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By Lin Hung-sun

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THE PRODUCTION AND NUCLEAR CAPTURE OF A K MESON OBSERVED IN A CLOUD CHAMBER

By Cheng Jen-ch'i, Lu Min,
Wang Kan-ch'ang,
Hsiao Chien

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BENDING OF CIRCULAR AND RING-SHAPED ELASTIC THIN PLATE UNDER ARBITRARY LATERAL LOAD*

-Communist China-

/Following is the translation of an article by Lin Hung-sun (2651 7703 5549), of the Institute of Mechanics, Academia Sinica, in Wu-li Hsüeh-pao (Journal of Physics), Vol. 12, No. 4, July 1956, pp. 360-372./

1. Introduction

The recent introductory report by Professor W. Nowacki of the Corresponding Institute, Academy of Science, Poland, written in Peiping, concerning his work on elasticity in thin plates, aroused both our interest and our attention. In the course of discussion, the author of this paper has tried to explain Professor W. Nowacki's basic method -- derived from the viewpoint of physics -- from the standpoint of finite transformations.

In what follows, we shall discuss the problem of bending of circular and ring-shaped elastic thin-plates under arbitrary, lateral distribution loads with clamped-edge, or simply supported-edge, in an effort to explain the application of finite Hankel transform.

In particular, the finite transforms of a function f(x) over the interval $0 \leqslant x \leqslant 1$ are defined as

$$\bar{f}(\xi_{m,i}) = \int_0^1 x\, f(x)\, J_m(\xi_{m,i}\, x)\, dx; \tag{1a}$$

where J_m represents the Bessel function of the first kind, m-th order, and $\xi_{m,i}$ represents the i-th root of the equation

$$J_m(x) = 0 \tag{1b}$$

* Received 3 February 1956.

Then, it may be proved,

$$f(x) = 2\sum_{i=1}^{\infty} f(\xi_{m,i})\ \frac{J_m(\xi_{m,i}\,x)}{[J'_m(\xi_{m,i})]^2}. \tag{1c}$$

The individual conditions of anti-symmetrical bending of elastic circular thin-plates has been discussed by W. Flugge, H. Reissner, H. Schmidt, A. I. Lur'ye, and others. The method suggested in this paper facilitates the simplified expression of the solution to the problem.

2. Bending of Circular Thin-Plates Under Arbitrary Lateral Load with Clamped-edge

As everybody knows, the fulfilled differential equation of deflection W, for the elastic circular plate, under the effects of arbitrary lateral load p(r, θ), and with thickness h, is

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)^2 w = \frac{p}{D}; \tag{1}$$

where $D = \frac{Eh^3}{12(1-\sigma^2)}$, is the bending rigidity, E is the Young's Modulus, and σ is Poisson's co-efficient. If the plate is clamped on the edge r a (a is the radius of the plate), then we find for the problem the following boundary condition:

$$w\Big|_{r=a} = 0, \qquad \frac{\partial w}{\partial r}\Big|_{r=a} = 0. \tag{2}$$

Introducing the dimensionless quantity

$$W = \frac{w}{a}, \quad x = \frac{r}{a}, \quad q = \frac{pa^3}{D}; \tag{2a}$$

Then, the boundary condition in equation (2), and differential equation (1) becomes

$$\left(\frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial x} + \frac{1}{x^2}\frac{\partial^2}{\partial \theta^2}\right)^2 W = q; \tag{3}$$

$$W\Big|_{x=1} = 0, \qquad \frac{\partial W}{\partial x}\Big|_{x=1} = 0. \tag{4}$$

First, expand $q(x, \theta)$ into

$$q = Q_0(x) + \sum_{m=1}^{\infty} Q_m(x) \cos m\theta + \sum_{m=1}^{\infty} R_m(x) \sin m\theta , \tag{5}$$

where

$$\left.\begin{aligned} Q_0(x) &= \frac{1}{2\pi} \int_0^{2\pi} q \, d\theta , \\ Q_m(x) &= \frac{1}{\pi} \int_0^{2\pi} q \cos m\theta \, d\theta , \\ R_m(x) &= \frac{1}{\pi} \int_0^{2\pi} q \sin m\theta \, d\theta . \end{aligned}\right\} \tag{6}$$

Now express W as

$$W = U_0(x) + \sum_{m=1}^{\infty} U_m(x) \cos m\theta + \sum_{m=1}^{\infty} V_m(x) \sin m\theta . \tag{7}$$

In regard to U_0, U_m, and V_m, we have the following type of boundary conditions:

$$\left(\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} - \frac{m^2}{x^2}\right)^2 X_m = P_m ; \tag{8}$$

$$X_m \Big|_{x=1} = 0 , \qquad \frac{dX_m}{dx}\Big|_{x=1} = 0 . \tag{9}$$

Now, utilizing the finite Hankel transform method, multiply both sides of equation (8) by $x J_m(\xi_{m,i} x)$; then, integrate with respect to x from 0 to 1:

Now, we have

$$\begin{aligned} &\int_0^1 x\left(\frac{d^2 X_m}{dx^2} + \frac{1}{x}\frac{dX_m}{dx} - \frac{m^2 X_m}{x^2}\right) J_m(\xi_{m,i} x)\, dx = \\ &= \left[x \frac{dX_m}{dx} J_m(\xi_{m,i} x)\right]_0^1 - \int_0^1 \frac{dX_m}{dx} x \xi_{m,i} J_m'(\xi_{m,i})\, dx - \int_0^1 X_m \left(\frac{m^2 J_m(\xi_{m,i} x)}{x}\right) dx = \\ &= \left[x \frac{dX_m}{dx} J_m(\xi_{m,i} x) - \xi_{m,i} x X_m J_m'(\xi_{m,i} x)\right]_0^1 - \xi_{m,i}^2 \int_0^1 x X_m J_m(\xi_{m,i} x)\, dx \end{aligned} \tag{9a}$$

Or,

$$\int_0^1 x\left(\frac{d^2 X_m}{dx^2}+\frac{1}{x}\frac{dX_m}{dx}-\frac{m^2 X_m}{x^2}\right)dx = \tag{10}$$
$$=\left[x\frac{dX_m}{dx}J_m(\xi_{m,i}x)-\xi_{m,i}x X_m J'_m(\xi_{m,i}x)\right]_0^1-\xi_{m,i}^2\bar{X}_m(\xi_{m,i}),$$

