PRESSURE IN A SHOCK WAVE
WITH AN INTENSE SPARK DISCHARGE

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SHOCK WAVE PRESSURE WITH AN INTENSE SPARK DISCHARGE IN WATER

Following is the translation of an article by I.M. Astrakhan entitled "Davleniya na Udarnoy Volne pri Silnom Iskrovom Razryade v Vode" (English version above) in Izvestiya Vysshikh Uchebnykh Zavedeniy - Neft' i Gaz (News of Higher Institutions of Learning - Petroleum and Gas), No. 10, Baku, 1959, pages 87 - 92.

A method is explained in this article for computing the pressure at a shock wave with a spark discharge in water, based on the solution of a self-modeled problem of a strong detonation in a compressible medium.

An underwater spark discharge is widely used as a source of pressure in water (1). The problem of the strong point explosion in an ideal gas and in an incompressible fluid has been solved by L.I. Ledov (2). The self-modeled singular problem of a point explosion in a compressible fluid, like water, has been examined by N.N. Kochina and N.S. Mel'nikova (3). S.A. Khristianovich has examined the shock wave in water, where the pressure at the wave front did not exceed 300 atm. (4).
At short distances from the discharge channel axis the shock wave is propagated with cylindrical symmetry corresponding to the symmetry of the discharge, while at large distances with spherical symmetry, as from a point source. It is in connection with this that we have solved the self-modeled problem of the detonation with a cylindrical wave and converted S.A. Krishianovich's solution for an explosive to the case of a spark discharge. In the intermediate region we will use the method of graphic interpolation.

As it has been shown in (1), in order for the problem of the strong detonation in a compressible medium to be self-modeled, it will suffice that the internal energy of the medium

$$s(p, p) = \frac{P}{\rho_0} \left( \frac{L}{L_0} \right) + \text{const},$$

(1)

where $\varphi$ is an arbitrary function of its argument. In this case the adiabatic equation

$$p = \psi(S) \rho \left( \frac{L}{L_0} \right),$$

(2)

where $\psi(S)$ is a certain function of entropy $S$.

The connection between $\varphi(R)$ and $\psi(R)$ is determined by the formulas

$$\varphi(R) = \frac{1}{\psi(R)} \left( c + \int \frac{x(R)}{R_x} dR \right),$$

$$x(R) = \frac{C}{\psi(R)} \exp \left[ \int \frac{dR}{R_x \psi(R)} \right],$$

(3)

where $c$ is an arbitrary constant,

$$R = \frac{p}{\rho_0}.$$

The equations of one-dimensional, nonsteady motion of the ideal compressible medium have the form

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{1}{\rho} \frac{\partial \rho}{\partial x} = 0,$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} + (v - 1) \frac{\rho v}{R} = 0,$$

(4)
\[
\frac{\partial}{\partial t} \left( \frac{p}{(p_0)^{\nu}} \right) + \nu \frac{\partial}{\partial x} \left( \frac{p}{(p_0)^{\nu}} \right) = 0,
\]

where \( \nu = 1 \) for plane waves, \( \nu = 2 \) for cylindrical and \( \nu = 3 \) for spherical waves.

We will consider that at moment \( t = 0 \) an explosion occurs (i.e. a finite energy \( E_o \) is instantaneously released) in a resting compressible medium at the center of symmetry and that the detonation is intense (i.e. the pressure \( p_o \) in the undisturbed medium can be neglected in comparison with the pressure at the shock front).

In crossing the surface of the intense explosion the conditions of conservation of mass, angular momentum and energy flow should be fulfilled. These conditions are the limits for equations (4):

\[
\begin{align*}
-\rho_1 c &= \rho_2 (v_1 - c), \\
\rho_1 c^2 &= \rho_2 (v_2 + c)^2, \\
\frac{1}{2} c^2 &= \frac{1}{2} (v_2 - c)^2 + \rho_2 + \rho_2 \frac{\rho_2}{\varphi} \left( \frac{r_2}{p_0} \right),
\end{align*}
\]

(5)

where \( c \) is the velocity of the shock wave.

Thus, there enter into the conditions of the problem only two constants with independent dimensions: \( p_0 \) and \( E_o \)

\[
[p_0] = ML^{-1} \quad [E_o] = ML^{2-1} T^{-2}.
\]

Consequently, the problem becomes self-modeled (1).

We will assume

\[
\begin{align*}
\varphi &= \frac{r}{l} V(\lambda), \\
\rho &= \rho_0 R(\lambda), \\
\varphi &= \frac{\rho}{\rho_0} \varphi(\lambda, \left( \lambda = \frac{r}{r_2} \right)),
\end{align*}
\]

(6)

where \( r_2 \) is the radius of the shock wave.

