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TECHNICAL NOTE 2725

INTERACTION OF OBLIQUE SHOCK WAVES WITH REGIONS OF VARIABLE PRESSURE, ENTROPY, AND ENERGY

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SUMMARY

Equations are derived for computing the form of an oblique shock wave as it passes through supersonic regions in which static pressure, stagnation pressure, stagnation temperature, or combinations of these are continuously variable. Rigorous portions of the analysis are limited to shock strengths for which the flow downstream of the shock remains supersonic. When no downstream waves other than those generated by the interaction process are present, the rate of change of shock angle with upstream Mach number is found to be a function only of the local shock angle and upstream Mach number; hence, the propagation through a nonuniform region depends only on the initial shock strength and Mach number. A procedure is described for computing the supersonic portion of the flow field downstream of the shock wave.

For the special cases of supersonic shear flow and Prandtl-Meyer flow, charts of the shock angle as a function of upstream Mach number are presented so that the passage of a shock wave through these types of nonuniform regions can be easily traced. For a prescribed initial shock strength and initial Mach number, a minimum upstream Mach number is found below which no physically realistic solution can be obtained with the equations for simple propagation. This result serves as a sufficient condition for the avoidance of separated flow, reversed flow, or other upstream effects. An example is computed of the propagation of a shock wave through a wake-type supersonic shear profile and the flow field downstream of the shock is constructed.

INTRODUCTION

The most frequently discussed example of the propagation of a shock wave into a nonuniform flow region appears, at present, to be the shock-wave boundary-layer interaction problem. Although much theoretical and experimental research has been devoted to this problem, it is still unsolved in the sense that no method exists whereby the effect of a shock of prescribed strength on a boundary layer with

prescribed profile can be predicted. The difficulty in formulating a theoretical approach can be attributed chiefly to the fact that the flow upstream of the shock is not independent of the presence of the shock. If the subsonic portion of the boundary layer were removed (by continuous suction, for example), it might be expected that this difficulty would be eliminated. Circumstances frequently arise, however, for which even this expectation is not fulfilled. It was shown qualitatively in reference 1, for example, that the presence of a supersonic wake upstream of a blunt body requires modification of the upstream flow in order that physically realistic pressure gradients can be obtained downstream of the detached shock. The fact that the flow upstream of the shock is completely supersonic does not, therefore, guarantee that the upstream flow will be uninfluenced by the presence of the shock. 2527

In order to formulate criteria for determining whether or not the flow upstream of a shock is independent of the presence of the shock, it is first necessary to analyze the propagation of shock waves through regions in which upstream flow conditions are variable over a wide range. Such an analysis was carried out for weak waves and for the particular case of a supersonic shear layer in reference 2.

In the present report, the propagation of oblique shock waves of arbitrary strength through supersonic regions in which static pressure, stagnation pressure, stagnation temperature, or combinations of these are variable over wide ranges is considered. In addition to providing criteria for the avoidance of upstream effects of shock waves, the analysis provides a method for tracing shock waves through jets or wakes of known profile or through other types of flow with large nonuniformities. An analysis of the propagation of weak waves in general nonuniform regions is also given, and a procedure is discussed for constructing the supersonic portion of the flow field downstream of shock waves or weak waves. This investigation was conducted at the NACA Lewis laboratory.

ANALYSIS

General Equations

The interaction of a shock wave with a nonuniform supersonic stream is first discussed for the most general conditions feasible. The resulting equations are then simplified for several special types of flow. Symbols used are defined in the appendix.

For the most general case considered, the following assumptions are made:

(1) In the vicinity of the interaction region, effects of viscosity turbulence are negligible. This assumption permits use of nonviscous shock and flow equations.

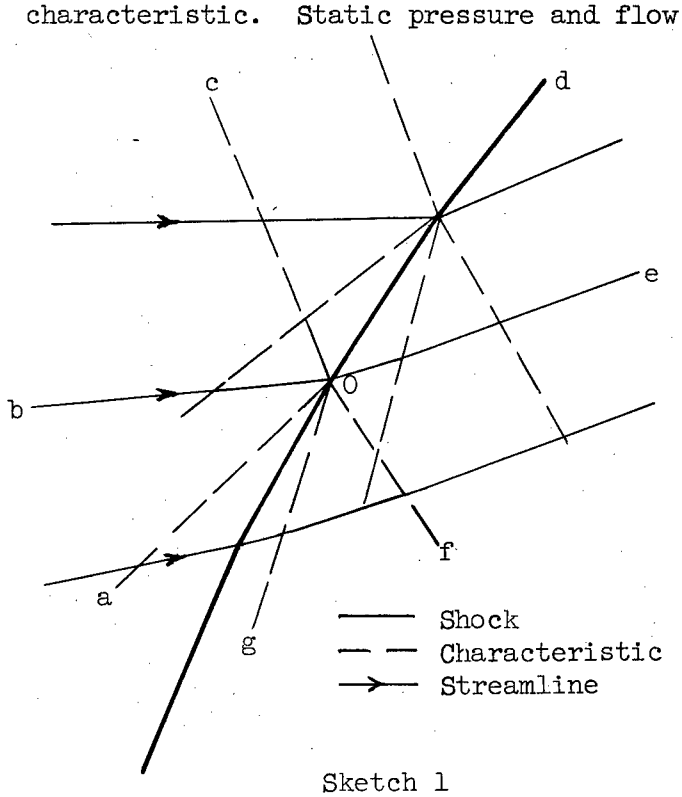
(2) The flow is everywhere supersonic and, consequently, changes in pressure and flow direction take place only across shock waves or characteristic lines (Mach waves).

(3) The flow field can be divided into stream tubes of small thickness. Stagnation temperature T and ratio of specific heats γ are constant in each stream tube but may change across the streamlines that bound each stream tube. (If the static temperature, and consequently γ, changes appreciably across the shock wave, an average value of γ can be used for each stream tube with slight additional complication.)

(4) Stagnation pressure remains constant in each stream tube except when that stream tube passes through the shock wave.

(5) Fluid properties upstream of the shock wave contain no large discontinuities. (If this condition is not imposed, the numerical work soon becomes prohibitively lengthy. Evaluation of the exact solution at the point of impingement of a shock wave on a contact surface that separates fluids whose properties differ by large amounts is discussed in references 3 and 4.)

With the preceding assumptions, the wave and streamline pattern near a point on the shock wave in the nonuniform flow field can be represented as in sketch 1. Lines $\overline{Oa}$ and $\overline{Oc}$ are incident waves or characteristics upstream of the shock; line $\overline{Og}$ is an incident wave or characteristic in the downstream field; and $\overline{Of}$ is a reflected wave or characteristic. Static pressure and flow direction may change across any characteristic or across the shock wave; stagnation pressure can change across any streamline or across the shock wave; and stagnation temperature and specific heat can change across any streamline. Assumptions (3) and (5) imply that all waves except the shock wave are sufficiently weak so that the relation between flow deflection and pressure change across the wave satisfies the weak-wave equation:



Sketch 1

$$\frac{p'}{p} = 1 \pm \frac{\gamma M^2}{\sqrt{M^2-1}}(\lambda'-\lambda) \quad (1)$$

where primes represent downstream values and unprimed symbols denote upstream

values. The plus and minus signs are used, respectively, for waves whose slope is positive or negative in sketch 1. All waves except the shock wave are inclined at the local Mach angle relative to the local flow direction.

The following relations between values upstream of a shock (unprimed) and values downstream of the shock (primed) are required for the analysis (see reference 5):

$$\frac{p'}{p} \equiv f_1(M,\gamma,\varphi) = \frac{2\gamma}{\gamma + 1} M^2\sin^2\varphi - \frac{\gamma - 1}{\gamma + 1} \tag{2}$$

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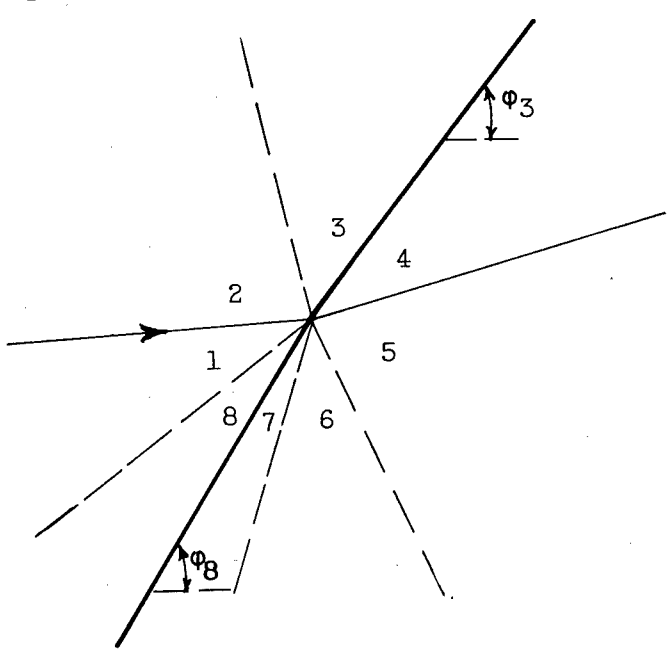
$$\lambda' - \lambda \equiv f_2(M,\gamma,\varphi) = \tan^{-1}\left[\frac{M^2\sin\varphi\cos\varphi - \cot\varphi}{1 + M^2\left(\frac{\gamma + 1}{2} - \sin^2\varphi\right)}\right] \tag{3}$$

$$f_3(M,\gamma,\varphi) \equiv \frac{\gamma M'^2}{\sqrt{M'^2 - 1}} \tag{4}$$

where

$$M'^2 = \frac{(\gamma + 1)^2M^4\sin^2\varphi - 4(M^2\sin^2\varphi - 1)(\gamma M^2\sin^2\varphi + 1)}{\left[2\gamma M^2\sin^2\varphi - (\gamma - 1)\right]\left[(\gamma - 1)M^2\sin^2\varphi + 2\right]} \tag{5}$$

The equations relating the quantities in each of the several regions of sketch 1 are derived with reference to the numbering shown in sketch 2, which is an expanded version of the vicinity of point 0 in sketch 1. The waves and the streamline separate the flow near point 0 into eight regions, in each of which all fluid properties are constant. Conditions in the upstream regions 1, 2, 3, and 8 are known; and the incident shock angle φ_8 is known. From the shock equations, conditions in region 7 are determined by φ_8, M_8, and γ_8. The unknowns are φ_3



Sketch 2

and conditions in regions 4, 5, and 6. Since pressure and flow angle are constant across streamlines, the following relations exist among the pressures and the flow angles:

$$\lambda_4 = \lambda_5 \tag{6}$$

$$\frac{p_4}{p_3} = \frac{p_5}{p_6}\,\frac{p_6}{p_7}\,\frac{p_7}{p_8}\,\frac{p_8}{p_3} \tag{7}$$

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Substitution from equations (1), (2), (3), (4), and (6) into equation (7) yields

$$f_1(M_3,\gamma_3,\varphi_3) = \left[1 - \frac{\gamma_8 M_6^2}{\sqrt{M_6^2 - 1}}(\lambda_4 - \lambda_6)\right]\left[1 + f_3(M_8,\gamma_8,\varphi_8)(\lambda_6 - \lambda_7)\right] f_1(M_8,\gamma_8,\varphi_8)\,\frac{p_8}{p_3} \tag{8}$$

