Interaction between Near-Wall Turbulent Flows and Compliant Surfaces

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Abstract

The general aim of this project is to get an improved understanding of the interaction between wall-generated turbulence and compliant surface coatings using analysis and direct numerical simulation in an integrated approach, with a view towards the reduction of turbulent sound production and turbulent drag.

For this purpose, in a first step that is targeted at identifying interesting domains in the space of parameters describing properties of a compliant wall coating, we are developing low-dimensional models based on Galerkin projection of the Navier-Stokes equations onto systems of eigenfunctions obtained via Proper Orthogonal Decomposition. Because of the relatively small effort involved in simulating and analyzing such models, this will allow us to scan large regions of parameter space, allowing us to find regions that lead to a reduction of turbulent drag and turbulent sound production.

Among the ultimate goals of this project are thus, first, to obtain a fundamental understanding of flow-structure interaction phenomena for the case of the compliant-wall/turbulence interaction, and second, to use this understanding to enhance the flight performance of air vehicles by increasing their lift-to-drag ratio.
1 Statement of Objectives

In this study, research is conducted into the issue of the interaction between a turbulent boundary layer and an adjacent compliant surface. This work has as one of its main objectives an improved understanding of the dynamical mechanisms at work in such a situation. Ultimately, this understanding then will be used to manipulate the turbulent flow such that some of its characteristics are altered in a desirable way. In this respect, we are mostly interested in two goals. We want to be able to reduce the turbulent drag generated by such boundary layers, and we also want to be able to attenuate the noise that is generated by the turbulent flow.

In order to achieve this, we will try to find regions in the space of parameters describing mechanical properties of compliant coatings (stiffness, damping) within which the interaction between the wall and the turbulent flow is such that the above goals can be achieved.

The research proposed here consists of two main components. To be able to determine regions in the parameter space of the wall coating that are of interest, we will construct low-dimensional models based on Galerkin projections onto Karhunen-Loève eigenfunctions. These models can only approximately describe the dynamics of the turbulent flow, but because of their low-dimensionality, they allow us to look at large regions of the parameter space. Once promising combinations of parameters are determined, we will then use accurate direct numerical simulations to assess the interaction between turbulence and the compliant wall in more detail.

2 Status of Effort

Our work on developing and incorporating models for the interaction between the turbulent flow and the compliant wall is proceeding as planned. As a reference for more refined models that we are working on now, we have developed a first simplified representation of the wall as a three-degree-of-freedom mass-spring system (see below). Results from a study of the properties of that model are encouraging, and are presented below. A refined version of this model, that removes most of the limitations of the previous one, is currently under investigation. In parallel, we are also working on a computer code for the direct numerical simulation of turbulence/compliant-wall interaction. That code will probably be functional this fall.

3 Accomplishments

3.1 Introduction

In the following, we will very briefly sketch the structure of our models, and highlight our most important results.
We have performed an analysis of the boundary conditions (for the case of small deformations of the wall as in the study of Choi et al. [3]) that suggests that for the purposes of our low-dimensional model, neglect of the nonlinear terms is usually quite justifiable—the only relatively large nonlinear term appears in a term for the streamwise velocity component, which makes no contribution within the approximations of our model. The boundary conditions become:

\[
\begin{align*}
  u_1 + U_{1,2} \xi_2 &= \dot{\xi}_1 \\
  u_2 &= \dot{\xi}_2 \\
  u_3 &= \dot{\xi}_3
\end{align*}
\]  

Note that if the velocity field is expressed as a linear combination of eigenfunctions \( u_i = \sum_{n,k} a_k^{(n)} \phi_{i_k}^{(n)} \exp(ikz) \), the boundary conditions above can be interpreted as a restriction on the combinations of \( a_k^{(n)} \), \( \xi \) and \( \dot{\xi} \) which are realizable.

In order to implement these boundary conditions in our model, we introduce a Fourier transform,

\[
 u_i(x, t) = \sum_{k_1, k_3} \hat{u}_i(k_1, y, k_3, t)e^{(ik_1 x + ik_3 z)},
\]

where we can also write \( \hat{u} \) using our POD modes \( \phi_i \),

\[
 \hat{u}_i = \sum_n a_n \phi_i^{(n)},
\]

where

\[
 a_m = \int_0^\infty \hat{u}_i \phi_i^{(m)*} dy.
\]

Neglecting the nonlinear terms in the wall-normal boundary condition, we obtain

\[
\begin{align*}
  \hat{u}_{2w} &= \dot{\xi}_2 \\
  \sum_n a_n \phi_i^{(n)} &= \dot{\xi}_2 \\
  a_1 \phi_2^{(1)} &\approx \dot{\xi}_2,
\end{align*}
\]

where it is assumed that the first eigenfunction is sufficient to represent the fluctuating velocity field\(^1\).