Here, use the symbol of the finite Hankel transform:

$$\bar{X}_m(\xi_{m,i})=\int_0^1 x X_m J_m(\xi_{m,i}x)\,dx. \tag{11}$$

Similarly,

$$\int_0^1 x\left(\frac{d^2}{dx^2}+\frac{1}{x}\frac{d}{dx}-\frac{m^2}{x^2}\right)\left(\frac{d^2 X_m}{dx^2}+\frac{1}{x}\frac{dX_m}{dx}-\frac{m^2 X_m}{x^2}\right)J_m(\xi_{m,i}x)\,dx = \tag{12}$$
$$=\left[x\frac{d}{dx}\left(\frac{d^2 X_m}{dx^2}+\frac{1}{x}\frac{dX_m}{dx}-\frac{m^2 X_m}{x^2}\right)J_m(\xi_{m,i}x)-\right.$$
$$\left.-\xi_{m,i}x\left(\frac{d^2 X_m}{dx^2}+\frac{1}{x}\frac{dX_m}{dx}-\frac{m^2 X_m}{x^2}\right)J'_m(\xi_{m,i}x)\right]_0^1-$$
$$-\xi_{m,i}^2\left[x\frac{dX_m}{dx}J_m(\xi_{m,i}x)-\xi_{m,i}x X_m J'_m(\xi_{m,i}x)\right]_0^1+\xi_{m,i}^4\bar{X}_m(\xi_{m,i}).$$

Suppose $\xi_{m,i}$ is taken as the i-th root of the equation

$$J_m(x)=0 \tag{1b}$$

Then, using boundary conditions in equation (9), we have

$$\int_0^1 x\left(\frac{d^2}{dx^2}+\frac{1}{x}\frac{d}{dx}-\frac{m^2}{x^2}\right)^2 X_m J_m(\xi_{m,i}x)\,dx = \tag{12a}$$
$$=-\xi_{m,i}A_m J'_m(\xi_{m,i})+\xi_{m,i}^4\bar{X}_m(\xi_{m,i}),$$

where

$$A_m=\left(\frac{d^2 X_m}{dx^2}\right)_{x=1} \tag{13}$$

is an undetermined quantity.

If we write

$$\bar{P}_m(\xi_{m,i}) = \int_0^1 x P_m(x) J_m(\xi_{m,i} x)\, dx, \tag{14}$$

Then, the result of the integration in equation (8) above, gives us the following

$$\xi_{m,i}^4 \bar{X}_m(\xi_{m,i}) - \xi_{m,i} A_m J'_m(\xi_{m,i}) = \bar{P}_m(\xi_{m,i}), \tag{14a}$$

Then, we obtain

$$\bar{X}_m(\xi_{m,i}) = \frac{\bar{P}_m(\xi_{m,i})}{\xi_{m,i}^4} + \frac{J'_m(\xi_{m,i})}{\xi_{m,i}^3} A_m; \tag{15}$$

And, according to the form of the finite Hankel transform, we have

$$X_m(x) = 2\sum_{i=1}^{\infty} \frac{\bar{P}_m(\xi_{m,i}) J_m(\xi_{m,i} x)}{\xi_{m,i}^4 [J'_m(\xi_{m,i})]^2} + 2 A_m \sum_{i=1}^{\infty} \frac{J_m(\xi_{m,i} x)}{\xi_{m,i}^3 J'_m(\xi_{m,i})}. \tag{16}$$

Obviously, $X_m \Big|_{x=1} = 0$. Then, in order to satisfy the condition $dY_n/dx \Big|_{x=1} = 0$, we must have

$$2\sum_{i=1}^{\infty} \frac{\bar{P}_m(\xi_{m,i})}{\xi_{m,i}^3 J'_m(\xi_{m,i})} + 2 A_m \sum \frac{1}{\xi_{m,i}^2} = 0. \tag{16a}$$

Whence, we have

$$A_m = -\frac{\sum_{i=1}^{\infty} \frac{\bar{P}_m(\xi_{m,i})}{\xi_{m,i}^3 J'_m(\xi_{m,i})}}{\sum_{i=1}^{\infty} \frac{1}{\xi_{m,i}^2}}. \tag{17}$$

Then, finally, the expression of the dimensionless deflection of the plate,

$$
\begin{aligned}
W(x,\theta) = {} & 2\sum_{i=1}^{\infty} \frac{\bar{Q}_0(\xi_{0,i})\, J_0(\xi_{0,i}\, x)}{\xi_{0,i}^4\, [J_0(\xi_{0,i})]^2} + 2\, A_0 \sum_{i=1}^{\infty} \frac{J_0(\xi_{0,i}\, x)}{\xi_{0,i}^3\, J_0'(\xi_{0,i})} + \\
& + 2\sum_{m=1}^{\infty}\sum_{i=1}^{\infty} \frac{J_m(\xi_{m,i}\, x)}{\xi_{m,i}^4\, [J_m'(\xi_{m,i})]^2}\, [\bar{Q}_m(\xi_{m,i}) \cos m\theta + \bar{R}_m(\xi_{m,i}) \sin m\theta] + \\
& + 2\sum_{m=1}^{\infty} (A_m \cos m\theta + B_m \sin m\theta) \sum_{i=1}^{\infty} \frac{J_m(\xi_{m,i}\, x)}{\xi_{m,i}^3\, J_m'(\xi_{m,i})} .
\end{aligned}
\tag{18}
$$

where

$$
\left.
\begin{aligned}
\bar{Q}_0(\xi_{0,i}) &= \int_0^1 x\, Q_0\, J_0(\xi_{0,i}\, x)\, dx , \\
\bar{Q}_m(\xi_{m,i}) &= \int_0^1 x\, Q_m\, J_m(\xi_{m,i}\, x)\, dx , \\
\bar{R}_m(\xi_{m,i}) &= \int_0^1 x\, R_m\, J_m(\xi_{m,i}\, x)\, dx .
\end{aligned}
\right\}
\tag{19}
$$

$$
\left.
\begin{aligned}
A_0 &= -\frac{\displaystyle\sum_{i=1}^{\infty} \frac{\bar{Q}_0(\xi_{0,i})}{\xi_{0,i}^3\, J_0(\xi_{0,i})}}{\displaystyle\sum_{i=1}^{\infty} \frac{1}{\xi_{0,i}^2}} \;{}^{1)} , \\
A_m &= -\frac{\displaystyle\sum_{i=1}^{\infty} \frac{\bar{Q}_m(\xi_{m,i})}{\xi_{m,i}^3\, J_m'(\xi_{m,i})}}{\displaystyle\sum_{i=1}^{\infty} \frac{1}{\xi_{m,i}^2}} , \\
B_m &= -\frac{\displaystyle\sum_{i=1}^{\infty} \frac{\bar{R}_m(\xi_{m,i})}{\xi_{m,i}^3\, J_m'(\xi_{m,i})}}{\displaystyle\sum_{i=1}^{\infty} \frac{1}{\xi_{m,i}^2}} .
\end{aligned}
\right\}
\tag{20}
$$

1) We can prove $\displaystyle\sum_{i=1}^{\infty} \frac{1}{\xi_{m,i}^2} = \frac{1}{4(m+1)}$,

thus, $\displaystyle\sum_{i=1}^{\infty} \frac{1}{\xi_{0,i}} = \frac{1}{4}$.

