A THEORETICAL INVESTIGATION OF THE EFFECT OF PARTIAL WING LIFT ON HYDRODYNAMIC LANDING CHARACTERISTICS OF V-BOTTOM SEAPLANES IN STEP IMPACTS

By Joseph L. Sims and Emanuel Schnitzer

Langley Aeronautical Laboratory
Langley Field, Va.
A THEORETICAL INVESTIGATION OF THE EFFECT OF PARTIAL WING LIFT ON HYDRO_DYNAMIC LANDING CHARACTERISTICS OF V-BOTTOM SEAPLANES IN STEP IMPACTS

By Joseph L. Sims and Emanuel Schnitzer

SUMMARY

A theoretical investigation is made of the motions and hydrodynamic loads experienced during the impact of prismatic V-bottom seaplanes in the step-landing condition where the wing lift is a constant fraction of the weight and the resultant velocity is normal to the keel. An approximate method is given for applying the results of this investigation to the more general case of oblique impact. This method involves obtaining an equivalent normal impact for any given oblique impact and then assuming that the percentage change in load due to a change in wing lift is the same in both oblique and normal impacts.

Equations are presented which relate the load and motion variables throughout a normal impact and it is shown that these variables may be expressed as dimensionless quantities which are related by a single parameter $\lambda$. This parameter depends on the unbalanced wing lift force, the initial conditions of the impact, and the hydrodynamic characteristics of the seaplane.

The results of the investigation are presented in the form of dimensionless plots which may be used directly to determine the loads, motions, and hydrodynamic pitching moments at any instant of the impact. These results suggest that the increase of hydrodynamic load is approximately 1.33 percent of the decrease in air load.

INTRODUCTION

The present paper is concerned with an evaluation of the effects of reduced wing lift during water landings of wide-beamed prismatic V-bottom seaplanes. Previously hydrodynamic theory was developed for oblique impacts in which the wing lift was equal to the weight of the seaplane for the entire range of trim and flight-path angles. (See, for example, ref. 1.) This theory has been checked experimentally and was found to
agree fairly well with the experimental results. A solution of seaplane impact equations for partial wing lift was presented in reference 2. This solution, however, was obtained by assuming the float to be of infinite length, to enter the water at zero trim, and to have the mass and wing lift uniformly distributed along its length.

The purpose of this paper is to present the solution of equations for seaplane impacts with constant partial wing lift where the resultant velocity is normal to the keel and to discuss the applicability of these results to the more practical case of oblique seaplane impact with partial wing lift.

The present investigation differs from the treatment given in reference 2 for partial wing lift in that the theoretical analysis of this paper is made for positive trims; whereas the analysis of reference 2 was made for 0° trim. The proposed normal-impact theory permits a closed-form solution to be made from which generalized curves can be constructed. Although a similar analysis could be made for the case of oblique impact with partial wing lift, such an analysis would require numerical methods for solution and would lead to a greater number of less general curves. The results of the normal-impact analysis can be used, however, in treating approximately the oblique case by the simple method described in the text. The use of this simple method would result in a substantial time saving over that required by the use of a detailed numerical procedure.

SYMBOLS

\( C_d \) nondimensional draft coefficient, \( z \Lambda \)

\( C_{F_v} \) nondimensional vertical hydrodynamic-force coefficient, \( \frac{F_{Vg}}{Wz_0^2 \Lambda} \)

\( C_t \) nondimensional vertical acceleration coefficient, \( -\frac{\ddot{z}}{z_0^2 \Lambda} \)

\( C_{m_s} \) nondimensional pitching-moment coefficient about the step-keel point, \( \frac{M_s (g \Phi (A))}{z_0^2 (W \varphi_m (A))} \sin \tau \cos \tau \)

\( C_t \) nondimensional time coefficient, \( t z_0 \Lambda \)
$F_v$ vertical component of hydrodynamic force

$g$ acceleration due to gravity

$L$ seaplane wing lift in vertical direction (a constant percentage of the seaplane weight)

