NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

OPTIMIZATION PROCEDURE FOR ELECTRIC PROPULSION ENGINES

by

John J. De Bellis

December 1999

Thesis Advisor: Oscar Biblarz
Co-Advisor: James Luscombe

Approved for public release; distribution is unlimited.

20000309 032
**OPTIMIZATION PROCEDURE FOR ELECTRIC PROPULSION ENGINES**

This thesis addresses the optimization of all types of space electrical propulsion thrusters. From the Langmuir-Irving payload mass fraction formulation, a “dual-optimum” solution is defined, yielding a minimum overall mass for a specified payload consistent with minimum transfer time. This solution fixes the ideal payload mass ratio ($m_p / m_0$) at a value of 0.45, establishing the ratios of effective exhaust velocity ($v / v_c$) and incremental change of vehicle velocity ($\Delta u / v_c$) to characteristic velocity at 0.820 and 0.327 respectively. The characteristic velocity ($v_c$) includes thrust time as well as engine efficiency ($\eta$) and specific power ($\alpha$). A range of mass ratios from 0.35 to 0.55 is used in order to allow the system designer some flexibility while remaining close to optimal. Nine examples are presented which demonstrate that mission profiles can be optimized by profile-to-thruster matching. A comprehensive list of currently available electric propulsion engines is provided. This list details important parameters such as the specific power, which “sizes” an engine in terms of power provided to the thruster at the cost of additional mass. Allowance is also made for a fuel tank mass penalty, and examples show that this can also noticeably influence the optimum design.
OPTIMIZATION PROCEDURE FOR ELECTRIC PROPULSION ENGINES

John J. De Bellis
Lieutenant, United States Navy
B.S., United States Naval Academy, 1991

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN APPLIED PHYSICS

from the

NAVAL POSTGRADUATE SCHOOL

December 1999

Author:

John J. De Bellis

Approved by:

Oscar Biblarz, Thesis Advisor

James Luscombe, Co-Advisor

William Maier II, Chair
Department of Physics
ABSTRACT

This thesis addresses the optimization of all types of space electrical propulsion thrusters. From the Langmuir-Irving payload mass fraction formulation, a "dual-optimum" solution is defined, yielding a minimum overall mass for a specified payload consistent with minimum transfer time. This solution fixes the ideal payload mass ratio ($m_{pl} / m_o$) at a value of 0.45, establishing the ratios of effective exhaust velocity ($v / v_c$) and incremental change of vehicle velocity ($\Delta u / v_c$) to characteristic velocity at 0.820 and 0.327 respectively. The characteristic velocity ($v_c$) includes thrust time as well as engine efficiency ($\eta_t$) and specific power ($\alpha$). A range of mass ratios from 0.35 to 0.55 is used in order to allow the system designer some flexibility while remaining close to optimal. Nine examples are presented which demonstrate that mission profiles can be optimized by profile-to-thruster matching. A comprehensive list of currently available electric propulsion engines is provided. This list details important parameters such as the specific power, which "sizes" an engine in terms of power provided to the thruster at the cost of additional mass. Allowance is also made for a fuel tank mass penalty, and examples show that this can also noticeably influence the optimum design.
This page intentionally left blank.
**TABLE OF CONTENTS**

I. **INTRODUCTION** .................................................................1
   A. SPACE ELECTRIC PROPULSION COMES OF AGE .....................1
   B. PHYSICS BEHIND ELECTRIC PROPULSION .........................3
      1. Electrothermal Thrusters ........................................4
      2. Electrostatic Thrusters .........................................7
      3. Electromagnetic Thrusters .....................................9

II. **THEORETICAL BACKGROUND** .............................................15
   A. FIRST PRINCIPLES ..................................................15
   B. ANALYSIS – DUAL OPTIMUM METHODOLOGY ......................20
   C. TANKAGE PENALTY ..................................................22

III. **CURRENT TECHNOLOGY – ELECTRIC ENGINES** ......................27
    A. NASA – DEVELOPMENT AND OBJECTIVES .......................27
    B. SUMMARY OF AVAILABLE ENGINE DATA ..........................30

IV. **PLANNING/OPTIMIZATION ALGORITHM – MATCHING THE MISSION, PAYLOAD, AND ENGINE** .........................39

V. **APPLICATION OF METHOD** ...............................................47
   A. “LIGHT” LEO TO GEO MISSION ....................................47
   B. “HEAVY” LEO TO GEO (COMM SAT) MISSION .....................49
   C. LEO TO MARS MISSION .............................................51

VI. **CONCLUSIONS AND RECOMMENDATIONS** .............................53
   A. CONCLUSIONS .....................................................53
   B. RECOMMENDATIONS ...............................................57
LIST OF FIGURES

1. Operating Principles of Various Electrical Propulsion Thrusters ..................14
2. Payload Mass Fraction Plot: $m_{pl} / m_0$ vs. $v / v_c$ .................................18
3. Payload Mass Fraction Plot: $\Delta u / v_c$ vs. $v / v_c$ .....................................19
4. Dual Optimum ......................................................................................................20
5. Three-dimensional Dual Optimum Surface .......................................................22
6. Payload Mass Fraction Plot with 10% Tankage Penalty: $\Delta u / v_c$ vs. $v / v_c$ ......24
7. Optimum Comparison with and without 10% Tankage Penalty: $\Delta u / v_c$ vs. $v / v_c$ at $m_{pl} / m_0 = 0.45$ .................................................................25
8. Specific Impulse ($I_s$) Ranges for Current Technology Engines .......................35
9. Comprehensive Plot of $\Delta u / v_c$ and $v / v_c$ for Ideal Systems .......................43
10. Comprehensive Plot of $\Delta u / v_c$ and $v / v_c$ for Systems Requiring “Tankage” of 10% ..................................................................................................................44
11. FLOWCHART – Optimization Procedure for Electric Propulsion Engines ......45
THIS PAGE INTENTIONALLY LEFT BLANK.
LIST OF TABLES

1. Typical Mission Velocity Requirements.........................................................2
2. Typical Performance Parameters of Various Types of Electrical Propulsion Systems..........................................................................................13
3. Optimal Values: $\Delta u / v_c$ and $v / v_c$ for Various Payload Mass Ratios........19
4. Optimal Values with 10% Tankage Penalty: $\Delta u / v_c$ and $v / v_c$...............25
5. Summary of Current Technology Engines.......................................................36
LIST OF SYMBOLS AND ABBREVIATIONS

\[ g_0 \quad (m/s^2) \] acceleration of gravity at sea level;

arbitrary constant that changes units of \( I_s \) to “seconds;”

\[ g_0 = 9.81 \, m/s^2 \]

\[ \Delta u \quad (m/s) \] incremental change of vehicle velocity

\[ v \quad (m/s) \] effective exhaust velocity of propellant

\[ v_c \quad (m/s) \] characteristic velocity; \( v_c^2 = 2 \, \alpha \, t_p \, \eta_t \)

\[ m_o \quad (kg) \] total initial vehicle mass; \( m_o = m_p + m_{pl} + m_{pp} \); \( m_o = m_f + m_p \)

\[ m_p \quad (kg) \] propellant mass expelled

\[ m_{pl} \quad (kg) \] payload mass

\[ m_{pp} \quad (kg) \] power-plant mass

\[ m_f \quad (kg) \] final mass; \( m_f = m_o - m_p = m_{pl} + m_{pp} \)

\[ m_{pl}/m_o \] payload fraction; \( m_o / m_f = e^{\Delta u/v} \)

\[ P_e \quad (W) \] electrical power supplied to propulsion system:

\[ \alpha \quad (W/kg) \] specific power or “state of the art”; \( \alpha = P_e / m_{pp} \)

\[ I_s \quad (sec) \] specific impulse; \( I_s = v / g_0 \)

\[ \eta_t \] thruster efficiency or “state of the art”

\[ t_p \quad (sec, \, days...) \] burn time

\[ t_m \quad (days) \] mission time – defined time for a particular \( \Delta u \)

\[ T \quad (N) \] thrust

GEO Geosynchronous Earth Orbit – approximately 42,227 km radius

LEO Low Earth Orbit – approximately 270 km

NSSK North-South Stationkeeping
THIS PAGE INTENTIONALLY LEFT BLANK.
ACKNOWLEDGEMENT

I would like to express my sincere thanks and appreciation to all the professors and faculty at the Naval Postgraduate School who helped make this one of my most enjoyable tours. I would especially like to thank Professor Oscar Biblarz for his guidance and for the countless office sessions during which I came to understand the principles of electric propulsion and rocketry. Thanks, as well, to Professor Jim Luscombe for his assistance, which was invaluable in the publication of this thesis.