At the center of the circular plate ($x=0$), we have

$$W(0) = 2\sum_{i=1}^{\infty} \frac{\bar{Q}(\xi_{0,i})}{\xi_{0,i}^{4}\,[J_0'(\xi_{0,i})]^2} + 2A_0 \sum_{i=1}^{\infty} \frac{1}{\xi_{0,i}^{3}\,J_0'(\xi_{0,i})}\,. \tag{21}$$

Now, we consider an important special load condition. Assume that at $x=c$ $(0 \leqslant c \leqslant 1)$, $\theta=0$, there is a 'unit' -- expressed in dimensionless quantity -- where the load is concentrated. Then, we can use the δ- function form:

$$q = \frac{\delta(x-c)\,\delta(\theta-0)}{x}\,, \tag{22}$$

Then,

$$\left.\begin{aligned} Q_0(x) &= \frac{1}{2\pi}\int_0^{2\pi} q\,d\theta = \frac{\delta(x-c)}{2\pi x}\,, \\ Q_m(x) &= \frac{1}{\pi}\int_0^{2\pi} q\cos m\theta\,d\theta = \frac{\delta(x-c)}{\pi x}\,, \\ R_m(x) &= \frac{1}{\pi}\int_0^{2\pi} q\sin m\theta\,d\theta = 0\,. \end{aligned}\right\} \tag{23}$$

However,

$$\left.\begin{aligned} \bar{Q}_0(\xi_{0,i}) &= \int_0^1 x\,Q_0\,J_0(\xi_{0,i}\,x)\,dx = \frac{J_0(\xi_{0,i}\,c)}{2\pi}\,, \\ \bar{Q}_m(\xi_{m,i}) &= \int_0^1 x\,Q_m\,J_m(\xi_{m,i}\,x)\,dx = \frac{J_m(\xi_{m,i}\,c)}{\pi}\,, \\ \bar{R}_m(\xi_{m,i}) &= \int_0^1 x\,R_m\,J_m(\xi_{m,i}\,x)\,dx = 0\,. \end{aligned}\right\} \tag{24}$$

Then,

$$\left.\begin{aligned} A_0 &= -\frac{\displaystyle\sum_{i=1}^{\infty} \frac{J_0(\xi_{0,i}\,c)}{\xi_{0,i}^{3}\,J_0(\xi_{0,i})}}{\displaystyle 2\pi\sum \frac{1}{\xi_{0,i}^{2}}}\,, \\ A_m &= -\frac{\displaystyle\sum_{i=2}^{\infty} \frac{J_m(\xi_{m,i}\,c)}{\xi_{m,i}^{3}\,J_m(\xi_{m,i})}}{\displaystyle \pi\sum_{i=1}^{\infty} \frac{1}{\xi_{m,i}^{2}}}\,, \\ B_m &= 0\,. \end{aligned}\right\} \tag{25}$$

Then, we have

$$W(x,\theta)=\frac{1}{\pi}\sum_{i=1}^{\infty}\frac{J_0(\xi_{0,i}c)\,J_0(\xi_{0,i}x)}{\xi_{0,i}^4\,[J_0'(\xi_{0,i})]^2}-\frac{1}{\pi}\frac{\left(\sum_{i=1}^{\infty}\frac{J_0(\xi_{0,i}c)}{\xi_{0,i}^3\,J_0'(\xi_{0,i})}\right)}{\sum_{i=1}^{\infty}\frac{1}{\xi_{0,i}^2}}\sum_{i=1}^{\infty}\frac{J_0(\xi_{0,i}x)}{\xi_{0,i}^3\,J_0'(\xi_{0,i})}+ \tag{26}$$

$$+\frac{2}{\pi}\sum_{m=1}^{\infty}\cos m\theta\sum_{i=1}^{\infty}\frac{J_m(\xi_{m,i}c)\,J_m(\xi_{m,i}x)}{\xi_{m,i}^4\,[J_m'(\xi_{m,i})]^2}-$$

$$-\frac{2}{\pi}\sum_{m=1}^{\infty}\frac{\left(\sum_{i=1}^{\infty}\frac{J_m(\xi_{m,i}c)}{\xi_{m,i}^3\,J_m'(\xi_{m,i})}\right)}{\sum_{i=1}^{\infty}\frac{1}{\xi_{m,i}^2}}\cos m\theta\sum_{i=1}^{\infty}\frac{J_m(\xi_{m,i}x)}{\xi_{m,i}^3\,J_m'(\xi_{m,i})},$$

And, at the center of the circular plate ($x=0$), we have

$$W(0)=\frac{1}{\pi}\sum_{i=1}^{\infty}\frac{J_0(\xi_{0,i}c)}{\xi_{0,i}^4\,[J_0'(\xi_{0,i})]^2}-\frac{1}{\pi}\left(\frac{\sum_{i=1}^{\infty}\frac{J_0(\xi_{0,i}c)}{\xi_{0,i}^3\,J_0'(\xi_{0,i})}}{\sum\frac{1}{\xi_{0,i}^2}}\right)\sum_{i=1}^{\infty}\frac{1}{\xi_{0,i}^3\,J_0'(\xi_{0,i})}. \tag{27}$$

If there is a line along $\theta=0$ with total intensity equal to load of 1, then

$$q=\frac{\delta(\theta-0)}{x}, \tag{28}$$

And,

$$Q_0=\frac{1}{2\pi x},\quad Q_m=\frac{1}{\pi x},\quad R_m=0; \tag{29}$$

And,

$$\left.\begin{aligned}\bar{Q}_0(\xi_{0,i})&=\frac{1}{2\pi}\int_0^1 J_0(\xi_{0,i}x)\,dx,\\ \bar{Q}_m(\xi_{m,i})&=\frac{1}{\pi}\int_0^1 J_m(\xi_{m,i}x)\,dx,\end{aligned}\right\} \tag{30}$$

$$\bar{R}_m(\xi_{m,i})=0.$$

Substituting into equation (18), we are able to obtain the solution. Of course, the solution can also be obtained by

integrating with respect to c from 0 to 1.

Nor, is it difficult to discuss the more general load conditions, such as

$$q = \frac{g(x)\,\delta(\theta - 0)}{x}, \tag{30a}$$

This is equivalent to variable load conditions along line $\theta = 0$, et cetera.

3. Bending of Circular Thin-Plates Under Arbitrary Lateral Load With Clamped-edge With Center Tension

Using the method suggested in the preceding section, we can solve the problem of bending of circular thin-plates under arbitrary lateral load with clamped-edge with center tension. This problem has been considered by W. G. Bickley, but he discussed only uniform loads and concentrated load conditions at arbitrary points on the plane plate. Furthermore, Bickley used different expansion methods.