But from equation (3)

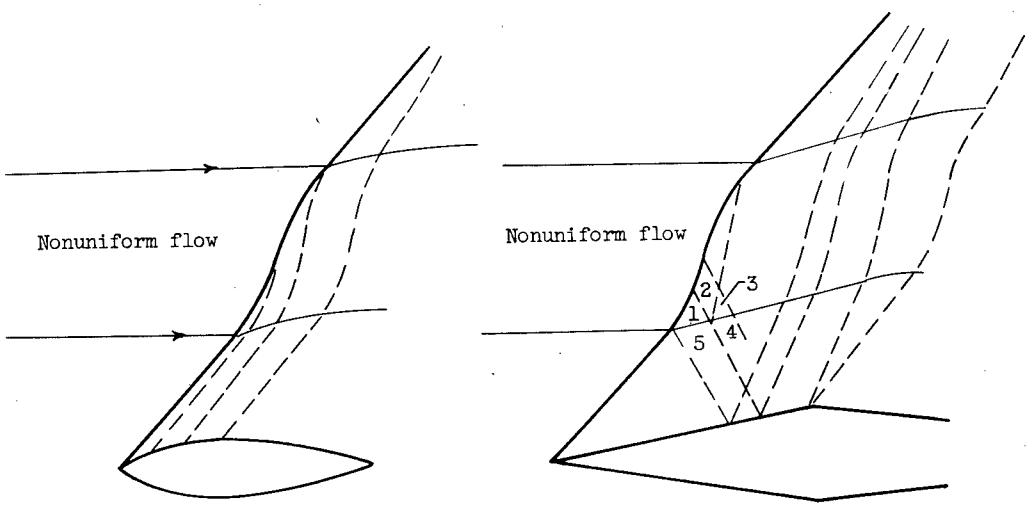
$$\lambda_4 = \lambda_3 + f_2(M_3,\gamma_3,\varphi_3) \tag{9}$$

$$\lambda_7 = \lambda_8 + f_2(M_8,\gamma_8,\varphi_8) \tag{10}$$

Hence, equation (8) becomes

$$f_1(M_3,\gamma_3,\varphi_3) = \left[1 - \frac{\gamma_8 M_6^2}{\sqrt{M_6^2 - 1}}(\lambda_3 - \lambda_6 + f_2(M_3,\gamma_3,\varphi_3)\right]\left\{1 + f_3(M_8,\gamma_8,\varphi_8)\left[\lambda_6 - \lambda_8 - f_2(M_8,\gamma_8,\varphi_8)\right]\right\} f_1(M_8,\gamma_8,\varphi_8)\,\frac{p_8}{p_3} \tag{11}$$

All quantities in equation (11) are known except φ_3, M_6, and λ_6. If waves of type $\overline{Og}$ in sketch 2 can be eliminated from the problem, the solution is greatly simplified, since then $\lambda_6 = \lambda_7$ and $M_6 = M_7$; and the only remaining unknown is the transmitted-shock angle φ_3. Conditions under which waves of type $\overline{Og}$ can be neglected can be ascertained with the aid of the following sketches:



Sketch 3(a). Sketch 3(b).

In sketch 3(a) waves of type $\overline{Og}$ are generated by the object that produces the shock wave and reach the shock before it passes through the nonuniform flow region. For this case, therefore, this type of incident wave is obviously not negligible. In sketch 3(b), on the other hand, waves of type $\overline{Og}$ that originate on, or are reflected from, the object reach the shock only after it has passed through the nonuniform region. There remains the possibility, however, (as illustrated in sketch 3(b)) that waves reflected within the nonuniform field itself will approach and appreciably influence the shock form before the shock leaves the nonuniform flow region. Conditions for which these waves are negligible can be determined by an analysis of the strength of the wave of type $\overline{Og}$ that separates regions 2 and 3 in sketch 3(b) relative to the strength of the wave of type $\overline{Of}$ that separates regions 1 and 2. Thus, in sketch 3(b), since $p_4 = p_3$, $p_5 = p_1$, $\lambda_3 = \lambda_4$, and $\lambda_1 = \lambda_5$, use of equation (1) yields $\left(\text{with } \dfrac{\gamma M^2}{\sqrt{M^2 - 1}} = f(M)\right)$

$$(\lambda_3 - \lambda_2)\left[f(M_2) + f(M_5) - f(M_2)\ f(M_1)(\lambda_2 - \lambda_1)\right] = (\lambda_2 - \lambda_1)\left[f(M_1) - f(M_5)\right]$$

from which is obtained

$$\frac{\lambda_3 - \lambda_2}{\lambda_2 - \lambda_1} = \frac{f(M_1) - f(M_5)}{f(M_2) + f(M_5) - f(M_1)\ f(M_2)(\lambda_3 - \lambda_1)} = \frac{\Delta f}{(2 - f\Delta\lambda)f}$$

Consequently, waves of type $\overline{Og}$, if they arise from internal reflection in the region of nonuniform flow, can be neglected without serious error if $f\Delta\lambda \ll 2$, where $\Delta\lambda$ can be interpreted as the strength of the strongest wave of type $\overline{Of}$. Since the strength of the $\overline{Of}$ waves is determined by the magnitude of the discontinuities upstream of the shock wave, this criterion, in effect, specifies the magnitude of the upstream discontinuities that can be tolerated if equation (11) is to be simplified. It will be assumed throughout the remainder of the report that the upstream flow satisfies this requirement, and that consequently M_6 and λ_6 can be set equal to M_7 and λ_7, respectively, if no waves other than those resulting from shock propagation are present. The modifications required when waves of type $\overline{Og}$ other than those resulting from internal reflection are present are considered in the section DISCUSSION AND APPLICATION.

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If $\lambda_6 = \lambda_7$ and $M_6 = M_7$, equations (8) or (11) become

$$f_1(M_3,\gamma_3,\varphi_3) = \left\{1 - f_3(M_8,\gamma_8,\varphi_8)\left[\lambda_3 - \lambda_8 + f_2(M_3,\gamma_3,\varphi_3) - f_2(M_8,\gamma_8,\varphi_8)\right]\right\} f_1(M_8,\gamma_8,\varphi_8)\frac{p_8}{p_3} \tag{12}$$

Since changes in flow variables from one stream tube to the next have been assumed small, functions of M_3, φ_3, and γ_3 differ only slightly from the same functions of M_8, φ_8, and γ_8. Consequently, let

$$f_n(M_3,\varphi_3,\gamma_3) = f_n(M_8,\varphi_8,\gamma_8) + df_n \qquad (n = 1,2,3) \tag{13}$$

and let

$$\begin{aligned} p_3 &= p_8 + dp \\ \lambda_3 &= \lambda_8 + d\lambda \end{aligned} \tag{14}$$

Equation (12) then becomes

$$\begin{aligned} f_1 + df_1 &= \frac{p_8}{p_8 + dp} f_1 \left[1 - f_3(d\lambda + df_2)\right] \\ &\approx 1 - \frac{dp}{p}\left[f_1 - f_1 f_3(d\lambda + df_2)\right] \end{aligned} \tag{15}$$

where the differentials correspond to changes in quantities upstream of the shock wave from one stream tube to the next in the direction of

propagation of the shock wave. If only first-order differential terms are retained, equation (15) becomes

$$\frac{df_1}{f_1} = - f_3(d\lambda + df_2) - \frac{dp}{p} \tag{16}$$

But

$$\left.\begin{aligned}
df_1 &= \frac{\partial f_1}{\partial \varphi} d\varphi + \left(\frac{\partial f_1}{\partial M} + \frac{\partial f_1}{\partial \gamma}\frac{d\gamma}{dM}\right) dM \\
df_2 &= \frac{\partial f_2}{\partial \varphi} d\varphi + \left(\frac{\partial f_2}{\partial M} + \frac{\partial f_2}{\partial \gamma}\frac{d\gamma}{dM}\right) dM \\
d\lambda &= \frac{d\lambda}{dM} dM \\
dp &= \frac{dp}{dM} dM
\end{aligned}\right\} \tag{17}$$

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Hence, equation (16) becomes

$$\frac{d\varphi}{dM} = - \frac{\frac{1}{f_1}\left(\frac{\partial f_1}{\partial M} + \frac{\partial f_1}{\partial \gamma}\frac{d\gamma}{dM}\right) + f_3\left(\frac{\partial f_2}{\partial M} + \frac{\partial f_2}{\partial \gamma}\frac{d\gamma}{dM} + \frac{d\lambda}{dM}\right) + \frac{1}{p}\frac{dp}{dM}}{\frac{1}{f_1}\frac{\partial f_1}{\partial \varphi} + f_3\frac{\partial f_2}{\partial \varphi}} \tag{18}$$

All quantities in the right member of equation (18) depend only on conditions upstream of the shock wave; hence, if the variations of γ, λ, and p with upstream Mach number are specified (as they will be if upstream conditions are known), then the rate of change of shock angle with upstream Mach number is a function only of φ and M. For any initial shock angle, and for arbitrary (but continuous) upstream Mach number distribution, equation (18) can therefore be integrated to obtain the form of the shock wave in regions of nonuniform supersonic flow. For discontinuous, but small, changes in upstream conditions, equation (18) must be evaluated in a step-by-step manner from one discontinuity to the next. The validity of equation (18) is limited to shock waves for which the flow downstream is supersonic and to segments of the shock wave that are free of interaction with downstream incident waves other than those resulting from the propagation process.

The partial derivatives required in equation (18) as evaluated from equations (2) and (3) are:

$$\frac{1}{f_1}\frac{\partial f_1}{\partial M} = \frac{4\gamma M \sin^2\varphi}{2\gamma M^2\sin^2\varphi - (\gamma - 1)} \tag{19}$$

$$\frac{1}{f_1}\frac{\partial f_1}{\partial \varphi} = \frac{4\gamma M^2\sin\varphi\cos\varphi}{2\gamma M^2\sin^2\varphi - (\gamma - 1)} \tag{20}$$

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$$\frac{1}{f_1}\frac{\partial f_1}{\partial \gamma} = \frac{2(M^2\sin^2\varphi - 1)}{(\gamma + 1)\left[2\gamma M^2\sin^2\varphi - (\gamma - 1)\right]} \tag{21}$$

$$\frac{\partial f_2}{\partial M} = \frac{(\gamma + 1)M\cot\varphi}{\left[1 + M^2\left(\frac{\gamma + 1}{2} - \sin^2\varphi\right)\right]^2 + \cot^2\varphi\,(M^2\sin^2\varphi - 1)^2} \tag{22}$$

$$\frac{\partial f_2}{\partial \varphi} = \frac{1 + \left(\frac{\gamma + 1}{2} - 2\sin^2\varphi\right)M^2 + \left(\frac{\gamma + 1}{2} - \gamma\sin^2\varphi\right)M^4\sin^2\varphi}{\sin^2\varphi\left\{\left[1 + M^2\left(\frac{\gamma + 1}{2} - \sin^2\varphi\right)\right]^2 + \cot^2\varphi\,(M^2\sin^2\varphi - 1)^2\right\}} \tag{23}$$

$$\frac{\partial f_2}{\partial \gamma} = \frac{-\frac{M^2}{2}\cot\varphi\,(M^2\sin^2\varphi - 1)}{\left[1 + M^2\left(\frac{\gamma + 1}{2} - \sin^2\varphi\right)\right]^2 + \cot^2\varphi\,(M^2\sin^2\varphi - 1)^2} \tag{24}$$

Special Cases

Supersonic shear flow. - If flow angle, static pressure, and ratio of specific heats are constant immediately upstream of the shock, equation (18) reduces to

$$\frac{d\varphi}{dM} = -\frac{\frac{1}{f_1}\frac{\partial f_1}{\partial M} + f_3\frac{\partial f_2}{\partial M}}{\frac{1}{f_1}\frac{\partial f_1}{\partial \varphi} + f_3\frac{\partial f_2}{\partial \varphi}} \equiv f_4(M,\varphi) \tag{25}$$

A contour plot of $d\varphi/dM$ as function of φ and M is shown in figure 1. Integral curves of figure 1 are presented in figure 2. The construction of these integral curves by an isocline method is aided by the fact that the curve of Mach angle against M $(M \sin\varphi = 1)$ is itself an integral curve of equation (25). This fact can be deduced from equation (25), which reduces for this case to

$$\left(\frac{d\varphi}{dM}\right)_{M \sin\varphi = 1} = - \frac{\tan \varphi}{M} = - \frac{1}{M\sqrt{M^2 - 1}} \tag{26}$$

which is exactly the slope of the curve $M \sin \varphi = 1$. Consequently, the slope of all integral curves at their intersection with an $f_4 = \text{constant}$ contour is the same as the slope of the $M \sin \varphi = 1$ curve at its intersection with that contour.

