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ON THE STABILITY OF PLANETARY SCALE ATMOSPHERIC MOTIONS

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The stability of the atmospheric motions is investigated by means of a two level model on a sphere. The dependence of the conditional stability and also the various characteristic motions on the difference of the circulation index at the chosen levels is investigated. An example of the prognosis of the pressure field is given on the basis of the preceeding theory.

The stability of atmospheric motions was examined in a series of studies, by the Soviets: E. N. Blinova (1), G. I. Marchuk (2), Ia. M. Keifets (3), C. V. Nemchinov, and by the foreign authors: Charney (4), Kuo (5), Eady (6), and Spar (11), by means of the solution of the linearized equations. In these, the majority of the cited investigators restricted themselves to only the qualitative (congruence) equivalence of the characteristic properties of the particular solutions of these equations with the typical pecularities of real pressure centers. In the capacity of such particular solutions the unstable waves are begun. The distribution of the stable and unstable waves, its influence on the motion, and the evolution of the pressure systems are not examined completely in this. Besides that, in the cited works either the complete meridional extents of the disturbances are not considered or are considered too crudely. (Phillips [7], Fleagle [8]). The stated restrictions do not permit the developed theory to be adapted to the quantitative investigations of the real pressure fields.

In the present work, we, just as stated by the Soviet authors, do not make similar restrictions which permits a possibility of adapting the theory to the prognosis of the pressure field. As long as we study the motion on the

planetary scale, it is necessary to examine the solution on a sphere. As in the work of E. N. Blinova [9], we take for the initial equations the following:

$$\frac{\partial \Delta\psi}{\partial t} + \frac{1}{a^2 \sin\theta}(\psi, \Delta\psi) + 2\omega \frac{\partial \psi}{\partial \lambda} = -\frac{2\omega\cos\theta}{\tilde{p}(0)} a^2 g \frac{\partial \tilde{\rho} V_z}{\partial \xi} \qquad (1)$$

$$\xi^2 \frac{\partial^2 \psi}{\partial t \partial \xi} + \frac{\xi^2}{a^2 \sin\theta}\left(\psi, \frac{\partial \psi}{\partial \xi}\right) = \frac{R^2 T_1 \; \gamma_a - \gamma}{2\omega \cos\theta \, \tilde{p}(0)} \tilde{\rho} V_z \qquad (2)$$

Here ψ is the streamfunction; a is the radius of the earth, V_z is the vertical velocity; $\tilde{\rho}(z)$ is a standard distribution of density; $\xi = p(z)/p(0)$ is a new independent variable; $\tilde{p}(z)$ is a standard distribution of pressure; T_1 is the mean vertical temperature. We shall take the boundary conditions in the form

$$V_z = 0 \qquad \text{for} \qquad z = 0$$

$$\tilde{\rho} V_z \to 0 \qquad \text{as} \qquad z \to \infty$$

We shall divide the whole thickness of the atmosphere by the variable ξ into four parts through the points $\xi_1 = 0.3$ (300 mb), $\xi_2 = 0.5$ (500) mb $\xi_3 = 0.7$ (700 mb). We approximate $\tilde{\rho} V_z$ by a parabola and take as boundary conditions $(\tilde{\rho} V_z)_0 = (\tilde{\rho} V_z)_4 = 0$. Here $(\tilde{\rho} V_z)_0$ is the value of $\tilde{\rho} V_z$ at infinity, $(\tilde{\rho} V_z)_4$ is the value of $\tilde{\rho} V_z$ on the earth's surface. Then we obtain that

$\tilde{\rho} V_z = 4 (\rho V_z)_2 \, \xi (1 - \xi)$, where $(\tilde{\rho} V_z)_2$ is the value of

$\hat{\rho} V_z$ at $p = 500$ mb . Then we can find $\frac{\partial \hat{\rho} V_z}{\partial \xi}$ and substitute it into equation (1). We shall write down equation (1) at $\xi = 0.3$ and $\xi = 0.7$, and equation (2) at $\xi = 0.5$. Into this we shall substitute for the height derivatives of Ψ the difference relations. Except that, in the term $\left(\psi, \frac{\partial \psi}{\partial \xi}\right)$ we put $\Psi_m = \frac{1}{2}\left(\psi_1 + \psi_3\right)$. Finally we obtain:

$$\frac{\partial \Delta \psi_1}{\partial t} + \frac{1}{a^2 \sin\theta}\left(\psi_1, \Delta \psi_1\right) + 2\omega \frac{\partial \psi_1}{\partial \lambda} = -\frac{1.6\, 2\omega \cos\theta\, a^2 g}{\hat{p}(0)} \left(\hat{\rho} V_z\right)_2$$

$$\frac{\partial \Delta \psi_3}{\partial t} + \frac{1}{a^2 \sin\theta}\left(\psi_3, \Delta \psi_3\right) + 2\omega \frac{\partial \psi_3}{\partial \lambda} = \frac{1.6\, 2\omega \cos\theta\, a^2 g}{\hat{p}(0)} \left(\hat{\rho} V_z\right)_2$$

$$\frac{\partial}{\partial t}\left(\psi_3 - \psi_1\right) + \frac{1}{a^2 \sin\theta}\left(\psi_1, \psi_3\right) = \frac{1.6\, R^2 T_1 \left(\gamma_a - \gamma\right)}{2\omega \cos\theta\, \hat{p}(0)} \left(\hat{\rho} V_z\right)_2$$

Later eliminating $\left(\hat{\rho} V_z\right)_2$ from the first two equations by means of the third we shall have:

$$\frac{\partial Q_1}{\partial t} + \frac{1}{a^2 \sin\theta}\left(\psi_1, Q_1 + 2\omega a^2 \cos\theta\right) = 0$$

$$\frac{\partial Q_3}{\partial t} + \frac{1}{a^2 \sin\theta}\left(\psi_3, Q_3 + 2\omega a^2 \cos\theta\right) = 0$$

here

$$Q_1 = \Delta\psi_1 + \frac{1}{\Gamma}\left(\psi_3 - \psi_1\right)$$

$$Q_3 = \Delta\psi_3 - \frac{1}{\Gamma}\left(\psi_3 - \psi_1\right)$$

$$\Gamma = \frac{R^2 T_1}{4\omega^2 a^2 g}\left(\frac{\gamma_a - \gamma}{\cos^2\theta}\right)_{AV}$$

We linearize our equations in relation to a west-coast transfer, as this appears in the work of E. N. Blinova [10].