The formula for loads under these circumstances is as follows:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial x} + \frac{1}{x^2}\frac{\partial^2}{\partial \theta^2}\right)^2 W - \tau\left(\frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial x} + \frac{1}{x^2}\frac{\partial^2}{\partial \theta^2}\right) W = q, \tag{31}$$

where $\tau = Ta^2/D$ is a dimensionless magnitude.

And, as before, q is written as

$$q = Q_0(x) + \sum_{m=1}^{\infty} Q_m(x)\cos m\theta + \sum_{m=1}^{\infty} R_m(x)\sin m\theta, \tag{32}$$

And, expanding W as

$$W = U_0(x) + \sum_{m=1}^{\infty} U_m(x)\cos m\theta + \sum_{m=1}^{\infty} V_m(x)\sin m\theta, \tag{33}$$

In regard to $U_0(x)$, $U_m(x)$, $V_m(x)$, we have the following type of boundary conditions:

$$\left(\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} - \frac{m^2}{x^2}\right)^2 X_m - \tau\left(\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} - \frac{m^2}{x^2}\right) X_m = P_m. \tag{34}$$

$$X_m\Big|_{x=1} = 0, \qquad \frac{dX_m}{dx}\Big|_{x=1} = 0. \tag{35}$$

Multiplying both sides of equation (34) by $x J_m(\xi_{m,i} x)$, integrating with respect to x from 0 to 1 and using equations (10) and (12), and boundary conditions in equation (35), we are able to obtain

$$\xi_{m,i}^4 \bar{X}_m(\xi_{m,i}) + \tau \xi_{m,i}^2 \bar{X}_m(\xi_{m,i}) - \xi_{m,i} A_m(\tau) J'_m(\xi_{m,i}) = \bar{P}_m(\xi_{m,i}); \tag{36}$$

where $\xi_{m,i}$ is the i-th root of equation

$$J_m(x) = 0 \tag{1b}$$

Then, the undetermined quantity is

$$A_m(\tau) = \left(\frac{d^2 X_m}{dx^2}\right)_{x=1} \tag{37}$$

And,

$$\bar{X}_m(\xi_{m,i}) = \frac{\bar{P}_m(\xi_{m,i})}{\xi_{m,i}^4 + \tau \xi_{m,i}^2} + \frac{J'_m(\xi_{m,i})}{\xi_{m,i}^3 + \tau \xi_{m,i}} A_m(\tau) \tag{38}$$

And,

$$X_m = 2 \sum_{i=1}^{\infty} \frac{\bar{P}_m(\xi_{m,i}) J_m(\xi_{m,i} x)}{(\xi_{m,i}^4 + \tau \xi_{m,i}^2)[J'_m(\xi_{m,i})]^2} + 2 A_m(\tau) \sum_{i=1}^{\infty} \frac{J_m(\xi_{m,i} x)}{(\xi_{m,i}^3 + \tau \xi_{m,i}) J'_m(\xi_{m,i})}. \tag{39}$$

Obviously, $X_m \Big|_{x=1} = 0$. Then, in order to satisfy the condition $dX_m/dX \Big|_{x=1} = 0$, we must have

$$A_m(\tau) = \frac{-\sum_{i=1}^{\infty} \frac{\bar{P}_m(\xi_{m,i})}{(\xi_{m,i}^3 + \tau \xi_{m,i}) J'_m(\xi_{m,i})}}{\sum_{i=1}^{\infty} \frac{1}{\xi_{m,i}^2 + \tau}}, \tag{40}$$

And, finally, we have

$$W(x,\theta) = 2 \sum_{i=1}^{\infty} \frac{\bar{Q}_0(\xi_{0,i}) J_0(\xi_{0,i} x)}{(\xi_{0,i}^4 + \tau \xi_{0,i}^2)[J'_0(\xi_{0,i})]^2} + 2 A_0(\tau) \sum_{i=1}^{\infty} \frac{J_0(\xi_{0,i} x)}{(\xi_{0,i}^3 + \tau \xi_{0,i}) J'_0(\xi_{0,i})} + \tag{41}$$

$$+ 2 \sum_{m=1}^{\infty} \sum_{i=1}^{\infty} \frac{J_m(\xi_{m,i}\, x)}{(\xi_{m,i}^2 + \tau\, \xi_{m,i}^2)\, [J_m(\xi_{0,i})]^2} \, [\bar{Q}_m(\xi_{m,i}) \cos m\theta + \bar{R}_m(\xi_{m,i}) \sin m\theta] +$$

$$+ 2 \sum_{m=1}^{\infty} [A_m(\tau) \cos m\theta + B_m(\tau) \sin m\theta] \sum_{i=1}^{\infty} \frac{J_m(\xi_{m,i}\, x)}{(\xi_{m,i}^2 + \tau\, \xi_{m,i})\, J_m(\xi_{m,i})} \; . \tag{41}$$

where

$$\left.\begin{aligned} \bar{Q}_0(\xi_{0,i}) &= \int_0^1 x\, Q_0\, J_0(\xi_{0,i}\, x)\, dx \, , \\ \bar{Q}_m(\xi_{m,i}) &= \int_0^1 x\, Q_m\, J_m(\xi_{m,i}\, x)\, dx \, , \\ \bar{R}_m(\xi_{m,i}) &= \int_0^1 x\, R_m\, J_m(\xi_{m,i}\, x)\, dx \, . \end{aligned}\right\} \tag{42}$$

And,

$$\left.\begin{aligned} A_0(\tau) &= - \frac{\sum_{i=1}^{\infty} \frac{\bar{Q}_0(\xi_{0,i})}{(\xi_{0,i}^2 + \tau\, \xi_{0,i})\, J_0(\xi_{0,i})}}{\sum_{i=1}^{\infty} \frac{1}{\xi_{0,i}^2 + \tau}} \, , \\ A_m(\tau) &= - \frac{\sum_{i=1}^{\infty} \frac{\bar{Q}_m(\xi_{m,i})}{(\xi_{m,i}^2 + \tau\, \xi_{m,i})\, J_m(\xi_{m,i})}}{\sum_{i=1}^{\infty} \frac{1}{\xi_{m,i}^2 + \tau}} \, , \\ B_m(\tau) &= - \frac{\sum_{i=1}^{\infty} \frac{\bar{R}_m(\xi_{m,i})}{(\xi_{m,i}^2 + \tau\, \xi_{m,i})\, J_m(\xi_{m,i})}}{\sum_{i=1}^{\infty} \frac{1}{\xi_{m,i}^2 + \tau}} \, . \end{aligned}\right\} \tag{43}$$