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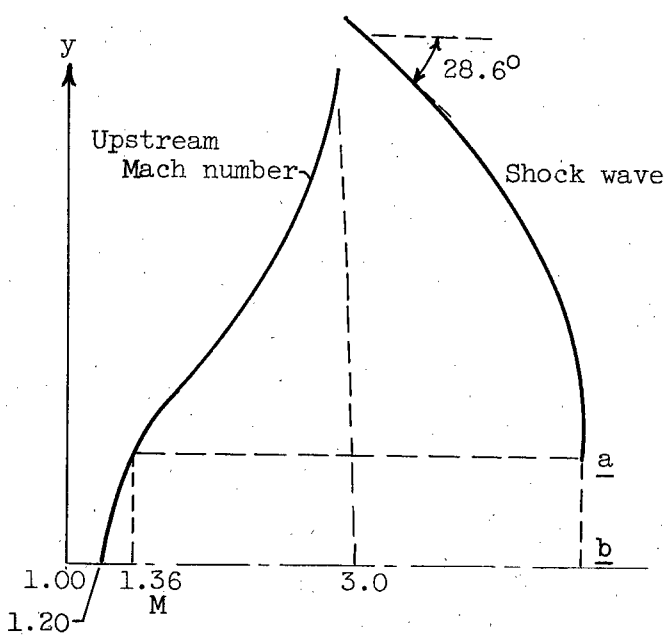
The integral curves of figure 2 yield the variation of shock angle with upstream Mach number when the reflected wave from each differential interface is taken into account. For comparison, figure 2 also contains curves obtained with the assumption that these reflected waves are negligible and, hence, that the pressure ratio across the shock (or $M \sin \varphi$ (see equation (2)) remains constant as the shock passes through the shear region. Since the slope of any $M \sin \varphi = \text{constant}$ curve is $d\varphi/dM = - (\tan \varphi)/M$, the curve $f_4(M,\varphi) = - (\tan \varphi)/M$ (fig. 2) is the locus of points for which the slope of the integral curves is equal to the slope of the constant-pressure-ratio curves. Comparison of the constant-pressure-ratio curves with the integral curves in figure 2 shows that the pressure ratio across the shock decreases as upstream Mach number decreases in the region to the left of the curve $f_4(M,\varphi) = - (\tan \varphi)/M$. To the right of this curve, pressure ratio increases as upstream Mach number decreases. The intersection of this curve with the Mach angle curve at $M = 1.41$ agrees with a result obtained in reference 2 for the interaction of weak waves with parallel supersonic shear layers. For a weak incident wave entering the region of decreasing Mach number, it was found that for $M = \sqrt{2}$ no reflected wave occured. The curve $f_4(M,\varphi) = - (\tan \varphi)/M$ in figure 2 thus represents a generalization of this result to the case of waves of arbitrary strength.

For fairly weak shocks, the assumption that the pressure ratio across the shock remains constant through the shear region appears to be an adequate approximation. For stronger shocks, however, the pressure gradient downstream of the shock can become quite large, and the actual shock angles may differ considerably from those obtained with the constant-pressure-ratio assumption.

The integral curves of figure 2 cannot be constructed beyond the $M' = 1$ curve, since the reflected-wave concept used to derive these curves is meaningless if the downstream flow is subsonic. For this region $(\varphi_s < \varphi < 90^\circ)$, however, it should be approximately valid in certain cases to assume that the pressure immediately downstream of the shock is constant along the shock, particularly if the flow downstream of the shock is expected to be almost parallel. A large pressure gradient normal to the subsonic streamlines would be inconsistent with

the expected nearly parallel flow. Consequently, in order to compute the form of the shock for $\varphi > \varphi_s$, the integral curves of figure 2 can probably be extended to $\varphi = 90^\circ$ along curves of constant-pressure ratio with no significant error.

When such extensions are made, it is seen from figure 2 that for any initial Mach number and shock angle there exists a minimum upstream Mach number below which no shock solution is obtained. If the Mach number profile of the shear region is such that the minimum Mach number is less than that for which a shock of specified initial strength can be traced through the region, some readjustment of the upstream flow appears to be required to obtain a solution that is physically realistic. For example, the situation shown in sketch 4 may be considered, wherein a shock of initial angle $\varphi_0 = 28.6^\circ$ and initial Mach number of 3.0 enters a shear region in which the minimum Mach number is 1.2. From figure 2, the incident shock is found to become normal at $M = 1.36$. If the shock remained normal from a to b, as indicated by the dotted extension of the shock, then the pressure ratio at b would be 1.52 as compared with the pressure ratio at a of 2.0. Since the normal shock implies initially parallel subsonic downstream flow, it seems unreasonable to expect that a pressure gradient normal to the streamlines such as required in sketch 4 will be established.



Sketch 4

A situation similar to that shown in sketch 4 was discussed in reference 1, where the modification required in the shock form when a shear layer occurs ahead of a blunt body was considered. It was argued that the pressure near the base or center line of the shear layer could be increased to a magnitude compatible with that outside the shear layer only if the stagnation point moved upstream to the vicinity of the shock wave. Hence, a separated-flow region was required ahead of the body to provide a mechanism for establishing pressure equilibrium in subsonic portions of the downstream flow. Although the nature of the modification required in the present case is less evident, the

formation of a separated-flow region with a consequent lambda shock outside of the shear layer appears to be the most likely possibility. In any event, the curves of figure 2 provide a criterion for determining whether simple propagation of a shock wave through the shear region can take place. If the shock can be traced through the shear region by means of figure 2, then the physical shock should correspond to that calculated. If, on the other hand, the shock cannot be traced through the shear layer, then a more complicated flow pattern will be obtained which may or may not involve reversed flow and flow separation.[a] This criterion yields no information, of course, on the magnitude of the separation that may result if simple propagation is impossible. 2527

Nonuniform isentropic flow. - If the stagnation pressure upstream of the shock is constant, then from the isentropic flow relation

$$p = P\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{-\gamma}{\gamma-1}}$$

there is obtained

$$\frac{1}{p}\frac{dp}{dM} = -\gamma M\left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1} \tag{27}$$

If γ is also constant, equation (18) becomes

$$\frac{d\varphi}{dM} = -\frac{\frac{1}{f_1}\frac{\partial f_1}{\partial M} + f_3\left(\frac{\partial f_2}{\partial M} + \frac{d\lambda}{dM}\right) - \gamma M\left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1}}{\frac{1}{f_1}\frac{\partial f_1}{\partial \varphi} + f_3\frac{\partial f_2}{\partial \varphi}} \tag{28}$$

Since the variation of λ with M depends, in general, on the structure of the flow upstream of the shock, the profile of M and λ must, in general, be specified to obtain the shock form from equation (28). For some types of flow, however, such as Prandtl-Meyer flow, the quantity $d\lambda/dM$ is a prescribed function of M so that equation (28) can be represented on a φ - M plot as for shear flow. Thus, for Prandtl-Meyer flow, since

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[a]Since spearated-flow regions unsupported by solid boundaries are not commonly observed experimentally, it may be helpful to point out that such regions can be maintained in equilibrium relative to the main stream by the high downstream pressure that is established in the separated region. The branched-shock configuration that results from flow separation can produce the required equilibrium conditions.

$$\pm\lambda = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1}\sqrt{\frac{\gamma - 1}{\gamma + 1}\left(M^2 - 1\right)} + \sin^{-1}\frac{1}{M} + C$$

there is obtained

$$\frac{d\lambda}{dM} = \frac{\pm\sqrt{M^2 - 1}}{M\left(1 + \frac{\gamma - 1}{2}M^2\right)} \tag{29}$$

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and equation (28) becomes

$$\frac{d\varphi}{dM} = -\frac{\frac{1}{f_1}\frac{\partial f_1}{\partial M} + f_3\frac{\partial f_2}{\partial M}}{\frac{1}{f_1}\frac{\partial f_1}{\partial \varphi} + f_3\frac{\partial f_2}{\partial \varphi}} - \frac{\left(-\gamma M \pm \frac{\sqrt{M^2 - 1}}{M} f_3\right)\left(1 + \frac{\gamma - 1}{2}M^2\right)^{-1}}{\frac{1}{f_1}\frac{\partial f_1}{\partial \varphi} + f_3\frac{\partial f_2}{\partial \varphi}} \equiv f_5(M,\varphi) \tag{30}$$