We obtain

$$v_\lambda = V_\lambda(\theta) + v_\lambda', \qquad V_\lambda(\theta) = \alpha a \sin\theta$$

$$v_\theta = v_\theta' \qquad \Psi(\theta) = -\alpha a^2 \cos\theta$$

$$\psi = \Psi(\theta) + \psi'$$

Here $\alpha(\theta)$ is the circulation index, which we take independent of latitude. Later we obtain by a simple transformation:

$$\left[\frac{\partial}{\partial t} + \alpha_1 \frac{\partial}{\partial\lambda}\right]\left[\Delta\psi_1' + \frac{1}{\Gamma}\left(\psi_3' - \psi_1'\right)\right] + \left[2(\omega + \alpha_1) - \frac{1}{\Gamma}\left(\alpha_3 - \alpha_1\right)\right]\frac{\partial\psi_1'}{\partial\lambda} = 0 \tag{3}$$

$$\left[\frac{\partial}{\partial t} + \alpha_3 \frac{\partial}{\partial\lambda}\right]\left[\Delta\psi_3' - \frac{1}{\Gamma}\left(\psi_3' - \psi_1'\right)\right] + \left[2(\omega + \alpha_3) + \frac{1}{\Gamma}\left(\alpha_3 - \alpha_1\right)\right]\frac{\partial\psi_3'}{\partial\lambda} = 0$$

Subsequently we ignore α in comparison with ω and we shall look

r a solution in the form:

$$\psi_1 = F_1(\theta)\, e^{im(\lambda+\sigma t)}$$

$$\psi_3 = F_3(\theta)\, e^{im(\lambda+\sigma t)} \tag{4}$$

Substituting (4) into (3) we will obtain a system of two ordinary fierential equations:

$$+\alpha_1)\left[\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dF_1}{d\theta}\right)-\frac{m^2}{\sin^2\theta}F_1+\frac{1}{\Gamma}(F_3-F_1)+\left[2\omega-\frac{1}{\Gamma}(\alpha_3-\alpha_1)\right]F_1=0\right.$$

$$+\alpha_3)\left[\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dF_3}{d\theta}\right)-\frac{m^2}{\sin^2\theta}F_3-\frac{1}{\Gamma}(F_3-F_1)+\left[2\omega+\frac{1}{\Gamma}(\alpha_3-\alpha_1)\right]F_3=0\right.$$

will satisfy this system if for F_1 and F_3 we take solutions of the uations as sums of Legendre functions

$$F_1 = C_1 P_n^m, \qquad F_3 = C_3 P_n^m$$

this form we obtain the system:

$$+\alpha_1)\left[-n(n+1)\Gamma F_1+F_3-F_1\right]+\left[2\omega\Gamma-(\alpha_3-\alpha_1)\right]F_1=0$$

$$+\alpha_3)\left[-n(n+1)\Gamma F_3-F_3+F_1\right]+\left[2\omega\Gamma+(\alpha_3-\alpha_1)\right]F_3=0 \tag{5}$$

The system has a non-trivial solution if the determinant of the unknowns is equal to zero. The last condition gives an equation for the determination of σ :

$$\sigma_n^{1/2} = -\alpha_2 + \frac{(1+\Lambda)B}{(\Lambda+2)\Lambda} \pm \sqrt{\frac{B^2 - \Lambda^2(4-\Lambda^2)V^2}{\Lambda(\Lambda+2)}} \tag{6}$$

$$B = 2\omega\Gamma ; \qquad \Lambda = n(n+1)\Gamma ; \qquad \alpha_2 = \frac{\alpha_1 + \alpha_3}{2}$$

$$V = \frac{1}{2}(\alpha_1 - \alpha_3) = \frac{1}{2}\Delta\alpha$$

The value of σ obtained in the work of Phillips (7) is similar in form, however, the parameters have a different meaning in our work. From the first equation (5) we find the relation between F_1 and F_3 . We obtain that:

$$F_1 = k_n^{1/2} F_3 , \quad \text{where} \quad k_n^{1/2} = 1 + \Lambda - \frac{B - 2V}{\sigma_n^{1/2} + \alpha_3} \tag{7}$$

We notice that $\sigma_n^{1/2} + \alpha_2$, just as $k_n^{1/2}$, depends solely on $\Delta\alpha$ (i.e. on the difference of the circulation indexes at our chosen levels and does not depend on the same indexes). This is evident because $\sigma_n^{1/2} + \alpha_3 = \sigma_n^{1/2} + \alpha_2 - V$. The expression for σ indicates the possibility of a complex value, i.e. the loss of stability. The condition of stability is determined by the inequality:

$$B^2 - \Lambda^2(4-\Lambda^2)V^2 > 0 \tag{8}$$

After fixing n the inequality (8) is either satisfied or not depending on $\Delta\alpha$, which can change from day to day. The remaining parameters (entering into B and Λ) are standard. We have n , an integer. Conse-

quently, Λ takes discrete values only. For more suitable representation of the results we introduce the variables ν, which varies continuously from 0 to ∞. Then the equation

$$B^2 - \Lambda^2 (4 - \Lambda^2) V^2 = 0 \tag{9}$$

where $\Lambda = \nu(\nu+1)\Gamma$ determines in the plane surface $(\nu, \Delta\alpha)$ the boundary of the regions of stability (see fig. 1).