My most sincere thanks to the many great friends that I have met here at school. Thanks to LCDR Jon Wood for the afternoon runs, and to all my good buddies and neighbors in Pacific Grove and Carmel.

I would most especially like to give a big thank you and huge hug to my beautiful sweetie, Ms. Susana Guzmán, whose love of life and diverse interests made this thesis particularly difficult to complete.
I. INTRODUCTION

A. SPACE ELECTRIC PROPULSION COMES OF AGE

The application of electric propulsion for space is not a recent revolution in the field. The idea was proposed as early as the 1950’s. One significant article appeared as a chapter in the text, *Space Technology*, entitled “Low Thrust Flight: Constant Exhaust Velocity in Field-free Space,” written by D. B. Langmuir (Langmuir, 1959, pg. 9-01). Aside from the evolution of technology, little has changed in the fundamentals of electric propulsion since that time.

In the last 35 years, more than 300 electric propulsion thrusters have flown on over 100 spacecraft, and in the next decade, a significant increase in electrical propulsion employment is expected (Filliben, 1997, pg. 5). Nonetheless, electrical propulsion is only now seeing rapid and wider-scale introduction into spacecraft propulsion programs.

The first electrical propulsion engines flew in the mid-1960’s. The former Soviet Union used a pulsed plasma thruster (PPT) to provide attitude control to their Zond-2/1 satellite, which launched in 1964 (Martinez-Sanchez and Pollard, 1998, pg. 692). Then, in 1965, the United States followed up their effort using a resistojet engine for orbit adjustment of the Vela/2 satellite (Martinez-Sanchez and Pollard, 1998, pg. 689).

If both of those missions proved successful, why has additional implementation of electrical propulsion for space applications been so seemingly slow, spanning almost 40 years? For this, there are both technical and doctrinal reasons.

First, engineers have only recently been able to package sufficient electrical power and power conditioning equipment onboard spacecraft in order to provide for the
demands of the electrical propulsion system. Unlike chemical propulsion, electrical requires energy accumulation and energy processing functions creating a power supply penalty at the expense of payload mass (Humble, Henry, and Larson, 1995, pg. 510). Likewise, many mission planners have been reluctant to replace proven chemical rockets with comparatively untested electrical systems. As a result, electrical systems have only gradually become accepted and only for less demanding tasks -- such as north-south station keeping (NSSK) and orbit insertion.

Nonetheless, electrical propulsion has several advantages over chemical rockets for near-zero gravity applications. Moreover, for deep-space missions, which require huge incremental changes in vehicle velocity (the factor \( \Delta u \)), electrical propulsion is the only realistic option. In fact, it is likely that the future of deep space exploration will depend largely upon electric propulsion systems as chemical systems will simply be unable to attain the high \( \Delta u \) required for these missions. Table 1 (Typical Mission Velocity Requirements) presents a summary of various mission \( \Delta u \) requirements.

<table>
<thead>
<tr>
<th>Flight Path</th>
<th>( \Delta u ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth to LEO(^a)</td>
<td>7,600</td>
</tr>
<tr>
<td>LEO to GEO(^b)</td>
<td>4,200</td>
</tr>
<tr>
<td>LEO to Earth escape</td>
<td>3,200</td>
</tr>
<tr>
<td>LEO to lunar orbit (7 days)</td>
<td>3,900</td>
</tr>
<tr>
<td>LEO to Mars orbit (0.7 yr)</td>
<td>5,700</td>
</tr>
<tr>
<td>LEO to Mars orbit (40 days)</td>
<td>85,000</td>
</tr>
<tr>
<td>LEO to Neptune orbit (29.9 yr)</td>
<td>13,400</td>
</tr>
<tr>
<td>LEO to Neptune orbit (50 yr)</td>
<td>70,000</td>
</tr>
<tr>
<td>LEO to solar escape</td>
<td>8,700</td>
</tr>
<tr>
<td>LEO to 1000 AU(^c) (50 yr)</td>
<td>142,000</td>
</tr>
<tr>
<td>LEO to Alpha-Centauri (50 yr)</td>
<td>(30 \times 10^6)</td>
</tr>
</tbody>
</table>

\(^a\) Low Earth Orbit = 270 km  
\(^b\) Geosynchronous Earth Orbit = 42,227 km radius  
\(^c\) Astronomical Units (1 AU = 149,558,000 km)

Table 1: Typical Mission Velocity Requirements. From Hill, Peterson, page 508.
Note however, that these mission velocity requirements are calculated for chemical propulsion and not electric propulsion systems. Regardless, Table 1 illustrates the typical magnitude of Δu required for various missions as the numbers do not change significantly. The values will be slightly higher for electrical systems.

On the down side, electrical propulsion systems cannot overcome high-gravity fields due to their low acceleration or low thrust. Therefore, the initial boost to attain escape velocity from Earth will need to be accomplished via chemical rockets, and it is evident that the types of missions slated for the future will be accomplished via a combination of both electrical and chemical propulsion. There are specific performance limitations to each. (Hill and Peterson, 1992, pg. 491)

Chemical rockets can be described as “energy limited” in that the fundamental chemical behavior of the propellant defines the quantity of energy (per unit mass of propellant) that can be released during combustion. High propellant energy for spacecraft is possible if a separate energy source is utilized — such as nuclear or solar energy. In that manner, the rate of conversion of nuclear or solar energy to electrical and then to propellant kinetic energy is limited by the mass of conversion equipment required. That mass can be a comparatively large portion of the total vehicle mass. As a result, the electrical rocket is generally said to be “power limited.” (Hill and Peterson, 1992, pg. 491)

B. PHYSICS BEHIND ELECTRIC PROPULSION

Space electrical propulsion systems consist of three basic subsystems. Those subsystems include the power processing unit, propellant management assembly, and the thruster assembly consisting of one or more thrusters.
The power processing unit (PPU) contains electrical interfaces that link the spacecraft power source and the propulsion system. The PPU sends power and command signals to the thruster(s) and propellant flow control signals to the propellant subsystem. Additionally, the PPU is responsible for converting various power sources (solar, batteries, fuel cells, nuclear, etc.) to the proper voltage, frequency, pulse rate, and current suitable for the system components.

The propellant management assembly (PMA) provides the means of storing, metering and delivering the propellant. The thrusters convert electrical energy to kinetic energy of a propellant in the form of exhaust or thrust.

Regardless of type, the common feature of electric propulsion systems is the addition of energy to a working fluid from an electrical source. Operating in a steady or pulsed mode, the exhaust is accelerated via one or more of the three fundamental types of thrusters. Those types of thrusters are (1) electrothermal, (2) electrostatic, and (3) electromagnetic. (Martinez-Sanchez and Pollard, 1998, pg. 688)

In reading the following sections, reference Figure 1, “Operating Principles of Various Electrical Propulsion Thrusters,” found at the end of this chapter on page 14. The figure will prove exceptionally useful in understanding the operation of several of the following thrusters as they are discussed.