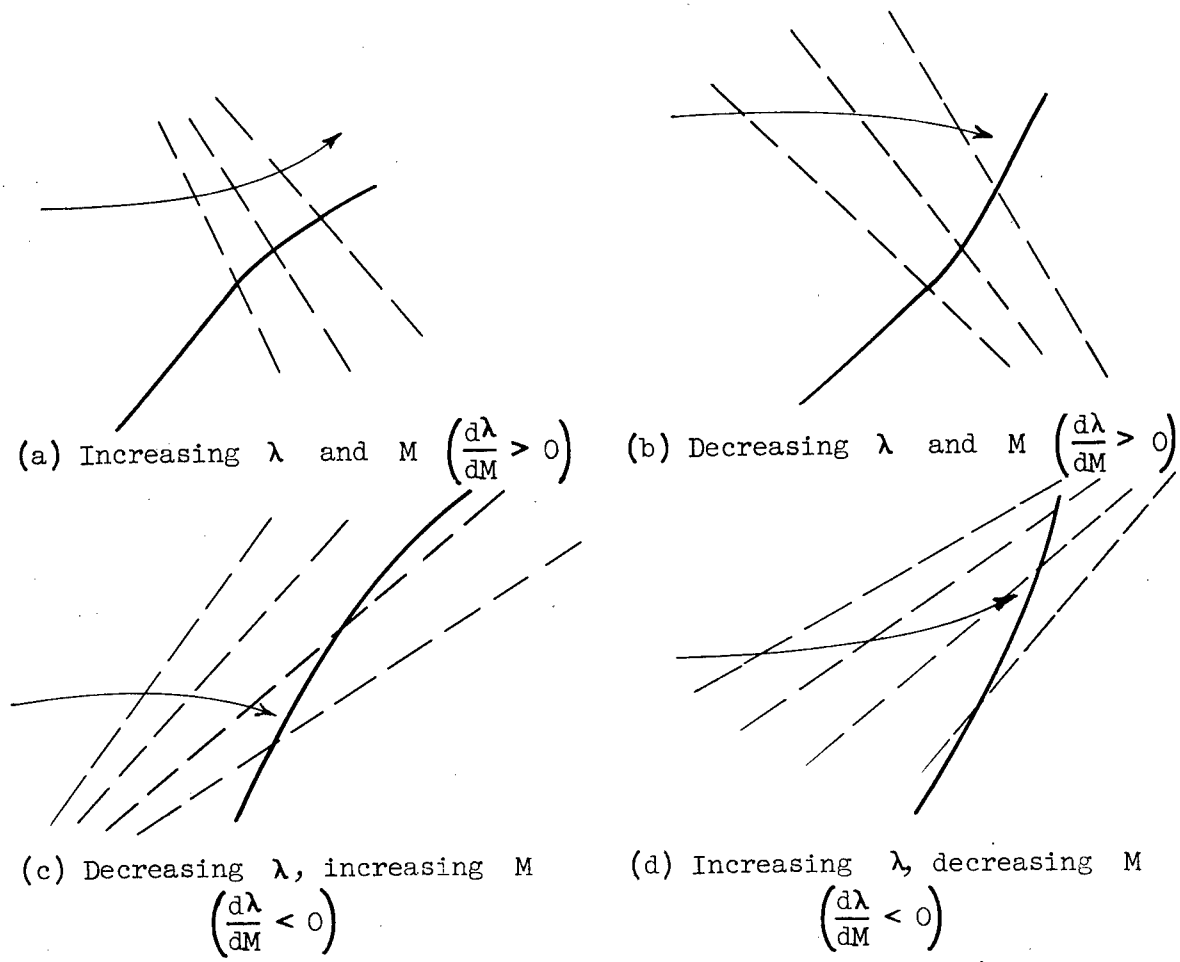
where the plus sign is used when $d\lambda/dM$ is positive and the minus sign when $d\lambda/dM$ is negative.

Positive values of $d\lambda/dM$ correspond to interactions such as those shown in sketches 5(a) and 5(b); negative values occur for interactions such as those shown in sketches 5(c) and 5(d) and arise less frequently in practice than positive values.

The compressive, or reversed, Prandtl-Meyer flow shown in sketches 5(b) and 5(d) is, of course, described by equation (29) only in regions in which the flow can be represented by a sequence of weak waves. When the weak waves form an envelope shock of sufficient strength to produce notable entropy increases, the flow angle and Mach number are no longer related by the Prandtl-Meyer equation.

The first term of the right member of equation (30) is the value of $d\varphi/dM$ for constant p, λ, and γ (equation (25)); the second term is the effect of static pressure and flow angle changes in Prandtl-Meyer flow.

A contour plot of $d\varphi/dM = f_5(M,\varphi)$ for Prandtl-Meyer flow is shown in figure 3 for the case in which the shock is entering a region in which both λ and M either increase or decrease ($d\lambda/dM > 0$). The curve $\sin\varphi = 1/M$ is again an integral curve of equation (30), since the last term in this equation is zero when φ is



Sketch 5. - Types of interaction of shock waves with Prandtl-Meyer flow regions.

the Mach angle. Comparison of figures 1 and 3 shows that for most values of M and φ the shock angle increases less rapidly with decreasing Mach number for Prandtl-Meyer flow than for shear flow.

Integral curves of figure 3 are plotted in figure 4, together with several constant-pressure-ratio curves. It is seen that the assumption that pressure ratio across the shock remains constant in passing through the nonuniform region is even less valid for this case than for shear flow. For fairly weak shocks $(\lambda' - \lambda < 10^{\circ})$, however, it is found that the integral curves follow quite closely curves of constant-flow deflection (constant f_2). The integral curves for shear flow, on the other hand, followed curves of constant-pressure ratio more closely than curves of constant-flow deflection. For very weak waves, the two types of curves become indistinguishable.

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The curve $f_5(M,\varphi) = -(\tan\varphi)/M$ separates the region in which pressure ratio decreases with decreasing Mach number from the region in which pressure ratio increases as Mach number is reduced. This curve again intersects the Mach angle curve at $M = \sqrt{2}$.

Strength Distribution for Weak Waves

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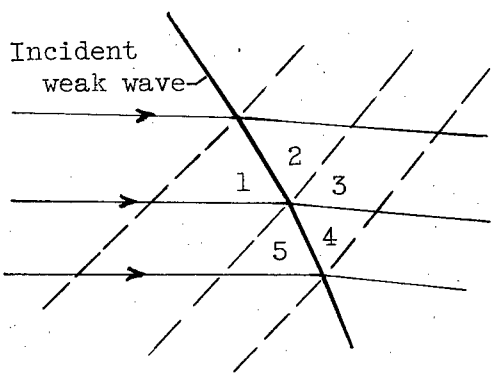
As mentioned previously, an analysis of the interaction of weak waves with a supersonic shear flow was developed in reference 2. In this analysis, the assumptions were made (as in the preceding analysis of shock interaction) that the flow upstream of the wave was uninfluenced by the presence of the wave and that this upstream flow could be divided into elementary stream tubes. An additional assumption was made that the Mach number in each stream tube was unchanged in passing through the wave. In the present section the interaction of a weak wave with more general nonuniform supersonic regions is considered, and the assumption that the Mach number remains constant in each stream tube is omitted.

A weak wave is herein defined as one across which pressure ratio and flow deflection are related by equation (1). When the upstream flow variables are specified, the local downstream flow variables are determined by means of a five-sector configuration (sketch 6) in which, since $p_3 = p_4$, the pressures are related by



Sketch 6

$$\frac{p_3}{p_2}\frac{p_2}{p_1} = \frac{p_4}{p_5}\frac{p_5}{p_1} \tag{31}$$

Use of equation (1) for the pressure ratios of equation (31) results in

$$\left[1+\frac{\gamma_1 M_2^2}{\sqrt{M_2^2-1}}(\lambda_3-\lambda_2)\right]\left[1-\frac{\gamma_1 M_1^2}{\sqrt{M_1^2-1}}(\lambda_2-\lambda_1)\right] = \frac{p_5}{p_1}\left[1-\frac{\gamma_5 M_5^2}{\sqrt{M_5^2-1}}(\lambda_4-\lambda_5)\right] \tag{32}$$

As for the shock wave, an average value of γ instead of the upstream value can be used for each wave with slight additional complication. In this manner the variation of γ with temperature can be taken into account as the computation proceeds.

Since all changes across the waves and across the streamlines are small, let

$$p_5 = p_1 + dp_1 \equiv p + dp$$

$$f(\gamma_1,M_2) = f(\gamma_1,M_1) + \delta f(\gamma_1,M_1) \equiv f + \delta f$$

$$f(\gamma_5,M_5) = f(\gamma_1,M_1) + df(\gamma_1,M_1) \equiv f + df$$

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where

$$f(\gamma,M) \equiv \frac{\gamma M^2}{\sqrt{M^2 - 1}}$$

Equation (32) can then be written (since $\lambda_4 \equiv \lambda_3$)

$$\left[1 + (f + \delta f)(\lambda_4 - \lambda_5 + \lambda_5 - \lambda_1 + \lambda_1 - \lambda_2)\right]\left[1 - f(\lambda_2 - \lambda_1)\right] = \left(1 + \frac{dp}{p}\right)\left[1 - (f + df)(\lambda_4 - \lambda_5)\right] \quad (33)$$

Let $d\lambda = \lambda_5 - \lambda_1$ and let θ be the change in flow angle across the weak wave; then

$$\theta = \lambda_2 - \lambda_1$$

$$d\theta = (\lambda_4 - \lambda_5) - (\lambda_2 - \lambda_1)$$

$$\theta + d\theta = \lambda_4 - \lambda_5$$

Equation (33) now becomes

$$\left[1 + (f + \delta f)(\theta + d\theta + d\lambda - \theta)\right]\left[1 - f\theta\right] = \left(1 + \frac{dp}{p}\right)\left[1 - (f + df)(\theta + d\theta)\right]$$

which, when rearranged, becomes

$$f\,d\theta\left[2 + \frac{dp}{p}\left(1 + \frac{df}{f}\right) - f\theta\left(1 + \frac{\delta f}{f}\right) + \frac{\delta f}{f} + \frac{df}{f}\right] = -\theta\,df + \frac{dp}{p}(1 - \theta f - \theta\,df) - d\lambda(1 - \theta f)(f + \delta f)$$

Neglect of all terms of higher order than the first yields

$$f\,d\theta(2 - \theta f) = -\theta\,df + \left(\frac{dp}{p} - f\,d\lambda\right)(1 - \theta f) \quad (34)$$

Calculations show that f is less than 10.1 for $1.01 < M < 5.0$. Consequently, in this range of Mach numbers, if θ is of the order of 1°, then θf can be considered negligible compared with unity and equation (34) becomes

$$\frac{d\theta}{\theta} = -\frac{df}{2f} + \frac{1}{2\theta f}\left(\frac{dp}{p} - f\,d\lambda\right) \tag{35}$$

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For the case of shear flow $(dp = 0,\ d\gamma = 0,\ d\lambda = 0)$, equation (35) reduces to

$$\frac{d\theta}{\theta} = -\frac{M^2 - 2}{4(M^2 - 1)}\frac{dM^2}{M^2} \tag{36}$$

which is the result derived by a different process in reference 2 with the assumption that Mach number is constant in each stream tube. As pointed out in reference 2, it is readily shown from equation (36) that a weak incident wave entering a region of decreasing Mach number produces a reflected wave of the same sense (compression or expansion) or opposite sense depending on whether M is less than or greater than $\sqrt{2}$. Integration of equation (36) yields, as in reference 2,

$$\frac{\theta}{\theta_0} = \frac{M_0}{M}\left(\frac{M^2 - 1}{M_0{}^2 - 1}\right)^{1/4} \tag{37}$$

where subscript zero refers to any initial or specified values. The function $(M^2 - 1)^{1/4}/M$ is plotted in figure 5. For shear regions with large changes in Mach number, it is evident that the cumulative effect of the reflected waves, which accounts for the change in θ in the interaction process, is negligible only if the initial weak wave is itself negligible.