From (9) we have

$$\Lambda^2 = 2 \pm \sqrt{4 - (B/V)^2}$$

From this it is evident that there exists a minimum value of V and consequently also $\Delta\alpha$, below which all the disturbances are stable. The quantity is $\Delta\alpha_{min} = B$ i.e. it depends in essence on the static stability.

It is easy to see that the stability curve has two asymptotes: for $\nu = 0$ and $\nu = \nu_u$ (the last corresponds with $\Lambda = 2$); for $n > \nu_u$ it is always unstable for any $\Delta\alpha$.

We shall take the following values of the parameters:

$$\gamma = 0.006 \quad \text{degrees/meter}$$
$$T_1 = 250^\circ \qquad \theta_u = 45^\circ$$

With such choices Γ is equal to 1.95×10^{-2}. And so

$$\Delta\bar{\alpha}_{min} = \bar{B} = 37 \left(\bar{\alpha} = \frac{\alpha}{\omega} \cdot 1000 \right)$$

For $\Delta\alpha > \Delta\alpha_{min}$ there exists an interval $\nu_1 < \nu < \nu_2$ within which integral values of n can be found. The unstable disturbances will correspond to it. In fig. 2 this is displayed, the behavior of $\bar{\sigma}_n$ and $\bar{\sigma}_n^2$ $\left(\bar{\sigma}_n = \frac{\sigma_n + \alpha_u}{\omega} \cdot 1000 \right)$ as a function of ν for $\Delta\bar{\alpha} = 23$ (denoted No. 1) and $\Delta\bar{\alpha} = 53$ (denoted No. 2).

For $\Delta\bar{\alpha} = 53$ the values of $n = 6, 7, 8, 9$ fall in the interval of instability. In this case the speed of propogation of both types of waves is one and the same and is equal to σ_r. Therefore the critical $\bar{\sigma}_n'$ and $\bar{\sigma}_n^2$ converge in the interval of instability.

Elementary solutions of the type $\cos m(\lambda + \sigma_n^{1/2} t)\, P_n^m(\cos\theta)$ will correspond to a real value of σ.

In the case of complex σ we will have:

$$e^{\nu_i t} \cos m(\lambda + \sigma_r t)\, P_n^m(\cos\theta)$$

here $\sigma = \sigma_r + i\sigma_i$; $\nu_i = m\sigma_i$

The value of k_n will also be complex

$$k_n^{1/2} = f_n e^{\mp i\varphi_n} \quad ; \quad f_n = \sqrt{\frac{2\Lambda V + B}{2\Lambda V - B}} \; ; \quad tg\,\varphi_n = \frac{\sqrt{\Lambda^2(4-\Lambda^2)V^2 - B}}{\Lambda^2 + V}$$

We see that the intensity of growth of the amplitudes ν_i depends on m and n (through σ_i). The quantity σ_i depending only on n, attains a maximum for a certain value of n. The computations show that this value of n is weakly dependent on the jumps of the index of circulation, and in the range $39 \leqq \Delta\bar{\alpha} \leqq 53$, σ_i takes a maximum value for $n = 8$. For this the possible wave number is $m = 7$ at most. (We have different even numbers so that $\Psi = 0$ on the equator). Since in this case $n - m = 1$, the waves with the maximum intensity of growth of amplitude will have a maximum latitudinal extent, [on the meridional] at $\theta = 45°$. The wave length of 4000 km corresponds to wave number $m = 7$. This is the same value Kuo obtained in the above referenced work. Phillips gets a larger value (6000 km).

If in the factor $e^{\nu_i t}$ the time is written for 24 hour [periods], then in the range $39 \leq \Delta\bar{\alpha} \leq 53$ the value ν_i for the wave with the most intensive amplitude growth will increase from 0.13 to 0.37. In the first instance the wave amplitude grows about 13% in the course of 24 hours and in the second about 45%. This dependence of ν_i on $\Delta\bar{\alpha}$ in general agrees with that which Phillips obtained [7]. Still it is proper to note that in the light of the criteria brought out by us, the instability in the atmosphere will be obtained more rarely than is obtained from the theory of Phillips. Phillips made use of the local values of the gradient of the vertical velocity while we took the vertical gradient of the index of circulation which is more expedient.

Now we shall write out the solution that satisfies the initial conditions.

In the stable case we get:

$$\psi_3 = \sum_{n=1}^{\infty} \sum_{m=1}^{n} \Big[C_{1n}^{m} \cos m(\lambda + \sigma_n' t) + D_{1n}^{m} \sin m(\lambda + \sigma_n' t) + C_{2n}^{m} \cos m(\lambda + \sigma_n^2 t) + D_{2n}^{m} \sin m(\lambda + \sigma_n^2 t) \Big] P_n^m(\cos\theta) \tag{4c}$$

$$\psi_2 = \sum_{n=1}^{\infty} \sum_{m=1}^{n} \Big[\bar{C}_{1n}^{m} \cos m(\lambda + \sigma_n' t) + \bar{D}_{1n}^{m} \sin m(\lambda + \sigma_n' t) + \bar{C}_{2n}^{m} \cos m(\lambda + \sigma_n^2 t) + \bar{D}_{2n}^{m} \sin m(\lambda + \sigma_n^2 t) \Big] P_n^m(\cos$$

