ON NONLINEAR PRESSURE COUPLING IN CYLINDRICAL SHELL ANALYSIS

Paul E. Wilson
Edward E. Spier

1 November 1963

ENGINEERING DEPARTMENT

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This work was supported under General Dynamics sponsored Research Program Number 111-9582-911
## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ILLUSTRATIONS</td>
<td>iii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. NOMENCLATURE</td>
<td>2</td>
</tr>
<tr>
<td>3. THEORY</td>
<td>2</td>
</tr>
<tr>
<td>4. ANALYSIS</td>
<td>3</td>
</tr>
<tr>
<td>5. ACKNOWLEDGMENT</td>
<td>5</td>
</tr>
<tr>
<td>6. BIBLIOGRAPHY</td>
<td>6</td>
</tr>
<tr>
<td>7. DISTRIBUTION</td>
<td>11</td>
</tr>
<tr>
<td>Figure</td>
<td>Title</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>Shell Juncture</td>
</tr>
<tr>
<td>2</td>
<td>Moment Comparison Curves</td>
</tr>
<tr>
<td>3</td>
<td>Shear Comparison Curves</td>
</tr>
</tbody>
</table>
ON NONLINEAR PRESSURE COUPLING IN CYLINDRICAL SHELL ANALYSIS

ABSTRACT

Two well known thin shell theories are used to evaluate and compare shears and moments at the juncture of two pressurized cylindrical shells. For highly pressurized cylinders with large radius-thickness ratios the numerical results indicate that nonlinear coupling effects of pressure significantly influence computed values of discontinuity shears and moments.

1. INTRODUCTION

Many structural problems in the aerospace industry involve highly pressurized shells with large radius-thickness ratios, and under these circumstances, as suggested by Hetenyi (1), the coupling effects of pressure often have a significant influence on discontinuity shears, moments, and stresses. Accordingly, a number of investigations have recently been focused on this problem for pressurized cylindrical (2-8), spherical (9-10), and arbitrary shells of revolution (11). For internally pressurized shells, comparisons made between discontinuity analyses that include and neglect the coupling effects of meridional load (7, 8) imply that use of the refined method generally yields lower calculated maximum stresses and results in a lighter-weight structure.

In this report two well known thin shell theories are used to evaluate and compare shears and moments at the juncture of two pressurized cylindrical shells. For thin highly pressurized cylinders the numerical results indicate that nonlinear coupling effects of pressure have a significant influence on computed values of discontinuity shears and moments.

1Numbers in parentheses designate references listed in the Bibliography.
2. NOMENCLATURE

\[ a = \text{radius of shell middle surface} \]
\[ A, B = \text{constants of integration} \]
\[ D = \text{flexural rigidity, } \frac{Eh^3}{12(1-\nu^2)} \]
\[ E = \text{modulus of elasticity} \]
\[ h = \text{shell thickness} \]
\[ i = \sqrt{-1} \]
\[ k = \text{shell parameter, } \frac{Eh}{a^2} \]
\[ m = \text{nondimensional load parameter} \]
\[ M_0, Q_0 = \text{moment and shear at discontinuity neglecting } N_x \frac{d^2w}{dx^2} \]
\[ M_N, Q_N = \text{moment and shear at discontinuity including } N_x \frac{d^2w}{dx^2} \]
\[ N_x = \text{axial stress resultant, positive when tensile} \]
\[ p = \text{internal pressure} \]
\[ w = \text{radial deflection measured positive inward} \]
\[ w_c, w_p = \text{complementary and particular solutions, respectively} \]
\[ x = \text{axial coordinate} \]
\[ \alpha, \beta, \gamma = \text{parameters entering into complementary solution} \]
\[ \delta_1, \delta_2 = \text{membrane expansions given by Eq. [7]} \]
\[ \eta = \text{shell thickness ratio, } h_1/h_2 \]
\[ \lambda^4 = \text{shell parameter, } k/hD \]
\[ \nu = \text{Poisson's ratio} \]

3. THEORY

The differential equation that governs small axisymmetric displacements of thin cylindrical shells may, with the notation in the Nomenclature, be written in the form (12)

\[
D \frac{d^4w}{dx^4} - N_x \frac{d^2w}{dx^2} + \frac{Eh}{a^2} w = -p + \nu \frac{N_x}{a} \quad [1]
\]
In many discontinuity analyses (13) it is permissible, as verified by test results (14, 15), to delete the pressure coupling term \( N_x \frac{d^2 w}{dx^2} \) from Eq. [1]. However, by parametric evaluation of a simple problem, it will be demonstrated that this simplification process is not generally valid for highly pressurized cylindrical shells with large radius-thickness ratios.

The general solution of Eq. [1] for a semi-infinite cylindrical shell is well known (1) and may be expressed in the form

\[
v = v_c + v_p
\]

where

\[
v_p = - \frac{e^2}{Eh} (p - \nu \frac{N}{a})
\]

\( N_x < 2\sqrt{kD} \), \( v_c = e^{-\alpha x} (A \sin \beta x + B \cos \beta x) \)

\( N_x = 2\sqrt{kD} \), \( v_c = e^{-\gamma x} (A + Bx) \)

\( N_x > 2\sqrt{kD} \), \( v_c = e^{-\alpha x} (A \sinh \beta x + B \cosh \beta x) \)

and

\[
\alpha = \sqrt{\lambda^2 + \frac{N}{4D}}, \quad \beta = -1 \quad \beta = \sqrt{\lambda^2 - \frac{N}{4D}}, \quad \gamma = \sqrt{\frac{N}{2D}}
\]