For problems in which dp/p or $d\lambda$ are of the same order as $\theta\,df$, equation (35) must be used instead of equation (37). If the flow upstream of the weak wave is of the Prandtl-Meyer type, equation (35) becomes, by use of equations (27) and (29),

$$\frac{1}{\theta}\frac{d\theta}{dM} = -\frac{M^2 - 2}{2M(M^2 - 1)} - \frac{\gamma M}{\left(1 + \frac{\gamma - 1}{2}M^2\right)}\frac{(1 \pm 1)}{2\theta f} \tag{38}$$

where the plus sign corresponds to $d\lambda/dM > 0$ (see sketches 5(a) and 5(b)). Since an incident weak wave is everywhere inclined at the local Mach angle, interactions of the type shown in sketches 5(c)

and 5(d) do not occur in this case $(d\theta = 0$ because $dM = 0)$. For the physically significant case $(d\lambda/dM > 0)$ equation (38) can be written

$$\frac{1}{\theta}\frac{d\theta}{dM} = -\left[\frac{M^2 - 2}{2M(M^2 - 1)} + \frac{\sqrt{M^2 - 1}}{\theta M\left(1 + \frac{\gamma - 1}{2}M^2\right)}\right] \tag{38a}$$

This equation is easily integrated numerically for any prescribed θ_0 and M_0. 2527

DISCUSSION AND APPLICATION

Construction of downstream flow by stepwise process. - When the form of a shock wave in a nonuniform region has been determined by equation (18) or its corollaries, and when the resulting downstream flow is completely supersonic, the method of characteristics for anisentropic flow can be used to complete the computation of the downstream flow field. This method, however, is quite tedious numerically; and it may be desirable, in cases where great accuracy is not required, to establish the essential features of the downstream flow by a simple method. The weak-wave procedure to be described constitutes such a simplification.

Before discussing the computation procedure, it is necessary to establish and define the order of magnitude of the various waves encountered. For this purpose, the interaction process shown in sketch 7 is considered, wherein a shock wave is shown entering a nonuniform region from the upper left. The flow upstream of the shock may be continuous or it may contain discontinuities sufficiently small so that the criterion for neglect of all except the primary reflected waves for computing the shock form is satisfied. This criterion was discussed in the section General Equations. After the shock form is determined, the upstream flow field can be divided into a sequence of streamtubes in each of which all fluid quantities can be assumed to be constant. This subdivision is required for stepwise computation of the downstream flow field even if the upstream flow is continuous. The subdivision produces a wave pattern of the type shown in sketch 7.

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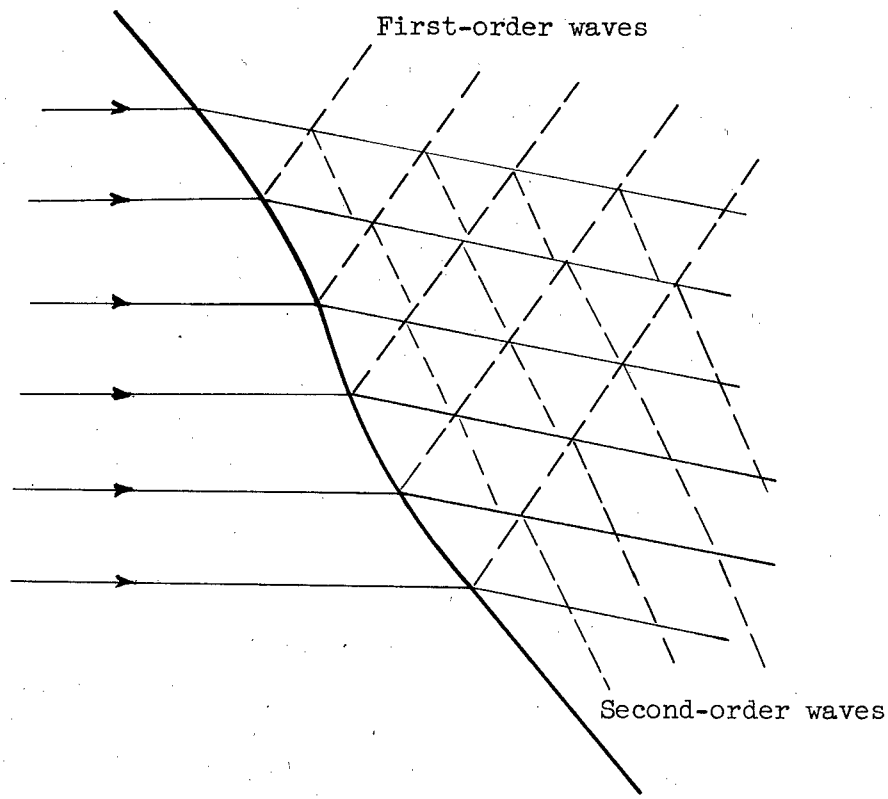
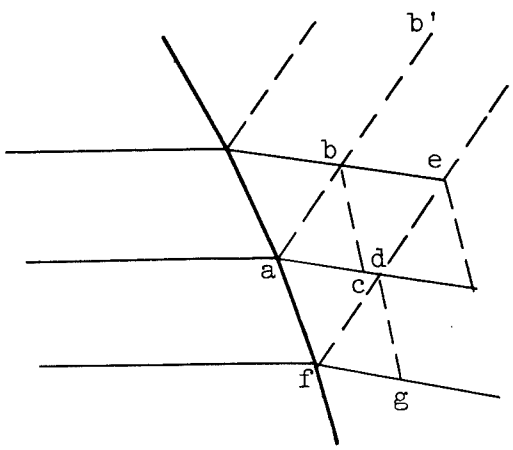


Sketch 7

The shock wave can be considered to be a zero-order wave which, in passing through the nonuniform region, produces a sequence of reflected first-order waves. The integrated effect of the first-order waves on the strength of the shock wave can be of the same order of magnitude as the initial strength of the shock. A zero-order solution to the interaction problem must therefore retain the integrated effect of first-order waves. Similarly, each of the reflected first-order waves, whose initial strength is known from conditions just downstream of the shock wave, can be considered to be a primary weak wave passing through a region of nonuniform flow. Each first-order wave produces a sequence of second-order reflected waves, whose integrated effect on the strength of the first-order waves can be of the first order. To compute accurately the strength of the first-order waves as they emerge from the nonuniform flow field, it is therefore necessary to use equation (35) for each first-order wave (since p, λ, and M are all variable upstream of these waves). The use of equation (35) to compute the strength of each first-order wave yields an approximate solution for conditions upstream of the next first-order wave. With these conditions, the strength distribution of each succeeding first-order wave can be computed.

Sketch 8 shows the precise manner in which the second-order waves are taken into account by this procedure. Conditions upstream of first-order reflected waves $\overline{ab}$ and $\overline{fd}$ are known from the shock form. The strength of the secondary reflected wave $\overline{bc}$ is obtained from equation (35), which gives the change in flow deflection of wave $\overline{abb'}$ at b. The strength of wave $\overline{bc}$, in turn, determines conditions upstream of segment $\overline{de}$, from which the strength of segments $\overline{de}$ and $\overline{dg}$ can be calculated by equation (35). By continuation of this procedure, the entire supersonic portion of the downstream flow field can be constructed. The only approximation made is that waves such as $\overline{bc}$ do not significantly affect the average flow variables upstream of primary wave segments such as $\overline{fd}$; in other words, that the ratio $\frac{\overline{ac}}{\overline{ad}}$ is close to unity. Since this ratio is very close to unity when all waves are inclined at the local Mach angle, the maximum error that can result from violations of this condition is equivalent to the error that could result from neglect of a single sequence of second-order waves (those nearest the shock wave), which, in turn, is approximately equivalent to the effect of a single first-order wave. In general, therefore, the use of equation (18) and its corollaries for shock waves and the use of equation (35) and its corollaries for first-order waves will yield an accurate first-order solution to the interaction problem.



Sketch 8

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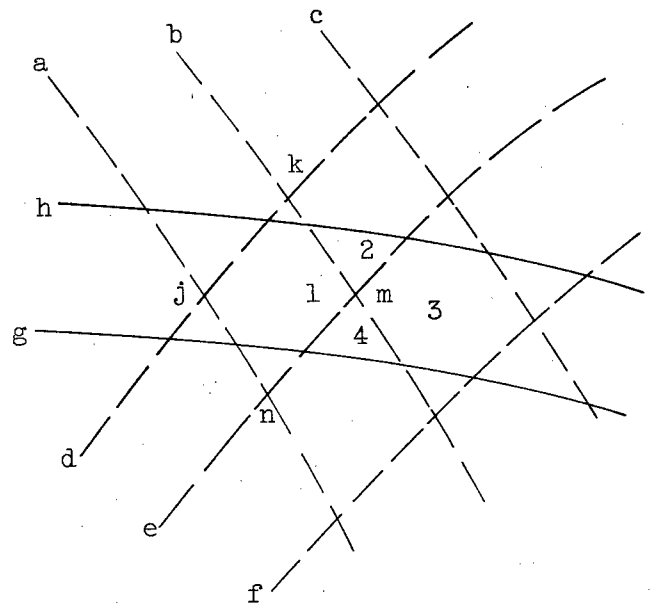
In many problems, the foregoing procedure may yield higher accuracy than is required; and it may be desirable, for the sake of simplicity, to ignore second-order waves entirely. In such a case, the pressure ratio or flow deflection across each first-order wave is considered constant until it intersects another first-order wave or a shock wave. The neglect of second-order waves is permissible, if only a few such waves are generated. Thus, in sketch 7, the first three or four first-order waves generate only a few second-order waves before they emerge from the nonuniform region. In general, if the upstream Mach number varies over a small range, so that only a few streamsheets are required to break the flow into sufficiently small differences, then the neglect of second-order waves will result in negligible error.

Effect of incident first-order waves. - If first-order waves other then those arising from the passage of a shock wave through the nonuniform region are present, then the computation of the downstream flow field by

the wave method is feasible only if second-order waves are entirely neglected. Even with this approximation, however, the precise construction of the flow is rather laborious because the intersections of waves with waves, and waves with streamlines occur at different positions. The situation locally is as shown in sketch 9, where a, b, and c may be first-order waves such as those shown in sketch 3(a); d, e, and f may be first-order reflected waves due to the passage of a shock wave; and g and h are streamlines. The simplest computation procedure consists in assuming that stagnation pressure and stagnation temperatures are constant in grids such as jkmn (rather than between streamlines). If the width of each grid is of the same order as the initially assumed streamtube width, then no appreciable error is introduced by this procedure. The streamlines can be ignored except insofar as they indicate the approximate local stagnation pressure and temperature, and each computation of properties in successive grid spaces is merely an application of the weak-wave relation (equation (1)) to the problem of interaction of two weak waves. Thus, if conditions in grid spaces 1, 2, and 4 are known from previous computations, conditions in region 3 are obtained from the relation



Sketch 9

$$\frac{p_3}{p_4}\frac{p_4}{p_1} = \frac{p_3}{p_2}\frac{p_2}{p_1} \tag{39}$$

or, with the use of equation (1) for these pressure ratios,

$$\left[1 - f(M_4)(\lambda_3 - \lambda_4)\right]\left[1 + f(M_1)(\lambda_4 - \lambda_1)\right] = \left[1 + f(M_2)(\lambda_3 - \lambda_2)\right]\left[1 - f(M_1)(\lambda_2 - \lambda_1)\right]$$

which, when solved for λ_3, yields

$$\lambda_3 = \frac{f(M_2)\lambda_2\left[1 - f(M_1)(\lambda_2 - \lambda_1)\right] + f(M_4)\lambda_4\left[1 + f(M_1)(\lambda_4 - \lambda_1)\right] + f(M_1)\left[\lambda_2 - \lambda_1 + \lambda_4 - \lambda_1\right]}{f(M_4)\left[1 + f(M_1)(\lambda_4 - \lambda_1)\right] + f(M_2)\left[1 - f(M_1)(\lambda_2 - \lambda_1)\right]} \tag{40}$$

Since all waves are considered to be weak, let

$$f(M_2) = f(M_1) + \delta f$$

$$f(M_4) = f(M_1) + df$$

If the steps are sufficiently small so that $f(M)\Delta\lambda$ is small compared with unity, then neglect of all except the lowest order terms in equation (40) yields 2527

$$\lambda_3 - \lambda_2 = \lambda_4 - \lambda_1 \tag{41}$$

or

$$\lambda_3 - \lambda_4 = \lambda_2 - \lambda_1$$

which shows that the assumption commonly made for isentropic flow, namely, that the flow deflection remains constant across each first-order wave in interactions with other first-order waves, is also applicable when continuous or stepwise small variations in entropy and energy occur.