In consequence of (7)

$$\bar{C}_{1n}^{m} = k_n' \, C_{1n}^{m}, \qquad \bar{D}_{1n}^{m} = k_n' \, D_{1n}^{m}$$

$$\bar{C}_{2n}^{m} = k_n^2 \, D_{2n}^{m} \qquad \bar{D}_{2n}^{m} = k_n^2 \, D_{2n}^{m}$$

Putting the expansions of the initial field into spherical functions we have the form:

$$\Psi_2(\theta,\lambda,0) = \sum_{n=1}^{\infty} \sum_{m=1}^{n} \left[A_n^m \cos m\lambda + B_n^m \sin m\lambda \right] P_n^m(\cos\theta)$$

(11)

$$\Psi_1(\theta,\lambda,0) = \sum_{n=1}^{\infty} \sum_{m=1}^{n} \left[\bar{A}_n^m \cos m\lambda + \bar{B}_n^m \sin m\lambda \right] P_n^m(\cos\theta)$$

Suppose in (10) $t = 0$ and do the same in (11) we get the following correlation for the determination of C_{1n}^m, C_{2n}^m and so on:

$$C_{1n}^m + C_{2n}^m = A_n^m, \qquad k_n' \, C_{1n}^m + k_n^2 \, C_{2n}^m = \bar{A}_n^m$$

$$D_{1n}^m + D_{2n}^m = B_n^m, \qquad k_n' \, D_{1n}^m + k_n^2 \, D_{2n}^m = B_n^m$$

Finally for the unknown coefficients we write the following formula:

$$C_{1n}^m = \beta_n' \, A_n^m + \beta_n^2 \, \bar{A}_n^m, \qquad \bar{C}_{1n}^m = \beta_1^4 \, A_n^m + \beta_n^3 \, \bar{A}_n^m$$

$$D_{1n}^m = \beta_n' \, B_n^m + \beta_n^2 \, \bar{A}_n^m, \qquad \bar{D}_{1n}^m = \beta_n^4 \, B_n^m + \beta_n^3 \, \bar{B}_n^m$$

$$C_{2n}^m = \beta_n^3 \, A_n^m - \beta_n^2 \, \bar{A}_n^m, \qquad \bar{C}_{2n}^m = \beta_n' \, \bar{A}_n^m - \beta_n^4 \, A_n^m$$

$$D_{2n}^m = \beta_n^3 \, B_n^m - \beta_n^2 \, \bar{B}_n^m, \qquad \bar{D}_{2n}^m = \beta_n' \, \bar{B}_n^m - \beta_n^4 \, B_n^m$$

Here

$$\beta_n^1 = -k_n^2 \beta_n^2; \quad \beta_n^2 = \frac{1}{k_n^1 - k_n^2}, \quad \beta_n^3 = k_n^1 \beta_n^2; \quad \beta_n^4 = -k_n^2 \beta_n^3$$

Instead of k_n^1 and k_n^2 substitute their value from (7). Then β_n^2 and β_n^4 change into the following form:

$$\beta_n^2 = \frac{B - 2\Lambda V}{2\sqrt{B^2 - \Lambda^2(4-\Lambda^2)V^2}}, \quad \beta_n^4 = \frac{B + 2\Lambda V}{2\sqrt{B^2 - \Lambda^2(4-\Lambda^2)V^2}} \tag{13}$$

In fig. 3 [is produced] the curve of β_n in its dependence on V for $\Delta\alpha = 29$ (figures 1,2,3,4 have to do with β_n^1, β_n^2, β_n^3, β_n^4). In fig. 4 is shown [evidenced] the behavior of the curves of β_n^1 (figures 1,2,3,4 designated the cases $\Delta\alpha = 23, 29, 34, 41$).

If in our formula, we let $V \to 0$, then we obtain a solution for an average level. For this

$$\sigma_n^1 \longrightarrow -\alpha + \frac{B}{\Lambda} = -\alpha + \frac{2\omega}{n(n+1)}$$

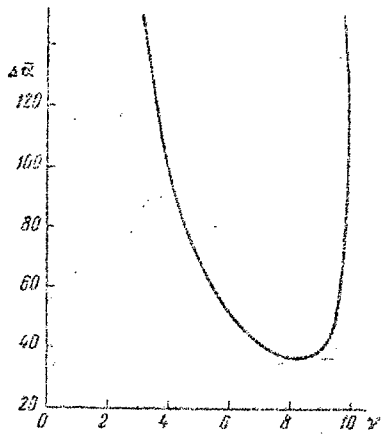
$$\sigma_n^2 \longrightarrow -\alpha + \frac{B}{\Lambda+2} =$$

Analagously $k_n^1 \to 1$, $k_n^2 \to 1$, and the coefficients β_n^1, β_n^2, β_n^3, β_n^4 converge toward 1/2. From this it is evident that

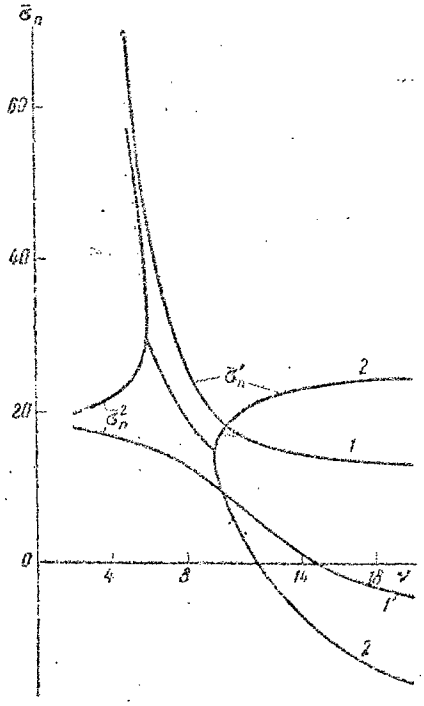
$$C_{1n}^m \to A_n^m, \quad D_{1n}^m \to B_n^m, \quad C_{2n}^m \to 0 \quad D_{2n}^m \to 0$$

although σ_n^2 does not vanish, corresponding terms in the solution converge to zero. The solution completely agrees with that obtained by E. N. Blinova [10].