$M$ pitching moment

$t$ time after contact

$W$ seaplane weight

$z$ draft of keel at step, normal to undisturbed water surface

$\frac{z}{z_0}$ vertical-velocity ratio

$\beta$ angle of dead rise, deg

$f(\beta)$ dead-rise function

$\gamma$ flight-path angle relative to undisturbed water surface, deg

$\kappa$ approach parameter, $\frac{\sin \tau}{\sin \gamma_0} \cos(\tau + \gamma_0)$

$\Lambda$ impact geometry constant, $\left( \frac{g}{W} \frac{f(\beta)}{\varphi(A) \rho} \right)^{1/3}$

$\lambda$ unbalanced-lift-force parameter, $\left( 1 - \frac{L}{W} \right) \frac{g}{z_0^{2/3} \Lambda}$

$\rho$ mass density of water

$\tau$ trim

$\varphi(A)$ aspect-ratio (end-flow) correction to total hydrodynamic load

Subscripts:

$e$ effective value

$f$ oblique-impact case
condition of wing lift equal to weight

condition of wing lift less than weight

referring to pitching-moment solution

initial conditions

conditions at step

A dot over a symbol represents differentiation of that parameter with respect to time.

ANALYSIS

Basis of Theory

In the present analysis the following assumptions are made: The impact is assumed to occur at constant trim with no penetration of the chines below the water surface; the resultant velocity is assumed to be normal to the keel with the wing lift vertical and equal to a constant fraction of the seaplane weight throughout the impact.

The vertical component of the hydrodynamic force for an impact where the resultant velocity is normal to the keel is given by the following relation:

\[ F_v = \frac{[f(\beta)]^2 \varphi(A) \rho \kappa (z^3 \dot{z}^2 + 3z^2 \ddot{z}^2)}{6 \sin \tau \cos^2 \tau} \]  

which was obtained from equation (12) of reference 1 for this case, where

\[ f(\beta) = \frac{\pi}{2\beta} - 1 \]

and

\[ \varphi(A) = 1 - \frac{\tan \tau}{2 \tan \beta} \]
Equations of Motion

If the foregoing assumptions regarding wing lift are used, the equation of motion is determined by the application of Newton's second law to this hydrodynamic-force equation. The equation of motion then becomes

\[- \frac{W}{g} \ddot{z} = \frac{f(\beta)}{6 \sin \tau \cos^2 \tau} \phi(A) \rho \left( 2 \frac{3z_0^2 + 3z_0^2}{z_0^2} \right) - W\left(1 - \frac{L}{W}\right) \]  

(2)

This equation expresses the general relationship that exists among the variables at any time during the impact. The equation may be expressed as

\[(1 + \Lambda^3 \dot{z}) \ddot{z} + 3\Lambda^3 \dot{z}^2 = g \left(1 - \frac{L}{W}\right) \]  

(3)

where

\[\Lambda = \left(\frac{g}{W} \frac{f(\beta)}{6 \sin \tau \cos^2 \tau} \phi(A) \rho \right)^{1/3}\]

and can be integrated to obtain the following relationship between the draft and the vertical velocity:

\[(1 + \Lambda^3 \dot{z}) \ddot{z}^2 = 2 \left(1 - \frac{L}{W}\right) g \left( z + \Lambda^3 \frac{z_0^4}{4} \right) + \dot{z}_0^2 \]  

(4)

When equation (4) is integrated, the relationship between draft and time is obtained:

\[z + \Lambda^3 \frac{z_0^4}{4} = \left(1 - \frac{L}{W}\right) \frac{g}{2} t^2 + t \dot{z}_0 \]  

(5)

Equations (3), (4), and (5) may be expressed in terms of the following nondimensional variables which were introduced in reference 1:

\[C_l = -\frac{\ddot{z}}{\dot{z}_0^2 \Lambda} \]  

(6)
\[ \text{Equations (3), (4), and (5) then become, respectively,} \]
\[ -\left(1 + C_d^3\right)C_l + 3C_d^2\left(\frac{\dot{z}}{\dot{z}_o}\right)^2 = \lambda \]  
\[ \left(1 + C_d^3\right)^2\left(\frac{\dot{z}}{\dot{z}_o}\right)^2 = 2\lambda C_d\left(1 + \frac{C_d^3}{4}\right) + 1 \]  
\[ C_d\left(1 + \frac{C_d^3}{4}\right) = \frac{\lambda}{2} C_t^2 + C_t \]  

where
\[ \lambda = \left(1 - \frac{L}{W}\right)\frac{g}{\dot{z}_o^2 2\Lambda} \]

For any given value of \( \lambda \) equations (9), (10), and (11) give the relationships that exist among the nondimensional variables during an impact.