With P_3 and γ_3 taken equal to P_1 and γ_1, respectively, equations (41) permit calculation of pressure and Mach number in region 3. The accuracy of this procedure depends on the negligibility of second-order reflections at the differential entropy and energy interfaces which, in turn, depends on the number of stream tubes required to divide the upstream flow into sufficiently small differences. If the upstream Mach number varies over a very wide range, use of the method of characteristics for anisentropic flow may be required in place of the first-order wave procedure if a high degree of accuracy is required downstream of the shock wave.

Interaction of incident first-order waves with shock wave. - When first-order waves other than reflected waves due to passage of the shock are present, it is likely that some of these waves will intersect the shock wave in the region of nonuniform flow. This possibility was pointed out in the first section of the ANALYSIS, and discussion was postponed because the analysis neglecting the waves was valid if such intersections were few and for segments of the shock between such intersections. The local modification of shock strength required at these intersection points is derived with the aid of sketch 10, in which an incident wave of type $\overline{Og}$ is assumed to intersect the shock wave

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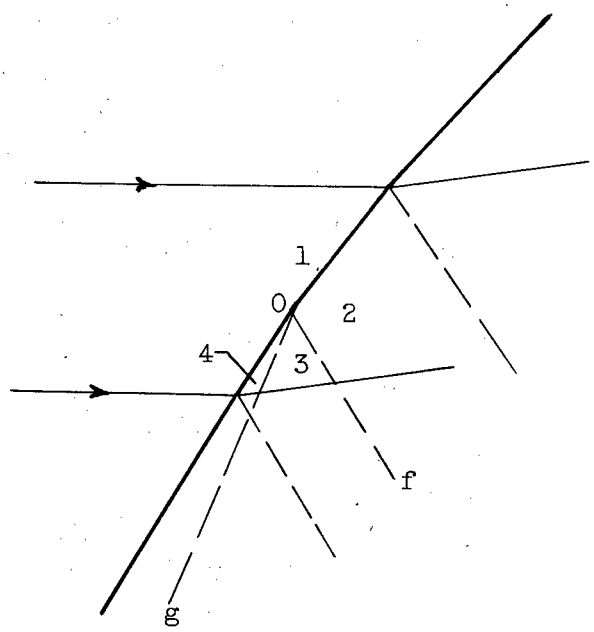

Sketch 10

between two successive reflected waves due to upstream flow variations. At the intersection point, an additional reflected wave $\overline{Of}$ can arise which is due only to the interaction of $\overline{Og}$ with the shock wave and not to variation in the upstream flow. The strength of $\overline{Og}$ at the intersection point is considered known from computations of its interactions with all first-order waves in its path. Although a solution for the strength of the wave $\overline{Of}$ resulting from the interaction with the shock wave can be obtained from equation (11), the procedure is rather laborious, and it seems adequate for most purposes to neglect the reflected wave $\overline{Of}$ entirely. The assumption that no reflected wave of type $\overline{Of}$ occurs at the intersection is the same assumption that is made with good accuracy in the shock-expansion theory for two-dimensional airfoils. The conditions for which the $\overline{Of}$ wave is negligible compared with the incident wave are fully discussed in reference 6. Specifically, the assumption is that in sketch 10 $\lambda_2 = \lambda_3$ and $p_2 = p_3$, where p_3 and λ_3 are known from the preceding step in which the formula for the intersection of two weak waves was used. The change in pressure ratio across the shock at point 0 transfers the shock to another integral curve of equation (18) (or fig. 2 or 4), which it follows until another intersection of this type occurs. The entire shock form is therefore determined to first order by its own propagation process and by first-order waves from external sources that intersect it.

The accuracy with which the flow field can be calculated by neglecting all except first-order waves and the initial shock wave depends on the strength of the second-order waves that are neglected. An estimate of the error can be made by determining the strength of the second-order waves that are produced by the strongest first-order wave in the flow field. The errors in the computed pressures and flow angles should be comparable with the changes in pressure and flow angle across these neglected waves.

EXAMPLE

As an example of shock propagation through supersonic shear regions, the form of a shock wave in a constant-pressure wake-type profile was computed. The profile assumed is shown in figure 6, together with the shock form traced with the aid of figure 2. The initial shock angle and Mach number were assumed to be 34.0° and 3.0, respectively. The streamlines and first-order reflected waves were computed under the assumption that second-order waves were negligible and that consequently the flow deflection and pressure ratio across each first-order reflected wave remain constant. 2527

The minimum Mach number in the wake was taken to be 1.75, which yields a region of subsonic flow downstream of the shock, but does not exceed the criterion for simple propagation. The region of subsonic flow was desired to illustrate an approximate procedure for handling such regions when they are imbedded in a supersonic stream. The procedure used was as follows: In the region of subsonic flow, the wave-type calculation, of course, is not valid; but it was assumed that the pressure difference and flow deflection across each first-order wave was impressed on the imbedded subsonic region and that these pressure differences propagate through the subsonic region normal to the flow direction and emerge at the opposite sonic line in the form of waves with the same strength as the incident waves. This process appears reasonable if the extension of the shock through the region $\varphi > \varphi_s$ by means of the constant-pressure criterion is approximately valid. In that case, the shock form determines the first-order wave pattern which, in turn, determines the streamwise pressure gradient in the supersonic streamlines adjacent to the subsonic streamlines. The assumption that this pressure gradient is transmitted normally through the subsonic region is consistent with the expectation that curvature of the subsonic streamlines will be small if the curvature of the adjacent supersonic streamlines is small.

With the assumption that no separation takes place downstream of the shock and that pressure is transmitted normally through the subsonic portion of the downstream shear layer, the interaction process is easily constructed in terms of the local Mach angles and flow angles. The resulting pattern of streamlines and first-order reflected waves is shown in figure 6 for the example chosen. The flow angle just downstream of the shock varied from 16.7° at the point of incidence to 19.2° near the center line. The pressure ratio across the shock varied from 3.12 at the point of incidence to 2.72 near the center line. Since the static pressure and flow angle are constant between any two reflected first-order waves (due to neglect of second-order waves), these figures represent the range of variation for the entire interaction region.

Although these variations in flow angle and pressure ratio are appreciable relative to the initial flow deflection and pressure ratio (about 15 and 13 percent, respectively), they are nevertheless fairly small and may be negligible for some purposes. The resulting variations in Mach number are indicated on figure 6. In the stream outside the shear layer the variation is only about 4 percent of the initial downstream Mach number. These ranges of variation can, of course, be predicted as soon as the form of the shock is known. The location of the variations, however, must be determined by constructing the first-order wave pattern.

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The self-consistency of the method used to trace changes in pressure and flow angle through the subsonic streamtubes can be checked to some extent by comparing the change in streamtube area required by one-dimensional equations with the change actually obtained. In the present case, the change in Mach number from 0.90 to 0.80 along the central streamtubes requires an area change of only about 3 percent, which is self-consistent with the area changes actually obtained within the error in tracing the streamlines.

CONCLUDING REMARKS

Analysis of the propagation of shock waves and first-order waves through nonuniform regions of supersonic flow showed that the strength distribution of an incident wave in such regions is determined by the upstream distribution of Mach number, flow angle, pressure, and specific-heats ratio provided that no waves other than the incident wave and reflected waves generated by its propagation are present. If the upstream flow can be divided into a sequence of small discontinuities, each incident wave can be considered to produce a sequence of reflected waves whose strength is an order of magnitude less than that of the primary wave. The integrated effect of these reflected waves, however, may be of the same order of magnitude as the strength of the primary wave.

When the primary wave is a shock wave, a minimum upstream Mach number is found below which no physically realistic solution can be obtained with the equations for simple propagation. A sufficient condition is thereby obtained for the avoidance of flow separation, reversed flow, or other nonsimple phenomena.

When first-order waves other than those reflected within the nonuniform region due to passage of a shock wave occur downstream of the shock, a first-order solution to the interaction problem is obtained by neglecting all second-order reflected waves. The accuracy of this procedure depends on the magnitude of the variations in fluid properties upstream of the shock and becomes less accurate when these variations

are large. If a high degree of accuracy is required, the flow downstream of the shock can be constructed by the method of characteristics for anisentropic supersonic flow.

Although the wave analysis used becomes invalid when a portion of the flow downstream of the shock wave becomes subsonic, approximate procedures are given for computing the downstream flow field, as well as the shock form, when the curvature of the streamlines in the downstream flow field is small.

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Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, March 13, 1952

APPENDIX - SYMBOLS

The following symbols are used in this report:

$f(M,\gamma)$	$\dfrac{\gamma M^2}{\sqrt{M^2 - 1}}$
$f_1(M,\gamma,\varphi)$	pressure ratio across shock, p'/p (equation (2))
$f_2(M,\gamma,\varphi)$	flow deflection across shock, $(\lambda' - \lambda)$ (equation (3))
$f_3(M,\gamma,\varphi)$	$\dfrac{\gamma M'^2}{\sqrt{M'^2 - 1}}$ (equation (4))
$f_4(M,\gamma,\varphi)$	$d\varphi/dM$ for shear flow (equation (25))
$f_5(M,\gamma,\varphi)$	$d\varphi/dM$ for Prandtl-Meyer flow (equation (30))
M	Mach number
P	stagnation pressure
p	static pressure
γ	ratio of specific heats
θ	flow deflection across shock or weak wave $(\lambda' - \lambda)$ (positive value for compression, negative value for expansion)
λ	flow angle relative to reference direction (positive sense indicated where needed)
φ	shock angle relative to local upstream flow direction
φ_s	value of φ for $M' = 1.0$

For λ, p, and M unprimed values represent conditions upstream of shock or weak wave and primed values refer to conditions downstream of shock or weak wave.

Number subscripts for λ, φ, p, γ, M refer to values in corresponding regions in accompanying sketches.

