Atomic Stabilization of Electromagnetic Field Strength Using Rabi Resonances

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**Abstract:**
For nearly fifty years, atomic resonances have been employed to stabilize electromagnetic field frequency. Here, a dynamical Rabi resonance, associated with an alkali atom's ground-state hyperfine transition, is used to stabilize electromagnetic field strength. A 7% sinusoidal variation was superimposed on a microwave field's strength, and through atomic stabilization, this variation was reduced to 0.1%. Although the method is demonstrated with microwave fields, the technique could also be employed to stabilize optical fields via Rabi resonances associated with electronic transitions in atoms or molecules.
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Transitions between eigenstates in atoms (or molecules) have great utility for stabilizing electromagnetic field frequency. For example, it is now possible to realize the second to fourteen significant figures by locking the frequency of a microwave field to the ground state hyperfine transition of Cs\(^{133}\) [1]. To no small extent, the utility of atoms in this regard is achieved because the transition probability between eigenstates is a resonant function of energy, and because the transition frequency is (in principle) a constant of nature. This, of course, is well known, and quantum mechanical transitions have been employed for frequency stabilization since 1948 [2].

The stabilization of electromagnetic field strength, however, is another matter, since the transition probability between two atomic eigenstates is, in general, a monotonically increasing function of intensity. Consequently, resonances between energy eigenstates are not particularly useful for field-strength stabilization (except in the case of dc fields, where the Zeeman and Stark effects can be exploited [3]), and other methods must be found. In the case of lasers, where field stability can be of importance for precision spectroscopy, field strength is typically stabilized by comparing a voltage measurement of laser intensity to some reference voltage [4]. A deviation of the laser intensity from the reference produces an error signal that can be exploited in a feedback control loop to stabilize the laser’s field strength. Identical techniques are employed for the stabilization of microwave field strengths [5]. Of course, one disadvantage with this approach is that the reference voltage is arbitrary, sometimes making it a cumbersome process to reference the field strength to an absolute value, for example in ac Stark shift measurements. Additionally, the reference voltage may drift over time, giving rise to long term field-strength instability. It should be noted that the achievement of stable microwave field strength over long times has technological implications, as it is known that microwave power variations in atomic clocks degrade timekeeping ability [6,7].

In the present paper, field-strength stabilization is achieved by “locking” the field strength to an atomic Rabi resonance [8–12]. In simplest terms, when an atom or molecule is subjected to a phase modulated resonant field, a Fourier component of the resulting population variations shows a resonant increase when the corresponding Fourier frequency matches the Rabi frequency (i.e., the electromagnetic field strength); hence the term “Rabi resonance.” It is to be noted, however, that even though the Rabi resonance appears in a quantum system it is not a typical resonance between energy eigenstates. (We are referring to the so-called “bare-atom” eigenstates as opposed to the quantum system’s “dressed-atom” eigenstates [13]). Rather, it is a dynamical resonance associated with a frequency match between the rate of a perturbation’s induced atomic variations (at harmonics of the phase modulation frequency, \(\nu_{\text{mod}}\) and an atom’s internal rate of response to that perturbation (i.e., the Rabi frequency, \(\Omega\)). Here, a microwave field (resonant with the ground state \((F = 2, m_F = 0) \rightarrow (1,0)\) hyperfine transition of Rb\(^{87}\), \(\Delta \nu_{\text{hfs}}\)) was phase modulated, and the resulting Rabi resonance was observed as an enhancement in the atoms’ second harmonic response to the modulated field. Using standard heterodyne detection methods, the change in the atoms’ response with microwave field strength was used to generate an error signal for field-strength stabilization. As a test of this atomic stabilization technique, a slow 1.2 dB (peak-to-peak) sinusoidal power variation (i.e., “drift”) was superimposed on the microwave field. By locking the microwave field strength to the Rabi resonance, this drift was reduced by nearly two orders of magnitude.