The dynamical equations for the POD amplitudes (derived from the Navier-Stokes equation via Galerkin projection onto the set of POD modes) become

\[
 \ddot{a}_m + \cdots = \frac{1}{\rho \nu} \frac{\partial_p}{a_1} \frac{\dot{\xi}_2}{a_1} + \cdots,
\]

\(^1\)Note that the "≈"-sign in that equation has to be taken seriously: At this level of truncation, it would in fact be inconsistent to require the boundary condition to be met using just one POD mode. Observe that Eq. (5) above (with a "≈" sign instead of the "≈"-sign), once \( \dot{\xi}_2 \) and \( \phi_2^{(1)} \) are given, uniquely determines \( a_1 \). In other words, the dynamics of the flow would be directly tied to the dynamics of the wall, with no fluid mechanics in between. From a dynamical systems point of view, it turns out that we need at least one additional degree of freedom to be able to accommodate both the wall boundary condition and the Navier-Stokes equation.
with the dots indicating the terms that are present in a model for a fixed wall, as in [1]. The pressure at the wall that is needed above can be obtained by using the instantaneous Navier Stokes equation, which finally allows us to write

\[ \frac{\ddot{p}_w}{\rho} = \frac{\nu a_1 \phi_{3,22w}^{(1)}}{i k_3}, \]  

which ultimately gives

\[ \dot{a}_m + \cdots = \frac{\nu \phi_{3,22w}^{(1)}}{i k_3} \xi_2. \]  

It may be a bit surprising at first to find that the lateral response of the surface apparently does not contribute. The reason for this is that—through our choice of considering only one POD mode—the boundary conditions at the wall are satisfied only in an approximate fashion. This is also the reason that it is only the wall normal motion that affects the equations: Once one has (implicitly) decided to approximate the effect of the wall motion on the boundary condition, and only include an effect through the pressure term, the influence of tangential motions is gone (since in the pressure integral for a domain with a horizontal boundary only, it is only the wall normal velocity at the boundaries that gives a contribution). So it is the one-mode truncation\(^2\) that causes the influence of tangential motions to go away, not the linearizations that we have introduced above. Note that this approximation is quite consistent with the overall accuracy of this type of model. These limitations will be removed in the refined model described below.

### 3.2 Low-dimensional Model of Flow Over a Compliant Wall

#### 3.2.1 Introduction

We have investigated a model for a turbulent boundary layer interacting with a deformable wall, using the approximations described above. The flow is modeled via a pair of complex-valued ODEs with O(2) symmetry, and the dynamics of the wall is described using a simple damped mass-spring system. The complete set of dynamical equation consists of the equations for the flow,

\[ \frac{d a_1}{d t} = \mu_1 a_1 + c_{21} a_1 a_2 + a_1(e_{11}|a_1|^2 + e_{12}|a_2|^2) + \frac{\nu \phi_{3,22w}^{(1)}}{i k_3} a_1, \]  

\[ \frac{d a_2}{d t} = \mu_2 a_2 + c_{11} a_1^2 + a_2(e_{21}|a_1|^2 + e_{22}|a_2|^2), \]

\(^2\)In conjunction with us insisting on \(a_1\) being determined by fluid mechanics. Note that if we had gone the alternative route of interpreting the last equation of Eq. (5) as an equation for \(a_1\), then the dynamics of the flow would have been determined entirely by the wall motions; in this case, we would have to decide which component of the wall motion to pick, because prescribing just one component of \(\xi\) would already determine \(a_1\).
and a second-order equation for the wall motion,

\[ m\ddot{\xi}_2 + c\dot{\xi}_2 + k\xi_2 = a_1 \frac{\nu \phi_{3,22w}^{(1)}}{t k_3}. \]  

(11)

where \( a_1 \frac{\nu \phi_{3,22w}^{(1)}}{t k_3} = 0.2996a_1 \nu \) (evaluated numerically) defines the pressure at the wall in Fourier space.

