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SOME DYNAMIC EFFECTS OF FUEL MOTION IN SIMPLIFIED
MODEL TIP TANKS ON SUDDENLY EXCITED
BENDING OSCILLATIONS

By Kenneth F. Merten and Bertrand H. Stephenson

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SUMMARY

An exploratory investigation of the dynamic effects of fuel sloshing in tip tanks on suddenly excited bending oscillations was conducted with two simplified model beam-tank systems. The larger system consisted of a cylindrical tank 7.5 inches in diameter and 10 inches long mounted on the tip of an 80-inch cantilever beam and the smaller system consisted of a cylindrical tank 4.2 inches in diameter and 6.9 inches long mounted on the tip of an 18-inch cantilever beam. Several fluids of different densities and viscosities (water, carbon tetrachloride, benzene, and linseed oil) were used in combination with various conditions of tank fullness. Recorded oscillations of the beams after sudden release from an initially deflected position showed the effects both of fluid damping and of the variation in effective mass on the beam motion. High-speed motion pictures were used to study the fluid motion.

Envelope curves of the beam-displacement time histories are compared to show the effects on the oscillations of variation in tank fullness, fluid density, fluid viscosity, and tank shape. The effective weight of fluid for the smaller test system is shown for each successive cycle of vibration, and the variation of effective fluid weight with tank fullness is presented. The results of this investigation indicate that after several cycles substantial damping may be obtained from fuel sloshing in a tip tank and the effective mass of the fuel may vary considerably under certain conditions of tank oscillation. For the fluids tested no effects attributable to viscosity were observed but for a given beam-tank system the density of fluid and tank fullness were found to be important parameters.

INTRODUCTION

For some airplanes the quantity of fuel carried in wing-tip tanks has become a large percentage of the total mass of the wing. It has
become evident that the sloshing of these large masses of fuel may cause important dynamic effects. For example, the question arises in flutter as to whether a partly full tank behaves like a solid mass or whether the sloshing and splashing of the fuel causes a reduction in the effective mass and perhaps even causes substantial damping. The same question arises with regard to the transient motion of an airplane encountering rough air.

Some initial theoretical and experimental investigations have been made concerning the effect of fuel sloshing on the dynamic stability of rigid airplanes (refs. 1 to 5); however, much more remains to be done. For example, no information as to the effects of fuel sloshing on wing vibrations such as occur in flutter or gusts is available. The theoretical analyses of references 1 to 4 consider the effects of small-amplitude fuel oscillations by means of an equivalent-pendulum concept. This equivalent-pendulum concept, however, appears unadaptable to the analysis of the violent turbulent motion of the fluid likely to be encountered in wing-tip tanks during flutter or gust-excited oscillations. Since any theoretical approach to this turbulent fluid motion appears impracticable at this time, it was considered expedient to make an exploratory experimental investigation of a turbulent case to gain an insight into some of the dynamic effects and fundamental parameters involved.

Although many types of tank motions (such as translatory or rotational, forced or free, steady-state or transient, and combinations thereof) and tank sizes and shapes could be investigated, the present exploratory investigation was limited to a study of suddenly excited bending oscillations by use of some simplified model tip tanks mounted on cantilever beams. Two different beam-tank systems were used. One beam-tank system, designated herein as the large beam-tank system, was used to study the damping effects of the fluid on the beam at a nearly constant frequency. The damping effects of several fluids of different densities and viscosities at various tank fullnesses were studied. The other beam-tank system, designated the small beam-tank system, was designed primarily to study the variations in effective mass of the fluid from cycle to cycle.