Figure 1 is a block diagram of the experimental arrangement. A Corning 7070 glass resonance cell containing isotopically pure Rb\(^{87}\) and 10 Torr of N\(_2\) was placed in a microwave cavity whose TE\(_{011}\) mode was resonant with \(\Delta \nu_{\text{hfs}}\) at 6834.7 MHz. The cylindrical cavity had a radius of 2.8 cm and a length of 5 cm, and the resonance cell filled the cavity volume. Braided windings, wrapped around the cavity, heated the resonance cell to approximately 40°C, and the entire assembly was centrally located in a set of three perpendicular Helmholtz coil pairs: Two pairs zeroed out the Earth’s magnetic field
while the third pair (~300 mG) provided a quantization axis for the atoms parallel to the cavity’s cylindrical axis. An AlGaAs diode laser (~3 mW) was tuned to the Rb $^3P_{1/2} - ^3S_{1/2}(F = 2)$ transition [14] and was attenuated by a 2.0 neutral density filter. The transmission of the laser light through the vapor was monitored with a Si photodiode, and the propagation direction of the laser was parallel to the cavity axis. In addition to optical pumping [15], the laser light monitored the $F = 2$ population density: In the absence of resonant microwaves, optical pumping reduced the $F = 2$ atom density and resulted in an increased level of transmitted light; when resonant microwaves were present, atoms returned to the $^3S_{1/2}(F = 2)$ state from the $F = 1$ state, thereby reducing the amount of transmitted light. As a result, any microwave field induced oscillation of the atomic population could be observed as oscillations in the transmitted light intensity.

The microwaves were derived from a voltage-controlled crystal oscillator (VCXO) which had a modulation bandwidth of 10 kHz, and its output frequency at ~107 MHz was multiplied up to $\Delta \nu_{\text{hfs}}$. The microwaves were attenuated by the combination of a voltage-controlled attenuator (VCA) and a fixed attenuator (labeled as $-\text{dB}$ in the figure) before being amplified by a $+30$ dB solid state amplifier. The attenuators were calibrated to Rabi frequency by measuring the hyperfine transition linewidth [16]. The output from a frequency synthesizer at 357 Hz and a dc voltage were added, and these provided the VCXO’s control voltage, $V_c$. The dc level of $V_c$ tuned the average microwave frequency to $\Delta \nu_{\text{hfs}}$, while the sine wave provided microwave phase modulation. The amplitude of the phase modulation was 2.28 radians, and this was chosen by maximizing the amplitude of the atoms’ Rabi resonance.

The Rabi resonance was manifested in the atoms’ second harmonic response to the phase-modulated microwave field, and thus occurred for $\Omega = 2\nu_{\text{mod}}$. The output of the photodiode was sent to a spectrum analyzer (25 kHz bandwidth) and a lock-in amplifier (labeled as #1 in Fig. 1) referenced to $2\nu_{\text{mod}}$. The spectrum analyzer was used primarily to tune the microwave frequency to $\Delta \nu_{\text{hfs}}$ by zeroing the atoms’ fundamental response to the phase-modulated microwaves. The photodiode–preamp–lock-in combination acted as a low-pass detector for the atoms’ second harmonic response. (The bandwidth of the preamp was 1 MHz, and the time constant of the lock-in amplifier depended on the experiment.)

Figure 2(a) shows the amplitude of the atoms’ second harmonic response as a function of the fixed attenuator setting. Two sets of data are shown corresponding to measurements made by the spectrum analyzer (open circles) and measurements obtained at the output of lock-in #1 (filled circles). Both measures of the second harmonic response show a resonance centered at about 31 dB ($\Omega = 850$ Hz) with a linewidth of about 10 dB; this is the Rabi resonance. Figure 2(b) shows that the phase of the second harmonic signal varies rapidly in the vicinity of the Rabi resonance [11], and explains the slight difference in width between the two resonances shown in Fig. 2(a).

To generate an error signal for field-strength stabilization, the fixed attenuator was set to 31 dB, and the microwave power was modulated by applying a sinusoidal signal to the VCA’s control voltage (i.e., 0.8 dB peak-to-peak modulation at 47 Hz). The field-strength modulation resulted in a modulation of the atoms’ second harmonic response, and this could be observed in the output of lock-in #1. The amplitude of atomic modulation was monitored in a heterodyne fashion with the aid of lock-in #2, whose output thus became a “field-strength discriminator.” This is illustrated in Fig. 3, where the output of lock-in #2 and the spectrum analyzer’s indication of second harmonic response are plotted as a function of fixed attenuator setting.

A field-strength feedback control loop was closed by adding the field-strength discriminator voltage to the VCA control voltage. (No efforts were made to optimize the
FIG. 2. (a) Rabi resonance associated with the ground state hyperfine transition of Rb⁸⁷ as observed via the atoms' second harmonic response to the modulated microwave field. Open circles correspond to spectrum analyzer measurements, while closed circles correspond to lock-in #1 measurements. (b) Rapid change in the phase of the atoms' second harmonic response to the modulated microwave field. Since a lock-in amplifier's output voltage depends on the relative phase between the reference and the signal, this rapid change in phase explains the slight difference in width between the two resonances shown in (a).