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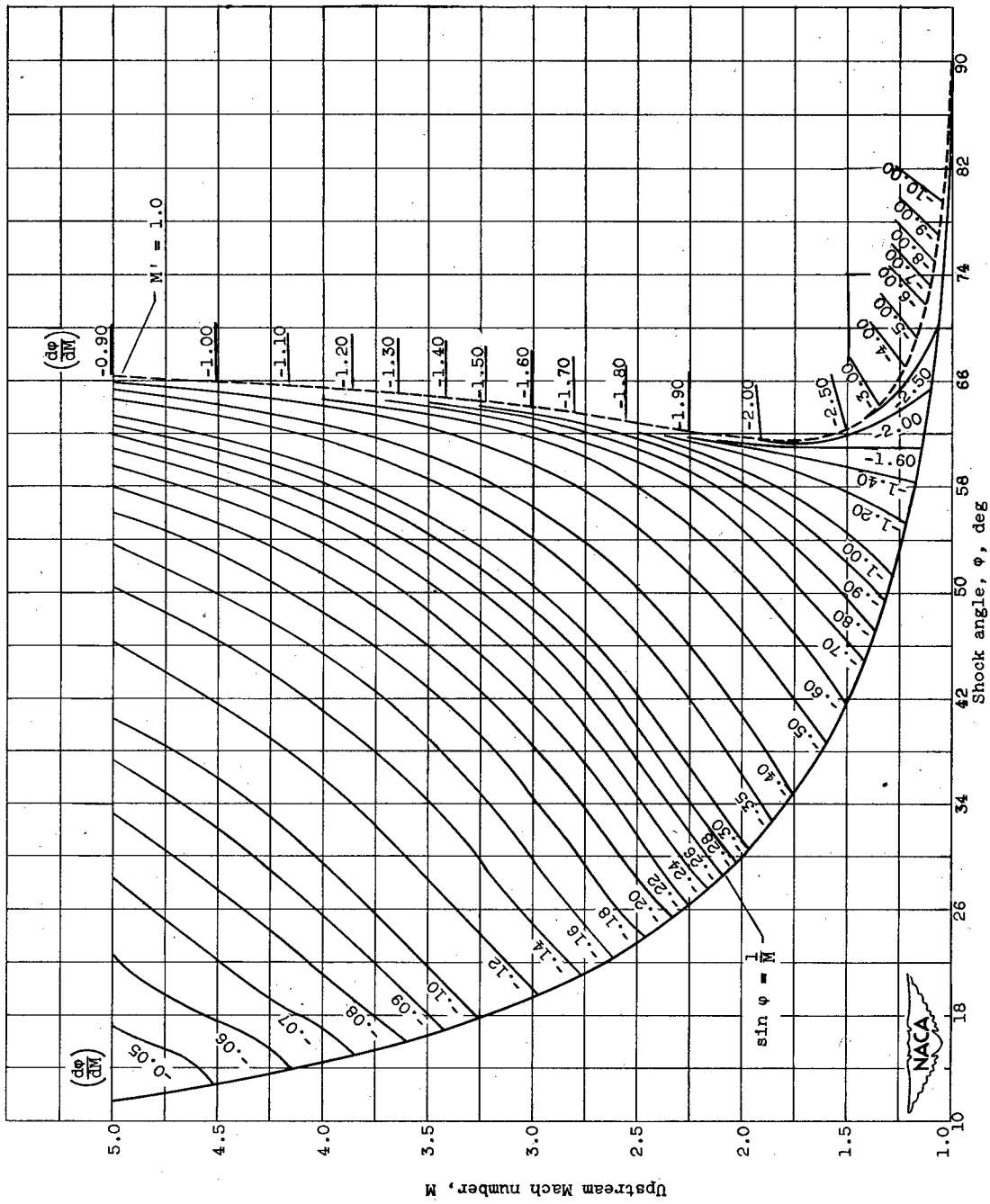

Figure 1. - Rate of change of shock angle with upstream Mach number. Constant upstream static pressure. Ratio of specific heats, 1.40.

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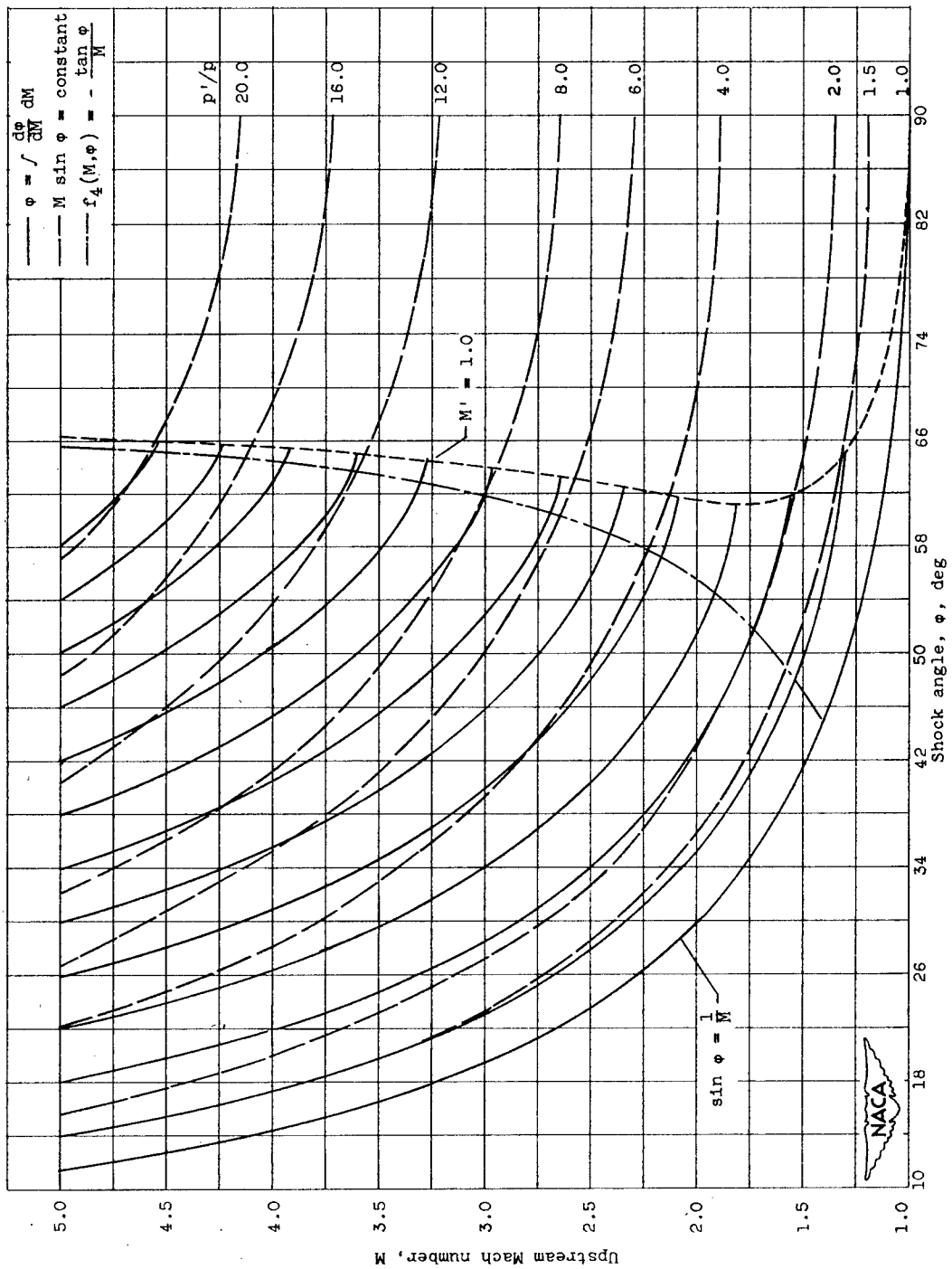

Figure 2. - Variation of shock angle with upstream Mach number. Constant upstream static pressure and flow angle. Ratio of specific heats, 1.40.

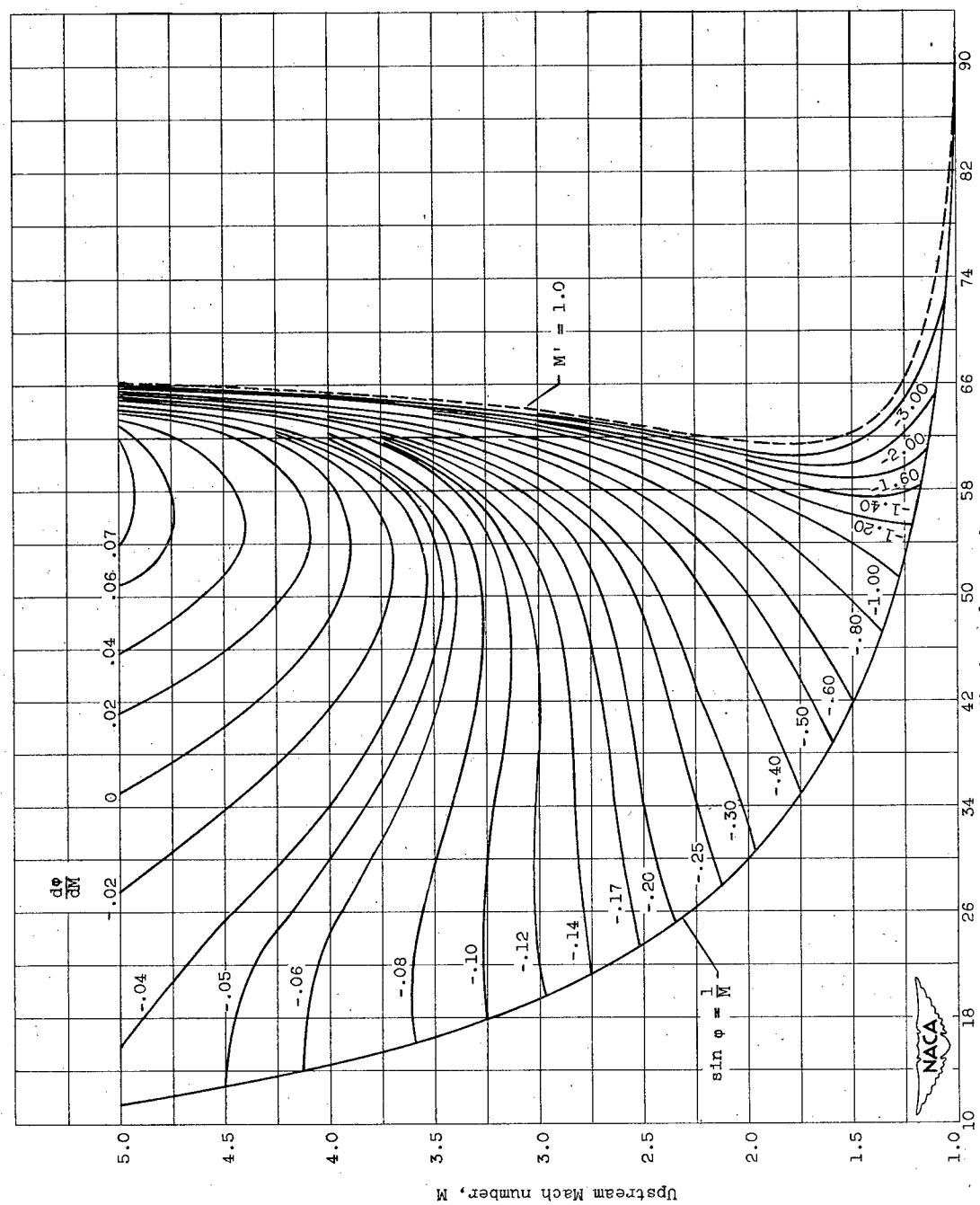

Figure 3. - Rate of change of shock angle with upstream Mach number in Prandtl-Meyer flow region. Ratio of specific heats, 1.40.