APPARATUS AND TEST PROCEDURE

Large Beam-Tank System

The larger of the two beam-tank systems was designed so that the empty mass of the system dominated in determining the frequency of the system; this design permitted damping data to be obtained at an essentially constant frequency throughout a test run in spite of variations
in the effective mass of the sloshing fluid. This system is shown in figure 1 and consists of a uniform steel beam 6 by 1 inches in cross section rigidly mounted on a concrete block with an overhang of 60 inches. The cylindrical tank shown is 7.5 inches in diameter and 10 inches long and is mounted with the tank center line 3 inches from the tip of the beam. The larger of the two rectangular tanks shown in figure 1 was also used. The dimensions of this tank are 10 inches by 9 inches by 5 inches. All tanks were transparent so that the fluid motion could be observed. The beam was set in motion from an upward-deflected position with the fluid at rest. Instantaneous release of the beam was obtained by means of a manually collapsed extension to a hydraulic jack.

The time history of the displacement of the beam at a point directly beneath the center line of the tank was recorded by means of a rigid connecting arm between the beam and a pointed element of the recorder that scribed a revolving drum of wax paper upon which timing marks were also being scribed.

A test run consisted of releasing the beam from an upward-deflected position and recording the ensuing displacement time history of the beam. A typical record is shown in figure 2. All the test runs were started from a 3-inch initial displacement of the tank. Tests were run for several different fluids, various quantities of fluid, different tank shapes, and for a solid mass replacing the tank of fluid. The beam frequency ranged from 3.55 to 4.10 cycles per second depending upon the weight of fluid in the tank and the initial accelerations ranged from 3.9g to 5.3g. This acceleration is computed as explained in the section "Reduction of Data." Table I contains the conditions of each test run.

High-speed motion pictures (124 frames per second) were taken of some test runs in the cylindrical tank. Frames selected at quarter-cycle intervals from the motion pictures of a typical test run are presented in figure 3 for the first 12 cycles of the run.

Estimated accuracy of measurements based on instrument and reading errors are as follows:

Beam displacement (center line of tank), inches ............ ±0.01
Frequency, cycles per second ......................... ±0.05

As previously mentioned, the large beam-tank system was used to study damping effects at a constant frequency. In order to investigate the variation in effective mass of the sloshing fuel and to obtain damping data under variable frequency conditions a second and somewhat similar beam-tank system was constructed. This system is described in the following section.
Small Beam-Tank System

The smaller of the two beam-tank systems was designed so that the frequency of the system when partly full of fluid was sensitive to variations in the effective mass of the fluid. Measurements of the frequency variation during a run allowed the variation in effective mass to be determined. (See section "Reduction of Data"). This system is shown in figure 4 and consisted of a steel beam with a uniform cross section 1 inch by 5/16 inch rigidly mounted on a concrete block with a fixed overhang of 18 inches. The cylindrical tank shown is 4.2 inches in diameter and 6.9 inches long and is mounted with the tank center line 1 inch from the beam tip. The beam was deflected downward 0.47 inch by the weight shown suspended from the beam tip by a small-diameter wire. Instantaneous release of the beam was achieved by cutting this wire. All the tests were made with carbon tetrachloride as the test fluid or with a solid weight on the tip. Carbon tetrachloride was used because its high density gave the greatest variation in frequency from the empty-to-full condition. The frequency of beam oscillations ranged from 8.2 to 19.7 cycles per second and the initial accelerations ranged from 3.2g to 18.7g. Table II lists the conditions of each test run.

The tank center-line deflections were determined by measuring the bending strains at the root of the beam by means of two pairs of electrical strain gages mounted on the top and bottom of the beam. Each pair of gages was incorporated into a Wheatstone bridge circuit and the output of the bridge was recorded together with 0.01-second timing marks. A typical record is shown in figure 5. The recorder galvanometer had a flat frequency response up to 62 cycles per second. The strain gages were calibrated with respect to tank center-line deflections by suspending weights of various size from the beam beneath the tank center line.