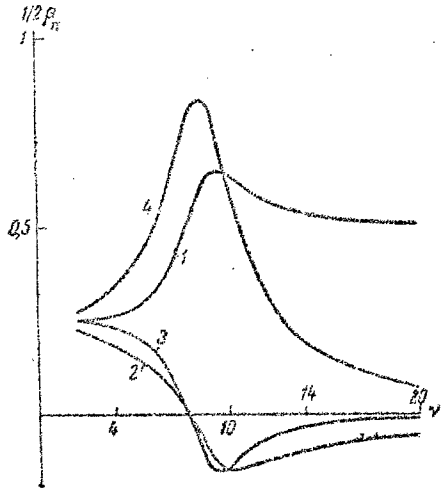
For instability of waves the groups of terms corresponding to fixed values of n and m have the form:



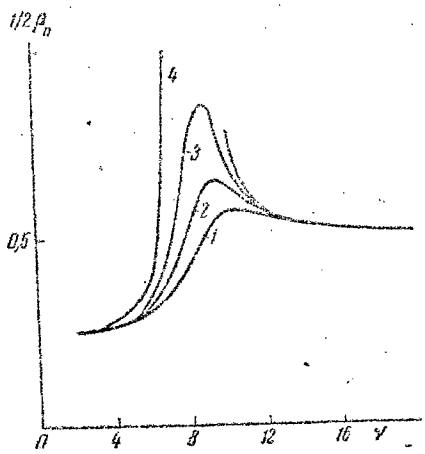
Фиг. 1
FIG.



Фиг. 2
FIG.



Фиг. 3
FIG.



Фиг. 4
FIG.

$$\psi_{3n}^{m} = \Big[\left(C_{1n}^{m} e^{-\nu_i t} + C_{2n}^{m} e^{\nu_i t} \right) \cos m(\lambda + \sigma_r t) +$$

$$+ \left(D_{1n}^{m} e^{-\nu_i t} + D_{2n}^{m} e^{-\nu_i t} \right) \sin m(\lambda + \sigma_r t) \Big] P_n^m$$

$$\psi_{1n}^{m} = f_n \Big\{ C_{1n}^{m} e^{-\nu_i t} \cos[m(\lambda + \sigma_r t) - \varphi] + C_{2n}^{m} e^{\nu_i t} \cos[m(\lambda + \sigma_r t) + \varphi] + \tag{14}$$

$$+ D_{1n}^{m} e^{-\nu_i t} \sin[m(\lambda + \sigma_r t) - \varphi] + D_{2n}^{m} e^{\nu_i t} \sin[m(\lambda + \sigma_r t) + \varphi] \Big\} P_n^m$$

Analogously to the stable case we find:

$$C_{1n}^{m} = \frac{1}{2} A_n^m + k_1 \bar{B}_n^m - k_2 B_n^m \qquad k_1 = \frac{1}{2 f \sin \varphi}$$

$$D_{2n}^{m} = \frac{1}{2} B_n^m + k_1 \bar{A}_n^m + k_2 A_n^m$$

$$C_{2n}^{m} = \frac{1}{2} A_n^m - k_1 \bar{B}_n^m - k_2 B_n^m \qquad k_2 = \frac{1}{2 \operatorname{tg} \varphi}$$

$$D_{1n}^{m} = \frac{1}{2} B_n^m - k_1 \bar{A}_n^m + k_2 A_n^m$$

The relations (14) show that the phase of the disturbance the unstable case changes with height and consequently the axis of the disrbance tilts from the vertical.

We shall show that inside of the region of stability or instability e determined coefficients do not have peculiarities. In the expression for f_n the denomination in the expression under the radical [never] does not come zero. In the region of instability it is always $2\Lambda V > B$, r from the inequality opposite (8) it follows that $2\Lambda V > \sqrt{B^2 + \Lambda^4 V^2}$. the coefficient k_n^2 the sum $\sigma_n^2 + \alpha_3$ can become zero. The same

coefficient then becomes infinite. We shall show that β_n' (in which k_n^2 enters) is maintained by this final value. Consequently, for $\sigma_n^2 + \alpha_3 = 0$ it is realized in accordance with (6), the equality

$$\frac{(1+\Lambda)B}{\Lambda(\Lambda+2)} - V = \sqrt{\frac{B^2 - \Lambda^2(4-\Lambda^2)\Lambda^2}{\Lambda(\Lambda+2)}}$$