**Hydrodynamic Force**

In contrast to the equations of reference 1 in which the hydrodynamic load was equal to the inertia force, the lift force and weight are not balanced for the equations developed herein. The load on the hull bottom is obtained from equation (1), which can be expressed in terms of the nondimensional variables as follows:

\[ C_{Fv} = \lambda + C_l \]
where

$$C_{F_V} = \frac{F_V}{W} \frac{g}{z_o^2 \sigma}$$

Hydrodynamic Pitching Moments

The hydrodynamic pitching moment about the step-keel point for a fixed-trim, step-landing impact of a seaplane is given by equation (16) in reference 3. For the case where the resultant velocity of the seaplane is normal to the keel this equation becomes

$$M_s = \frac{\bar{f}(\beta)}{6 \sin^2 \tau \cos^3 \tau} \left( \frac{z_k^2}{4} + z_k^2 \right)$$

(13)

where $\varphi_m(A)$ is similar to $\varphi(A)$. Equation (13) may be written in terms of the nondimensional variables as

$$C_{m_s} = C_d^3 \left[ \left( \frac{z}{z_o} \right)^2 - \frac{C_d C_l}{4} \right]$$

(14)

where the pitching-moment coefficient about the step-keel point is defined by

$$C_{m_s} = \frac{M_s}{z_o^2 \varphi(A)}$$

(15)

In this paper $\varphi_m(A)$ is assumed to be equal to $\varphi(A)$.

RESULTS AND DISCUSSION

The equations presented in the analysis permit the preparation of dimensionless plots which may be used directly to determine the loads, motions, and hydrodynamic pitching moments at any instant of the impact. The relations between the nondimensional variables describing the loads and motions during normal impacts are given in figures 1 to 6 for a range
of values of the nondimensional unbalanced-lift-force parameter \( \lambda \) from 0 to 2. These plots indicate that the acceleration, velocity, draft, hydrodynamic force, and pitching moment increase with decreasing wing lift (since \( \lambda \) increases with decreasing wing lift).

Figure 3 shows that the draft approaches infinity as time increases without limit because the buoyancy of the float is neglected. The dashed line gives the value of the nondimensional draft at the time of maximum nondimensional acceleration.

In figure 7 the maximum nondimensional vertical hydrodynamic force \( CF_{V_{\text{max}}} \) is plotted against the unbalanced-lift-force parameter \( \lambda \). The dashed line on this figure is a straight line approximating the function. Thus, the agreement between the two lines shows that the maximum value of this force is an increasing approximately linear function of the unbalanced-lift-force parameter.

The physical significance of the nondimensional plots can be visualized from figure 8. In this figure the difference between the ratio of maximum vertical hydrodynamic force to weight for the case where the wing lift is less than the weight and for the case where the wing lift is equal to the weight (as obtained from the straight-line approximation of fig. 7) is plotted against a function of the lift-to-weight ratio. The incremental hydrodynamic load factor \( \Delta \left( \frac{F_Y}{W} \right)_{\text{max}} \) is seen to vary from 0 to 1.33 as the wing lift goes from a value equal to the weight to 0. This curve is valid for all values of \( \left( \frac{F_Y}{W} \right)_{\lambda_{\text{max}}} \) where \( \left( \frac{F_Y}{W} \right)_{\lambda_{\text{max}}} \) is the maximum nondimensional vertical impact-load factor for the condition where the wing lift is equal to the weight. This curve indicates that the increase in hydrodynamic load due to a decrease in wing lift is approximately 133 percent of the decrease in air load. Thus, the effect of wing lift on landing loads is greatest for large seaplanes that have low design impact loads and could result in a 44-percent increase in load for a large flying boat landing with an impact-load factor of 3 if the wing lift is varied between a value equal to the weight and 0. For lower design impact accelerations the effect of wing lift is greater. The variation of lift that appears in the abscissa of figure 8 can be expressed as a function of \( \lambda \), the maximum load factor for lift equal to weight, and the maximum generalized load factor of 0.6 for the condition of normal impact in reference 1. Inasmuch as the usual range of interest of impact-load factors is from 1 to 10, the relationship expressed in the abscissa of figure 8 gives an indication of the ranges of \( \lambda \) that will be encountered. If the wing lift is 0, this range varies from a value of \( \lambda \) of 0.06
is 10 to a value of \( \lambda \) of 0.6 where \( \left( \frac{F_v}{W} \right)_{l_{max}} \) is 1. If the ratio of wing lift force to weight is between 0 and 1 the values of this maximum range of \( \lambda \) will be decreased. When the wing lift balances the weight, the value of \( \lambda \) is 0.