Estimated accuracy of measurements based on instrument, calibration, and reading errors are as follows:

Beam displacement (center line of tank), inches . . . . . . . ±0.01
Period of oscillation, seconds . . . . . . . . . . . . . . . . . . . . . ±0.002

REDUCTION OF DATA

From each test record the amplitude, frequency, and acceleration were obtained for each cycle of oscillation. For the large beam-tank system the energy losses per cycle and a viscous damping constant were computed. For the small beam-tank system the energy losses per cycle and the effective mass of the fluid during each cycle were computed. The present section describes the manner in which these quantities were determined.
Large Beam-Tank System

From each recorded time history of the displacement of the large beam-tank system, the maximum upward displacements from the neutral position for successive oscillations of the beam were obtained together with the frequency of the oscillations. Within reading accuracy, the frequency from cycle to cycle was constant (see fig. 2, for example); therefore, the frequency of the oscillations was taken as the average over about ten cycles. The recorded time histories of the displacement were assumed to be damped sinusoidal oscillations, and the incremental accelerations imposed on the tank were computed for the initial downswing of the beam by the following formula

\[ n = \frac{(2\pi f)^2 x}{g} \]

where

- \( n \) acceleration, g units
- \( f \) frequency
- \( x \) peak beam displacement
- \( g \) acceleration of gravity

The measured frequency and initial deflection together with the computed initial acceleration are included for each test in Table I.

The energy lost by the beam during each cycle of the test runs was determined as follows: From the spring constant of the beam (\( K = 90 \) lb/in.) and the peak beam displacements \( x \), the energy stored in the beam at each peak displacement was computed from the formula \( \frac{Kx^2}{2} \). The difference in the energy content of the beam for successive cycles was then obtained. These differences in energy content of the beam represented the total energy taken from the beam during each cycle by the fluid, and by the inherent damping of the beam (both structural and aerodynamic) and recorder. The energy per cycle dissipated by the beam and recorder alone was determined by making similar calculations from the displacement curve of the beam with the tank empty. The energy per cycle taken from the beam by the fluid alone was then found by subtracting from the total energy loss per cycle of the beam the energy dissipated by the beam and recorder alone at amplitudes corresponding to the partly filled tank displacements.
Although the test results show that the damping is not viscous, the quantity $\delta$, which is based on the viscous-damping concept, was computed for each cycle of some of the runs by the approximate formula

$$\delta = \frac{x_n - x_{n+1}}{x_n},$$

where $x$ is the amplitude of vibration at the beginning of each cycle and $n$ is a subscript denoting cycle number. For small values of damping, $\delta$ is a good approximation to the logarithmic decrement.

Small Beam-Tank System

From solid-weight test records the frequency of the beam for various solid weights was obtained and a plot of the frequency against tip weight was constructed (fig. 6). The zero-weight condition used in this plot is taken as the weight present when the empty tank is attached to the beam. The tank fullnesses in percent for carbon tetrachloride corresponding to the solid tip weights are also indicated along the lower abscissa.

For each record obtained from the partly filled tank tests (a typical record is presented in fig. 5), the peak downward displacements of the beam and the time between these peak displacements were read and the frequency for each cycle computed. The peak upward displacements of the beam were also read and, a damped sinusoidal motion being assumed for each cycle, were used in conjunction with the frequency of that cycle to compute the peak acceleration. The initial acceleration was computed from the frequency of the first cycle together with the initial displacement, which was 0.47 inch for all the test runs. The initial acceleration, the frequency of the system with the fluid replaced by a solid weight equal to the weight of fluid, and the maximum frequency obtained for any cycle during the test run are presented in table II.

The solid weight corresponding to the measured frequency of each cycle during the tests with carbon tetrachloride was obtained from the plot of figure 6. This solid weight is equivalent to that part of the total fluid weight which is influential in determining the frequency of the system and is hereinafter called the effective weight. For example, assume that the tank is 40 percent full of fluid. According to the plot, if all the fluid acted as a solid weight, the frequency of the beam-tank system should be 11.6 cycles per second; however, the frequency obtained from the strain-gage record during the third cycle was 14.3 cycles per second. This frequency corresponds to the solid-weight equivalent of the tank only 20 percent full. This result is interpreted to mean that the fluid in the tank was only 50 percent effective during the third cycle.
The total energy lost by the beam during each cycle, the amount of this energy lost to the fluid, and the amount lost to inherent beam damping were computed in the same way as for the large test system.