This is possible if $V = B/2A$ or $V = B/2$. We consider the first case. From (13) it is clear then that $\beta_n^2 = 0$. In order that one may find β_n', we take $V = \frac{B}{2\Lambda} + x$, we substitute in the expression for β_n', and we pass to the limit for $x \to 0$. Then $\beta_n' \to 1$. Analogously the second case can also be examined. The numerical evaluations show that $\sigma_n' + \alpha_3$ cannot take the value zero. We pass to the study of the properties of the short waves, i.e. the disturbances with a large number n. If for these large-values, n is greater than m, then the disturbance has a lesser longitudinal dimension. But if m is not large, then $n-m$ is large. And, since $n-m$ is a quantity equal to zero on the meridian, then the disturbance has a small cross dimension. Therefore the larger value of n corresponds to the disturbances which have either small longitudinal or small transverse dimension. From the formulas (6) and (13) it is clear that for $n \to \infty$

$$\sigma_n' \to -\alpha_3, \quad \sigma_n^2 \to -\alpha_1, \quad \beta_n^2 \to 0, \quad \beta_n^4 \to 0$$

because $\Lambda \to \infty$ as n^2, but the remaining parameters do not depend on n. In order to find the limit of β_n' we represent this value in the form:

$$\beta_n' = \frac{(1+\Lambda)\cdot(2\Lambda V - B)}{2\sqrt{B^2 - \Lambda^2(4-\Lambda^2)V^2}} + \frac{B - 2V}{\sigma_2 + \alpha_3}\beta_n^2$$

The limit of the first component is 1 and the second rapidly converges to zero.

Consequently $\beta_n^1 \to 1$ as $n \to \infty$. Since by (12) $\beta_n^3 = 1 - \beta_n^1$, then it is clear that $\beta_n^3 \to 0$ for $n \to \infty$. From this last it follows that $C_{1n}^m \to A_n^m$, $D_{1n}^m \to B_n^m$, $\bar{C}_{2n}^m \to \bar{A}_n^m$, $\bar{D}_{2n}^m \to \bar{B}_n^m$. At the same time, C_{2n}^m, D_{2n}^m, $\bar{C}_{1n}^m$, D_{1n}^m converge rapidly to zero.

Thus, the short waves move, without changing their shapes, with the speed of the west-east transport on an equivalent level. The elaborated theory is adapted by us to the prognosis of the pressure fields of 700 mb and 300 mb. In the series (14) we take $m = 12$ and $n = 20$. The latitude step is $\Delta\theta = 5^\circ$, the longitude step is $\Delta\lambda = 15^\circ$. For the simplification of the computation we have considered $\Psi = g z / f$ where z is the absolute topography, f is an average value of the Coriolis parameter for the hemisphere. The initial situation taken was December 9, 1957. The prognosis was given for one day, $\Delta\alpha$ was computed from the initial map and was equal to 53 (an unstable case); $\bar{\alpha}_2 = 72$. As is seen in figures 5-10 the results obtained reflect the evolution and movement of the large scale cyclones. The motion of the cyclone from the Stockholm region to the Leningrad region and its deepening was correctly predicted. Also a good prediction of the development of an Icelander cyclone. By the calculation there is a pressure fall of 15 decameters ~~and~~ that is actually 16 decameters. It was worse in the western hemisphere. Thus, in the territory of the USA the pressure formation showed movements to the East. Separate errors could be partially explained by thesmall number of terms of the series and also large steps for the longitude and latitude. Therefore it is necessary to take in the first specification the [enlistment] of a greater number of terms in the series for the spherical functions and the selection of smaller grid intervals for the horizontal coordinates.

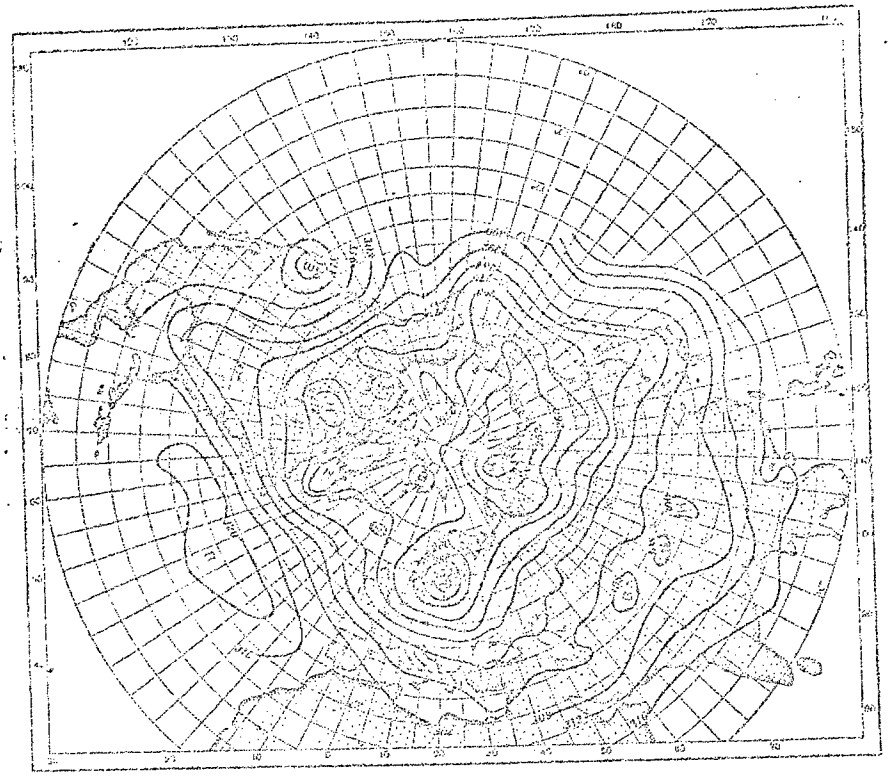


Fig. 5. Map AT - 700 for 03Z December 9 1957.

