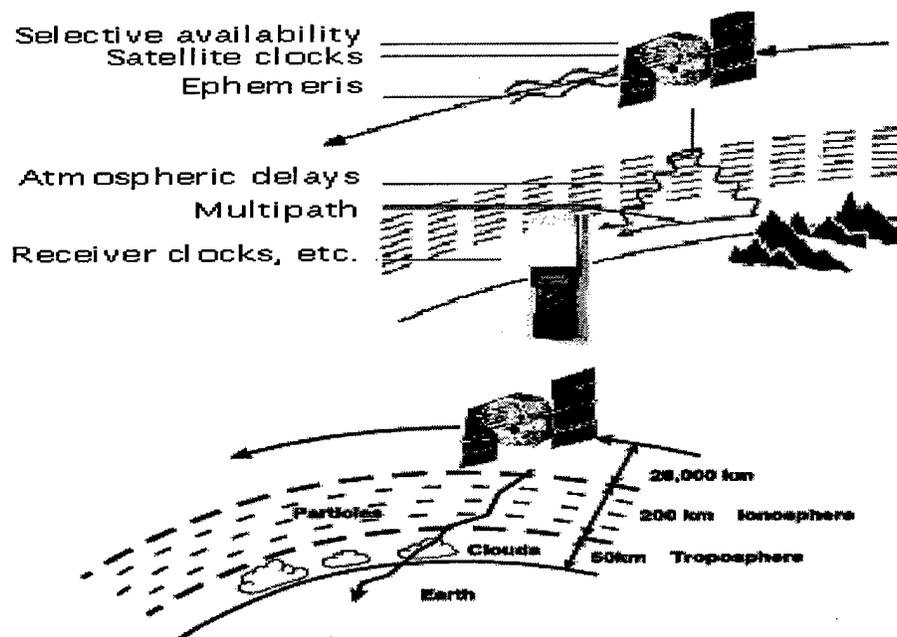
Figure 4: Visual of Major Error Sources that can Disturb GPS Performance (copied from Bak, 1999).

All errors can create a substantial amount of uncertainty in determining an accurate solution. These error source values may seem small in magnitude, but the

resulting position errors may be an order of magnitude greater (Parkinson, 1994). In other words, a little error in space can create a lot of error on earth.

Dilution of Precision

Dana (1999) explains that Dilution of Precision (DOP) depends only on the positions of the GPS satellites relative to the GPS receiver's location. Without even using the GPS system, we can calculate the satellite positions in advance and determine the quality of the user's position in advance. The user finds the satellite geometry by determining how high the satellites are in the sky, the orientation towards the satellites, and how many satellites the receiver can see. Since the satellites move, the geometry varies with time. Good satellite geometry results in low (or good) DOP. Figure 5 demonstrates this concept.

Good DOP
(Receiver selects well-oriented satellites.)

Poor DOP
(Receiver selects bunched-up satellites.)
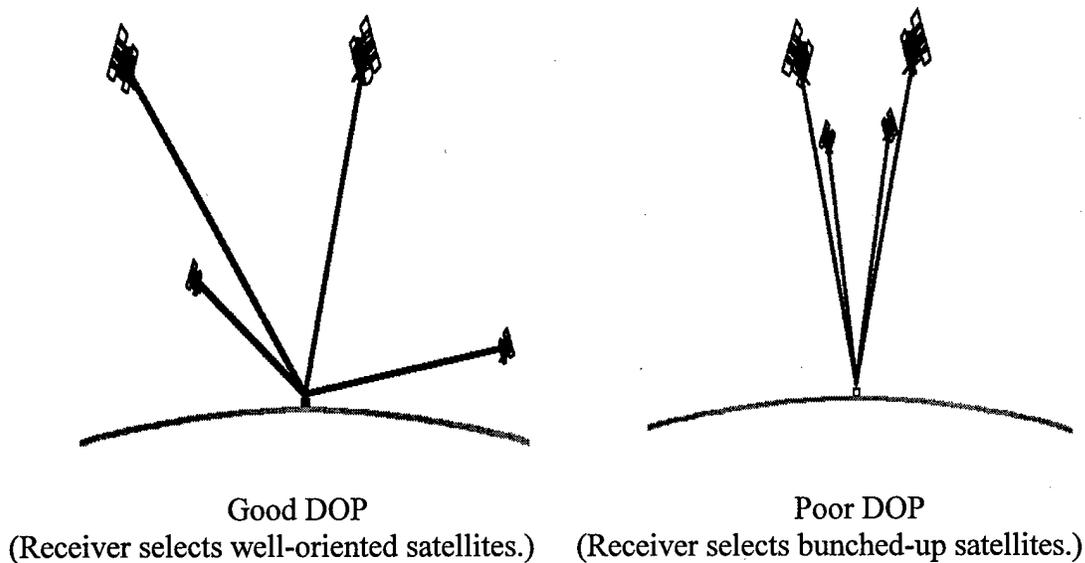
Figure 5: Examples of DOP (copied from Dana, 1999).

Dana (1999) further discusses that we divide DOP up into several components. We use these distinct components because the accuracy of the GPS system varies. For example, GPS provides a better measure for horizontal positioning than for vertical positioning. The input errors are the same, but the geometry may favor one direction over another. GPS analysts define VDOP as vertical DOP (altitude in the Up direction), HDOP as horizontal DOP (latitude in the East direction and longitude in the North direction), and TDOP as time DOP. They also use PDOP (Position Dilution of Precision) for three-dimensional position. GDOP stands for geometric DOP, which is the culmination of all the previously mentioned "DOPs".

Current Error Prediction Models

Ever since space-based navigational systems became operational, receivers have incurred position errors in their solution. In order to optimize GPS performance, modelers would like to accurately predict the magnitudes of these errors.

Current error prediction models estimate the magnitude of solution error that the user should observe at a given place and time in the future. While these models provide sensible predictions, modelers can still achieve better error prediction. Improved estimation procedures or algorithms may allow for better predictions. For example, modelers may be able to obtain a more precise weather prediction from a better understanding of the troposphere's condition and its effects on GPS performance for a particular place and time of day. If modelers can improve weather predictions, they can improve the ability to predict the position errors more accurately.

Currently, the Space Warfare Center (SWC) uses the Operational Model to Exploit GPS Accuracy (OMEGA) and Space Information Distributed Architecture (SPIDAR) models for predicting error accuracy. While these models show some advances in error prediction accuracy, they have to achieve a better prediction algorithm in order to better assess errors. A better prediction algorithm is necessary because if the error predictions are more accurate, then the military can perform missions that use GPS with a higher level of confidence than before (Brottlund and Harris, 1997).

Predicting GPS accuracy is an important concern for mission planners. The accuracy of the GPS system directly affects the effectiveness of military systems. Air Operation Centers, in producing Air Tasking Orders for combat missions, previously used OMEGA to predict GPS position accuracy. OMEGA estimates how good of a GPS solution can be obtained for predicting errors over the next several days for a given point and time (Lucia and Storz, 1997).

Lucia and Storz (1997) point out that in order to simulate a generic receiver's algorithm, OMEGA selects four satellites in order to generate a solution. The first satellite that OMEGA chooses is the one located most overhead of the user's position. OMEGA then selects the other three satellites that produce the best Position Dilution of Precision (PDOP). Based on this PDOP, OMEGA generates an estimated error.

That is to say if OMEGA predicts a poor PDOP, then the PDOP is probably poor. On the other hand, if OMEGA predicts a good PDOP, then the actual PDOP may or may not be good. Because OMEGA does not accurately predict when the satellite geometry is good, OMEGA is inadequate for meaningful mission planning (Lucia and Storz, 1997).