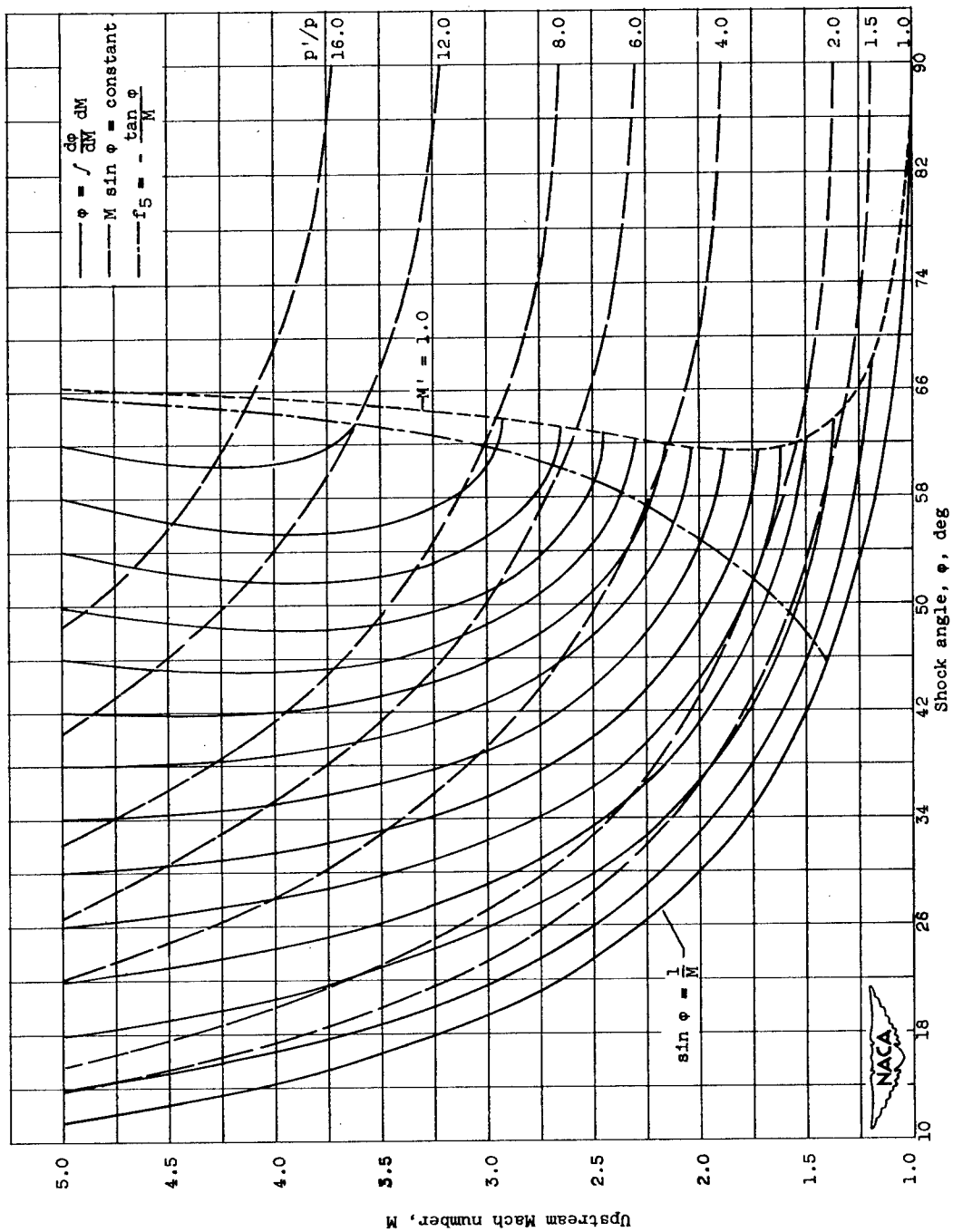

Figure 4. - Variation of shock angle with upstream Mach number in propagation through Prandtl-Meyer flow region. Ratio of specific heats, 1.40.

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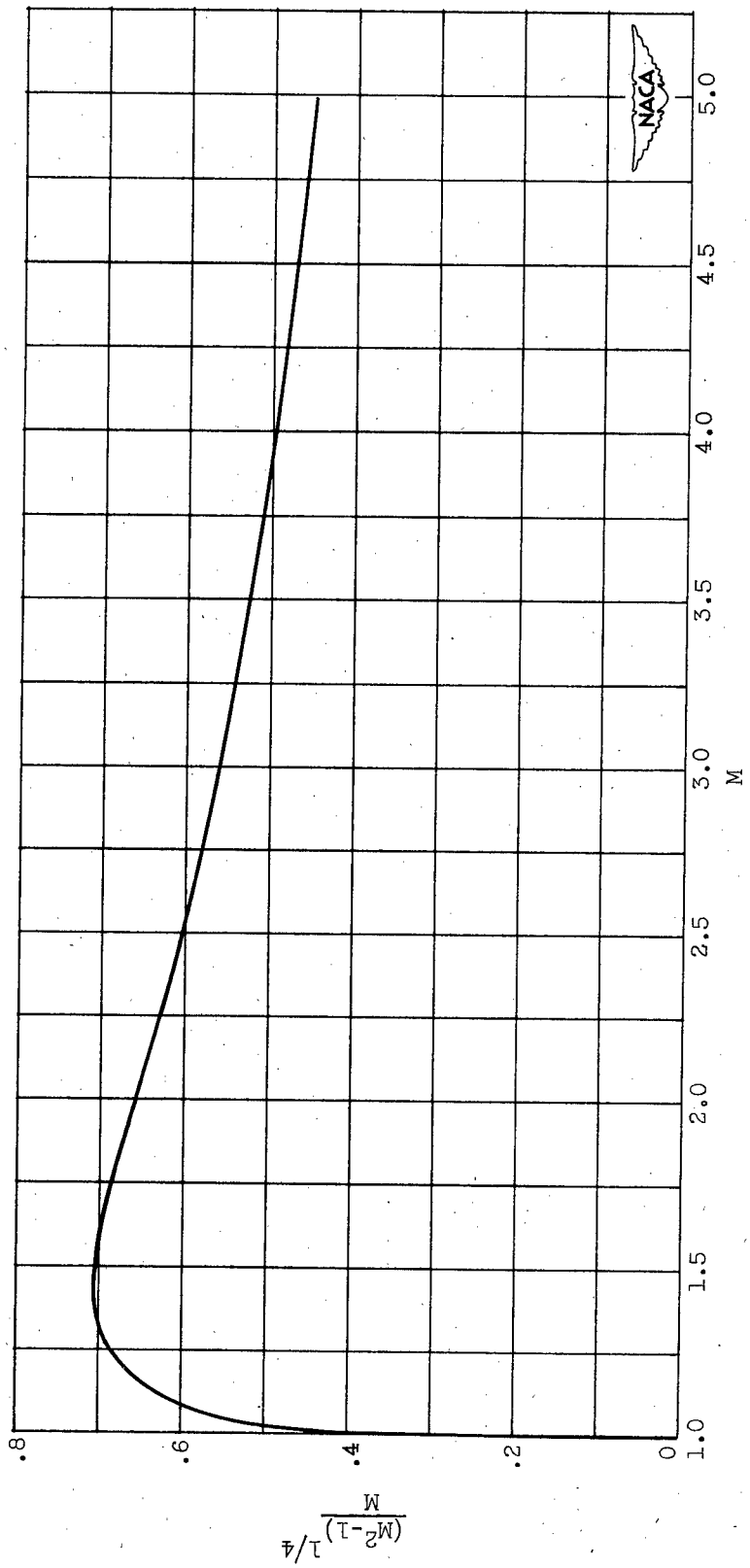

Figure 5. - Function for computing flow deflection across weak wave in shear region.

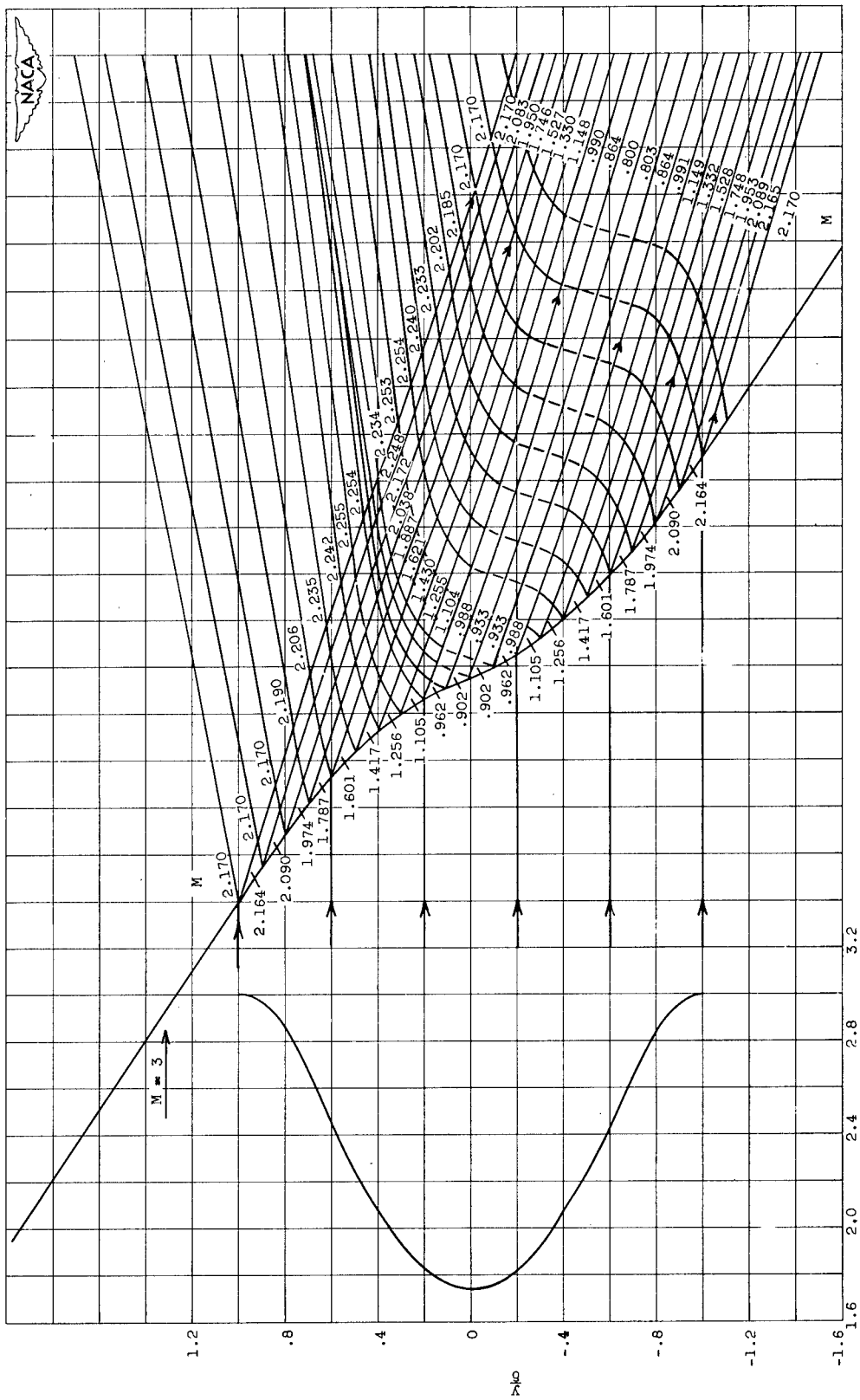

Figure 6. - Interaction of shock wave with wake-type nonuniform supersonic flow region. Initial Mach number, 3.0; initial shock angle, 34.0°.

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