PRESENTATION AND DISCUSSION OF RESULTS

Large Beam-Tank System

As mentioned previously, the test conditions for each test of the large beam-tank system are presented in table I. In order to indicate the nature of the fluid motion and damping obtained during the test, the results of a representative test are presented in the following several ways; the envelope curve for the displacement time history, the energy loss per cycle, an approximate logarithmic decrement for each cycle, and high-speed motion pictures of the fluid motion are each presented. Comparisons of envelope curves showing the effects of tank fullness, fluid density and viscosity, and tank shape are also presented.

An envelope curve for a representative recorded oscillation (test 1) for the large beam-tank system is shown in figure 7(a). Peak displacements of the beam from the static position are plotted against cycle number for the beam released from an initial displacement of 3 inches with the cylindrical tank 40 percent full of water by volume. In order to show the consistency of the test results the peak displacements from the record presented in figure 2 (test 2) and another comparable run (test 3) are also presented. Except for the low beam amplitudes the results are very consistent. Also shown is the envelope curve obtained when this tank of fluid was replaced by an equivalent solid weight. This solid-weight curve shows the effects of the inherent damping (beam hysteresis, aerodynamic effects, and recorder friction) of the system on the peak displacements. The more rapid decay of the curve for the case of the tank partly filled with fluid as compared with the decay for the solid-weight curve is indicative of the additional damping caused by the fluid.

Figure 7(b) shows the energy lost by the beam during each cycle for the envelope curve for the partly filled tank presented in figure 7(a). The total height of the bars indicates the total energy lost by the beam. The hatched portions of the bars indicate the energy dissipated by the inherent damping of the beam at amplitudes corresponding to the displacement curve for the partly filled tank. The open portions of the bars therefore represent the energy imparted to the fluid by the motion of the beam.

The results shown in both figure 7(a) and figure 7(b) indicate that during the first cycle the tank of fluid absorbed only a small
amount of energy; that is, almost all the energy loss was due to the inherent damping of the system. The high-speed motion pictures accounted for this effect (see frames for a typical run presented in fig. 3) by showing that in spite of the 4 g acceleration imposed on the fluid during the initial downswing of the beam, the fluid adhered to the bottom of the tank and remained essentially undisturbed during the first cycle. During the next two cycles, the fluid became increasingly more turbulent and absorbed maximum energy at the third cycle even though the amplitude had decreased appreciably. Thereafter, the energy imparted to the fluid decreased as the amplitude decreased.

It should be noted that the cycle-by-cycle energy losses indicated in figure 7(b) are the energies imparted to the fluid by the beam and are not necessarily the energies actually dissipated by the fluid in the form of heat. The amount of the energy dissipated by the fluid and the amount still in the fluid in the form of turbulence could not be ascertained. During the second cycle, when the fluid is first becoming turbulent, a large proportion of the energy is probably only stored and not dissipated, but the regular decrease of energy imparted to the fluid with decrease in amplitude shown after the third cycle and the rapidity with which the fluid became quiet when an incremental acceleration below 1 g was reached indicate that the energy dissipated by the fluid cycle by cycle is probably very near to that shown in figure 7(b), the first two cycles being ignored. Under transient conditions, therefore, very little damping can be expected from the fluid during the first cycle because the fluid does not become agitated immediately. Also, after the fluid is fully agitated at a given frequency, the energy absorbed depends upon the amplitude or accelerations imposed on the fluid by the beam. Although this curve was obtained for one set of test conditions, all the test runs exhibited similar characteristics.