1. Electrothermal Thrusters

In the electrothermal thruster, the propellant is electrically heated and thermodynamically expanded. The heated gas then achieves supersonic velocity as it passes through a nozzle. Overall, these thrusters have one major drawback in that, as a class, they produce limited high exhaust velocities. Additionally, their material
characteristics limit their performance to values that are generally in line with those exhibited by chemical rockets. At present, there are two basic types of electrothermal thrusters -- resistojets and arcjets.

The resistojet is the more elementary of the two in its class. In essence, it operates by using an electric current to directly heat metal components (coiled wire, tubing, or fins) which then transfer heat to the gaseous propellant through radiation and/or convection. As a result, the resistojet's performance is limited by the capacity of its structural components to withstand high temperatures, and resistojets achieve only a modest specific impulse of 300-310 seconds. This is due to the relatively high molecular mass of the propellant gas in addition to material limitations of the heating wall, which can sustain temperatures up to approximately 2000 K. (Martinez-Sanchez and Pollard, 1998, pg. 689)

Even with its comparatively lower value of specific impulse, the resistojet's superior efficiency contributes to far higher values of thrust/power than other varieties of electrical propulsion. Additionally, these engines possess the lowest overall system "dry mass" since they do not require a power processor and their plumes are uncharged. (Sackheim and Byers, 1998, pg. 672)

Resistojets have seen successful employment on such craft as Intelsat V, Satcom 1-R, GOMS, Meteor 3-1, Gstar-3, and Iridium S/C in addition to older satellites and test flights (Martinez-Sanchez and Pollard, 1998, pg. 689). They are most attractive to the mission planner for low-to-modest energy applications. In particular, they are most suitable where power limits, thrusting times, and plume impacts are mission drivers. (Sackheim and Byers, 1998, pg. 672)
The other form of the electrothermal thruster, the arcjet, manages to overcome the thermal problems encountered by the resistojet, and it enables a significant increase (approximately double) in specific impulse over the resistojet. The arcjet accomplishes this by forming an attached arc within the nozzle that directly heats the propellant stream to temperatures much greater than those encountered at the thruster body. Essentially, the power is deposited internally in the form of the electric arc, which typically operates between a concentric upstream rod cathode and a downstream anode that serves as a supersonic nozzle. (Martinez-Sanchez and Pollard, 1998, pg. 689)

The arc core reaches temperatures of 10-20,000 K while the buffer layer near the wall is maintained at less than 2000 K. As a consequence of the temperature gradient, there is practically no propellant flow through the arc core. This results in high specific impulse at the cost of reduced propulsive efficiency. In addition, the flow structure at the throat is extremely non-uniform, and the thruster is primarily limited by arc instabilities and erosion of the nozzle. (Martinez-Sanchez and Pollard, 1998, pg. 689) Generally, arcjets exhibit significant decreases (approximately six times) in thrust/power relative to the resistojet. This is due to an increased specific impulse coupled with relatively low values of efficiency ranging from 0.3 to 0.4. (Sackheim and Byers, 1998, pg. 672)

One significant disadvantage to the arcjet is that the required PPU is substantially more complex than that encountered in the resistojet as the complex plasma arc must be controlled. In fact, the PPU may be several times heavier than the thruster itself. Still, however, the arcjets exhibit relatively low dry masses when compared with some of the other electrical propulsion systems in use or development. (Martinez-Sanchez and Pollard, 1998, pg. 689)
With its intermediate specific impulse of 600 - 650 seconds, the arcjet is a very viable option for short burn duration missions. The lower specific impulse implies higher thrust for a given power, and arcjets have been used for geostationary applications, including the Telstar 4 and GE-1 satellites. (Martinez-Sanchez and Pollard, 1998, pg. 689) The engines are relatively simple to integrate, and they are the least costly and complex of any plasma propulsion device (Sackheim and Byers, 1998, pg. 672).

2. Electrostatic Thrusters

Electrostatic thrusters rely on electrostatic fields in order to exert forces on the propellant which is throttled into the field in the form of charged particles. Such thrusters can only operate in a vacuum, and all the created particles must be of the same sign -- i.e. all positive or all negative. Likewise, those particles must be neutralized after passing through the engine. Otherwise, the thruster would eventually render itself ineffective as a net charge would build up on the spacecraft setting up the return of oppositely charged ions to the craft. As a result, any thrust would be cancelled. In addition, sensitive system components could be damaged by the returning ions. (Humble, Henry, and Larson, 1995, pg. 532)

Naturally, electrons would seem to be the most likely candidate particles for these engines. They are easy to produce and to accelerate; however, their small mass renders them impractical. They are, in fact, so extremely light that the momentum imparted to them is negligible even with their high velocity.

Consequently, electrostatic thrusters operate using heavy atoms charged as positive ions or as colloids (liquid droplets). Neutralization of the positive ions is easily accomplished with electrons, and the ions can be 240,000 times as heavy as the electron.
The charged colloids are generally at least 10,000 times as massive as the ions. Thus, substantial momentum can be generated.

The implementation of heavy atoms has other advantages. The thrust per unit area increases as the square of the particle mass to the charge ratio in addition to the highly desirable characteristic of high voltage and low current as opposed to low voltage and high current (Sutton, 1992, pg. 580).

Electrostatic thrusters are categorized by the source of charged particles, and all three types exhibit the same three basic stages: ion production, acceleration, and neutralization. The five types of thrusters that have been studied are as follows:

1. Electron Bombardment Ion Thruster. Positive ions are produced via the bombardment of vaporized or gaseous propellant (such as xenon or mercury) with electrons emitted from a heated cathode.

2. Ion Contact Thruster. Positive ions are created by passing a propellant vapor (cesium) through a hot contact ionizer (usually tungsten). Although it was intensely investigated 15 to 30 years ago, the ion contact thruster was abandoned as impractical.

3. Field Emission (Colloid) Thruster. This engine generates positive or negative particles by passing tiny liquid droplets of propellant through an intense electric field (corona discharge). (Sutton, 1992, pg. 580)

4. Radio Frequency Ion Thruster. This thruster produces a high specific impulse at the price of complexity and mass. Essentially, it consists of a discharge chamber that is surrounded by an inductive coil connected to a radio frequency generator. Neutral propellant atoms are ionized by the
bombardment of electrons which are accelerated by the induced high-frequency eddy fields in the discharge chamber.

5. Electron Cyclotron Resonance / Microwave Discharge Ion Thrusters. Here a circularly polarized microwave beam is projected from a conventional waveguide via a suitable dielectric window in order to ionize the propellant in the discharge chamber. (Filliben, 1997, pg. 5)

At the present time, ion engines are the “thruster of choice” for deep missions such as interplanetary transfers. These missions require a high Δv and must tolerate long thrusting times. (Martinez-Sanchez and Pollard, 1998, pg. 692) A typical xenon ion thruster can generate an average mission specific impulse of 2,800 - 3,500 seconds with a lifetime of greater than 8000 hours (333 days).

3. Electromagnetic Thrusters

There are several variations to the electromagnetic type of thruster. All make use of a propellant gas that is heated to a plasma state. That plasma is then part of a current-carrying electric circuit which interacts with combined electric and magnetic fields thereby generating thrust.

The four types of electromagnetic thrusters which we will discuss include the Hall-effect thruster, stationary plasma thruster (SPT), the magnetoplasmadynamic (MPD) thruster, and the pulsed plasma thruster (PPT).