4. Bending of Circular Thin-Plate Under Arbitrary Load With Simply-Supported Edge

Now we consider the conditions of a simply-supported edge. We do not need to discuss in further detail the basics, we need only state here that we have secondary-type boundary conditions:

$$\left(\frac{d^2}{dx^2} + \frac{1}{x} \frac{d}{dx} - \frac{m^2}{x^2} \right)^2 X_m = P_m \, , \tag{44}$$

$$X_m|_{x=1}, \tag{45}$$

$$\left[\frac{d^2 X_m}{dx^2} + \frac{\sigma}{x}\frac{d X_m}{dx}\right]_{x=1} = 0. \tag{46}$$

Equation (46) represents the edge bending moments equivalent to zero. Assume that $\xi_{m,i}$ is the i-th root of the equation

$$J_m(x) = 0 \tag{1b}$$

Then, we have (see Eq. 12):

$$\int_0^1 x\left(\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} - \frac{m^2}{x^2}\right)^2 X_m J_m(\xi_{m,i}x)\,dx = \tag{46a}$$

$$= \left[-\xi_{m,i}x\left(\frac{d^2 X_m}{dx^2} + \frac{1}{x}\frac{dX_m}{dx} - \frac{m^2 X_m}{x^2}\right)J'_m(\xi_{m,i}x)\right]_0^1 +$$

$$+ \xi_{m,i}^3 x X_m J'_m(\xi_{m,i}x)\Big|_0^1 + \xi_{m,i}^4 \bar{X}_m(\xi_{m,i});$$

Then, using boundary conditions in equations (45) and (46), we get

$$\int_0^1 x\left(\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} - \frac{m^2}{x^2}\right)^2 X_m J_m(\xi_{m,i}x)\,dx = \tag{47}$$

$$= -\xi_{m,i}(1-\sigma)C_m J'_m(\xi_{m,i}) + \xi_{m,i}^4 \bar{X}_m(\xi_{m,i}),$$

where

$$C_m = \left(\frac{dX_m}{dx}\right)_{x=1} \tag{48}$$

is the undetermined magnitude.
Hence, from equation (44), we are able to obtain

$$\bar{X}_m = \frac{\bar{P}_m(\xi_{m,i})}{\xi_{m,i}^4} + (1-\sigma)C_m\frac{J'_m(\xi_{m,i})}{\xi_{m,i}^3}. \tag{49}$$

And

$$X_m = 2\sum_{i=1}^{\infty} \frac{\bar{P}_m(\xi_{m,i})}{\xi_{m,i}^4} \frac{J_m(\xi_{m,i}x)}{[J'_m(\xi_{m,i})]^2} + 2(1-\sigma)\, C_m \sum_{i=1}^{\infty} \frac{J_m(\xi_{m,i}x)}{\xi_{m,i}^3 J'_m(\xi_{m,i})}, \tag{50}$$

Constant C_m is determined from the following equations:

$$C_m = \left(\frac{dX_m}{dx}\right)_{x=1} = 2\sum_{i=1}^{\infty} \frac{\bar{P}_m(\xi_{m,i})}{\xi_{m,i}^3 J'_m(\xi_{m,i})} + 2(1-\sigma)\, C_m \sum_{i=1}^{\infty} \frac{1}{\xi_{m,i}^2}, \tag{50a}$$

Or,

$$C_m = \frac{2\sum \frac{\bar{P}_m(\xi_{m,i})}{\xi_{m,i}^3 J'_m(\xi_{m,i})}}{1-2(1-\sigma)\sum_{i=1}^{\infty} \frac{1}{\xi_{m,i}^2}}. \tag{51}$$

Finally, we get the plates' dimensionless deflection W in the following manner:

$$\begin{aligned} W(x,\theta) = {} & 2\sum_{i=1}^{\infty} \frac{\bar{Q}_0(\xi_{0,i})\, J_0(\xi_{0,i}x)}{\xi_{0,i}^4\, [J'_0(\xi_{0,i})]^2} + 2\, C_0 \sum_{i=1}^{\infty} \frac{J_0(\xi_{0,i}x)}{\xi_{0,i}^3 J'_0(\xi_{0,i})} + \\ & + 2\sum_{m=1}^{\infty}\sum_{i=1}^{\infty} \frac{J_m(\xi_{m,i}x)}{\xi_{m,i}^4\, (J'_m(\xi_{m,i}))^2} [\bar{Q}_m(\xi_{m,i}) \cos m\theta + \bar{R}_m(\xi_{m,i}) \sin m\theta] + \\ & + 2(1-\sigma)\sum_{m=1}^{\infty}\sum_{i=1}^{\infty} \frac{J_m(\xi_{m,i}x)}{\xi_{m,i}^3 J'_m(\xi_{m,i})} [C_m \cos m\theta + D_m \sin m\theta]; \end{aligned} \tag{52}$$

where

$$\left.\begin{aligned} \bar{Q}_0(\xi_{0,i}) &= \int_0^1 x\, Q_0\, J_0(\xi_{0,i}x)\, dx, \\ \bar{Q}_m(\xi_{m,i}) &= \int_0^1 x\, Q_m\, J_m(\xi_{m,i}x)\, dx, \\ \bar{R}_m(\xi_{m,i}) &= \int_0^1 x\, R_m\, J_m(\xi_{m,i}x)\, dx. \end{aligned}\right\} \tag{53}$$

And,

$$\left.\begin{aligned} C_0 &= \frac{2\sum_{i=1}^{\infty} \frac{\bar{O}_0(\xi_{0,i})}{\xi_{0,i}^3 J_0(\xi_{0,i})}}{1-2(1-\sigma)\sum_{i=1}^{\infty}\frac{1}{\xi_{0,i}^2}}, \\ C_m &= \frac{2\sum_{i=1}^{\infty} \frac{\bar{O}_m(\xi_{m,i})}{\xi_{m,i}^3 J_m'(\xi_{m,i})}}{1-2(1-\sigma)\sum_{i=1}^{\infty}\frac{1}{\xi_{m,i}^2}}, \\ D_m &= \frac{2\sum_{i=1}^{\infty} \frac{\bar{R}_m(\xi_{m,i})}{\xi_{m,i}^3 J_m'(\xi_{m,i})}}{1-2(1-\sigma)\sum_{i=1}^{\infty}\frac{1}{\xi_{m,i}^2}}. \end{aligned}\right\} \tag{54}$$