In figure 7(c), the quantity $\delta$ is plotted against cycle number for the three tests of figure 7(a). For viscous damping $\delta$ would be constant with cycle number. Figure 7(c) shows that $\delta$ varies considerably from cycle to cycle, and results from tests made for other tank fullnesses and frequencies show an even greater variation of $\delta$ from cycle to cycle. Thus, it is obvious that the damping present in the beam-tank system cannot be described by the viscous-damping concept.

Figure 8(a) shows the envelope curves obtained for various quantities of water with the beam released from the same initial deflection. The quantity of water was varied from empty to full in intervals of 10 percent. As can be seen from the key in figure 8 and from table I, the frequency of the system varied somewhat from test to test because of the change in quantity of water and, since the initial deflection was constant, the initial acceleration also varied from test to test. These variations, however, probably had only a small effect on the trend presented.
For clarity of presentation, these envelope curves are replotted in figure 8(b) as the amplitude of vibration against tank fullness. The various curves refer to different cycles of oscillation. On this plot the lower the amplitude attained after a given number of cycles the more effective has been the fluid in damping the beam. Thus, the effectiveness of the fluid in damping the beam increased rapidly as the quantity of fluid was varied from 10 percent to 30 percent full but for 30-, 40-, and 50-percent-full tanks the beam was damped about the same amount for each depth. As the tank fullness was varied from 60 to 90 percent full the effectiveness of the fluid in damping the beam decreased. Thus, as might be expected, tank fullness is an important factor in the damping of a beam-tank system.

Although it has been shown in figure 7(c) that the damping obtained with the partly filled tank does not follow the law of viscous damping from cycle to cycle, a perspective as to the over-all magnitude of the damping being obtained might be gained by computing the viscous-damping coefficient required to decrease the beam amplitude a given amount in the same number of cycles as that required by the experimental beam-tank system. For example, figure 8(b) shows that the tank 30 percent full of fluid has damped the beam from an amplitude of 3.0 inches to 0.4 inch in eight cycles. The nondimensional viscous-damping coefficient required to damp the beam to the same amplitude in the same number of cycles is \( \frac{C}{C_C} = 0.08 \). If the beam-tank system were a scale model of a full-scale wing-tip tank system, this nondimensional damping coefficient of \( \frac{C}{C_C} = 0.08 \) would apply to the full-scale system. According to references 6 and 7, the structural damping of a metal airplane wing can be expected to be within the limits of \( \frac{C}{C_C} = 0.01 \) to \( \frac{C}{C_C} = 0.05 \), with most values on the low side; reference 8 presents test results which show much lower values of \( C/C_C \), ranging from 0.002 to 0.006 depending on the amplitude of wing vibration. Thus, the damping obtained for the beam-tank (or wing-tip tank) system when the tank is 30 percent full would correspond to an over-all viscous-damping coefficient several times that usually obtained from structural damping in an airplane wing. Since the scale laws governing the beam-tank system partly filled with fluid are not known, such a comparison of damping coefficients at this stage is not valid and a full-scale beam-tank system may have more or less damping; however, possibilities of substantial damping for a full-scale partly filled beam-tank system are indicated.

In order to study the effects of the viscosity and density of the fluid upon damping, three fluids other than water were tested. The fluids tested were benzene, linseed oil, and carbon tetrachloride. The key of figure 9 shows the values of viscosity and density for each of these fluids. The results from three tests for each fluid with the tank 40 percent full are presented in figure 9 in the form of envelope curves. The tests for the various fluids were all started from the same initial
amplitude and, except for carbon tetrachloride, all had approximately the same frequency and initial acceleration. The large density of the carbon tetrachloride resulted in a lower frequency of the beam-tank system which, in turn, caused the lower initial acceleration. For the purposes of this paper, however, this test is considered comparable with the others.

The curves of figure 9 indicate that at the end of a given cycle (except at the lower amplitudes) the greater the fluid density the more effective has been the fluid in damping the beam. Thus, in spite of the large variation in the viscosity of the fluids (approximately 0.5 that of water for benzene to 33 times that of water for linseed oil) the spread of the curves seems to be due entirely to the differences in fluid density with the viscosity having no apparent effect.