The Hall-effect thruster is named for its closed circular electron drift that is exhibited between the cathode and anode of the thruster. This effect is analogous to the Hall Effect. The SPT operates on similar principles.
In the Hall-effect thruster, an axial electric field and radial magnetic field are established in the discharge chamber. Meanwhile, neutral propellant atoms are fed into the discharge chamber and ionized by the bombardment of electrons. The perpendicular electric and magnetic fields then force the electrons, emitted from an external hollow cathode, to become trapped in azimuthal drift motion. At the same time, the radial magnetic field is not sufficiently strong to affect the propellant ion trajectory, and they are consequently accelerated axially by the electric field, producing thrust. (Filliben, 1997, pg. 5)

The first use of a Hall thruster electric propulsion system in space took place in October of 1998. The U. S. Navy’s Hall thruster, installed on an experimental spacecraft owned by the National Reconnaissance Office, began operating as part of the electric propulsion demonstration module (EPDM). The engine provides for orbit raising and long-term station keeping. (Space Tracks, 1999, pg. 13)

The particular engine onboard the EPDM is a TAL (thruster with anode layer) type Hall thruster, using xenon gas. It is a gridless device producing greater than twice the thrust of an ion engine of the same operating power. (Space Tracks, 1999, pg. 13)

Although still in development, the magnetoplasmadynamic thruster (MPD) is generally regarded as the leading potential candidate for future deep space missions such as a heavy-lift Mars transfer, utilizing a nuclear powerplant. (Martinez-Sanchez and Pollard, 1998, pg. 693)

The engine operates by generating a current along a conducting bar. The current consequently yields a self-induced azimuthal magnetic field that interacts with the current of an arc that travels from the point of the bar to a conducting wall. The resulting force
has two components. One component is a radially inward force that constricts the flow, and the other force acts along the axis, producing directed thrust. As expected, one of the great problems with this thruster is the substantial erosion that occurs at the point of contact between the arc current and electrodes. (Santarius, 1997) Additionally, the MPD thruster is more difficult to optimize due to the fact that several physical effects are involved in its operation.

The Pulsed Plasma Thruster (PPT) is “different from all other concepts” in two fundamental ways. This device operates in short pulses of approximately 10 microseconds duration, and in its most developed form uses a portion of solid propellant feedstock (typically Teflon®). As a result, no propellant tanks (referred to as “tankage”) are required in this device. (Martinez-Sanchez and Pollard, 1998, pg. 692)

In the PPT, a portion of the feedstock is ablated and ionized by an electrical arc discharge sheet initiated between two electrodes by a discharging capacitor. The resulting propellant plasma is accelerated by the interaction of the arc and the self-induced magnetic field of the current loop. (Filliben, 1997, pg. 5)

The PPT design is comparatively simple and reliable. However, one of the inherent problems with this thruster is that the force falls off as the current loops get large. In addition, PPT’s generally suffer from low efficiencies on the order of 8 to 13 percent. (Santarius, 1997)

PPT’s offer two additional distinct advantages in that they integrate a non-toxic propellant feed system with a thruster in a single compact unit as well as offering a variable pulse repetition rate. As a result, they possess a flexibility of operation over a wide range of mean power or thrust, and therefore, they have found employment for
precision orbital or attitude-control tasks since the late 1960's with the LES-6 satellite and the U. S. Navy's NOVA constellation. (Martinez-Sanchez and Pollard, 1998, pg. 692)

The optimal implementation of these three fundamental types of electric propulsion engines (electrothermal, electrostatic, and electromagnetic) will be the focus of our analysis. As illustrated in this chapter, there are several variants of each of the engines. Some of those engines are currently in use or in various stages of development. Please reference Table 2 on page 13 for a summary of typical electrical propulsion performance parameters.
<table>
<thead>
<tr>
<th>Type</th>
<th>Thrust (mN) / Duration</th>
<th>Iₜ (sec)</th>
<th>Efficiencyᵃ</th>
<th>Specific Power (W/mN)</th>
<th>Typical Propellants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrothermal:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resistojet</td>
<td>200 - 300 / months</td>
<td>200 - 350</td>
<td>0.65 - 0.9</td>
<td>0.5 - 6</td>
<td>NH₃, N₂H₄, H₂</td>
</tr>
<tr>
<td>Arcjet</td>
<td>200 - 1000 / months</td>
<td>400 - 1000</td>
<td>0.3 - 0.5</td>
<td>2 - 3</td>
<td>H₂, N₂, N₂H₄, NH₃</td>
</tr>
<tr>
<td>Electrostatic:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ion Engine</td>
<td>0.01 - 200 / months</td>
<td>1500 - 4000</td>
<td>0.6 - 0.8</td>
<td>10 - 70</td>
<td>Xe, Ar, Kr</td>
</tr>
<tr>
<td>Electromagnetic:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid Pulsed Plasma</td>
<td>0.05 - 10 / years</td>
<td>1000 - 2000</td>
<td>0.2 - 0.3</td>
<td>10 - 50</td>
<td>Teflon</td>
</tr>
<tr>
<td>MPD Arcjet</td>
<td>0.001 - 2000 / weeks</td>
<td>1000 - 8000</td>
<td>0.1</td>
<td>100</td>
<td>Ar, Xe, H₂</td>
</tr>
<tr>
<td>Hall Effect</td>
<td>0.01 - 2000 / months</td>
<td>2000 - 5000</td>
<td>0.3 - 0.5</td>
<td>100</td>
<td>Xe</td>
</tr>
<tr>
<td>Monopropellant rocketᵇ</td>
<td>30 - 100,000 / hrs., mins.</td>
<td>200 - 230</td>
<td>0.87 - 0.97</td>
<td>N/A</td>
<td>N₂H₄</td>
</tr>
</tbody>
</table>

ᵃ Efficiency = thrust-power output / electrical-power input
ᵇ Listed for comparison only.

Table 2: Typical Performance Parameters of Various Types of Electrical Propulsion Systems. Adapted from Sutton, page 567.
Figure 1. Operating Principles of Various Electrical Propulsion Thrusters.

From Martinez-Sanchez and Pollard, page 691.

(a) resistojets
(b) arcjets
(c) Hall thrusters
(d) ion engines
(e) pulsed plasma thrusters
(f) field-effect electrostatic propulsion thrusters
(g) self-field magnetoplasmadynamic thrusters
II. THEORETICAL BACKGROUND

A. FIRST PRINCIPLES

Electrical propulsion systems have four general yet unique characteristics. These characteristics are exploited in analyzing and understanding the electric propulsion system, and they are delineated in the following paragraph.

Electric propulsion engines (1) incorporate a comparatively heavy mass of required power generation equipment, and they (2) produce low thrust. As a result of that low thrust, (3) very low accelerations (on the order of $10^{-4}$ to $10^{-6} \text{g}_0$) are attained (4) requiring long operating times. In the following development of the equations governing electrical engines, the electrical rocket’s performance will be characterized in terms of power and mass as first proposed by Langmuir and Irving. (Sutton, 1992, pg. 596)

First, the mass aspects of electrical propulsion will be analyzed.

The major components integral to an electrically powered space vehicle are broken down as follows:

$$m_o = m_p + m_{pl} + m_{pp}. \quad (1)$$

In words, this equation states that the initial total mass of the vehicle ($m_o$) is equivalent to the sum of the masses of the propellant ($m_p$), payload ($m_{pl}$), and powerplant ($m_{pp}$). Refer to the list of symbols and abbreviations (on page xiii) for the comprehensive table of the following abbreviations and their respective units.

The determination of $m_{pp}$ for an assigned mission is the most amorphous. This term incorporates a wide variety of components that make up the empty (or un-fueled) propulsion system. Those components include the thruster, the propulsion storage and
feed system, the energy source with its conversion system and auxiliaries as well as the associated structure (Sutton, 1992, pg. 597).