5. Bending of Ring-shaped Plates Under Arbitrary Lateral Load

Assume that the ring-shaped boundary is composed of the concentric circles $x=1$ and $x=\lambda(\lambda>1)$. Under this type of condition, we can use the following (other) type of finite Hankel transform with respect to the function $f(x)$ of $1 \leqslant x \leqslant \lambda$, we introduce the transform equation

$$\bar{f}(\xi_{m,i}) = \int_1^{\lambda} x f(x) \left[J_m(\xi_{m,i} x) Y_m(\xi_{m,i} \lambda) - Y_m(\xi_{m,i} x) J_m(\xi_{m,i} \lambda)\right] dx, \tag{55}$$

Where Y_m is the Bessel function of the second kind, m-th order, and $\xi_{m,i}$ is the i-th root of the equation

$$J_m(x) Y_m(\lambda x) = Y_m(x) J_m(\lambda x) \tag{56}$$

Then, it may be proved that

$$f(x) = \frac{\pi^2}{2} \sum_{i=1}^{\infty} \frac{\xi_{m,i}^2 J_m^2(\xi_{m,i}\lambda) \bar{f}(\xi_{m,i})}{J_m^2(\xi_{m,i}) - J_m^2(\xi_{m,i}\lambda)} \left[J_m(\xi_{m,i} x) Y_m(\xi_{m,i}\lambda) - Y_m(\xi_{m,i} x) J_m(\xi_{m,i}\lambda)\right]. \tag{57}$$

First, consider the internal-external, two-sided

clamped situation, where we have the following kind of boundary conditions:

$$\left(\frac{d^2}{dx^2}+\frac{1}{x}\frac{d}{dx}-\frac{m^2}{x^2}\right)^2 X_m = P_m . \tag{58}$$

$$X_m = 0 , \qquad \frac{dX_m}{dx} = 0 \quad \text{at } x = 1 , \tag{59}$$

$$X_m = 0 , \qquad \frac{dX_m}{dx} = 0 \quad \text{at } x = \lambda \tag{60}$$

For the sake of simplicity, we can use the following symbol:

$$S_m(\xi_{m,i}\, x;\lambda) = J_m(\xi_{m,i}\, x)\, Y_m(\xi_{m,i}\, \lambda) - Y_m(\xi_{m,i}\, x)\, J_m(\xi_{m,i}\, \lambda) . \tag{61}$$

Multiplying both sides of equation (58) by $x\, S_m(\xi_{m,i}\, x;\lambda)$, and integrating with respect to x from 1 to λ, we have (see Eq. 12):

$$\begin{aligned}
&\int_1^\lambda x\left(\frac{d^2}{dx^2}+\frac{1}{x}\frac{d}{dx}-\frac{m^2}{x^2}\right)^2 X_m\, S_m(\xi_{m,i}\, x;\lambda)\, dx = \\
&= \left[x\frac{d}{dx}\left(\frac{d^2 X_m}{dx^2}+\frac{1}{x}\frac{dX_m}{dx}-\frac{m^2 X_m}{x^2}\right) S_m(\xi_{m,i}\, x;\lambda) - \right. \\
&\quad \left. - \xi_{m,i}\, x\left(\frac{d^2 X_m}{dx^2}+\frac{1}{x}\frac{dX_m}{dx}-\frac{m^2 X_m}{x^2}\right) S'_m(\xi_{m,i}\, x;\lambda)\right]_1^\lambda - \\
&\quad - \xi_{m,i}^2 \left[x\frac{dX_m}{dx}\, S_m(\xi_{m,i}\, x;\lambda) - \xi_{m,i}\, x\, X_m\, S'_m(\xi_{m,i}\, x;\lambda)\right]_1^\lambda + \\
&\quad + \xi_{m,i}^3\, X_m(\xi_{m,i}) .
\end{aligned} \tag{61a}$$

Using boundary conditions in equations (57) and (60), we get

$$\begin{aligned}
&- \xi_{m,i}\,\lambda\left(\frac{d^2 X_m}{dx^2}\right)_{x=\lambda} S'_m(\xi_{m,i}\, x;\lambda) + \xi_{m,i}\left(\frac{d^2 X_m}{dx^2}\right)_{x=1} S'_m(\xi_{m,i};\lambda) + \\
&\quad + \xi_{m,i}^3\, X_m(\xi_{m,i}) = \bar{P}_m(\xi_{m,i}) ,
\end{aligned} \tag{61b}$$

Whence, we obtain

$$\bar{X}_m(\xi_{m,i}) = \frac{\bar{P}_m(\xi_{m,i})}{\xi_{m,i}^3} + \frac{\lambda\left(\frac{d^2 X_m}{dx^2}\right)_{x=\lambda} S'_m(\xi_{m,i}\,\lambda;\lambda) - \left(\frac{d^2 X_m}{dx^2}\right)_{x=1} S'_m(\xi_{m,i};\lambda)}{\xi_{m,i}^3} ; \tag{62}$$

where

$$\bar{X}_m(\xi_{m,i}) = \int_1^\lambda x\, X_m\, S_m(\xi_{m,i}\, x;\, \lambda)\, dx\,, \tag{63}$$

$$\bar{P}_m(\xi_{m,i}) = \int_1^\lambda x\, P_m\, S_m(\xi_{m,i}\, x;\, \lambda)\, dx\,. \tag{64}$$

Referring to equation (57), we get

$$\begin{aligned} X_m = \frac{\pi^2}{2} \Bigg\{ & \sum_{i=1}^{\infty} \frac{J_m^2(\xi_{m,i}\,\lambda)}{[J_m^2(\xi_{m,i}) - J_m^2(\xi_{m,i}\,\lambda)]} \frac{\bar{P}_m(\xi_{m,i})}{\xi_{m,i}^2} S_m(\xi_{m,i}\, x;\, \lambda) + \\ & + \lambda \left(\frac{d^2 X_m}{dx^2}\right)_{x=\lambda} \sum_{i=1}^{\infty} \frac{J_m^2(\xi_{m,i}\,\lambda)}{[J_m^2(\xi_{m,i}) - J_m^2(\xi_{m,i}\,\lambda)]} \frac{S_m(\xi_{m,i}\, x;\, \lambda)}{\xi_{m,i}} - \\ & - \left(\frac{d^2 X_m}{dx^2}\right)_{x=1} \sum_{i=1}^{\infty} \frac{J_m^2(\xi_{m,i}\,\lambda)}{[J_m^2(\xi_{m,i}) - J_m^2(\xi_{m,i}\,\lambda)]} \frac{S_m(\xi_{m,i}\, x;\, \lambda)}{\xi_{m,i}} \Bigg\}. \end{aligned} \tag{65}$$

Constants $\left(\frac{d^2 X_m}{dx^2}\right)_{x=\lambda}$ and $\left(\frac{d^2 X_m}{dx^2}\right)_{x=1}$, can be determined from conditions in equations (59) and (60). That is to say, they can be determined by the equation group.