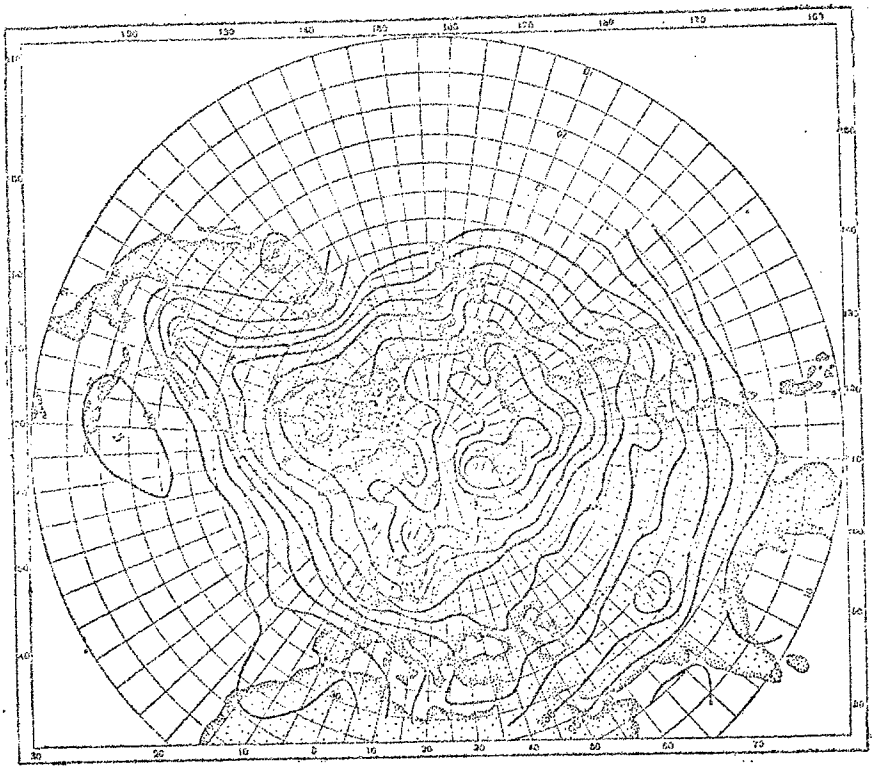


Fig. 6. Map AT - 300 for 03Z December 9 1957.

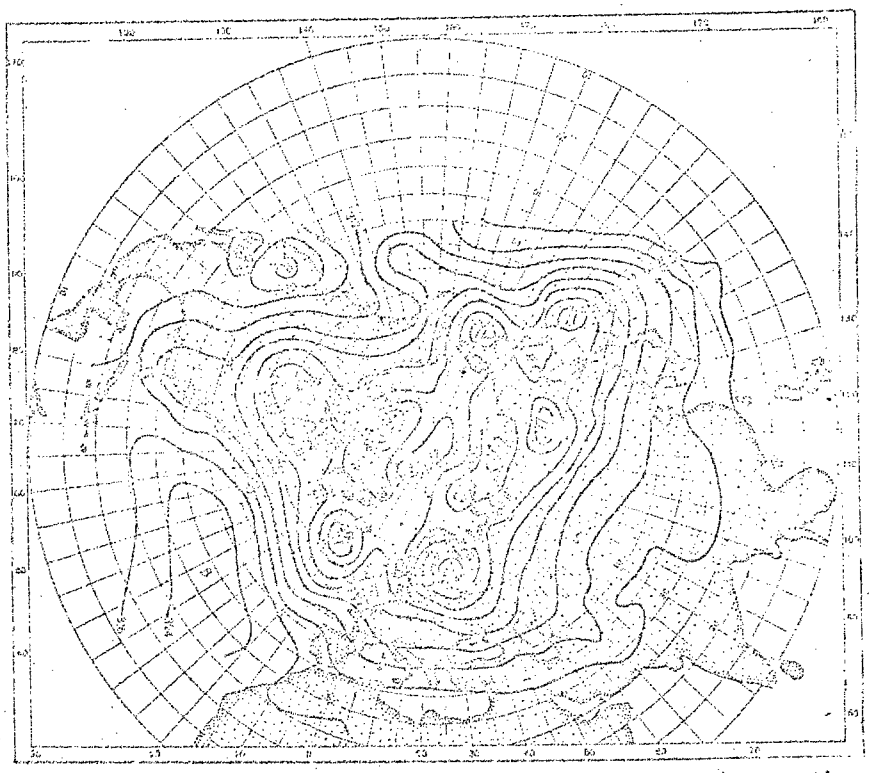


Fig. 7. Map AT - 700 for 03Z December 10 1957 (Actual)