Now, let us look at the power aspects.

Regardless of the energy source (battery, nuclear, solar, fuel cells, etc.) to the input power supply, the input energy must always be greater than the required electrical power output, and power conversion efficiencies typically average approximately 70 percent. This aspect is accounted for in the engine parameter known as specific power, \( \alpha \), or the “state-of-the-art.” It is defined as the ratio of electrical power output to the mass of the power plant, and it serves as an indication of the system’s ability to effectively and efficiently generate or convert to electrical power. The equation for \( \alpha \) is defined as follows:

\[
\alpha = \frac{P_e}{m_{pp}},
\]

(2)

where \( P_e \) is the electrical power that is actually supplied to the propulsion system.

Typical high “alpha” values that are currently attainable range from approximately 100 to 200 W/kg. It is a singular goal in electrical propulsion to strive to push this number to the highest attainable values. (Sutton, 1992, pg. 597)

In an electrical propulsion thruster, the electrical power input is converted to the kinetic energy of the propellant exhaust. Thus, \( P_e \) can also be defined by accounting for losses due to thruster efficiency, \( \eta_t \), where

\[
\eta_t = \frac{P_{jet}}{P_e} = (1/2) \frac{m}{P_e} \frac{v^2}{P_e} \quad \text{(Sutton, 1992, pg. 571)},
\]

(3)

and, consequently, we may redefine \( P_e \) as the following:

\[
P_e = \alpha m_{pp} = (1/2) \frac{m}{\eta_t} \frac{v^2}{P_e} = (2 t_p \eta_t).
\]

(4)
In this equation, \( m \) is the propellant mass flow rate, \( v \) is the effective exhaust velocity, and \( t_p \) is the duration of operation in which the propellant mass, \( m_p \), is ejected from the thruster. It is important to note that this equation assumes a uniform rate of propellant expulsion. (Sutton, 1992, pg. 597)

Using the previous definitions of \( m_0 \), \( \alpha \), and \( P_e \), we can form the equation for the payload mass fraction, which is written,

\[
\frac{m_p}{m_0} = \left(1 - \frac{v^2}{(2\alpha t_p \eta_s)}(e^{\Delta u/v} - 1)\right) \frac{e^\Delta u}{e^{\Delta u/v}}.
\]

This equation assumes gravity-free and drag-free flight. Additionally, by defining the characteristic velocity, \( v_c \), as

\[
v_c^2 = 2\alpha t_p \eta_s,
\]

we can re-write the payload mass fraction as follows:

\[
\frac{m_p}{m_0} = \left(1 - \frac{v}{v_c}\right)^2(1 - \frac{\Delta u}{v_c}) \frac{e^\Delta u}{e^{\Delta u/v}}.
\]

In this form, the payload mass fraction is more manageable. (Sutton, 1992, pg. 597)

A plot of this equation is illustrated in Figure 2. The equation is graphed over the range of \( \frac{v}{v_c} \) for various values of \( \Delta u/v_c \). It is evident that a maximum exists such that the payload mass ratio, \( \frac{m_p}{m_0} \), for a given velocity increment, \( \Delta u/v_c \), is maximum at a particular value of exhaust velocity, \( v/v_c \), or specific impulse, \( I_s/v_c \). Specific impulse and effective exhaust velocity are related via the equation,

\[
I_s = v/g_0 \text{ where } g_0 = 9.81 \text{ m/s}^2. \] (Sutton, 1992, pg. 597)

The maximum described in these curves exists for two reasons. First, the inert mass of the power plant, \( m_{pp} \), increases almost linearly with the specific impulse. However, the mass of consumed propellant decreases with specific impulse. The vehicle velocity increment, \( \Delta u \), increases with higher \( I_s \), however, \( \Delta u \) is also decreased by the
lower mass ratio due to the increased inert mass. Consequently, there is a unique optimum value of $I_e$ (or $v$) for every electric propulsion mission as defined by the corresponding required value of $\Delta u$. (Sutton, 1992, pg. 597)

Again, refer to Table 1 for a brief summary of various mission $\Delta u$ values.

![Figure 2. Payload Mass Fraction Plot: $m_p / m_o$ vs. $v / v_c$.](image)

The payload mass ratio can also be re-written to a somewhat more useful form by solving for $\Delta u / v_c$. In this case, the equation takes on the following form:

$$\Delta u / v_c = (v / v_c) \ln \left[ (1 + (v / v_c)^2) / (m_{pl} / m_o + (v / v_c)^2) \right],$$

and it is plotted in Figure 3. In this case, the equation is graphed over the same range of
v / v_c for various values of m_{pl} / m_o.

![Diagram showing payload mass fraction plot](image)

**Figure 3.** Payload Mass Fraction Plot: $\Delta u / v_c$ vs. $v / v_c$.

This graph shows the optimum curves for three different mass payload ratios.

From this plot and the defining equations, we note that the optimal values for $\Delta u / v_c$ and $v / v_c$ are as follows:

<table>
<thead>
<tr>
<th>$m_{pl} / m_o$</th>
<th>$\Delta u / v_c$</th>
<th>$v / v_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>0.404</td>
<td>0.782</td>
</tr>
<tr>
<td>0.45</td>
<td>0.327</td>
<td>0.820</td>
</tr>
<tr>
<td>0.55</td>
<td>0.257</td>
<td>0.864</td>
</tr>
</tbody>
</table>

**Table 3.** Optimum Values: $\Delta u / v_c$ and $v / v_c$ for Various Payload Mass Ratios.
These values will be an essential aspect of our engine selection (or engine-mission matching) algorithm, which is developed in Chapter IV.

B. **ANALYSIS – DUAL OPTIMUM METHODOLOGY**

Incorporating the previous development via “first principles,” it is now advantageous to establish a “joint” or “dual” optimum, which will optimizes the payload mass ratio and yields the minimum transfer time. It can be calculated by multiplying the payload mass ratio and the previously defined equation for $\Delta u / v_c$.

The plot of the resulting equation (for various payload ratios) is illustrated in Figure 4, and it shows that the dual-optimum payload mass ratio peaks at a value of $m_{pl} / m_0 = 0.45$.

![Figure 4. Dual Optimum.](image-url)
This plot indicates that the optimum payload ratio is 0.45. However, due to the fact that it may be difficult for the designer to achieve this mass ratio, an approximate design range of payload mass ratios of 0.35 to 0.55 is investigated. As illustrated in Figure 4, these values do not lie too far below the optimum for 0.45.

It should be restated that for constant propellant mass flow \( (m) \) constant thrust \( (F) \) and negligibly short start and stop transients:

\[
I_s = \frac{F}{m g_0} \quad \text{and} \quad : F = m g_0 I_s. \tag{10}
\]

Hence, for a higher specific impulse, \( I_s \), less fuel is required. However, as the specific impulse is increased, we may drive through the optimum due to the equation for \( \Delta u / v_c \), which incorporates the applicable "efficiencies" of the engines in the forms of \( \alpha, t_p, \) and \( \eta_t \). As a result, an engine with an arbitrarily high \( I_s \) may not be optimum for a particular application or mission. For example, the Teflon PPT has relatively high values for \( I_s \) at the expense of comparatively poor values for \( \alpha \) and \( \eta_t \).

Figure 5 illustrates a three-dimensional optimum surface. The plot is illustrated across the axis values of...

- \( x: \frac{v}{v_c} \) range: 0.5 to 1.3
- \( y: \frac{m_{pl}}{m_o} \) range: 0.3 to 0.6
- \( z: \left( \frac{m_{pl}}{m_o} \right) \left( \Delta u / v_c \right) \) range: 0.115 to maximum at 0.148

The plot clearly shows the establishment of a optimum or maximum point at the payload mass ratio of precisely \( \frac{m_{pl}}{m_o} = 0.45 \).
Figure 5. Three-dimensional Dual Optimum Surface.