$$\begin{aligned} & \left(\frac{d^2 X_m}{dx^2}\right)_{x=1} \sum_{i=1}^{\infty} \frac{J_m^2(\xi_{m,i}\,\lambda)}{[J_m^2(\xi_{m,i}) - J_m^2(\xi_{m,i}\,\lambda)]} S'_m(\xi_{m,i};\, \lambda) - \\ & \quad - \lambda \left(\frac{d^2 X_m}{dx^2}\right)_{x=\lambda} \sum_{i=1}^{\infty} \frac{J_m^2(\xi_{m,i}\,\lambda)}{[J_m^2(\xi_{m,i}) - J_m^2(\xi_{m,i}\,\lambda)]} S'_m(\xi_{m,i}\,\lambda;\, \lambda) = \\ & \quad = \sum_{i=1}^{\infty} \frac{J_m^2(\xi_{m,i}\,\lambda)}{(J_m^2(\xi_{m,i}) - J_m^2(\xi_{m,i}\,\lambda)]} \frac{\bar{P}_m(\xi_{m,i})}{\xi_{m,i}} S'_m(\xi_{m,i};\, \lambda)\,, \end{aligned} \tag{66}$$

$$\begin{aligned} & \left(\frac{d^2 X_m}{dx^2}\right)_{x=1} \sum_{i=1}^{\infty} \frac{J_m^2(\xi_{m,i}\,\lambda)}{[J_m^2(\xi_{m,i}) - J_m^2(\xi_{m,i}\,\lambda)]} S'_m(\xi_{m,i}\,\lambda;\, \lambda) - \\ & \quad - \lambda \left(\frac{d^2 X_m}{dx^2}\right)_{x=\lambda} \sum_{i=1}^{\infty} \frac{J_m^2(\xi_{m,i}\,\lambda)}{[J_m^2(\xi_{m,i}) - J_m^2(\xi_{m,i}\,\lambda)]} S'_m(\xi_{m,i}\,\lambda;\, \lambda) = \\ & \quad = \sum_{i=1}^{\infty} \frac{J_m^2(\xi_{m,i}\,\lambda)}{[J_m^2(\xi_{m,i}) - J_m^2(\xi_{m,i}\,\lambda)]} \frac{\bar{P}_m(\xi_{m,i})}{\xi_{m,i}} S'_m(\xi_{m,i};\, \lambda) \end{aligned} \tag{67}$$

After we have solved these equations, we can begin putting down the complete solutions to the problems discussed above.

Therefore, it is not difficult to apply the methods discussed herein to both sides simply-supported, and the inside (outside) edge simply-supported, outside (inside) edge clamped conditions. We need not delve into any further detailed discussions here.

The advantage of the finite Hankel transform method, it should be pointed out, is in reducing the clamped edge or simply-supported problem to one which uses a fixed procedure. The one possible disadvantage is in convergence -- especially in ring-shaped plate conditions.

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THE PRODUCTION AND NUCLEAR CAPTURE
OF A K MESON
OBSERVED IN A CLOUD CHAMBER*

/Following is the translation of an article by Cheng Jen-ch'i (6774 0088 0967), Lu Min (0712 2404), Hsiao Chien (5618 0256), Wang Kan-ch'ang (3769 3227 2490), of the Institute of Physics, Academia Sinica, in Wu-li Hsüeh-pao (Journal of Physics), Vol. 12, No. 4, July 1956, pp. 376-378./

At a mountain location -- situated 3,185 meters above sea-level -- a cloud chamber is used to select the high-energy nuclear activity of cosmic radiation. The cloud chamber has sides 50 cm. long, 27.5 cm. high, with an effective volume of 40x40x12 cubic cm. Seven lead plates -- each 1.2 cm. in thickness -- are placed in the chamber (in some cases, each plate has a thickness of 0.64 cm.). In most cases, lead layers of 10 cm. (thickness) are placed on the cloud chamber. An array of counting tubes, above and below the chamber, are placed with the coincidence of the single upper counting tube and the lower array of three counting tubes as a condition for the selection of these examples. Solid pictures are taken by a pair of cameras. The angle of inclusion in the optical axis of the camera is 15°.

During seven months' work with the cloud chamber, we took 30,000 pairs of pictures, of which 8,206 pair displayed nuclear activity. Initially, we observed approximately 200 heavy mesons and hyperons three of which each produced two V^0 particles from a single nuclear action. In addition to these, one example produced, simultaneously, from a single nuclear activity, one V^* particle and one S particle (in this example, the thickness of each lead plate in the cloud chamber was 1.2 cm.). The S particle in this particular example stops inside the seventh lead plate,

* Received 11 May 1956.

and emits at the end a V^0 particle and a charged particle.

Figure 1 is a photograph of this example, while Figure 2 is a descriptive diagram of the main contents of the photograph just mentioned (see page 20). Table 1 gives the ionization ratio values for different tracks. These data represent a composite tabulation based upon the observations of four researchers. Table 2 shows the different relative angles and the thickness of the sixth lead plate threaded by tracks ADE and HIJ.

Table 1. The Ionization Ratio of Different Tracks.

Track	Ionization Ratio $I(XI_{min})$
AB	>6
BC	1.5-3
AD	1.5-3
D' E	3-6
E' F	~ 1
GH	>10
HI	~ 1
I' J	<2

From track ADE, which pierced through the sixth lead plate -- that is, through the actual thickness DD' of the lead plate -- and reached ionization ratio values I_{AB}, $I_{D'E}$ on the front and back lead plates, we gathered our calculation data at the point in the seventh lead plate where the track finally terminated. The particle which produced particle track ADE should have a mass heavier than a π meson, but lighter than that of a proton, and close to the K meson mass. The magnitude of this particle, taken as the K meson, and the angle of scattering $\angle$ ADE, produced when it pierced through the sixth lead plate, also are not contradictory.

In regard to the track analysis of GH and HIJ, we recognize that they are the track left by protons and π mesons after the H point decay of a Λ^0 hyperon. The kinetic energy of the Λ^0 hyperon is about 40-MEV.