APPLICATION TO OBLIQUE IMPACT

The effect of wing lift on the vertical acceleration and the vertical hydrodynamic load experienced during a normal impact may be used in an approximate method to find the effect of wing lift during an oblique impact. As a first approximation, the effect of wing lift is assumed to be the same for normal and oblique impacts having the same values of maximum acceleration and the same time to maximum acceleration. This assumption appears to be valid up to the point of maximum acceleration, since in this region the time histories of the normal and oblique impacts have similar shapes (see fig. 8 of ref. 1). Consequently, by the use of this assumption, the time histories of the vertical acceleration and the vertical hydrodynamic load for any oblique impact are computed as follows:

1. Obtain a value of the approach parameter \( \kappa \) for the oblique impact from the equation

\[
\kappa = \frac{\sin \tau}{\sin \gamma_0} \cos (\tau + \gamma_0) \tag{16}
\]

where \( \tau \) is the trim and \( \gamma_0 \) is the initial flight-path angle.

2. Compute an effective initial vertical velocity \( \dot{z}_{oe} \) from the equation

\[
\dot{z}_{oe} = \dot{z}_0 \left( \frac{C_{l_{max}} C_{t_{max}}}{C_{l_{max}} C_{t_{max}}}_{\kappa=0} \right) = 2.31 \dot{z}_0 \left( C_{l_{max}} C_{t_{max}} \right)_{\kappa=0} \tag{17}
\]

where \( C_{l_{max}} \) and \( C_{t_{max}} \) are obtained from figures 9(c) and (d) of reference 1. The symbol \( C_{l_{max}} \) designates the maximum vertical acceleration coefficient and \( C_{t_{max}} \) designates the corresponding time coefficient.
(3) Compute an effective value of the impact geometry constant $\Lambda_e$ from the equation

$$\Lambda_e = \Lambda \left( \frac{(C_{l_{\text{max}}}^2l_{\text{max}}^2)}{\mu} \right)_{\kappa=0} = \frac{0.305\Lambda}{\left[ (C_{l_{\text{max}}}^2l_{\text{max}}^2) \right]_{r}}$$

where

$$\Lambda = \left[ \frac{g}{W} \left( \frac{f(\beta)}{6 \sin \tau \cos^2 \tau} \right)^{2/3} \right]$$

$$f(\beta) = \frac{\pi}{2\beta} - 1$$

$$\varphi(A) = 1 - \frac{\tan \tau}{2 \tan \beta}$$

(4) Compute an effective value of the unbalanced-lift-force parameter $\lambda_e$ from the equation

$$\lambda_e = \left( 1 - \frac{L}{W} \right) \frac{g}{\dot{z}_{o_e}^2 \Lambda_e}.$$  \hspace{1cm} (19)

(5) Use these effective values of $\dot{z}_{o_e}$, $\Lambda_e$, and $\lambda_e$ in the plots of this paper to obtain time histories of the acceleration and hydrodynamic load during oblique impact at constant partial wing lift up to the point of maximum load factor.