Figure 1 shows a comparison of the behavior of this model for the cases of a compliant and a fixed wall, respectively (in the case of the compliant wall, the last term in (9) is omitted, together with (11)). One can see that for the small value of \( c \) (wall damping) that

![Heteroclinic Connection](image1)

(a) Fixed wall.

![Wall Normal-Real Subspace](image2)

(b) Compliant wall. Behavior for small wall damping, \( \alpha = 1.4, \nu = 1.5, \nu^* = 3, c = 1.0 \cdot 10^{-5}, k = 1 \).

Figure 1: Phase portraits of low-dimensional models.

was chosen in the figure, the lower fixed point is no longer attracting in all directions (it is a saddle point). Solutions which are initially attracted to this point oscillate around it for an extended amount of time before diverging to infinity. This has the effect of reducing the bursting frequency and thus potentially reducing turbulent drag.

3.3 Five mode model with wall-normal compliance

To remove the limitations of the model described above, we have derived a refined model, that can take into account wall motions in all three coordinates, and that fully satisfies
the interface conditions between the moving wall and the flow. To do this, we model the compliant surface as a simple mass-spring-damper system driven by the fluid stress at the wall. In the streamwise and spanwise directions, the deformations of the wall are driven by the shear stress which is easily accessible in the models as a linear term in the model coefficients when the deformations are decomposed into Fourier space:

\[
M_w \ddot{\xi}_{1k} + D_w \dot{\xi}_{1k} + K_w \xi_{1k} = \mathring{\tau}_{1k} |_{\text{wall}} = \sum_{n} a^{(n)}_{k} \phi^{(n)}_{1,k_2} \\
M_w \ddot{\xi}_{3k} + D_w \dot{\xi}_{3k} + K_w \xi_{3k} = \mathring{\tau}_{3k} |_{\text{wall}} = \sum_{n} a^{(n)}_{k} \phi^{(n)}_{3,k_2}
\]  

(We employ capital letters to describe the properties of the compliant surface for clarity.) The wall-normal deformations of the compliant surface are driven by the pressure fluctuations at the boundary.

\[
M_w \ddot{\xi}_{1k} + D_w \dot{\xi}_{1k} + K_w \xi_{1k} = \mathring{p}_k |_{\text{wall}}
\]  

Unfortunately, the pressure at the boundary is not as easily accessible in the models as the shear stress.

The pressure at the wall in the models may be determined by examining the Galerkin projection of the pressure gradient term in the models.

\[
\langle -\mathring{p}_{k,i}, \phi^{(n)}_{ik} \rangle = \left( \mathring{p}_k \phi^{(n)*}_{2,k} \right) |_{\text{wall}} + \int_{0}^{\infty} \mathring{p}_k \phi^{(n)}_{1,k} \, dy
\]

\[
= \left( \mathring{p}_k \phi^{(n)*}_{2,k} \right) |_{\text{wall}}
\]

The eigenfunctions are defined to be divergence free and are non-zero only close to the wall, leaving us only with the pressure term at the wall. As a result, the model equations now have an additional term:

\[
\dot{a}^{(n)}_k = \sum_{p} \left( b^{(n)\text{meauntel}}_{kp} + (1 + 6.28\alpha) b^{(n)\text{oscil}}_{kp} \right) a^{(p)}_k + \sum_{k',p,q} c^{(n)}_{k,k',k'}^{(p)} a^{(p)}_{k'} a^{(q)}_{k}
\]

\[
+ \sum_{r,k',p,q} d^{(n)}_{r,k',p,q} a^{(r)} \text{Re} \left( a^{(p)}_{k'} a^{(q)*}_{k} \right) + \frac{1}{\rho} \left( \phi^{(n)*}_{2,k} \mathring{p}_k \right) |_{\text{wall}}
\]  

Our previous models have focused exclusively on the rigid-wall case with \( \phi^{(n)}_{2,k} = 0 \), so that the pressure term disappears. (If the eigenfunctions are defined only in a region close to the wall, an additional term appears which was incorporated into our previous models as a forcing term. The magnitude of this term is small relative to the others and will be neglected here.) However, to incorporate the compliance of the wall and satisfy the boundary conditions in equation 1, we must include eigenfunctions in our model which are non-zero at the wall.