The latest error prediction model, SPIDAR, was created to account for some of OMEGA's shortcomings. The two models perform similar operations in predicting satellites used by the actual receiver and output the same types of measures (such as PDOP and error probables). SPIDAR takes the process a step further by modeling the ephemeris and satellite clock error sources (Beers, 1999). SPIDAR accomplishes this by using an exponentially weighted algorithm to take into consideration the satellite error growth rate and time since the last upload from the control stations. SPIDAR factors in the past errors of the satellite and models a generic receiver satellite selection algorithm. It predicts when the satellite uploads will occur and informs the user of how much error to expect at a given place and time (Beers, 1999). The intent of SPIDAR was to improve the capability to predict the satellite clock and ephemeris errors by modeling each individual satellite's estimated range deviation (ERD) value in calculating the spherical error probable/circular error probable (SEP/CEP) values. The SEP/CEP is the smallest radius of a sphere/circle that captures 50% of the error distribution when centered at the correct error-free location (Kaplan, 1996). For example, if a navigation solution has a CEP of 15 meters and we receive 10 readings to determine the actual position, then 5 of those reading should be within or on the 15-meter radius of the circle and the other 5 readings should be outside this radius. Figure 6 demonstrates this example of CEP.

With this background information on GPS together with an understanding of the current prediction models, we are prepared to investigate the error sources that impact GPS solutions and the potential to predict them. The next chapter will begin this process explaining the major error sources in detail and evaluating whether or not these error sources are worth modeling in future prediction models.

14
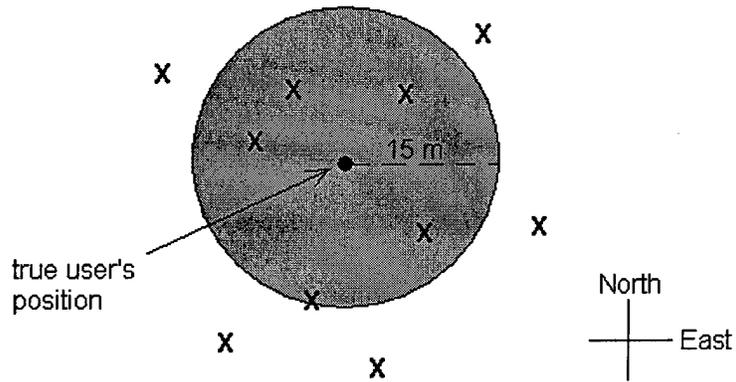
Figure 6: Example of a Navigation Solution with an Actual CEP of 15 Meters and 10 Measurement Readings Computed.

III. Error Sources

This chapter provides a thorough discussion of the major GPS error sources. This discussion explains each source's characteristics and modeling capabilities in depth. To distinguish each source's potential for inclusion in future error prediction models, the sources are categorized based on each source's modeling capability.

Categorization of Error Sources

A categorization of the major GPS error sources distinguishes the sources by their attributes. In particular, several GPS texts commonly classify these sources in three distinct categories: signal-in-space errors, propagation errors, and receiver errors. For Storz's (1999) study concerning covariance matrices, he distinguished the GPS error sources into four categories:

1. Satellite ephemeris and clock
2. Ionosphere
3. Troposphere
4. Receiver and multipath.

In order to support the purpose of this research, we distinguish the error sources into categories that explain the sources' modeling capabilities. Since our objective is to decide which error sources deserve prompt consideration in error prediction models, we distinguish the major GPS error sources using the following categories:

1. Error sources not affecting military receivers
2. Error sources currently modeled
3. Error sources possessing modeling potential.

The errors classified in this fashion are displayed in Table 2. Since the first category of error sources does not affect military receivers, only a brief discussion about why this is so is required. For the second category, this research recommends modeling

16

improvements for the currently modeled errors sources. The third category suggests error sources that have not yet been modeled, but possess good potential for model implementation. The next three sections explore these three categories of error sources.

Table 2: Categorization of the Major Error Sources.

Category	Error sources
No Affect	Selective Availability, Ionosphere
Currently Modeled	Satellite Clock, Ephemeris
Model Potential	Troposphere, Multipath, Receiver

Error Sources Not Affecting Military Receivers

This section addresses the errors classified in the first category. These errors are negligible for military receivers. It is important to note, however, that these error sources still disrupt GPS activity for unauthorized users and so are included here for completeness.

<u>Selective Availability</u>

Selective Availability (SA) is the deliberate distortion of the civilian GPS signal in order to avoid hostile exploitation of the United States and its allies. The Department of Defense (DoD) implemented SA in order that the United States and its allies could preserve a prediction accuracy advantage over unauthorized users. By design, SA is the dominant error source for unauthorized users (Lehmkuhl, 1999).

SA produces intentional noise added to the GPS signal that leaves the satellite. What makes SA so difficult to model for unauthorized users is that SA is uncorrelated between satellites. This lack of correlation results in limited position accuracy. Figure 7

exhibits the difference in horizontal position accuracies between stationary receivers where SA was turned on and those where SA was turned off in the satellites. In both cases, the receivers computed their solutions from the same location.
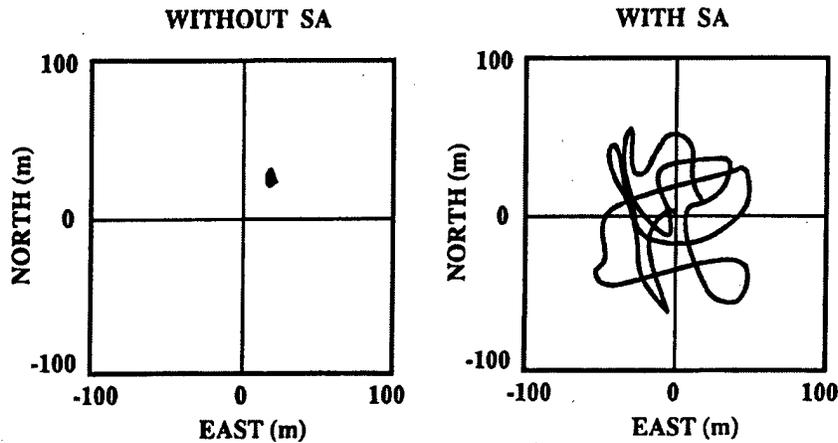
Figure 7: Horizontal Position Errors with SA and without SA for Data Collected during a 1-Hour Period for a Stationary Receiver (copied from van Graas, 1994).

When the first satellites were launched, the military did not immediately implement the SA feature. When early testing of the C/A-code revealed accuracies that were much better than projected (as good as those tested using P-code), the DoD decided to intentionally corrupt the accuracy available to unauthorized users. The DoD originally set the SA level at 500 meters and reduced it to 100 meters in 1983. When GPS became fully operational at the beginning of 1990, the DoD also officially implemented SA into GPS. SA levels have typically been less than 100 meters for most of the 1990s (van Graas, 1994).

In an effort to modernize GPS, the President of the United States directed the end of SA early in the year 2000 in order to encourage civilian confidence in GPS. Since SA always remains an option for the DoD, SA is not currently applicable to any users. Since

SA does not affect military receivers whether SA is turned on or off, it does not require

modeling in future prediction models.

Ionosphere

When analyzing GPS, researchers typically refer to the ionosphere as the

atmospheric region occupied by freely-charged space particles. While the exact range of

this region fluctuates constantly, it is generally located 50 kilometers to more than 1000

kilometers above the earth's surface (Klobuchar, 1993). Figure 8 shows that the

ionosphere is located beyond the troposphere in the earth's atmosphere.
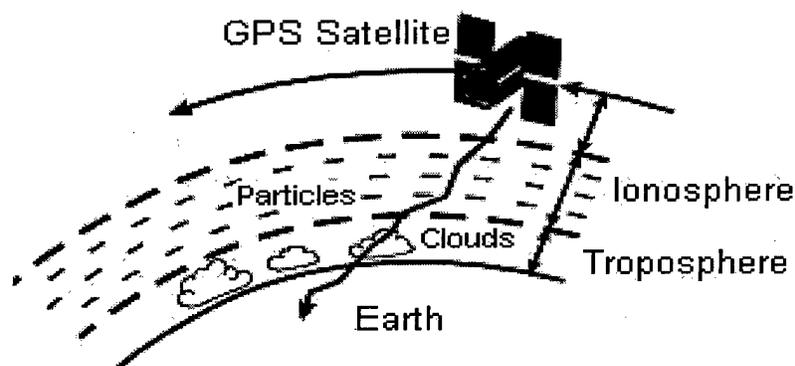
Figure 8: Composition of Atmosphere Used for GPS Signal Delay Analysis (copied from Trimble, 1999).