C. TANKAGE PENALTY

The parametric curves that have been presented up to this point are inherently ideal in nature. They generally assume that all spacecraft mass can fall neatly into the three categories of mass of propellant, payload, or power-plant. However, as previously noted, it can be difficult to assess where exactly certain masses (such as shared power conditioning equipment) should be charged, and as a result, the calculation of the specific power, $\alpha$, for the respective electric propulsion engine can have high variability. In the
same manner, one can assess somewhat of a "tankage penalty" in proportion to the amount of propellant required as a result of the following reasoning.

The typical propellant tanks are composed of overwrapped titanium or aluminum. Their loading varies depending upon several factors including satellite mass, expected life on orbit, mission (orbit raising, NSSK, etc.) and cant angle of thruster. As a result, the actual size and mass of tanks will vary regardless of engine type or manufacturer.

An accurate figure of merit for the required tankage revolves around the following assumptions. First, that a propellant tank that holds zero propellant mass is assessed at one kilogram. Second, that the tank has a mass fraction of approximately ten percent as a function of loaded propellant. It is important to note that all of the listed electrical propulsion engines, except the pulsed MPD type, require tankage. (Clauss, 1999)

For example, a tank that holds 40 kg of xenon would have a dry mass of...

$$1.0 \text{ kg} + 0.1 (40 \text{ kg}) = 5 \text{ kg}.$$ 

If this 10% tankage "penalty" and one kilogram of dead dry mass are incorporated into the previous analysis, the equation for the initial mass and components is altered from

$$m_o = m_p + m_{pl} + m_{pp}$$

to

$$m_o = 1.1 m_p + m_{pl} + m_{pp}.$$ 

However, the equation for final mass remains unaltered as follows:

$$m_f = m_o - m_p.$$ 

The 0.1 m_p is residual mass that remains within the system.

Using these definitions, the equation for the rewritten payload mass ratio, solved for $\Delta u / v_c$, takes the following form:

$$\Delta u / v_c = (v / v_c) \ln \left[ \left( (1.1 + (v / v_c)^2) \right) \left( \frac{m_{pl} / m_o}{m_o} + (v / v_c)^2 + 0.1 \right) \right]. \quad (11)$$

This equation is plotted as $\Delta u / v_c$ vs. $v / v_c$ in Figure 6.
Figure 6. Payload Mass Fraction Plot with 10 % Tankage Penalty:

\[ \frac{\Delta u}{v_c} \text{ vs. } \frac{v}{v_c}. \]

Note that the curve is essentially similar to the plot illustrated in Figure 3; however, it is adjusted for the ten percent tankage "penalty." As a result, the optimum curves have shifted down and right from their original values. This is illustrated in Figure 7 which shows the optimum curves with and without the tankage "penalty" for the payload mass ratio \( \frac{m_{pl}}{m_o} \) of 0.45.

For a ten percent tankage "penalty," the following optimal values for \( \frac{\Delta u}{v_c} \) and \( \frac{v}{v_c} \) result:
Table 4. Optimal Values with 10% Tankage Penalty: $\Delta u / v_c$ and $v / v_c$.

<table>
<thead>
<tr>
<th>$m_p / m_o$</th>
<th>$\Delta u / v_c$</th>
<th>$v / v_c$</th>
<th>$\text{opt}$</th>
<th>$v / v_c$</th>
<th>$\text{opt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>0.375</td>
<td>0.850</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.45</td>
<td>0.306</td>
<td>0.895</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.55</td>
<td>0.240</td>
<td>0.945</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 7. Optimum Comparison with and without 10% Tankage Penalty: $\Delta u / v_c$ vs. $v / v_c$ at $m_p / m_o = 0.45$.

For all of the plotted "tankage" penalty curves, the values plotted are for a 10% penalty. Typical tankage fractions range from 0.05 for the hydrazine ($N_2H_4$) resistojet and arcjet to 0.15 for the $H_2$ arcjet. Xenon Hall thrusters and ion thrusters have tankage fractions of about 0.12. (Martinez-Sanchez and Pollard, 1998, pg. 690)
III. CURRENT TECHNOLOGY – ELECTRIC ENGINES

A. NASA DEVELOPMENT AND OBJECTIVES

Today, there are many organizations pursuing the research and development of space electric propulsion. Various private companies (including Primex, Hughes, ARC, and Daimler Chrysler) presently offer electric propulsion engines for sale and use in various applications. Additionally, several research facilities -- such as NRL (Naval Research Labs) and NASA’s JPL (Jet Propulsion Labs) -- have also developed various engines or components of engines. However, the one department of NASA that is primarily responsible for electric propulsion research is the On-board Propulsion Branch.

NASA’s On-board Propulsion Branch is based at the John H. Glenn Research Center located on Lewis Field in Cleveland, Ohio. The branch is responsible for developing advanced on-board spacecraft propulsion technologies -- both chemical and electrical -- for future NASA missions. The electrical propulsion technologies currently under investigation at Lewis Field include resistojets, arcjets, gridded ion xenon thrusters, Hall xenon ion thrusters, and pulsed plasma thrusters (PPT’s). (Oleson, 1999)

In October of 1998, the on-board propulsion branch celebrated two major milestones in the realm of electrical propulsion. Deep Space One (DS1) was successfully launched on October 24, 1998 from Florida’s Cape Canaveral Air Station. This craft is intended to validate new space technologies, including its ion propulsion primary engine, which was developed in the NSTAR (NASA Solar Electric Propulsion Technology Application Readiness) program. NASA Glenn developed the engine and power processors for DS1, and this flight was the first-ever application of such a technology to a
Deep space mission. DS1 was to rendezvous with the near-Earth asteroid, 1992 KD, in July 1999, and it passed within 15 miles of the asteroid Braille on July 28 at a speed of greater than 35,000 mph (15.5 km/s). (Oleson, 1999)

DS1’s NSTAR ion engine utilizes 81 kilograms of xenon propellant, which it ionizes and accelerates to 68,000 mph. The engine, which is 12 inches in diameter, can operate at half throttle for greater than 20 months, generating a continuous thrust of 0.02 lbf. As a result, the engine is ten-times more fuel efficient than a comparable chemical system, and it is the prototype for smaller / lower-cost space vehicles. (Oleson, 1999)

On October 3, 1998, a STEX spacecraft was launched, carrying the EPDM (Electric Propulsion Demonstration Module). This module was developed under joint effort on the part of NASA, Naval Research Labs (NRL) and private industry. The Glenn Research Center led the program, which was intended to demonstrate the first use of a high specific impulse, low-power Hall thruster system on a U. S. spacecraft. (Oleson, 1999)

That particular Hall engine is a TAL (Thruster with Anode Layer) D-55 Hall-effect thruster. It was developed by the Russian Central Research Institute of Machine Building (TSNIIMASH), and it successfully operated, firing ten times from 23 to 24 October. The engine provided 40 mN of thrust and raised the satellite’s orbit 0.35 NM. (Oleson, 1999)

As electrical propulsion evolves, NASA Glenn has clearly defined its objectives. For fiscal year 1999, four major product targets have been defined. Those targets include a next generation ion system for near- and further-term space science and a
pulsed plasma thruster (PPT) technology for precision imaging. In addition, NASA would like to develop both a high specific impulse, low-mass Hall system and a miniature electrothermal thruster for Earth science. (Oleson, 1999)

NASA Glenn has also defined what it calls “grand challenges” in the development of electrical propulsion. Those challenges are delineated for each of the three types of electrical propulsion systems, and they are outlined as follows: (Oleson, 1999)

Electrothermal Thrusters:
- integrated electronics / thruster package for micro-spacecraft applications

Electrostatic Thrusters:
- lightweight, high total impulse ion systems for space science
- very high total impulse ion system for far-term space science

Electromagnetic Thrusters:
- low-cost, simple, low-power Hall for miniature Earth-orbital spacecraft
- high specific impulse \( (I_s) \), direct drive Hall for space science and Earth orbital
- increase thruster efficiency by factor of two to 16%
- increase capacitor life to 40,000,000 pulses and beyond
- lower electromagnetic emissions to below present standard specifications
- predict contamination on spacecraft surfaces in design phase

Nonetheless, the ability and timeliness of electric propulsion to meet these challenges is unpredictable. This is due inherently to the fact that the “state of the art” in electric propulsion is due and determined largely by the “state of the art” of the electrical components that comprise various aspects of the electrical propulsion system. Primarily, the concern is the required additional mass of electrical components. This fact will
become evident in the forthcoming discussion of the specific power or “alpha” (α) parameter.