To illustrate, consider for example the case of a flying boat landing with the following geometric characteristics and initial conditions at water contact:

- $W = 50,000$ lb
- $\beta = 25^\circ$
- $\tau = 9^\circ$
- $\gamma = 6^\circ$
- $\dot{z}_o = 10$ ft/sec
- $\rho = 1.97$ slugs/cu ft
- $L = 25,000$ lb
A theoretical investigation is made of the motions and hydrodynamic loads experienced during the impact of prismatic V-bottom seaplanes in the step-landing condition where the wing lift is a constant fraction of the weight and the resultant velocity is normal to the keel. An approximate method is given for applying the results of this investigation to the more general case of oblique impact. This method involves obtaining an equivalent normal impact for any given oblique impact and then assuming that the percentage change in load due to a change in wing lift is the same in both oblique and normal impacts.

Equations are presented which relate the load and motion variables throughout a normal impact and it is shown that these variables may be expressed as dimensionless quantities which are related by a single parameter \( \lambda \). This parameter depends on the unbalanced wing lift force, the initial conditions of the impact, and the hydrodynamic characteristics of the seaplane.

The results of the investigation are presented in the form of dimensionless plots which may be used directly to determine the loads, motions, and hydrodynamic pitching moments at any instant of the impact. These results suggest that the increase of hydrodynamic load is approximately 133 percent of the decrease in air load.

INTRODUCTION

The present paper is concerned with an evaluation of the effects of reduced wing lift during water landings of wide-beamed prismatic V-bottom seaplanes. Previously hydrodynamic theory was developed for oblique impacts in which the wing lift was equal to the weight of the seaplane for the entire range of trim and flight-path angles. (See, for example, ref. 1.) This theory has been checked experimentally and was found to
REFERENCES


Figure 1.- Theoretical time histories of acceleration coefficient for various values of unbalanced-lift force parameter.
Figure 2.- Theoretical time histories of vertical-velocity ratio for various values of unbalanced-lift-force parameter.
Figure 3.- Theoretical time histories of draft coefficient for various values of unbalanced-lift-force parameter.
Figure 4.- Theoretical time histories of hydrodynamic-force coefficient for various values of unbalanced-lift-force parameters.
Figure 5. Variation of theoretical hydrodynamic-force coefficient with draft coefficient.
Figure 6. - Theoretical time histories of pitching-moment coefficient for various values of unbalanced-lift-force parameter.
Figure 7.- Variation of theoretical maximum hydrodynamic-force coefficient with unbalanced-lift-force parameter.
Figure 8. - Variation of incremental hydrodynamic load factor with unbalanced gravity parameter.
<table>
<thead>
<tr>
<th>NACA TN 2815</th>
<th>1. Hydrodynamic Theory (2.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Sims, Joseph L. II. Schnitzer, Emanuel III. NACA TN 2815</td>
<td>A theoretical investigation is made of the loads and motions in water-landing impacts of wide prismatic V-bottom seaplanes for constant partial wing-lift conditions where the resultant velocity of the seaplane is normal to the keel. An approximate method is given for applying the results of this investigation to the more general case of oblique impact. The increase in vertical hydrodynamic load factor due to wing-lift reduction is shown to be approximately 133 percent of the decrease in air load.</td>
</tr>
</tbody>
</table>

Copies obtainable from NACA, Washington

<table>
<thead>
<tr>
<th>NACA TN 2815</th>
<th>1. Hydrodynamic Theory (2.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Sims, Joseph L. II. Schnitzer, Emanuel III. NACA TN 2815</td>
<td>A theoretical investigation is made of the loads and motions in water-landing impacts of wide prismatic V-bottom seaplanes for constant partial wing-lift conditions where the resultant velocity of the seaplane is normal to the keel. An approximate method is given for applying the results of this investigation to the more general case of oblique impact. The increase in vertical hydrodynamic load factor due to wing-lift reduction is shown to be approximately 133 percent of the decrease in air load.</td>
</tr>
</tbody>
</table>

Copies obtainable from NACA, Washington
1. Hydrodynamic Theory
2. Loads, Landing - Impact, Water

A theoretical investigation is made of the loads and motions in water-landing impacts of wide prismatic V-bottom seaplanes for constant partial wing-lift conditions where the resultant velocity of the seaplane is normal to the keel. An approximate method is given for applying the results of this investigation to the more general case of oblique impact. The increase in vertical hydrodynamic load factor due to wing-lift reduction is shown to be approximately 135 percent of the decrease in air load.

Copies obtainable from NACA, Washington