The free electrons in the ionosphere frequently contribute significant errors that

lead to inaccuracies in a user's position. Ideally, a GPS signal travels at the vacuum

speed of light from the satellite to the receiver. However, because these charged

electrons distort the GPS signal, the signal is delayed while traveling from the satellites to

the receiver. The resulting signal delay is proportional to the total electron content (TEC)

(or the total number of free electrons) in the ionosphere. The ionosphere's behavior also varies with the user's latitude position. The ionosphere is stable and predictable in the temperate zones, but becomes increasing unsteady and less predictable as the user draws closer toward the equator or either of the magnetic poles (Klobuchar, 1993).

Fortunately, military users automatically account for the ionosphere delay effects. The P-code receivers possess dual frequencies (L1 and L2) that measure the GPS signal's arrival time. By comparing the arrival times of the two different carrier frequencies of the GPS signal, the user solves for the ionospheric effects using algebra. Once the user knows the amount of ionosphere delay, it is a simple matter to correct this error. The effective ranging accuracy for dual-frequency P-code users is typically well below 1 meter of range error. Therefore, errors caused by the ionosphere have a negligible affect on military users.

Error Sources Currently Modeled

As previously mentioned, error prediction models are used to assess the amount of solution error for a given place and time in the future. The prediction model currently used by the Space Warfare Center is OMEGA and the model currently in development is SPIDAR. These prediction models attempt to address two of the major error sources: satellite clock and ephemeris. These two signal-in-space error sources were modeled before the other error sources because of their significant impact on the GPS solution and their similar attributes. These error sources are discussed in detail next.

Satellite Clock

Satellites contain atomic clocks that control all onboard timing operations including broadcast signal generation. The ability to predict clock behavior depends on the quality of the satellite's atomic clock. Atomic clocks are highly stable, with accuracy to the nanosecond. While accuracy to the nanosecond may seem impressive, a millisecond of error in GPS time translates to a solution error of 300,000 meters. The nanosecond of accuracy results in about 3.5 meters per day if the satellites had not been uploaded within a 24-hour period.

Modelers can predict the satellite clock error most accurately immediately after an upload occurs. When the mission control station sends an upload to the satellites, the satellite clock errors are reset to zero. Standard deviations of this error grow quadratically with time since the last upload. The master control station determines and transmits predicted clock correction coefficients a_{fo}, a_{f1}, and a_{f2} to the satellites for rebroadcast in the navigation message to be uploaded. Kaplan (1996) states that the receiver uses the following second-order polynomial implements these predicted coefficients:

$$dt = a_{fo} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^2 + dt_r$$

dt	=	computed correction at time t (seconds)
a_{fo}	=	clock bias (seconds)
a_{f1}	=	clock drift (seconds per second)
a_{f2}	=	frequency drift (seconds per second squared)
t	=	current time epoch (seconds)
t_{oc}	=	clock data reference time (seconds)
dt_r	=	correction due to relativistic (or gravitational) effects (seconds).

Some residual error remains in the satellite clock since the parameters are "fitted" estimates of the actual satellite clock errors (Kaplan, 1996).

In order to address the error in the satellite clock, the ground control stations upload all the satellites at least once a day with updated clock information (to reset the satellite clocks to the correct time). The current prediction models estimate the time since the last upload and the rate at which the clocks are deviating from the actual time to account for the estimated error that results in the receiver. If uploads occur twice as frequently (about every 12 hours), then the maximum amount of error would be less than the maximum error at 24-hour uploads.

Current error prediction models explain the satellite clock error well. The current models address this source as well as can be expected at this time. Perhaps, the only possible improvement would be to actually update the predictions. Given that ground control stations upload approximately every 24-hour, modelers probably have the best prediction that they can attain for the satellite clock error for now.

Ephemeris

Ephemeris errors are the differences between the satellite's actual location and the location predicted in the satellite orbital data (Kalafas *et al.*, 1986). Satellites characteristically travel along long smooth arcs in space. Figure 9 shows the position components that are affected when the satellite's orbit is off its mark, in particular: the radial, tangential, and cross-track components. Of these, the radial error has the biggest effect on ranging accuracy (Kaplan, 1996).
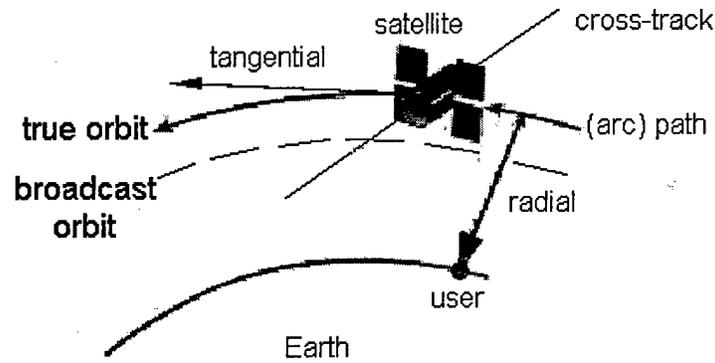
Figure 9: Components of Satellite Orbit (adapted from Kaplan, 1996).

The ephemeris error is most predictable immediately after the navigation data

upload takes place. These errors tend to grow slowly with time since the last upload

(Parkinson, 1994). The mission control station computes and uploads the optimal

estimates of the ephemerides to all of the satellites with other navigation data message

parameters for rebroadcast to the user. The control segment generates the broadcast

ephemeris in real-time using data from the five GPS monitor stations around the world.

This computed broadcast ephemeris typically has 3 meters of accuracy. Hundreds of

reference stations worldwide generate the precise orbits using several days of data; the

reference stations calculate these precise orbits with an average accuracy of 6

centimeters. This data, which can be obtained from the National Geodetic Survey, serves

as useful truth reference for broadcast ephemeris errors (Raquet, 1999).

The ephemeris error generally ranges from 2 to 15 meters. Figure 10 supports

this error range for satellites #11, #18, #19, and #28 for data collected in April of 1993.

Satellite #31 experienced error outside this error range because Selective Availability was

turned on for that particular satellite (Lachapelle, 1997).
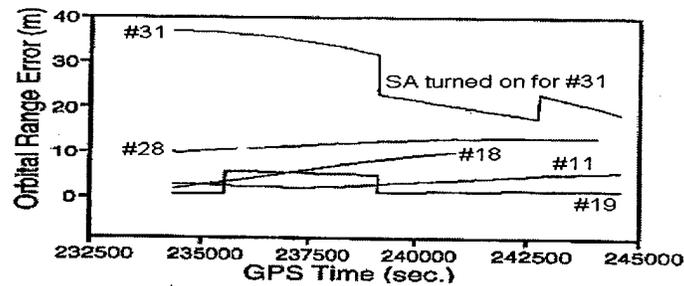
Figure 10: 3.5-Hour Test Performed in April of 1993 that Compares Orbital Range Error Versus GPS Time (copied from Lachapelle, 1997).

Improving the Signal-in-Space Error Models

The satellite clock and ephemeris errors are currently modeled because both of these error sources are subject to uploads daily. Both the satellite clock and location drift in the time that transpires between uploads (up to 24 hours). If modelers better estimate how far off these drifts are, then they can implement this estimation in a future prediction model. Current prediction models account for both of these signal-in-space error sources. At this time, these error sources appear to be modeled well, but there may be some improvements necessary after the receiver algorithm has been modeled more accurately, as we will discuss in the next chapter.

Error Sources Possessing Modeling Potential

The error sources addressed to this point either generate little to no effect on military receivers or are modeled in existing prediction algorithms. The remaining sources of error arise from the receiver, multipath, and the troposphere. We explain each of these sources' causes and modeling capabilities in detail.

Receiver

Most receiver algorithms initially compute similar GPS solutions. The major distinction transpires when one of the four initially selected satellites "sets" or falls out of the receiver's view. How are new solutions computed? Different receiver algorithms handle recalculation in different ways. The number of tracking channels a receiver possesses often characterizes different receiver algorithms.