B. SUMMARY OF AVAILABLE ENGINE DATA

One of the contributions of this thesis is to obtain and summarize the most current reported data regarding present-day electric propulsion engine performance characteristics. This summary, which includes data from various vendors as well as the above research sites, can be found on page 36 in Table 5: Summary of Current Technology Engines, and the various sources of this information are listed at the bottom of the table. Figure 8, on page 35, provides an overall map of current engines and their respective values of specific impulse, \( I_s \).

The electrical engine parameter that has certainly been the most difficult to define has been an accurate representation of the parameter α, specific power or “state of the art.” This parameter is so named because it defines the relation of the power conversion unit with respect to the system’s mass. In other words, as previously defined, 
\[
\alpha = \frac{P_e}{m_{pp}}, \text{ where } P_e \text{ is the power supplied to the electrical propulsion system and } m_{pp} \text{ is the power-plant mass (Sutton, 1992, pg. 597).}
\]

The apparent controversy surrounding the “state of the art” parameter is evidenced in the article “Optimization of Electric Propulsion Systems Considering Specific Power as Function of Specific Impulse” by M. Auweter-Kurtz et al. This article, found in the Nov.-Dec. 1988 issue of the AIAA Journal of Propulsion and Power, presents the argument that specific power, \( \alpha \), is a strong function of specific impulse, \( I_s \), or exhaust velocity, \( v \). Specific impulse and exhaust velocity are related via the equation,
\[
v = I_s g_0.
\]
The article notes that several different organizations have conducted optimizations for various missions and that those optimizations assumed a constant specific power of the propulsion system. This assumption is arguably a potential source of error in that it implies the following about variations in power levels and specific impulse. First, it assumes that constant thrust and system efficiency could be evident with a changing specific impulse. Additionally, it implies that constant specific power could be maintained with changing power levels during optimization. (Auweter-Kurtz et. al., 1988, pg. 512)

In undertaking this optimization analysis, this point was certainly considered. However, due to the fact that the trace of an engine's efficiency is not continuous, average values were utilized over limited specific impulse ranges thereby assuming no strong dependence on specific impulse or exhaust velocity.

Still, it is not elementary to assign $\alpha$ values to a particular engine.

Items of concern in determining specific power include such things as the number of engines that will be necessary for a specific mission and whether or not those same engines can share various components of the power conditioning equipment. If power conditioning components can be shared, then $\alpha$ improves as mass is decreased. Likewise, $\alpha$ would be detrimentally affected if more mass were required such as additional solar cell panels to power required thrusters.

Specific power also can be improved if the electrical propulsion system can be powered through the systems already integral to the payload. As such, the question arises as to where that mass should be billed. If it is attributed to the payload mass ($m_{pl}$), then again the value for specific power is improved.
In summary of the above concerns, it should be evident that the \( \alpha \) parameter is not only engine specific but also configuration (number of engines and shared components or redundancy) and design (payload compatible power sources) specific.

In order to demonstrate this aspect of the determination of the value for specific power, the following example is illustrated.

Atlantic Research Corporation is one of the current suppliers of one of the commercially available Hall-type thrusters. In their brochure for the SPT-100, they depict the configuration for a “typical SPT-100 propulsion system for GEO satellites.”

The system depicted consists of the following major components: four SPT-100 thrusters, four pairs of xenon flow controllers (XFC’s), two PPU-100 power processing units, the propellant management assembly (PMA), and two xenon storage tanks. The entire configuration allows for redundancy; therefore, we would assume that the designed mission would require the use of only two of the four thrusters. (ARC brochure, 1998)

In determining specific power, it is necessary to account for the masses of all components that encompass the power-plant mass \( (m_{pp}) \). This does not include the actual mass of the propellant \( (m_p) \). Thus, it is often referred to as the “dry mass” -- probably hailing from the early days of liquid rocket propulsion.

The masses of all of the components are as follows:

SPT-100 (4)..........................3.5 kg / SPT
XFC (4 pairs)..........................0.64 kg / pair
PPU-100 (2)..........................6.2 kg / PPU
PMA........................................3.25 kg

However, the mass of the xenon tanks is mission dependent. (Clauss, 1999)
The tank mass can be estimated in the manner demonstrated in Chapter II, C. Again, assuming that a tank which holds zero propellant mass weighs 1 kg and that the tank has a mass fraction of 0.1 to 0.12 (as a function of the mass of xenon), then for a 40 kg of xenon, a 5 kg tank should be required as 1 kg + (0.1)(40 kg) = 5 kg. (Clauss, 1999)

In the brochure example for a GEO satellite, we pick a usable example of a 15-year GEO satellite for NSSK. Such a mission would require approximately 120 kg of xenon to be held in two tanks. Hence, 60 kg of xenon would be held in each tank. Accordingly, one tank would then weigh 7 kg as 1 kg + (0.1)(60 kg) = 7 kg.

Nowhere in any current publications is there a discussion as to how to “charge” mass values in determining specific power or “alpha.” In this case, all “dead” mass is attributed to power-plant mass ($m_{pp}$), and all mass that is proportional to power is assigned to payload mass ($m_{pi}$). Hence, we would have to include 2 kg of tankage in our value for power-plant mass.

With the value for tank mass, it is now possible to calculate the power-plant mass as follows:

$$m_{pp} = 4 \times 3.5 \text{ kg}_{SPT} + 4 \times 0.64 \text{ kg}_{XFC} + 2 \times 6.2 \text{ kg}_{PPU} + 3.25 \text{ kg}_{PMA} + 2.0 \text{ kg}_{tanks}.$$

Thus, $m_{pp} = 34.2 \text{ kg}$.

Each of the SPT-100 thrusters requires 1.35 kW of power delivered at 300 V and 4.5 A. With a known PPU efficiency of $\eta_{PPU} = 0.93$, we are able to calculate that a total of 5,808 W must be delivered to the thruster bank. Hence, we are now able to calculate the specific power for the configuration. (Clauss, 1999)

$$\alpha = \frac{P_e}{m_{pp}} = \frac{5,808 \text{ W}}{34.2 \text{ kg}} = 169.8 \text{ W/kg}.$$
It can not be overemphasized that this calculated value of specific power for the SPT-100 is only for the illustrated four-thruster configuration as designed for the mission of a 15-year GEO satellite. Otherwise, the masses change, and specific power must be re-calculated.
Table 5: Summary of Current Technology Engines.