We choose to base our low-dimensional model for the turbulent boundary layer over a compliant surface on the eigenfunctions of the rigid-walled boundary layer. We will introduce additional eigenfunctions to account for the motion of the compliant surface and allow
the boundary conditions to be satisfied. In the absence of surface compliance, our models will revert to the rigid-walled case. Since we do not have an experimental or computational database on the turbulent flow over a compliant surface available, we must derive our additional eigenfunctions in an *ad hoc* fashion. We choose our additional eigenfunctions as solutions of the Stokes equation with periodic motion of the wall:

\begin{equation}
\left( \frac{\partial}{\partial t} - \nu \Delta \right) u = 0
\end{equation}

\begin{equation}
\hat{u}_k(y=0,t) = \cos(\beta t)
\end{equation}

where \( \beta \) corresponds to the natural frequency of the compliant surface. The Stokes equation for the streamfunction \( \psi \) is employed to determine the additional eigenfunction for the wall-normal motion of the compliant surface. In this manner, we generate three additional eigenfunctions — one for each direction of surface motion — and then orthogonalize them with respect to each other and the rigid-walled eigenfunctions and normalize them.

We now have a set of evolution equations for the coefficients of the eigenfunctions as well as evolution equations for the motion of the wall and the simplified boundary conditions. We choose to determine the coefficients for the additional eigenfunctions from the boundary conditions at the wall. If only one or at most two eigenfunctions has a particular component which is non-zero at the wall, these coefficients may be found easily. The pressure may then be determined from the evolution equation for the additional eigenfunction representing the wall-normal motion of the wall \( \phi^{(2)}_k \):

\begin{equation}
\hat{p}_k \big|_{y=0} \approx \frac{\rho}{\phi^{(2)\ast}_{2k}} \bigg|_{y=0} \left[ \hat{\xi}^{(2)}_{2k} - \sum_p \hat{b}^{(2)}_{kp} \hat{a}^{(p)}_k - \sum_{k',p,q} \hat{c}^{(2)}_{(k',k-k')pq} \hat{a}^{(p)}_k \hat{a}^{(q)}_{k-k'} \right]
\end{equation}

The cubic terms disappear because this additional eigenfunction is defined to have no streamwise component, and there is no streamwise variation in our model. When this expression is substituted into the equation for the wall-normal motion of the wall, the time derivative term merges into the \( \hat{\xi}_{2k} \) term resulting in an new effective mass of the wall \( \hat{M}_w \). (In fact, \( \hat{\xi}_{2k} = a^{(2)}_k \phi^{(2)}_{2k} \) since \( \phi^{(2)} \) is the only eigenfunction with a non-zero vertical velocity at the wall.) Normalizing be this new mass results in:

\begin{equation}
\ddot{\hat{\xi}}_{2k} + \bar{D}_w \dot{\hat{\xi}}_{2k} + \bar{K}_w \hat{\xi}_{2k} \approx \bar{\rho} \left[ \sum_p \hat{b}^{(2)}_{kp} \hat{a}^{(p)}_k + \sum_{k',p,q} \hat{c}^{(2)}_{(k',k-k')pq} \hat{a}^{(p)}_k \hat{a}^{(q)}_{k-k'} \right]
\end{equation}

Having determined the coefficient and pressure in this way, we have a complete low-dimensional model for the flow over a compliant surface.

\begin{equation}
\hat{a}^{(1)}_k = \sum_p \hat{b}^{(1)}_{kp} \hat{a}^{(p)}_k + \sum_{k',p,q} \hat{c}^{(1)}_{(k',k-k')pq} \hat{a}^{(p)}_k \hat{a}^{(q)}_{k-k'} + \sum_{r,k',p,q} \hat{d}^{(1)}_{r,k'pq} \hat{a}^{(r)}_k \hat{a}^{(p)}_k \hat{a}^{(q)}_{k-k'}
\end{equation}