Up to thirteen satellites can be in a receiver's view at any given time from which to calculate a user's position. A receiver frequently views five to ten satellites at any given point on the earth. From these satellites in view, the receiver selects four satellites from which to compute a solution. The selection of these satellites depends on the algorithm the receiver uses for satellite selection. For the common military receivers in current use, the first satellite that the receiver selects is usually the one most overhead and the next three satellites chosen are the ones that combine to generate the best (or lowest) PDOP. When a receiver initially fixes on (or selects) four satellites to calculate a GPS solution, the error magnitudes for most receivers are approximate in value. As time increases, the amount of receiver error increases as well. We cannot assume that these error increases are equal among all receivers. The increase in error depends on a receiver's design, quality, algorithm, and number of tracking channels. Table 3 shows several different receivers used by today's military.

The number of tracking channels in a receiver determines how many satellites that a receiver can receive signals from concurrently. When the receiver is stationary, the number of channels in a receiver is not a major issue in determining position accuracy.

The greatest impact in solution error results after the initial calculation because the different algorithms recalculate solutions differently.

Table 3: Today's Military Receivers, Number of Tracking Channels They Have and Their Primary Application (copied from JSSMO, 2000, TRADOC, 2000 and Trimble, 2000).

Receiver	# Tracking Channels	Application
Rockwell-Collins PLGR	5	Ground
Rockwell-Collins MAGR	5	Air
Receiver 3A	5	Air
Receiver 3S	5	Water
SAGR	6	Ground
Trimble CUGR	6	Air
Trimble TASMAN ARINC 12	12	Air
Trimble Force 19 module	12	Ground, Air, and Water
Trimble Force 5 GRAM-S GPS module	12	Air
Trimble Force 18 module	12	Air

When a satellite "sets", the satellite goes below the earth's horizon and is consequently out of the receiver's view. When one of the first four initially selected satellites "sets" (or no longer produces an optimal GPS solution), different receiver algorithms handle recalculating a new optimal solution differently at this point. High-dynamic military receivers are often continuous or all-in-view (AIV). These distinct algorithms depend on the number of tracking channels the receiver possesses.

Continuous receivers possess at least four channels in order that a receiver simultaneously tracks four satellites. The most common continuous receivers are 5-channel receivers. Four channels track four different satellites for three-dimensional position solutions. The fifth channel reads the navigation message of the next satellite in

26

the selected constellation and performs dual-frequency measurements to account for the ionospheric delay.

When the full constellation of GPS satellites is in orbit, most users have at least six satellites in view at all times. Most receivers are programmed to select the four satellites that offer the best satellite geometry (lowest PDOP) to provide the best three--dimensional position. All-in-view receivers possess at least twelve channels to simultaneously monitor all the GPS satellites in the receiver's view and to quickly acquire satellites that move into view while the satellites in view are in use. Typically, the user determines solutions using data from all the satellites in view and the software in the receiver filters results to display the most accurate solution to the user. An advantage of all-in-view receivers is that operators would not notice a change in performance even if dense trees, nearby steep hills, buildings, or other obstacles temporarily blocked signals from one of two satellites. Figure 11 demonstrates a receiver attempting to select satellites with good satellite geometry, but the receiver has some signals blocked due to obstructions in the environment, which results in poor satellite visibility. In the past, all-in-view receivers have been expensive, however, continued development and integration of digital-signal processing components make them more affordable (TRADOC, 2000).
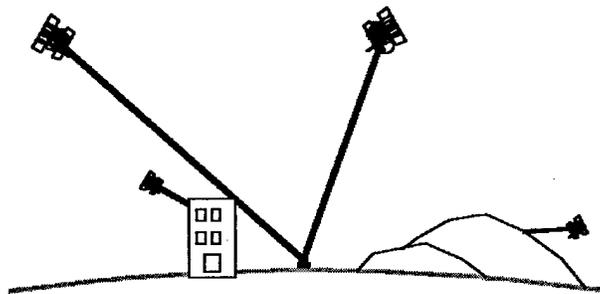
Figure 11: Receiver with Poor Satellite Visibility (copied from Dana, 1999).

We recognize that many different algorithms are in use by today's military receivers. A 5-channel receiver and a 12-channel receiver compute similar error magnitudes initially, but as time increases, the 12-channel receiver does not increase as much in solution error as the 5-channel receiver increases. A prediction model algorithm estimates the solution error most accurately when the modeled algorithm is receiver specific, not generic. The more channels a receiver has, the more accurate its solution is. If modelers correctly simulate several different receiver algorithms by the number of channels that different receivers possess, then the error prediction should be more accurate than what the modeled generic receiver algorithm predicts.

Both OMEGA and SPIDAR model the standalone GPS receiver in a generic sense. The satellite selection algorithms in these error prediction models are generic in that they do not model any several different receiver algorithms. Generic algorithms minimize PDOP when selecting the satellites, but not all receivers perform this same algorithm to compute a GPS solution. The advantages of current models are that they serve as excellent foundations for modeling all receivers and they accurately predict when a solution is poor. The limitations are that these current models are not receiver specific and do not accurately predict when a solution is good (Beers, 1999).

Different receiver algorithms frequently select different satellites and compute different solutions for the same position and time. Solutions are often the same for static receivers; the solutions vary distinctly for kinetic receivers with time. The solution accuracy depends on the number of tracking channels a receiver possesses.

Even though solution errors vary among different receiver types, most receivers incur similar sources of error within their units, particularly errors in the receiver's clock

and noise. Fortunately for the user, the errors do not need to be predicted due to their ability to be corrected within the receiver. In modeling the different receiver algorithms, the significant gain is in accurately predicting the overall solution error, which is why modelers attempt to accurately predict the other error sources in the first place. In using several different algorithms to imitate different receivers, modelers will probably correctly predict all four of the satellites that the receiver selects to compute its solution more often. When these four satellites are correctly predicted, modelers can expect a more accurate prediction of the total solution error and better assess the effectiveness of the previously modeled error sources.

Multipath

Many effects influence multipath, particularly the user's environment. Anything in the environment can cause deflections: buildings, mountains, flat surfaces, water, planes, etc. No satisfactory models have prevailed from the many years of research in this area. Multipath is simply the corruption of the direct GPS signal by one or more signals reflected from the local surroundings that enter the front end of the receiver's antenna.

The line of sight (LOS) signal is the direct signal from the satellite to the receiver. Reflected (or deflected) signals are indirect signals that reflect (or bounce) off of different surfaces in the environment. Signal deflections can bounce off almost anything in any environment. These effects tend to be more evident in a static receiver near large reflecting surfaces. Figure 12 displays an example of how multipath occurs. In this

example, the ground deflects a signal from the satellite to the receiver. At times, the deflected signal may be stronger than the direct signal (Braasch, 1995).
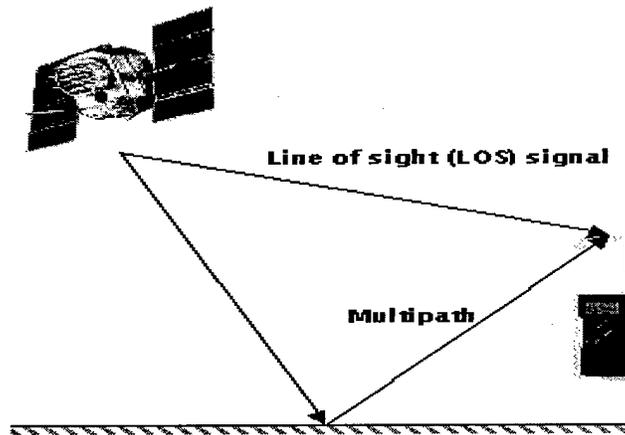
Figure 12: Demonstration of the Ground Deflecting a Signal in GPS Causing Multipath to Occur (copied from Bak, 1996).

With proper siting and antenna selection, the net impact to a moving user should be less than 1 meter under most circumstances. In extreme cases, the maximum ranging error is 1.5 meter for military receivers (Braasch, 1995).

Multipath is very difficult to model accurately. Existing models make an attempt to estimate this error source, but are not effective enough at this point to implement in future prediction models. Researchers have not been able to assess multipath behavior effectively. Researchers have tried to account for multipath, but the issue is so complex that no models have been generated so far. Researchers sometimes offer suggestions on where the receiver's antenna should be placed on top of a system so as to minimize the probability of incurring a multipath problem.

