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ANALYTICAL SOLUTION OF THE PROBLEM ON THERMAL EXPLOSION FOR THE CYLINDRICAL CASE

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- USSR -

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ANALYTICAL SOLUTION OF THE PROBLEM ON THERMAL EXPLOSION FOR THE CYLINDRICAL CASE

/Following is the translation of an article by D. A. Frank-Kamenetskiy entitled "Analiticheskoye Resheniye Zadachi o Teplovom Vzryve dlya Tsilindricheskogo Sluchaya" (English version above) in Zhurnal Fizicheskoy Khimii (Journal of Physical Chemistry), Vol. XXXII, No. 5, 1958, pages 1182-1183./

In the article [1] it was shown that the critical condition of thermal ignition is reduced to the condition of existence in the equation

$$\Delta^2\theta = -\delta e^{\theta} \tag{1}$$

of a solution satisfying the boundary conditions.

The analytical solution of equation (1) was found for the plane case, while the critical condition was reduced to the condition of the existence of radicals in the transcendental equation:

$$a = \left(\operatorname{ch}\sqrt{\frac{a\delta}{2}}\right)^2. \tag{2}$$

For the cylindrical and spherical cases, the critical δ values were found by means of numerical integration.

It is interesting that for the cylindrical case the critical δ value, with all the accuracy of numerical calculation, was found to be equal to the integral number two. But it was not clear whether this was an accidental coincidence or whether in the cylindrical case the problem of the critical condition actually has a simple integral solution.

Recently, Tonnemann and Cowigh [2], while examining the equation of type (1) in connection with a completely different problem, have found a substitution with the aid of which the cylindrical case is reduced to a plane one. For this purpose, it is sufficient to effect the change of variables:

$$u = r\left(\frac{r}{R}\right)^2 e^{\theta}; \quad \xi = \ln \frac{r}{R}, \tag{3}$$

after which (1) for the cylindrical case will assume the same form as for the plane one.

The general form of the solution for the plane and cylindrical cases may be expressed thus:

$$u = a\left[\operatorname{ch}\left(b - \sqrt{\frac{a\delta}{2}}\,\xi\right)\right]^{-2}, \tag{4}$$

where the values u and ξ for the plane case, according to [1] are:

$$u = e^{\theta}; \quad \xi = \frac{x}{d}, \tag{5}$$

and for the cylindrical case they are expressed by (3).

Accordingly, the boundary conditions for the plane case are:

$$u(\xi) = u(-\xi); \quad \text{when} \quad \xi = 1,\ u = 1, \tag{6}$$

and for the cylindrical case:

$$\text{when} \quad \xi = -\infty, \; u \sim r^2, \quad \text{when } \xi = 0, \; u = 1. \tag{7}$$

In the plane case, the boundary conditions (6) give for a the transcendental equation (2).

In the cylindrical case, the finding of the constant of integration of a and of the critical condition appears considerably simpler than in the plane case: all results are obtained in the final form and it is not necessary to solve a transcendental equation.

When $r \longrightarrow 0$ ($\xi_0 \longrightarrow -\infty$), (4) gives

$$u \sim 4ae^{-2b}\left(\frac{r}{R}\right)^{\sqrt{2ab}}. \tag{8}$$

In order that the temperature remain on the finite axis, the following condition should be fulfilled:

$$\sqrt{2ab} = 2. \tag{9}$$

Whereupon, dimensionless temperature on the axis is expressed as:

$$\theta_0 = \ln 4a - 2b. \tag{10}$$

The second boundary condition at the limit (at $\xi_0 = 0$) gives

$$\frac{a}{\operatorname{ch}^2 b} = 1. \tag{11}$$

From this, it is at once obvious that the minimal value of a is obtained at $b = 0$, and is equal to unity. Substituting in (9), we obtain:

$$\delta_{critical} = 2, \tag{12}$$

in exact agreement with the results of numerical calculation.

The maximal pre-explosion heatup is, according to (10):

$$\theta_m = \ln 4 = 1{,}38. \tag{13}$$

The numerical calculation produced practically the identical value of 1.37.

Thus, although in the plane case the equation integrates directly by the standard methods, the critical condition has a simpler character in the cylindrical case, where the properties of the solution are essentially determined by the singularity on the axis.

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