In order to get an indication of the sensitivity of the results thus far presented to tank shape, the cylindrical tank was turned from its normal horizontal position (fig. 1) to an upright position and tested. The larger of the two rectangular tanks shown in figure 1, which had the same volume as the cylindrical tank, also was tested. The envelope curves for both tanks 40 percent full are presented in figure 10. The envelope curves are noted to be affected only slightly. Thus, the damping obtained from the sloshing fluid does not appear to be very sensitive to the tank configuration. Of course, more radical changes in shape, addition of baffles, and so forth, might change these results.

Small Beam-Tank System

The test conditions for each test run of the small beam-tank system are presented in table II. As in the case of the large beam-tank system, results are presented for a representative test. A comparison of envelope curves is made for various conditions of tank fullness. Plots showing the variation in effective weight from cycle to cycle for various conditions of tank fullness and a summary plot showing the minimum effective weight obtained for each tank fullness are also presented.

A typical envelope curve for the smaller test system (for which the test conditions are the same as for the record presented in fig. 5) is shown in figure 11(a). The results presented in the figure are for the tank 40 percent full of carbon tetrachloride. Also shown is the envelope curve obtained when this tank of fluid was replaced by an equivalent solid weight. This solid-weight curve shows the effects of the inherent damping (beam hysteresis and air-damping effects) of the system on the peak displacements. Thus, a comparison of the two curves shows the effects of the damping caused by the fluid.
Figure 11(b) shows the energy lost by the beam during each cycle for the partly filled tank envelope curve presented in figure 11(a). The total height of the bars indicates the total energy lost by the beam. The hatched portions of the bars indicate the energy dissipated by the inherent damping of the beam at amplitudes corresponding to the displacement curve for the partly filled tank. Thus, the open portions of the bars represent the energy imparted to the fluid by the motion of the beam.

The action of the small system was similar to the action of the large system in that the fluid adhered to the tank during the first cycle, took additional cycles to become fully turbulent, and then absorbed energy at a rate which decreased with decreases in the amplitude of the beam. All the test runs for the smaller system exhibited similar characteristics.

Figure 12 shows a cross-plot of the envelope curves obtained for other quantities of carbon tetrachloride with the beam released from the same initial deflection. As in the cross plot for the larger system, the ordinate is the amplitude, the abscissa is the tank fullness and the various horizontal curves refer to different cycles of oscillation. A comparison of figure 12 with figure 8(b) indicates that, after a given number of cycles, the amplitude did not vary as much with tank fullness for the small system as it did for the large system. It should be noted, however, that, whereas for the large system the frequency was essentially constant for all quantities of fluid (fig. 8(a)), in the small system the frequency varied considerably with tank fullness.

Plots showing the variation in effective weight of the fluid (that part of the total fluid weight which is influential in determining the frequency of the system) from cycle to cycle for the tank filled to various percentages full are presented in figure 13. For each tank fullness, the effective weight of the fluid in percent of the total fluid weight present is plotted against cycle number. The peak incremental accelerations occurring during each cycle are also indicated along the abscissa. The points from three tests are presented on each plot and it can be seen that a consistent trend of the results was obtained from run to run. For the case of the tank 40 percent full (fig. 13(d)), it can be seen that during the first cycle the fluid was almost 100 percent effective. This result checks with the observation, made previously for the damping tests, that the fluid adheres to the tank during the first cycle. During the third cycle, however, the fluid became only about 50 percent effective because of the splashing and turbulence. In subsequent cycles, although the amplitude of the beam was decreasing from cycle to cycle because of the damping, the same percentage effectiveness was maintained until the acceleration became less than 1 g. For accelerations below 1 g the fluid became almost 100 percent effective again.
This trend was noted for all tests, but the percentage effectiveness of the fluid reached during the cycles following the first changed with the tank fullness.