\begin{equation}
\hat{M}_w \ddot{\hat{\xi}}_{1k} + \bar{D}_w \dot{\hat{\xi}}_{1k} + \bar{K}_w \hat{\xi}_{1k} = \bar{\tau}_{1k} |_{wall} = \sum_n \hat{a}^{(n)}_k \phi^{(n)}_{1,2k}
\end{equation}
\[
\ddot{\xi}_{2k} + \dot{D}_w \ddot{\xi}_{2k} + \dot{K}_w \ddot{\xi}_{2k} \approx \ddot{\rho} \left[ \sum_p \ddot{b}^{(2)}_{k,p} a_k^{(p)} + \sum_{k',p,q} \ddot{c}^{(2)}_{k',k-p} a_{k'}^{(p)} a_k^{(q)} \right] \\
M_w \ddot{\xi}_{3k} + D_w \ddot{\xi}_{3k} + K_w \ddot{\xi}_{3k} = \ddot{\tau}_k |_{wall} = \sum_n a_k^{(n)} \ddot{\phi}_{3k}^{(n)}
\]

(25)

\[
a^{(2)}_k = \frac{\ddot{\xi}_{2k}}{\phi_{2k}^{(2)} |_{y=0}}
\]

(26)

\[
a^{(3)}_k = \frac{\ddot{\xi}_{1k} - \ddot{\xi}_{2k} \frac{\partial U}{\partial y} |_{y=0}}{\phi_{1k}^{(3)} |_{y=0}}
\]

(27)

\[
a^{(4)}_k = \frac{\ddot{\xi}_{3k} - a_k^{(2)} \ddot{\phi}_{3k}^{(2)} |_{y=0}}{\phi_{3k}^{(4)} |_{y=0}}
\]

(28)

We are in the process of using this low-dimensional model to evaluate the potential of compliant surfaces for modification of the dynamics of near-wall turbulence and its resulting effect on the drag at the wall. In addition to our work with the models, we have begun the development of a direct simulation code for simulating turbulent channel flow with a compliant boundary. Because of the complication introduced by the linearized boundary conditions, we have chosen to implement the compliant boundary using the immersed boundary technique which was developed by a previous member of our group [6].

4 Personnel Supported

Faculty: John Lumley, Dietmar Rempfer
Graduate Students: Peter Blossey, Gad Reinhorn, Kiran Bhaganagar, Sheng Xu, Vejapong Juttijudata, Louise Parsons

5 Publications

5.1 Journal Articles:


5.2 Research reports (submitted, in review, in press):

- **Haeusser, T.; Leibovich, S.** (1999)

- **Leibovich, S.; Yang, G.** (1999)

- **Podvin, B.; Lumley, J.L.; Berkoza, G.** (1999)


5.3 Expository Articles, Books and Other Publications

- **Blossey, P.N.** (1999)

- **Blossey, P.N.; Lumley, J.L.** (1999)

- **Lumley, J.L.** (1999)

- **Lumley, J.L.; Blossey, P.N.** (1999)

- **Lumley, J.L.; Blossey, P.N.; Podvin-Delarue, B.** (1999)


6 Interactions

6.1 Conference Presentations


6.2 Technology Transitions or Transfer

None.

6.3 New Discoveries, Inventions, or Patent Disclosures

None.

6.4 Honors/Awards

John Lumley is a Fellow of the American Academy of Arts & Sciences, the American Physical Society, the American Academy of Mechanics, the American Institute of Aeronautics and Astronautics, and was a Guggenheim Fellow from 1973–74.

Dietmar Rempfer was supported by a Heisenberg Grant from the German National Science Foundation (Deutsche Forschungsgemeinschaft).

Sidney Leibovich is a Fellow of the American Academy of Arts & Sciences and the American Physical Society.

References


The general aim of this project is to get an improved understanding of the interaction between wall-generated turbulence and compliant surface coatings using analysis and direct numerical simulation in an integrated approach, with a view towards the reduction of turbulent around production and turbulent drag. For this purpose, in a first step that is targeted at identifying interesting domains in the space of parameters describing properties of a compliant wall coating, we are developing low-dimensional models based on Galerkin projection of the Navier-Stokes equations onto systems of eigenfunctions obtained via Proper Orthogonal Decomposition. Because of the relatively small effort involved in simulating and analyzing such models, this will allow us to scan large regions of parameter space, allowing us to find regions that lead to a reduction of turbulent drag and turbulent around production. Among the ultimate goals of this project are thus, first, to obtain a fundamental understanding of flow-structure interaction phenomena for the case of the compliant-wall/turbulence interaction, and second, to use this understanding to enhance the flight performance of air vehicles by increasing their lift-to-drag ratio.