The military frequently uses high-dynamic systems such as aircraft and guided missiles. Multipath in aircraft is often limited to just the aircraft itself. In other words,

the aircraft is the only terrain deserving consideration for modeling multipath; the terrain below an aircraft does not need consideration because it is not in the signal's path. Aircraft are mostly made from aluminum and other highly conductive materials. Jet shapes are often built in the same manner, in that most jets have a nose, wings, and a tail fin positioned in the same areas. Multipath modeling may be possible in aircraft because of the fixed features (such as conductivity and shape) of most aircraft. Perhaps, also modeling environments that change very little with receiver movement is possible. For instance, modelers may be able to model multipath effects of oceans or flatlands.

Multipath is a very complex issue and probably not the best error source to start modeling right away. Modelers will probably best predict multipath in portions, such as first modeling multipath in aircraft and then incorporate some digital terrain information for lower-dynamic systems such as submarines and tanks. Although modeling multipath is complex, it seems attainable. In my opinion, multipath will probably be moderately modeled within the next 20 years. If modelers accomplish an effective model, then they can explain up to approximately 20% of the total GPS error.

Troposphere

The troposphere is a region of the atmosphere where a moderate amount of GPS error originates. From the extensive research conducted in this area, researchers have constructed many credible models. This error source appears promising for model implementation in future error prediction models.

The troposphere is the atmospheric region that delays GPS signals due to weather effects. It is located between the earth's surface and the ionosphere, approximately 0 to

50 kilometers above the earth's surface. Figure 13 displays the location of the
troposphere in the earth's atmosphere. The "actual" troposphere is located 0 to 10
kilometers above the earth's surface and contributes about 75% of the total tropospheric
error in GPS. This is where weather affects the speed of light radio waves via
temperature, pressure, and humidity. The tropopause is located 10 to 16 kilometers
above the earth's surface, and the stratosphere is located 16 to 50 kilometers above the
earth's surface. Together, the tropopause's and the stratosphere's atmospheric gases
contribute about 25% of the total tropospheric delay in GPS. The combined weather
conditions frequently contribute significant delays in the GPS signal that lead to
inaccuracies in a user's position. Although these effects are huge at times, the
troposphere is generally stable and predictable (Raquet, 1999).
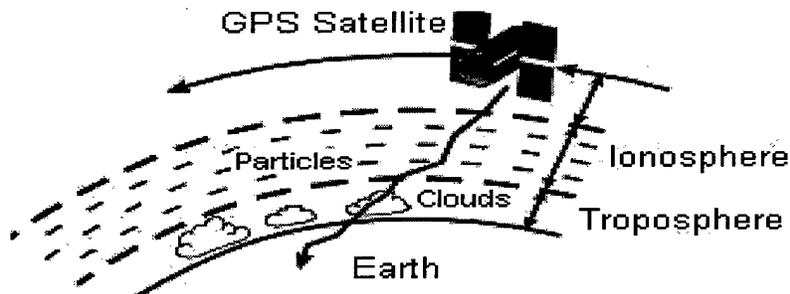
Figure 13: Composition of Atmosphere Used for GPS Delay Analysis (copied from Trimble, 1999).

Although modelers should consider many effects when properly modeling the
tropospheric error, Kaplan (1996) simply expresses the troposphere error (measured in
meters) as

$$\Delta S = \int_{satellite}^{user}(n-1)\partial s = 10^{-6}\int_{satellite}^{user}N\partial s$$

n = refractive index = c_v/c_m

c_v = vacuum speed of light = $3*10^{-8}$ meters per second

c_m = speed of GPS signal though air (meters per second)

N = refractivity = 10^{-6}(n-1)

ΔS = tropospheric error (meters)

∂s = change in signal's path length due to tropospheric effects (meters).

This simple model is accurate in determining signal delay through the troposphere, but the refractivity is difficult to estimate. Many researchers established their own techniques to computing refractivities, error corrections, and consequently signal delay. These computations are based on such parameters as pressures, temperatures, speeds, empirical constants, heights, radii, path distances, elevation angles, and other variables and constants (Raquet, 1999). These variations of the simple tropospheric model contribute additional accuracy in predicting the behavior of the troposphere. Some of the more popular models that evolved from the simple model are the Saastamoinen total delay, the Hopfield two quartic, the Black and Eisner, and the Marini and Murray models (Spilker, 1994).

Tropospheric models consist of dry and wet components. The dry component is usually located from 0 to 40 kilometers above the earth's surface. The dry term produces 80% to 90% of the total tropospheric error, yet we can predict it very accurately, (predictable up to 1% accuracy at the zenith). The wet term arises from water vapor in the atmosphere and produces 10% to 20% of the total tropospheric error. The wet component is more difficult to predict than the dry term due to uncertainties in the atmospheric distribution. The wet term error can be predicted to within 10% to 20% of

the actual wet term error and is located from 0 to 10 kilometers above Earth's surface, in the "actual" troposphere. Most tropospheric models are accurate at elevation angles greater than 15 degrees and inaccurate at elevation angles less than 15 degrees (Raquet, 1999).

Tropospheric effects vary mainly with satellite elevation angle (in degrees) and the temperature (in degrees Celsius) of where the receiver is located, as we see in Figure 14. If left uncompensated, the range error for a satellite at the zenith can be as low as 0.01 meters. Under extreme circumstances, the range delay for a satellite at a 5-degree elevation angle can equal approximately 33 meters (where 25 meters from the dry term + 8 meters from the wet term at 40 degrees Celsius = 33 meters of total tropospheric error). Figure 14 further suggests that the tropospheric error in GPS may behave exponentially. The typical error incurred is 0.7 meters. For most users and circumstances, a simple tropospheric model should be effectively accurate to 1 meter or better (Spilker, 1994).
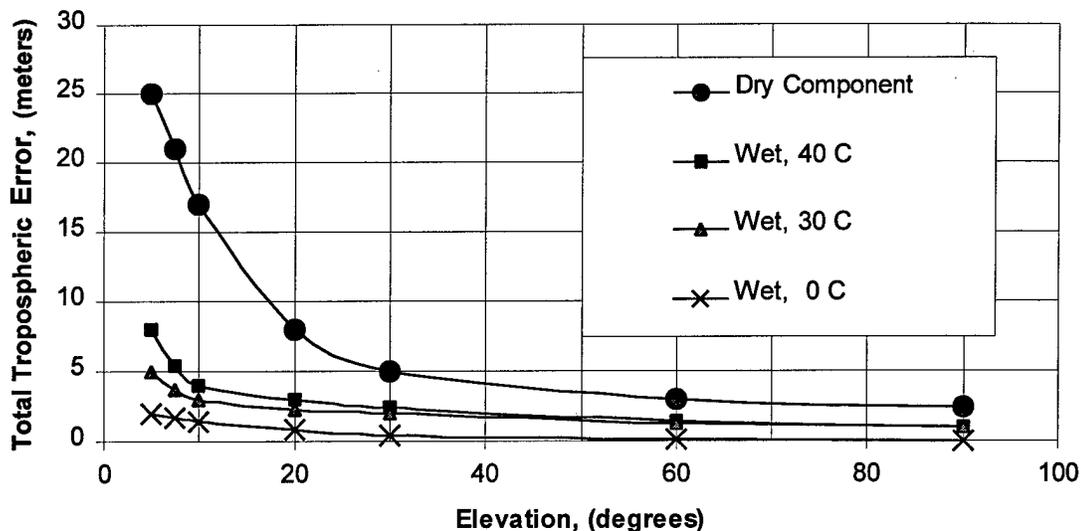
Figure 14: Typical Dry and Wet Tropospheric Errors (copied from Raquet, 1999).

Spilker (1994) compared many of the tropospheric delay models in practical use today to each other in order to determine which models were most accurate. For the zenith delay (or the delay directly overhead), the predictability of the dry component was within several millimeters of ray trace delay for the Saastamoinen model and within several millimeters of ray trace delay for the Hopfield model. For the wet component at the zenith, the predictability was within 30 millimeters for the Saastamoinen model and within 20 millimeters for the Hopfield model. At the zenith, both of these models are very accurate and comparable to each other when considering only the dry term, but the Hopfield model is more accurate for wet-term calculations. Generally, the Hopfield model calculated more accurate results at the zenith. At an elevation angle of 5 degrees, the predictability of the dry component was within 6 millimeters accuracy for the Saastamoinen model and within 5 centimeters accuracy for the Hopfield model. The Saastamoinen model is accurate for the dry term at low elevation angles (Spilker, 1994).