The efficacy of this atomic stabilization technique was tested by adding a slowly varying voltage to the VCA control voltage, and was accomplished with the aid of the "slow-drift" function generator shown in Fig. 1.

Referring to Fig. 1, relative field-strength variations were measured by splitting the microwave signal just prior to the VCA and just after the VCA, and then combining the two signals in a microwave mixer. As illustrated by the solid line in Fig. 4, the slow-drift function generator produced a 7.2% variation of the microwave field strength when the feedback control loop was open. The filled circles in Fig. 4 show the microwave field strength with the feedback control loop closed: The amplitude of the field-strength variations was considerably reduced, and the average value of the field strength was changed. The change in average field-strength value corresponds to a -0.12 dB change in microwave power attenuation, and arose because of lock-in #2 (closed circles); the time constant of lock-in #1 was set to 1 msec. For comparison, the open diamonds show the Rabi resonance obtained by measuring the second harmonic amplitude with the spectrum analyzer.

While we have demonstrated atomic field-strength stabilization with microwaves, nothing precludes its use with optical fields. Rabi resonances have been observed in the optical regime [10], and one could employ an electro-optical modulator as a VCA to stabilize laser intensity.

FIG. 3. Field-strength discriminator obtained from the output of lock-in #2 (closed circles); the time constant of lock-in #1 was set to 1 msec. For comparison, the open diamonds show the Rabi resonance obtained by measuring the second harmonic amplitude with the spectrum analyzer.

the fixed attenuator (having units of integer dB) could not be set to the exact center of the Rabi resonance. Examining the cumulative probability distribution of filled-circle deviations from their average, 90% of the microwave field-strength variations were within ±0.12% of the average field strength with the feedback control loop closed. Thus, by stabilizing the field strength to an atomic Rabi resonance, there was factor of 60 reduction in the field-strength variations.

FIG. 4. Demonstration of the efficacy of field-strength stabilization using a Rabi resonance. Relative microwave field-strength variations were measured via the IF output port of the mixer shown in Fig. 1. The solid line corresponds to the microwave field-strength variations that were externally imposed by the slow-drift function generator shown in Fig. 1. The filled circles correspond to the field-strength variations after atomic stabilization. Percentages are referenced to the average microwave field strength.
Moreover, if one employed square-wave phase modulation instead of sine-wave phase modulation, a spectrum of Rabi resonances would be generated [9], and each Rabi resonance in the spectrum could be employed to stabilize laser intensity at a different value.

With regard to microwave stabilization, the Rabi resonance method may have relevance to atomic clocks, where microwave power variations are known to give rise to atomic clock frequency variations [6,7]. In such an application, the atoms’ fundamental response to a phase-modulated microwave field would be used to stabilize field frequency (as is presently done in atomic clocks), while the atoms’ second harmonic response could be used to stabilize field strength. In the case of the gas-cell atomic clock, where different atoms in a vapor contribute to the clock signal to varying degrees, an attractive feature of the Rabi resonance method would be that those atoms dominating the frequency stabilization signal would also dominate the field-strength stabilization signal.

It should be noted that the Rabi resonance condition depends on the exact value of $\nu_{\text{mod}}$: Should the phase-modulation frequency change, the Rabi resonance condition will occur for a different value of field strength. Consequently, in this atomic stabilization technique, the field-strength stability can only be as good as the stability of $\nu_{\text{mod}}$. However, not only is the output frequency of a synthesizer typically quite stable, but the synthesizer can be referenced to an external atomic frequency standard. Hence, the Rabi resonance condition (via $\nu_{\text{mod}}$) has the potential to be as stable as the output of an atomic clock.

Referencing $\nu_{\text{mod}}$ to a cesium atomic clock is an intriguing notion, as it suggests a direct link between field strength and a fundamental definition of time interval. Note that the Rabi frequency has a well-defined relationship with field strength, and that the Rabi resonance condition defines $\Omega$ in terms of $\nu_{\text{mod}}$. Since the phase-modulation frequency would be traceable to a definition of the second via the cesium atomic clock, the field strength would also be traceable to a definition of the second. In other words, Rabi resonance stabilization of field strength could provide a means of defining the units of field strength in terms of the second at the atomic level.

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