Summing up what we have discussed above, we can state that this is a Λ^0 hyperon and a π^- meson emitted after a K^- meson has undergone nuclear capture. The following formula can be used to describe this activity:

$$K^- + n \longrightarrow \Lambda^0 + \pi^- .$$

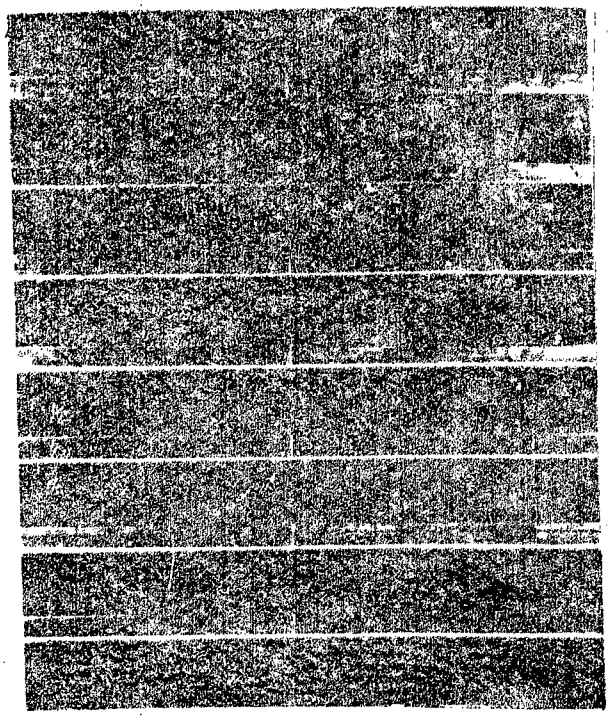

Figure 1. Photograph of Example 38844.

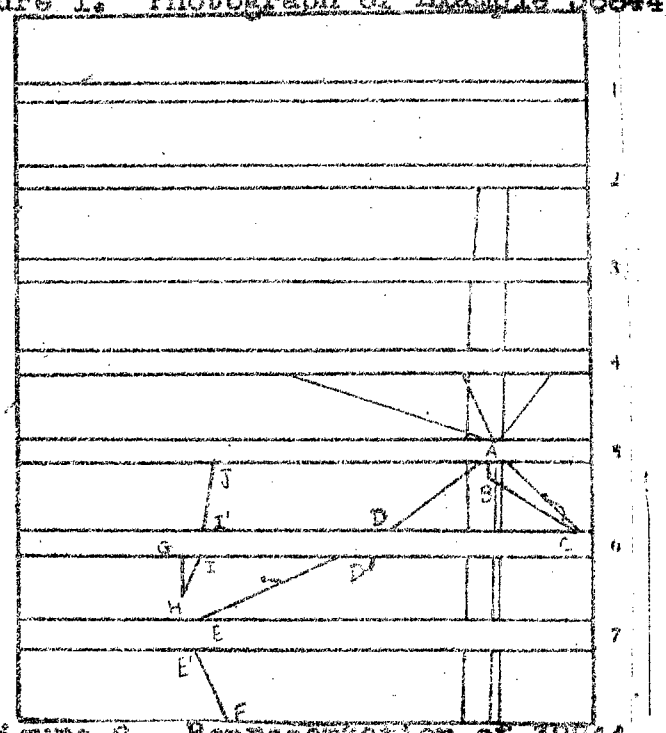

Figure 2. Representation of 38844.

Table 2. Angle and Lead Plate Thickness.

Angle	Threaded Lead Plate Thickness
∢ABC=116°	DD' = 30 g/cm.^{2}
∢ADE=166°	II' = 15 g/cm.^{2}
∢GHI=16°	
∢HIJ=163°	
∢HFF'=127°	

Track E' F represents the path of the π^- meson. In the photograph, however, the intersecting angle of the kinetic direction between Λ^0 hyperon and π^- meson is 127° and not 180°.

The situation of Λ^0 hyperon and π^- meson emission after nuclear capture of a K^- meson was also observed by De Staebler in a cloud chamber of several lead plates. In this example also, the intersecting angle of the kinetic direction between Λ^0 hyperon and π^- meson was not equal to 180°. Concerning this point, Rossi was of the opinion that it could be explained by considering the fact that the Fermi motion of the nucleon and the recoil nucleon in the nucleus carry away a portion of the momentum.

In our example here, in addition to the nuclear capture of the K^- meson, we also can see the nuclear activity which produced this K^- meson. Furthermore, this nuclear activity produces a $V^{\pm}$ particle or an ABC track.

In emulsion work, we observed an example of pairing production with three K^+ mesons and K^- mesons. From the theory proposed by Gell-Mann and Mapkob, we also can deduce that the K^- meson can only be produced (simultaneously) with other K mesons.

We have tried to take the $V^{\pm}$ particle as a clue to the explanation of the K^+ meson. However, we encountered the following difficulties:

1. When taking K mesons as two-body decay -- i.e., $K_{\pi 2}$ or $K_{\mu 2}$ -- then at a condition where angle ABC equals 116°, the ionization ratio between I_{AB} and I_{BC} of tracks AB and BC should bear the relationship as shown in the curve in Figure 3 (see page 22). The shaded portion of Figure 3 represents the estimation value range of I_{AB} and I_{BC}. From Figure 3 it can be seen that it is very difficult to have consistency.

2. Although it may happen to be the K meson of three-body decay, the probability of methodic decay of K^+ meson according to $K_{\mu 3}$ and $K_{\pi 3}$ (τ') is only about 4%.

3. It already has been determined that the life-

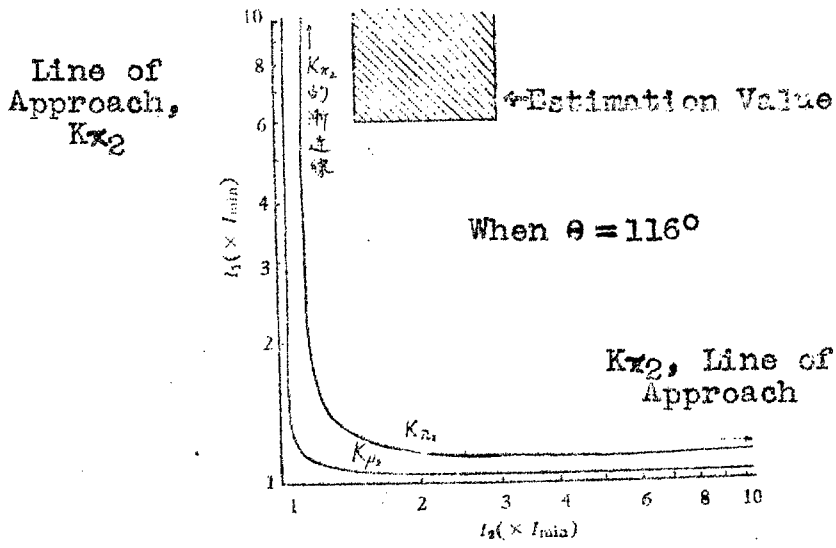

Figure 3. Relationship Between I_{AB} and I_{BC}.

time of an artificially-produced K^+ meson is about 1.2 X 10^{-8} seconds, and that the order of magnitude of lifetime for this $V^{\pm}$ particle is 10^{-10} seconds.

Of course, it is probably not a propos to give a comprehensive explanation of the nature of this $V^{\pm}$ particle from the example presented here; moreover, more examples are needed in order to illustrate the true phenomena of things.

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END