For cylindrical symmetry
\[ r_2 = \left( \frac{E}{p_0} \right)^{\frac{1}{2}} t^{1/2}. \]  

The velocity of the shock front

\[ c = \frac{1}{2} \left( \frac{E}{p_0} \right)^{1/4} t^{1/4} = \frac{1}{2} \sqrt{\frac{E}{p_0}} \frac{1}{r_2^{1/4}}, \]  

where \( E \) is a certain constant with energy dimensions proportional to the energy of the explosion

\[ E_0 = \alpha E. \]

From (5), (6) and (7) we obtain

\[ R_1(R_2) = \frac{1}{2} \left( 1 - \frac{R_2}{R_1} \right), \]

\[ V_1 = \frac{1}{2} \left( 1 - \frac{R_2}{R_1} \right), \]

\[ P_1 = \frac{1}{2} R_1 V_1. \]  

The first of these conditions gives

\[ R_2 = \frac{R_1}{1 + 2 R_1 R_2(R_2)}. \]  

At the shock wave we obtain

\[ p_2 = p_1 \left[ 1 + 2 R_1 R_2(R_2) \right], \]

\[ v_2 = \frac{2 c R_1 R_2(R_2)}{1 + 2 R_1 R_2(R_2)}, \]

\[ p_2 = \frac{2 c R_1 R_2(R_2)}{1 + 2 R_1 R_2(R_2)}. \]  

Usually the equation of the state of water is presented in the form (2)

\[ p = \varphi(S)(p^0 - p^0). \]  

Then \( \varphi(R) = R^N - 1 \) and from (3) we obtain an expression for \( 0 < R \) which can to a certain degree of accuracy approximate the function
\[ q(R) = \frac{R - 1}{2R^2}. \]  

(13)

We now compute the pressure at the shock wave. We assume

\[ \rho_0 = 0.938 \text{ g/cm}^3, \quad \rho_1 = 0.999 \text{ g/cm}^3. \]  

(14)

This means \( R_1 = 1.066, \quad R_2 = 1.816. \)  

(15)

Under these conditions \( \alpha = 0.01. \)

Considering (10), (11) and (13) we obtain

\[ \rho_2 = \rho_0 \frac{c^2 R_1}{R_2} \frac{R_2 - 1}{R_2} = \frac{\rho_0 c^2 (R_2 - 1)}{2 R_2 - 1}. \]

(16)

From (8)

\[ \rho_2 = \frac{\rho_0 E_0 (R_2 - 1)}{4 \alpha_0 (2 R_2 - 1) r_2}. \]

Substituting the data from (14) and (15) here, we obtain

\[ \rho_2 = \frac{8.2 E_0}{r_2^2}. \]  

(17)

We will compute the detonation energy \( E_0 \) for the data of an experiment made by the Chair of "Trade Machines and Mechanisms" of Moscow Institute of the Petroleum and Geos Chemical Industries imeni Academician I.M. Gubkin, when

\[ C = 1.5 \mu \text{f}, \quad U = 50 \text{ kv}, \quad d = 6 \text{ cm}. \]  

(18)

where \( C \) - the capacity of the capacitor; \( U \) - is the voltage at the electrodes; \( d \) - is the distance between the electrodes, we will assume

\[ E_0 = \frac{C U^2}{2d} = 0.31 \times 10^{10} \text{ erg/cm}. \]  

(19)

Thus, according to (17)

\[ \rho_2 = 0.28 \times 10^5 r_2^{-2} \text{ atm}. \]  

(20)

The results of computing \( p_2 \) by formula (20) are as follows:

\[ r_2, \text{ cm} = 3; \quad 5; \quad 8; \]

\[ p_2, \text{ atm} = 3000; \quad 1133.3; \quad 442.6. \]
At considerable distances from the place of the explosion, the shock wave is propagated with spherical symmetry. With low pressures, the equation of state for water is taken in the form (3)

\[ p = B \left[ \left( \frac{p}{p_0} \right)^{n} - 1 \right]. \]  

(21)

where \( p_0 \) is the density at pressure equal to zero,

\[ B = 3045 \text{ kg/cm}^2, \quad n = 7.15. \]

\[ \frac{1}{r} \frac{\partial}{\partial t} \left( rv \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( rv \right) + \frac{1}{r} \frac{\partial}{\partial t} \left( r \frac{\partial \theta}{\partial r} \right) = 0. \]

(22)

The velocity of propagation of sound is equal to

\[ a = \sqrt{\frac{\rho}{p}} = a_0 \left( \frac{p}{p_0} \right)^{-\frac{1}{2}}, \]

where

\[ a_0 = \sqrt{\frac{Bn}{p_0}}. \]

(23)

Equations (22) are linear for the case of low pressures. The solutions of these equations which depend solely on \( \frac{r}{t} \) when \( p_1 = 0 \) have the form

\[ v = a_0 \frac{p}{Bn}, \]

\[ \frac{r}{t} = a_0 \left[ 1 + \frac{n + 1}{2} \frac{p}{Bn} \ln \frac{p_0}{p} \right]. \]

(24)

where \( p_0 \) is an arbitrary constant.