Combining tropospheric models and mapping functions frequently attains even more error accuracy. A mapping function is a factor that depends on satellite elevation angle that provides additional accuracy to the predicted tropospheric error. Mapping functions are specifically useful at low satellite elevation angles (20 degrees or less). Mapping functions are used to relate troposphere error at a particular elevation with tropospheric error at the zenith. Raquet (1999) demonstrates the use of the mapping function along with the tropospheric error at the zenith in the following equation to determine the actual tropospheric error at the satellite elevation angle E:

$$\Delta S = F_T(E) \times \Delta S_{zenith}$$

$F_T(E)$ = mapping function at satellite elevation angle

ΔS = tropospheric error (meters)

ΔS_{zenith} = tropospheric error at the zenith (meters)

E = satellite elevation angle (degrees).

The following equation is the simplified mapping function:

$$F_T(E) = 1 / \sin (E).$$

Some variations of this simplified mapping function that are in practical use today are the Chao, Davis, Black and Eisner, and Saastamoinen mapping functions (Raquet, 1999). These mapping functions are not necessary for most models, but they frequently provide additional accuracy to most models.

Spilker (1994) compared the tropospheric delay mapping functions to each other in order to determine which functions were most accurate. At an elevation angle of 20 degrees, the predictability was within 8 millimeters for the Saastamoinen mapping function and 8 millimeters for the Black and Eisner mapping function. Both of these mapping functions are very accurate and comparable to each other at an elevation angle of 20 degrees. At an elevation angle of 10 degrees, the predictability was within 50 millimeters for the Saastamoinen mapping function and 50 millimeters for the Black and Eisner mapping function. Both of these mapping functions are very accurate and comparable to each other at an elevation angle of 10 degrees. At an elevation angle of 5 degrees for only the dry term, the predictability was within 1.2 meters for the Saastamoinen mapping function, 10 centimeters for the Black and Eisner mapping function, and only 6 centimeters for the Davis mapping function. The Davis mapping

function appeared to be the most accurate mapping function for the dry term at this angle (Spilker, 1994).

Meteorologists provide accurate weather forecasts up to several days in the future. Forecasted temperatures and pressures are frequently precise. Modelers can use these forecasted values to compute refractivities. Once they compute the refractivities, they only need to know the path of the signal in order to compute the tropospheric delay. From this sample plan, we can see that accurately modeling the troposphere in an error prediction model seems feasible.

A recommended approach to for modeling the troposphere would be to first design an algorithm of a simplified tropospheric model, which we discussed previously in this section. We then recommend modeling the dry term of the troposphere before proceeding to model the wet term. Properly modeling the dry term should explain at least 80% of the total troposphere error. Once a simple dry model is operational, modelers could manipulate the simple model easily into a more accurate dry Saastamoinen model. Once modelers have this more efficient dry model running correctly, then they should include the simplified mapping function. This simple mapping function would add another degree of accuracy to the dry model. Once the simple mapping function is operational, modelers could manipulate the simple mapping function into a more precise Saastamoinen mapping function. If the dry term alone is modeled properly, then the tropospheric model should account for at least 7.5% of the overall GPS error. If the dry term of the tropospheric error is completely modeled and more than 80% tropospheric error accuracy is desired, then modelers can proceed to model the wet term in a similar manner. The wet term is more difficult to accurately model than the dry term, but

37

fortunately the overall contribution of the wet term error is not as large as the dry term. Remember, the wet term only makes up at most 20% of the overall tropospheric error. These resulting models and mapping functions should optimize the prediction accuracy for the tropospheric error. If an overall accurate tropospheric model (with both dry and wet terms) is operational, then the algorithm should account for over 10% of the total GPS error.

Having looked at all these sources of error, the next step is to evaluate the error sources. This assessment determines the resulting benefits and efforts required to properly model each error source.

IV. Analysis and Results

In this chapter, we attempt to analyze the modeling efficiency of the signal-in-space error sources in existing models and we also determine an inclusion order for the remaining error sources in future prediction models.

Previously we categorized each of the major error sources as either a source that does not affect military (PPS) receivers, a source that has already been predicted in the latest error prediction model (SPIDAR), or a source that possesses good potential for model consideration. The Selective Availability (SA) and ionospheric error sources do not warrant further prediction consideration at this time since neither of these error sources affects military receivers.

As previously stated, both OMEGA and SPIDAR model a generic receiver's algorithm. Although SPIDAR only correctly predicts two of the four satellites used by the receiver to compute its solution most of the time, solution error magnitudes are often similar in value for all of the satellites within the receiver's view, with only some regard to which four satellites the receiver selects.

OMEGA partially models the ephemeris error whereas SPIDAR predicts both the ephemeris and satellite clock errors. If SPIDAR completely models the signal-in-space error sources, then nearly 60% of the overall GPS error could be explained, but this is not the case. From the data file containing OMEGA data, SPIDAR data, and truth data, we computed that OMEGA explains approximately 24% of the total GPS error and that SPIDAR accounts for about 60% to 70% of the total GPS error. These percentages were estimated using the average predicted and actual 3-dimensional radial errors. To determine SPIDAR's accuracy, the deviations from a known location were measured.

These included errors in the north, east, and up directions. From these direction error

values, a 3-dimensional radial error was determined using

$$\text{3-dimensional radial error} = \sqrt{North^2 + East^2 + Up^2}$$

The following equation explains this calculation more clearly.

% of GPS error explained $= 100\% - \dfrac{|\text{ average 3D radial true error} - \text{average 3D radial predicted error }|}{\text{average 3D radial true error} + \text{average 3D radial predicted error}} \cdot 100\%$

For most cases in SPIDAR, the predicted error is slightly (and consistently) higher than

the actual error. This slightly higher estimation is probably due to the unpredicted

portion of the signal-in-space error sources.

Modelers presumed that simultaneously estimating the signal-in-space error

sources in SPIDAR was advantageous because the signal-in-space error sources are

highly related. An error growth rate (EGR) model (based on exponential smoothing)

successfully models most of the signal-in-space error sources by predicting navigational

upload times to the satellites from the mission control station and estimating the rate of

error growth between uploads. The EGR seems to accurately model the ephemeris error,

but modelers should remain suspicious of the satellite clock error. The mission control

station determines and transmits predicted clock correction coefficients a_{fo}, a_{f1}, and a_{f2} to

the satellites for rebroadcast in the navigation message. Kaplan (1996) explains that the

standard deviations of the satellite clock error tend to grow quadratically with time since

the last upload. SPIDAR does not appear to directly model this suggested "quadratic

model" to account for the satellite clock error. While the current approach to modeling

the satellite clock error is better than no model at all, some improvement may be gained

by also considering the quadratic growth model. The ability to further improve modeling

these error sources is low at this time, but by correctly predicting all four satellites that the actual receiver uses to compute a solution, increased error accuracy of these sources may be achieved.

Since it appears that very little work can be done to improve the signal-in-space error source models until the receiver algorithm is modeled more accurately, we need to look at the other error sources to consider implementing in future prediction models. Having discarded the error sources that have no impact, and considered enhancements to the error sources currently modeled, we are left with the task of deciding an order for implementing the remaining GPS error sources in future error prediction models. The remaining error sources are the receiver, multipath, and troposphere. All of these sources warrant further consideration in future error prediction models, so we need to determine a priority for model implementation.

In evaluating errors for possible inclusion in future models, we examine several criteria. In particular, we want to know the predictability and modeling capability of each error source. To accomplish this, we establish a benefit-to-cost ratio using the information presented earlier in the error sources' chapter. The error source that provides the greatest benefit-to-cost ratio warrants research precedence. Table 4 reveals our suggested order for modeling the remaining error sources as well as the anticipated benefits and degree of modeling difficulty.