A summary plot showing the variation in the minimum effective weight of the fluid with tank fullness is presented in figure 14. For each tank fullness, the data for the two or three cycles for which the effective weight was lowest are presented. Except for the upper and lower parts of the curve the percentage effectiveness of the fluid increased almost linearly with tank fullness.

SUMMARY OF RESULTS

An exploratory investigation of the dynamic effects of fuel sloshing on wing bending motion was made by use of simplified model tip tanks mounted on cantilever beams which were allowed to oscillate freely after release from a deflected position. The results of this investigation indicated that partly full beam-tank systems may be damped substantially by fuel sloshing and that the effective weight of the fuel may vary considerably under certain conditions of tank oscillation. No effects of fluid viscosity were observed, but for a given beam-tank system the density of fluid and tank fullness were found to be important parameters. High-speed motion pictures showed that the fluid remained relatively undisturbed during the first cycle, entered violent sloshing motion thereafter, and reached maximum turbulence at about the third cycle. Correspondingly, the damping varied from nearly zero for the first cycle to its maximum at about the third cycle and the effective mass varied from nearly 100 percent during the first cycle to its minimum effectiveness at about the third cycle.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., June 3, 1952.
REFERENCES


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TABLE II
TEST CONDITIONS AND DATA FOR SMALL BEAM-TANK SYSTEM

[Initial deflection, 0.47 in.; fluid used is carbon tetrachloride]

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<th>Test</th>
<th>Tank fullness, percent</th>
<th>Equivalent solid-weight frequency, cps</th>
<th>Maximum frequency, cps</th>
<th>Initial acceleration, g units</th>
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¹Frequency of system with fluid replaced by solid weight equal to weight of fluid or for fluid 100 percent effective.
Figure 2.- Typical record for large beam-tank system. Test 2; tank 40 percent full.
Figure 3.- Motion picture record of fluid motion for large beam-tank system 40 percent full.
Cycle number

7
8
9
10
11
12

Figure 3.- Concluded.
Figure 4 - Small beam-tank system.
Figure 6.- Variation of frequency with tip weight for small beam-tank system.
(a) Envelope curves for partly filled tank and equivalent solid weight.

(b) Energy lost by beam during each cycle.

Figure 7.- Typical result of damping test for large beam-tank system.
(c) Variation of $\delta$ for large beam-tank system.

Figure 7.- Concluded.
Figure 8.- Effects of tank fullness on damping.
Figure 8. - Concluded.
Figure 9.- Effects of fluid viscosity and density.
Figure 10. Effects of tank shape, large beam-tank system. (All tanks 40 percent full except as noted.)
(a) Envelope curves for partly filled tank and equivalent solid weight.

(b) Energy lost by beam during each cycle.

Figure 11.- Typical result of damping test for small beam-tank system.
Figure 12.- Cross plot of envelope curves. Small beam-tank system.
Figure 13.- Variation of effective fluid weight with cycle number.
Figure 13.- Continued.
Figure 13. - Continued.
Approx. incremental acceleration, g

(h) Tank 80 percent full.

(g) Tank 70 percent full.

Test 22

Test 21

Figure 13.- Concluded.
Figure 14.- Variation of effective weight of fluid with tank fullness.
(Test points are included for correlation with data from fig. 13.)
An exploratory investigation of the dynamic effects of fuel sloshing in tip tanks on wing bending motion was conducted with two simplified model beam-tank systems. Envelope curves to beam-displacement time histories obtained after release from a delected position are compared and show the effects of variation in tank fullness, fluid density, fluid viscosity, and tank shape. Some variations of fluid weight effective from cycle to cycle are also presented. The results of these tests indicate that after several cycles substantial damping may be obtained from

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(over)
fuel sloshing in a tip tank and that the effective mass of the fuel may vary considerably under certain conditions of tank oscillation. The viscosity of the fluid did not affect the damping or inertia characteristics obtained but, for a given beam-tank system, the density of fluid and tank fullness were important parameters.

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