4. ANALYSIS

Figure 1 shows a longitudinal section of the juncture of two semi-infinite cylindrical shells that are subjected to internal pressure \( p \). Take \( h_1 < h_2 \) and let \( x_1 \) and \( x_2 \) be directed as shown. With the compatibility and equilibrium conditions at the juncture, along with Eqs. [2-5], it may be shown that the discontinuity moment \( M_N \) and shear \( C_N \) act as shown in Fig. 1 and can be expressed in the following form:

\[
M_N = \frac{2D_1D_2\lambda_1^2\lambda_2^2(D_2\lambda_2^2 - D_1\lambda_1^2)(\delta_1 - \delta_2)}{(D_1\lambda_1^2 + D_2\lambda_2^2)^2 + 2D_1D_2\alpha_1\alpha_2(\lambda_1^2 + \lambda_2^2) + \frac{N}{2}(D_1\lambda_1^2 + D_2\lambda_2^2)}
\]

\[
C_N = \frac{4D_1D_2\lambda_1^2\lambda_2^2\left[\alpha_2\lambda_2^2D_2 + \alpha_2\lambda_1^2D_1 + \frac{N}{2}(\alpha_1 + \alpha_2)\right](\delta_1 - \delta_2)}{(D_1\lambda_1^2 + D_2\lambda_2^2)^2 + 2D_1D_2\alpha_1\alpha_2(\lambda_1^2 + \lambda_2^2) + \frac{N}{2}(D_1\lambda_1^2 + D_2\lambda_2^2)}
\]
where

\[ \delta_i = \frac{g^2}{Eh_1} \left( \nu - \frac{N}{a} \right); \quad i = 1, 2 \]  

\[ N_x = \frac{p a}{2} \]

and the subscript \( i (i = 1, 2) \) denotes quantities evaluated in the region \( 0 < x_i < \infty \). When written in their present form, Eqs. [6] are valid for all values of \( N_x \).

If effects of pressure coupling are neglected the moment \( M_0 \) and shear \( Q_0 \) at the discontinuity may, by suitable reduction of Eqs. [6], be shown to be

\[ M_0 = \frac{2D_1D_2\lambda_2^2\lambda_3^2\left(D_2\lambda_2 - D_1\lambda_1\right)(\delta_1 - \delta_2)}{\left(D_1\lambda_1^2 + D_2\lambda_2^2\right)^2 + 2D_1D_2\lambda_1\lambda_2\left(\lambda_1^2 + \lambda_2^2\right)} \]

Eqs. [8] agree with results given earlier by Johns (13, 14).

An interesting quantitative comparison of these results may be obtained by forming the dimensionless ratios \( \frac{M_N}{M_0} \) and \( \frac{Q_0}{Q_N} \) as follows:

\[ \frac{M_N}{M_0} = \frac{(1+\eta)^2 + 2\eta^{3/2}(1+\eta)}{(1+\eta^2)^2 + 2\eta^{3/2}(1+\eta)(1+m)(1+m\eta^2) + 2m\eta^2 (1+\eta^2)} \]

\[ \frac{Q_0}{Q_N} = \frac{1 + \eta^{3/2}}{(1+2m\eta^2)(1+\eta^2)(1+m\eta^2) + 2m\eta^2 (1+\eta^2)} \cdot \frac{M_0}{M_N} \]

where

\[ \eta = \frac{n_1}{n_2}, \quad m = \frac{\sqrt{3(1-\nu^2)}}{2} \cdot \frac{p a}{Eh_1^2} \]

Plots of \( \frac{M_N}{M_0} \) and \( \frac{Q_0}{Q_N} \) versus \( \eta \) for various values of the nondimensional load parameter \( m \) are shown in Figs. 2 and 3, respectively. Note that results obtained including effects of \( N_x \) on local bending imply that computed values of the discontinuity bending moment and shear are significantly reduced and
increased, respectively, over corresponding values obtained by neglecting pressure coupling effects. For example, consider the following case:

\[
\begin{align*}
    a &= 20" \\
    h_1 &= 0.10" \\
    h_2 &= 0.20" \\
    p &= 200 \text{ psi} \\
    E &= 10 \times 10^6 \text{ psi} \\
    \nu &= 0.3
\end{align*}
\]

Thus

\[
\eta = 0.5, \quad m = 0.661
\]

and from Figs. 2 and 3

\[
M_N = 0.75 M_0, \quad Q_N = 1.39 Q_0
\]

Consequently in this instance the discontinuity moments and shears computed by neglecting coupling effects of meridional load are in error by approximately 25\% and 39\%, respectively.

The stresses have not been discussed in this report. However, preliminary calculations indicate that the maximum stress always occurs in the thinner shell. For small values of \( \eta \) the longitudinal stress governs the design, whereas for large values of \( \eta \) the hoop stress governs. Also, it was found that computed values of the maximum stress are reduced when pressure coupling effects are included in the analysis. In practice it is recommended that the maximum stress be computed by using Eqs. [6] in conjunction with the techniques outlined by Grossman (7) and Smith (8).

5. ACKNOWLEDGMENT

The authors extend their appreciation to Mr. D. R. Cropper and others at General Dynamics/Astronautics for assisting in the preparation of this report.
6. BIBLIOGRAPHY


Fig. 1 Shell juncture
Fig. 2 Moment comparison curves
Fig. 3 Shear comparison curves
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<td>2</td>
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<td>2</td>
</tr>
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<td>6-157</td>
<td>1</td>
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<td>3</td>
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<td>T-38</td>
<td>2</td>
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<td>3</td>
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<td>1</td>
</tr>
<tr>
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<td>H. A. Swift</td>
<td>6-56</td>
<td>2</td>
</tr>
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