If a spherical shock wave is propagated in a resting fluid, then at short distances the spread velocity of the shock front will equal
and the velocity of the particles directly behind the wave front
\[ u = \frac{a_v}{B_n}. \]

Differentiating (24) we have
\[ dr = a_v \left[ 1 + \frac{n + 1}{2} \frac{p}{B_n} \ln \frac{p_0}{p} \right] dt + a_v \frac{n + 1}{2B_n} \ln \frac{p_0}{p} \left[ \ln \frac{p_0}{p} - 1 \right] dp. \]

Together with this at the shock front
\[ dr = a_v \left[ 1 + \frac{n + 1}{2} \frac{p}{4B_n} \right] dt. \]

To determine the spread velocity of the front, we have consequently the equation
\[ \frac{1}{2} \rho c dt = \rho \ln \frac{p_0}{p} dt + t \ln \frac{p_0}{p} \left[ \ln \frac{p_0}{p} - 1 \right] dp. \]

Whence
\[ t = \frac{C}{\rho \ln \frac{p_0}{p} - 0.5}, \]

where \( C \) is an arbitrary constant.

Substituting in (24) we obtain
\[ r = C a_v \left( 1 + \frac{n + 1}{2} \frac{p}{B_n} \ln \frac{p_0}{p} \right) \frac{p}{\rho \ln \frac{p_0}{p} - 0.5}. \]

We will assume
\[ C a_v = D\beta \left( \frac{p_0}{\rho_0} \right)^{\frac{1}{3}}, \]

where \( \beta \) is a dimensionless constant.

The expression \( \left( \frac{E_0}{\rho_0 a_0^2} \right)^{1/3} \) has the dimension of length.
Then

\[
\frac{r}{\beta}\left(\frac{E_0}{\rho_0 a_0^2}\right)^{1/3} = \frac{D}{\rho} \left(1 + \frac{\beta + 1}{\rho} \frac{P}{\rho} \ln \frac{P}{\rho} \right)
\]

\[
\beta \left(\frac{E_0}{\rho_0 a_0^2}\right)^{1/3} = r_0
\]

We will designate \(\beta \left(\frac{E_0}{\rho_0 a_0^2}\right)^{1/3} = r_0\).

We assume \(p_0 = 17000\) atm., \(D = 16200\) atm. according to (3).

Then

\[
r = \frac{r_0}{\rho_0} = 16200 \frac{1 + 1.87 \cdot 10^{-4} \rho \ln \frac{17000}{p_2}}{\rho_0}
\]

or approximately

\[
p_2 = \frac{16200}{r} \left(1 + \frac{4(\ln r)^{1/3}}{r} \right)
\]

The parameter of 3 may be determined, knowing \(p_2\) for any single distance \(r_2\) from the center of the detonation with a given \(E_0\). For the above mentioned experiment made at the Moscow Institute of Petroleum and Gas Chemical Industries im. I.M. Gubkin, when \(E_0 = 1.87 \cdot 10^{10}\) erg, \(p_0 = 0.938\) g/cm\(^3\), \(a_0 = 1500\) m/sec, \(\beta = 0.3\), we have \(r_0 = 0.28\) cm.

Thus

\[
r = \frac{r_0}{p_0} = 16200 \frac{1 + 1.87 \cdot 10^{-4} p_2 \ln \frac{17000}{p_2}}{p_2}
\]

\[
\frac{1}{\sqrt{\ln \frac{17000}{p_2} - 0.5}}
\]

(30)
The results of computing $p_2$ according to formula (30) are the following:

$r_2$, cm: 8.4; 11.2; 14.0; 16.8; 25.2; 33.6; 50.4; 67.2; 154.0

$p_2$, atm: 364.0; 250.0; 188.0; 150.0; 91.0; 65.0; 40.5; 29.2; 19.0.

The calculated dependence of pressure at the shock wave on its radius for this particular experiment is depicted in the figure.

BIBLIOGRAPHY


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