The troposphere error deserves serious model consideration. Although the average error magnitude of the troposphere is not great, the range of values it can assume varies extensively. The tropospheric error ranges from 0.01 to 33 meters in error. Fortunately, several effective tropospheric models exist that researchers have adequately

tested, some of which are not too complicated to understand. As noted previously, the simple tropospheric model should effectively model tropospheric error for most applications and circumstances. Modelers can predict the dry term, which explains 80% of this error source, with great accuracy. The wet term, which explains only 20% of this source, is more difficult to accurately model due to unstable weather conditions.

Table 4: Suggested Order of Inclusion: Largest-to-smallest Benefit-to-Cost Ratio.

Rank	Error Source	Benefit of Modeling	Difficulty of Modeling
1	Troposphere	1. Many accurate models exist, particularly for dry term 2. A simple model accurate to 1 meter	1. The wet term hard to precisely model, but some fair wet term models exist 2. Weather conditions greatly vary
2	Receiver	1. Should offer respectable contribution to prediction accuracy 2. Flexible for further improvement	1. Most algorithms compute solutions similarly 2. Variation of existing generic algorithm
3	Multipath	1. Substantial increase in position accuracy if modeled properly 2. Could better explain related error sources	1. Amount of deflection varies significantly 2. If even possible, this may probably take years to accurately model

The receiver also warrants significant consideration for model implementation. This error source has great potential, but is potentially complex. Even though many different algorithms are currently used by the military, many of these receiver algorithms operate similarly. A complete prediction model would require separate submodels of every military receiver in use; an incremental approach could be pursued, however.

The biggest difference between the several different receiver algorithms in use today seems to be the number of tracking channels a receiver takes advantage of. By simply identifying the number of channels a receiver has available (as opposed to specific receiver type), we can anticipate much about the algorithm and its expected error growth. For example, if modelers could input the number of tracking channels that a receiver

possesses in the next version of SPIDAR, then they could use the respective modeled algorithm for computing a more accurate error prediction. Once a basic "channel-algorithm" model is established, variations of the algorithm could be added for different subclasses, if desired. These enhancements might include the receiver brand, application, or other criteria. Proper modeling of the receiver has an added benefit in that the receiver and multipath errors are related. Therefore, if the number of channels that a receiver utilizes is known, the more predictable the multipath error should be. Finally, by properly modeling the many specific receiver algorithms, modelers should expect near perfect prediction of the overall solution error accuracy to increase since all four of the actual receiver's satellites should be correctly predicted more often.

Multipath seems to be the worst quantified error source in GPS. Multipath depends on the environment, siting, antenna selection, and receiver used. The environment alone offers a host of problems, particularly with deflection. Most surfaces deflect signals to an uncertain degree. Environments change with every move; therefore, the dynamic environment is especially difficult to model. While an accurate model of the multipath error source would significantly increase error prediction accuracy, it may take years to accurately model this difficult error source. Proper modeling of multipath may not even be feasible. The moderate error contribution and complex modeling requirements, put multipath low on the inclusion priority list.

V. Conclusion

The troposphere deserves serious consideration for implementation in future prediction models due to its ease of modeling and impact on the total GPS error. Following this addition, modelers should model several different receiver algorithms. Modelers cannot fully assess the modeling effectiveness of the signal-in-space error sources or modeling of any other error sources until several different receiver algorithms are accurately modeled. Modelers might be able to better predict the signal-in-space error sources, but the reward for additional modeling of these particular sources is unknown at this time because of the lack of data. Correctly predicting the four satellites in a GPS solution would be a big step to take in better assessing errors more effectively. Since different receivers select different satellites when recalculating solutions, we should notice improvement in error accuracy if several different receivers are modeled accurately. Once different receiver algorithms are suitably modeled, we recommend that modelers model the remaining error sources in the order suggested in this chapter.

Review

We listed the seven major error sources that distort GPS operation and classified these sources into three groups that distinguish each source's modeling capability and possible implementation in future error prediction models. If accounting for an error source results in better error prediction, then a prioritization of inclusion helps direct research efforts. The error source that deserves highest priority provides the most benefit with the least effort for improved error prediction accuracy.

Recommendations for Improving SPIDAR's Successors

SPIDAR appears to be an improvement to its predecessor OMEGA. The implementation of an error growth model has partially modeled the actual ephemeris's behavior (Lanning and McIntyre, 2000). Although this is an accomplishment in itself, modelers can still improve SPIDAR, particularly in estimating CEP. The SPIDAR predicted CEP is too conservative (too high).

We may be able to improve the signal-in-space error sources modeled in SPIDAR, but it is hard to tell at this time since the four satellites used to compute the navigation solution are predicted correctly only half the time. The generic modeling of the receiver algorithm makes it difficult to fully assess error source modeling. Except for the tropospheric error source, we should first deal with properly modeling different receiver algorithms before improving or implementing any of the error sources. Probably the most efficient way to improve the previously modeled sources may be to revise the satellite selection algorithms in prediction models in order that these models predict the correct four satellites used in the navigation solution more often. Once different algorithms are modeled, complete modeling of the signal-in-space error sources may prove to offer the most benefit since progress has already been made in this area. Although modelers can work on several error sources simultaneously in the prediction model, they may benefit most by completing work on the previously modeled sources (to the desired level of satisfaction) if this is not too difficult to further model. Once they effectively model the signal-in-space error sources, then implementing other error sources should further improve error prediction accuracy.

Recommendations for Error Source Priority

We examined the GPS error sources in order to correctly estimate the amount of solution error a receiver will incur. Fortunately, modelers may be able to predict all of the error sources to a certain extent. Table 5 shows our suggested order in which to pursue modeling each error source. A very important item to note is the currently unpredicted error values for the satellite clock, ephemeris, and receiver error sources. The remaining error magnitudes of these sources are based on our intuition and are loosely approximated. Based on the opinions of experts responsible for validating SPIDAR, SPIDAR seems to address the satellite clock and ephemeris error sources very well (Lanning and McIntyre, 2000). We interpret "modeled very well" to represent that around 90% of the satellite clock and ephemeris error sources were correctly modeled. If our assumptions are reasonable, then it would appear that only about 55% of the total GPS error is explained by modeling these two error sources. SPIDAR seems to account for 60% to 70% of the total GPS error. We assume that the generic receiver algorithm models the remaining percentage of the total GPS error. These percentages are strictly assumed.

Table 5: Typical Error Magnitudes and Overall Contribution to GPS Error in a Military User's Solution.

GPS Error Source Model Ranking	Error, (m)	Percentage of Total GPS Error, (%)	Unpredicted Error, (m)	Unpredicted Percentage of Total GPS Error, (%)	Effort to Model
1. Tropo. Dry	0.56	7.5%	0.56	~22%	Easy
2. Receiver	1.0	13.5%	~0.15	~6%	Easy to Med.
3. Tropo. Wet	0.14	2%	0.14	~5.5%	Easy to Med.
4. Sat. Clock	2.1	28.5%	~0.21	~6%	Medium
5. Ephemeris	2.1	28.5%	~0.21	~6%	Medium
6. Multipath	1.4	20%	1.4	~55%	Hard
7. Ionosphere	0.01	0.2%	0.01	~0.5%	N/A
8. Sel. Avail.	0	0%	0	0%	N/A
Total	7.31	100%	2.68	100%	N/A

46

Modelers should be able to easily and quickly implement a tropospheric model of the dry term. This improved prediction model would explain over 7% of the total GPS error. We recommend that they initially model the simple dry tropospheric model. Once established, variations of the simple dry model could be implemented to further improve error prediction accuracy. The dry model should deliver high error prediction accuracy, predictable up to 1% accuracy at the zenith. The dry troposphere's error prediction potential is great. Several highly efficient dry-term models are in common use by members of the GPS community. If modelers properly implement a tropospheric model such as the dry Saastamoinen model in a prediction model, then 80% of the total tropospheric error and nearly 22% of the currently unpredicted GPS error should be accounted for.