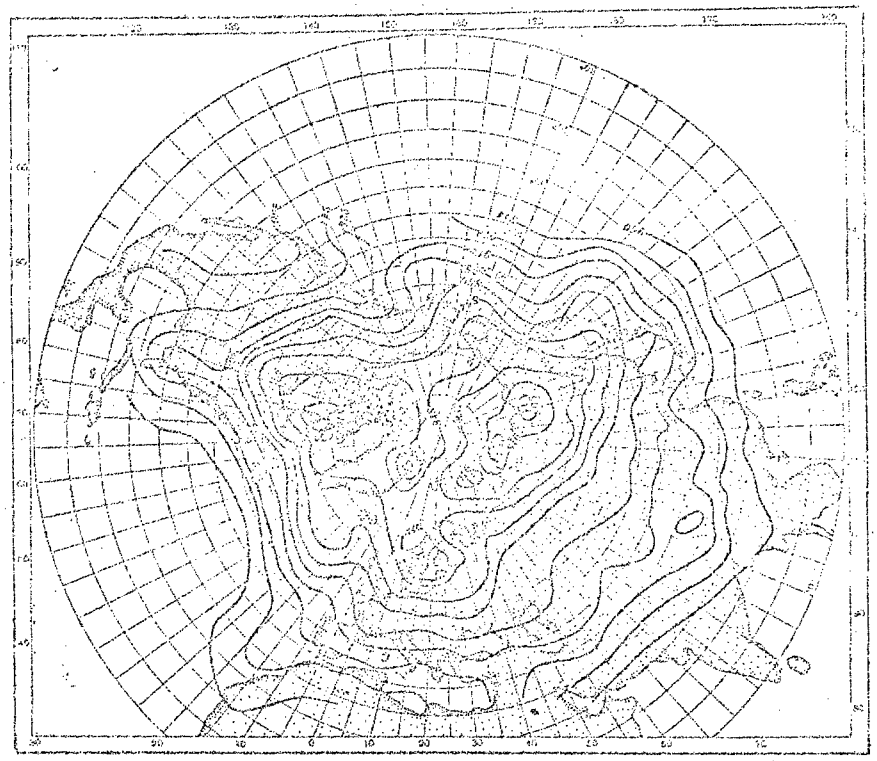


Fig. 8. Map AT - 300 for 03Z December 10 1957 (Actual)

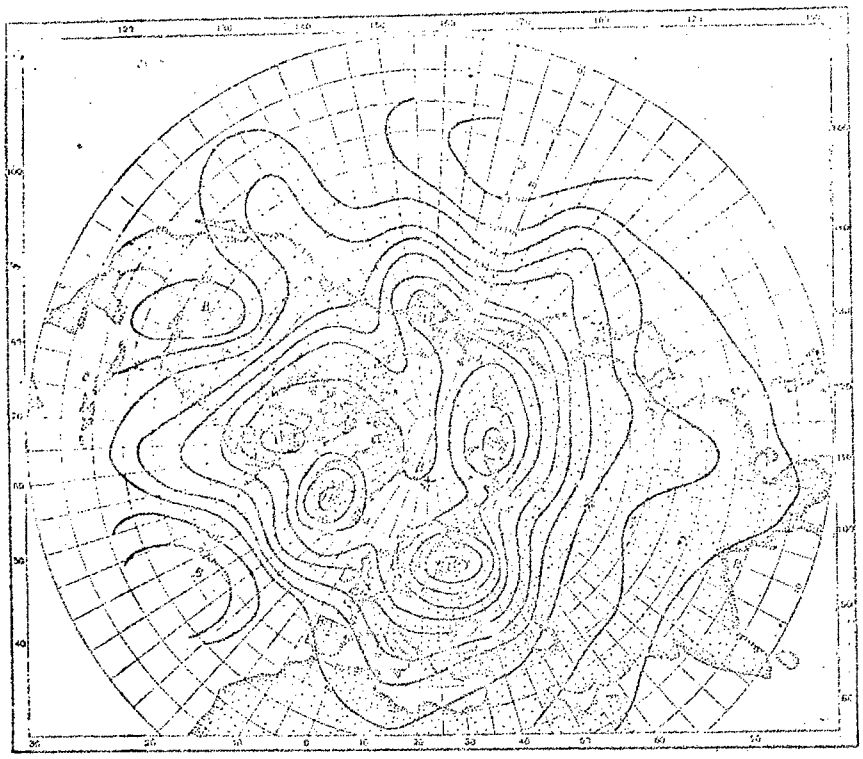


Fig. 9. Map AT - 700 for 03Z December 10 1957 (Forecast)

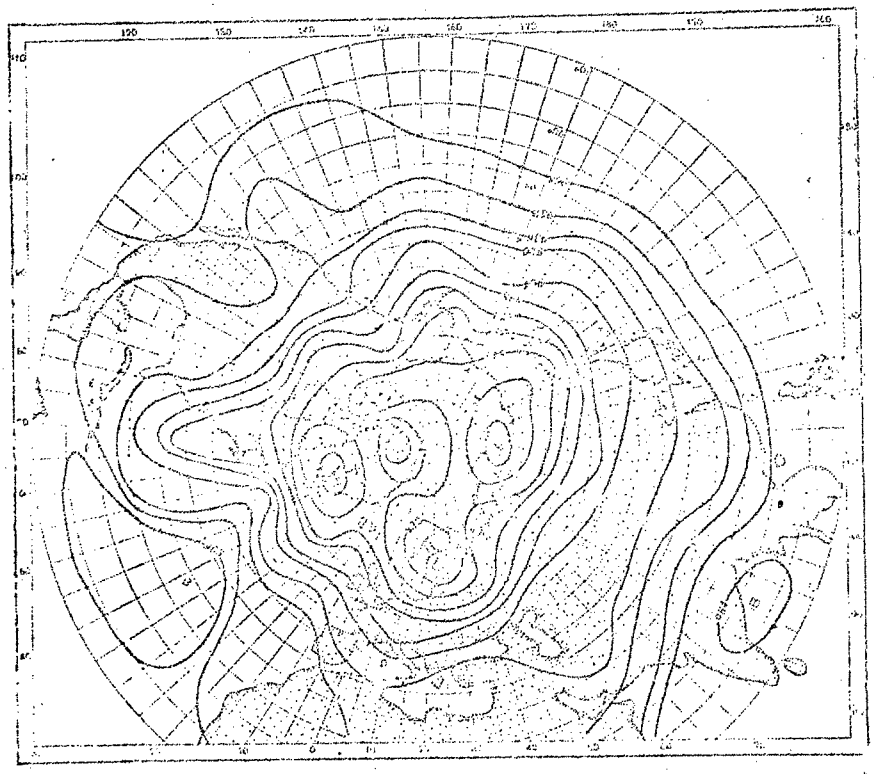


Fig. 10. Map AT - 300 for 03Z December 10 1957 (Forecast)

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