Once the dry tropospheric model is accurately modeled, the generic receiver algorithm in SPIDAR should be improved upon. Different receiver algorithms often initially compute similar solutions. As time progresses, these different algorithms recalculate different solutions to the same set of satellites in view. Since SPIDAR bases error prediction models on a generic receiver's algorithm, we cannot perform an accurate assessment of how well most receivers compute and recalculate solutions over time; this is why the receiver deserves attention in error modeling. Because the generic receiver algorithm supposedly represents most military receiver algorithms, SPIDAR offers limited prediction capability. At this time, SPIDAR correctly predicts only half of the four satellites used by the actual receiver. We recommend that modelers improve SPIDAR to correctly predict all four satellites used by the receiver more often. Modeling

other types of military receiver algorithms should correctly predict all four satellites used by the actual receiver more often. By accurately predicting the same four satellites that the receiver selects, modelers can almost guarantee instant improvement in error prediction accuracy. Once SPIDAR properly models other receiver algorithms, it should be easier to assess the effectiveness of the error sources currently modeled and to be modeled. Instituting other algorithms from common military receivers should explain the receiver's errors better and increase the accuracy of solution error estimates. Around 6% of the currently unpredicted GPS error should be accounted for by fully modeling the receiver.

Once modelers investigate and accurately model several different receiver algorithms, they should model the wet term of the troposphere. The troposphere's wet-term prediction is not as reliable as the dry term, but the wet term has fair models in current applications. The error prediction accuracy for the wet term ranges from 10% to 20%, so GPS users would gain moderate accuracy. This error source only explains 1.92% of the total GPS error and around 5.5% of the currently unpredicted GPS error could be explained by fully modeling the wet term of the troposphere.

The next option in error modeling is to reevaluate the ephemeris and satellite clock error sources currently modeled in SPIDAR. Modelers should decide if further modeling of these error sources would enhance error prediction accuracy. If they fully model these error sources, then nearly 60% of the total GPS error could be accounted for. The remaining unpredicted portion of these error sources contributes around 12% of the currently unpredicted GPS error.

Attempts by researchers to accurately model multipath have been unsuccessful thus far. Useful models have not emerged to accurately predict the behavior of the multipath error source. Modeling a stationary environment may be possible, but modeling a changing environment may be infeasible. The conductivity and shape of the environment change immensely every time the receiver moves. Modeling environments that appear to change very little with receiver movement may be possible. In general, accurately modeling multipath would be very challenging, but if modelers could successfully model multipath effects, then they could account for over 19% of the total GPS error and nearly 55% of the currently unpredicted GPS error should be accounted for.

We noted earlier that the Selective Availability error source and the ionospheric error source produce negligible error to the receiver's solution. Both of these error sources have no affect on military GPS receivers because military users utilize special dual-frequency, P-code receivers, which eliminate these errors completely. After forming models of all the other GPS error sources first, modelers should then consider modeling the major error sources in order that C/A-code (civilian) receivers (that are sometimes used by the military) could benefit from accurate error predictions as well. SA does not require error modeling at all.

Further Research

In this research, it was important to better understand the error sources that distort GPS activity in order to accurately predict error magnitudes. If nothing else, the user is well aware of performance inaccuracies. Of course, the amount of error that modelers

can accurately predict has a limit. One may want to test the exact accuracy of existing models under different applications, such as for the tropospheric error models.

Although error reduction is always a concern for the GPS community, the ideal next step in this particular research would be to implement one of the error sources in the next error prediction model and assess its modeling effectiveness. Testing the model for improved prediction accuracy would be a significant task.

Bibliography

Bak, Thomas. "GPS Tutorial Content,"
http://www.control.auc.dk/~tb/gps_view_graph/index.htm/: (September 1999).

Beers, Suzanne, "SPIDAR," Lecture. Directorate of Analysis and Engineering, Space
Warfare Center, Schriever AFB, Colorado: (November 1999).

Blackwell, Earl G. "Overview of Differential GPS Methods," Global Positioning
System, Volume III, Institute of Navigation, Washington, D.C.: 89-100 (January
1985).

Braasch, Michael. S. "Multipath Effects," Global Positioning System: Theory and
Applications Volume I: Volume 163, American Institute of Aeronautics and
Astronautics, Inc., Washington, D.C.: (1995).

Brottlund, Brian and Cliff Harris. "Global Positioning System Effectiveness Model
Improvement Feasibility Study," Directorate of Analysis and Engineering, Space
Warfare Center, Schriever AFB, Colorado, Technical Report: (June 1997).

Corvallis Micro Technology, Inc. "Introduction to the Global Positioning System for
GIS and TRANSVERSE," http://www.cmtinc.com/gpsbook/: (September 1999).

Dana, Peter H. "Global Positioning System Overview,"
http://www.utexas.edu/depts/grg/gcraft/notes/gps/gps.html: (September 1999).

Joint Service System Management Office (JSSMO). "Receivers,"
http://www.gps-ue.robins.af.mil/suprcvr.htm: (January 2000).

Kalafas, Rudolf M., Al J. Van Dierendonck, and Nevin A. Pealer. "Special Committee
104 Recommendations for Differential GPS Service," Global Positioning
System, Volume III, Institute of Navigation, Washington, D.C.: 101-116
(June 1986).

Kaplan, Elliot D. Understanding GPS Principles and Applications. Artech House Inc.,
Boston: (1996).

Klobuchar, J. A. "Ionospheric Effects on GPS," Global Positioning System: Theory and
Applications Volume I: Volume 163, American Institute of Aeronautics and
Astronautics, Inc., Washington, D.C.: (1993).

Kruczynski, Leonard R. "Global Positioning System," Microsoft Encarta 98
Encyclopedia. Microsoft Corporation: (1993-1997).

Lachapelle, G. "GPS Theory and Applications," Seminar Notes, Department of Geomatics Engineering, University of Calgary, Calgary, Canada: 4.0-6.60 (May 1997).

Lanning, Jeffrey W. and Gregory A. McIntyre, "SPIDAR Validation Report," Department of Operational Sciences, Air Force Institute of Technology, Wright-Patterson AFB, Ohio: (March 2000).

Lehmkuhl, Lee. "Signal from Space: The Next-Generation Global Positioning System," Directorate of Analysis and Engineering, Space Warfare Center, Schriever AFB, Colorado: (June 1999).

Lucia, David J. and Mark F. Storz, "Operational Model to Exploit GPS Accuracy (OMEGA) Validation Report," Directorate of Analysis and Engineering, Space Warfare Center, Schriever AFB, Colorado, Technical Report 97-01: (January 1997).

McGregor, Stephen. "Improving GPS Accuracy," http://www.cpc.unc.edu/services/spatial/gpsacc.htm: (September 1999).

Navstar GPS Joint Program Office. "GPS Interference and Discrepancy Reporting," http://gps.laafb.af.mil/interference/: (September 1999).

Parkinson, Bradford W. "GPS Error Analysis," Global Positioning System: Theory and Applications Volume I: Volume 163, American Institute of Aeronautics and Astronautics, Inc., Washington, D.C.: (1994).

Raquet, John. "EENG 533: Navigation Using the GPS Course Handouts, Spring 1999," Department of Electrical Engineering, Air Force Institute of Technology, Wright-Patterson AFB, Ohio: (Spring 1999).

Sennott, J.W. and D. Pietraszewski. "Experimental Measurement and Characterization of Ionospheric and Multipath Errors in Differential GPS," Global Positioning System, Volume IV, Institute of Navigation, Washington, D.C.: 19-32 (January 1987).

Spilker, J. J. "Tropospheric Effects on GPS," Global Positioning System: Theory and Applications Volume I: Volume 163, American Institute of Aeronautics and Astronautics, Inc., Washington, D.C.: (1994).

Storz, Mark. "General Expression for Parameter Covariance Matrix," Facsimile transmission to Raychelle Derbyshire, Space Warfare Center, Schriever AFB, Colorado: (March 8, 1999).

Training and Doctrine Command (TRADOC). "Section III: Position/Navigation Satellite Systems," http://www-tradoc.army.mil/dcscd/chap07c.htm: (January 2000).

Trimble Navigation Limited. "Trimble - All About GPS," http://www.trimble.com/gps/index.htm: (August 1999).

van Graas, Frank. "Selective Availability," Global Positioning System: Theory and Applications Volume I: Volume 163, American Institute of Aeronautics and Astronautics, Inc., Washington, D.C.: (1994).

Weimer, Norris. "DOP, the Non-Technical Description," http://joe.mehaffey.com/dopnontech.htm: (September 1999).