<table>
<thead>
<tr>
<th>Engine Type</th>
<th>Max. Thrust (lbf)</th>
<th>Weight (lb)</th>
<th>Length (ft)</th>
<th>Height (ft)</th>
<th>Wingspan (ft)</th>
<th>Max. Speed (knots)</th>
<th>Max. Range (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-16A Block 55</td>
<td>22,000</td>
<td>16,200</td>
<td>39</td>
<td>20</td>
<td>35</td>
<td>630</td>
<td>1,500</td>
</tr>
<tr>
<td>F-16C Block 50</td>
<td>22,000</td>
<td>16,200</td>
<td>39</td>
<td>20</td>
<td>35</td>
<td>630</td>
<td>1,500</td>
</tr>
<tr>
<td>F-16D Block 60</td>
<td>22,000</td>
<td>16,200</td>
<td>39</td>
<td>20</td>
<td>35</td>
<td>630</td>
<td>1,500</td>
</tr>
<tr>
<td>F-16E Block 70</td>
<td>22,000</td>
<td>16,200</td>
<td>39</td>
<td>20</td>
<td>35</td>
<td>630</td>
<td>1,500</td>
</tr>
<tr>
<td>F-16F Block 80</td>
<td>22,000</td>
<td>16,200</td>
<td>39</td>
<td>20</td>
<td>35</td>
<td>630</td>
<td>1,500</td>
</tr>
</tbody>
</table>

In addition to the above engines, there are several older models in service, including:

- F-16A Block 50
- F-16C Block 50
- F-16D Block 60
- F-16E Block 70
- F-16F Block 80

These engines are typically used for training and testing purposes.
<table>
<thead>
<tr>
<th>Model</th>
<th>Range</th>
<th>0.40 - 0.45</th>
<th>1600 - 1630</th>
<th>150 - 210</th>
<th>0.008 - 0.011</th>
<th>lab</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPI T-40 (11)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPI T-140 (11)</td>
<td>&gt; 0.55</td>
<td></td>
<td></td>
<td>2000 - 4000</td>
<td>0.200 - 0.210</td>
<td>8000 lab</td>
</tr>
<tr>
<td>SPI T-220 (11)</td>
<td>TBD</td>
<td></td>
<td></td>
<td>10K - 20K</td>
<td>TBD</td>
<td>development</td>
</tr>
<tr>
<td>T-100 SPT (10)</td>
<td>0.49, 0.52</td>
<td>1630</td>
<td>1350</td>
<td>0.083 &gt; 8000</td>
<td>lab</td>
<td></td>
</tr>
<tr>
<td>TAL D-55 (Russia) (5,10)</td>
<td>~ 50.9</td>
<td>1600, 950 - 1950</td>
<td>600-1500</td>
<td>0.082 &gt; 5000</td>
<td>operational</td>
<td></td>
</tr>
<tr>
<td>Primex BPT Hall (8)</td>
<td>0.5</td>
<td>1500 - 1800</td>
<td>500 - 6000</td>
<td></td>
<td></td>
<td>development</td>
</tr>
<tr>
<td>MPD -- steady</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>applied field (2)</td>
<td>0.5</td>
<td>2000 - 5000</td>
<td>1-100 K</td>
<td></td>
<td></td>
<td>lab</td>
</tr>
<tr>
<td>self-field (2)</td>
<td>0.3</td>
<td>2000 - 5000</td>
<td>200-4000 K</td>
<td></td>
<td></td>
<td>lab</td>
</tr>
<tr>
<td>MPD -- pulsed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teflon PPT (2)</td>
<td>1</td>
<td>0.07</td>
<td>1000</td>
<td>1-200</td>
<td>4000 N-s</td>
<td>&gt;10^7 pulses</td>
</tr>
<tr>
<td>LES 8/9 PPT (10)</td>
<td>0.0068, 0.009</td>
<td>836, 1000</td>
<td>25, 30</td>
<td></td>
<td></td>
<td>0.0003 &gt;10^7 pulses</td>
</tr>
<tr>
<td>NASA/Primex EO-1 (8)</td>
<td>~ 20</td>
<td>0.098</td>
<td>1150</td>
<td>up to 100</td>
<td>3000 N-s **</td>
<td>1.4 mN, 2 Hz</td>
</tr>
<tr>
<td>Primex PRS-101 (8)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EPEX arcjet (Jap) (10)</td>
<td>0.16</td>
<td>600</td>
<td>430</td>
<td></td>
<td>0.023</td>
<td></td>
</tr>
</tbody>
</table>

**last update: 29 October 99**

**SOURCES:**


(3) ARC (Atlantic Research Group) portfolio re: Stationary Plasma Thruster.


(5) Mr. Mike Osborn of Naval Research Labs (mosborn@space.nrl.navy.mil).


(9) Hughes Space and Communications: "XIPS: The Latest Thrust in Propulsion Technology"...

(10) *Electric Thruster Systems Enter 'Era of Application',* CPIA Bulletin / Vol. 23, No. 5, September 1997

(11) brochure: "Hall Effect Thrusters," Space Power, Incorporated

**NOTES:**

* alpha value calculated for 4 SPT-100 configuration as advertised for typical GEO satellites

** additional data: thrust / mass = 250 micro-N / kg (1.4 mN at 2 Hz); fuel = 0.3 kg Teflon; PPE efficiency > 0.80

~ = estimated parameter
THIS PAGE INTENTIONALLY LEFT BLANK.
IV. PLANNING / OPTIMIZATION ALGORITHM – MATCHING THE MISSION, PAYLOAD, AND ENGINE

OPTIMUM DESIGN PROCEDURE:

1. Specify the Mission.

   Define the mission profile in terms of the velocity requirements (Δu) and payload mass (m_p). List additional constraints such as mission time (t_m), power (P_e), cost, etc.

2. Estimate the payload mass ratio, m_p / m_0.

   Based upon past operational and flight-test experience, an initial estimate of suitable candidate electric propulsion thrusters may yield an approximate expected payload mass ratio. Note: This initial “ballpark” estimate is primarily based upon mission profile (velocity requirement) and suitable engines, looking at cost, mission time, and operational flight experience.

   In order to remain near the optimum for engine performance (as illustrated in earlier plots), a potential payload mass ratio range (from m_p = 0.35 to 0.55) is selected with the corresponding numbers for Δu / v_c and v / v_c.

IDEAL: without “tankage” penalty (i.e., particularly for Teflon PPT):

   m_p / m_0 = 0.35: Δu / v_c = 0.404; v / v_c = 0.782.
   m_p / m_0 = 0.45: Δu / v_c = 0.327; v / v_c = 0.820.
   m_p / m_0 = 0.55: Δu / v_c = 0.257; v / v_c = 0.864.

“TANKAGE”: with 10 % tankage penalty:

   m_p / m_0 = 0.35: Δu / v_c = 0.375; v / v_c = 0.850.
   m_p / m_0 = 0.45: Δu / v_c = 0.306; v / v_c = 0.895.
   m_p / m_0 = 0.55: Δu / v_c = 0.240; v / v_c = 0.945.
If none of the listed payload ratios will suffice (or if further iteration shows that an intermediate value for payload ratio is appropriate), the required numbers for \( \Delta u / v_c \) and \( v / v_c \) can be obtained from the plots in Figures 8 and 9. These graphs are for ideal systems and systems requiring "tankage" respectively.

3. **Calculate the corresponding specific impulse, \( I_s \), based upon payload mass ratio**... using the tabular data below and the equation,

\[
I_s = \frac{v}{g_0}.
\]

**IDEAL:** without "tankage" penalty:

\[
\begin{align*}
\frac{m_{pl}}{m_o} &= 0.35: & I_s &= 1.936 \frac{\Delta u}{g_0}. \\
\frac{m_{pl}}{m_o} &= 0.45: & I_s &= 2.51 \frac{\Delta u}{g_0}. \\
\frac{m_{pl}}{m_o} &= 0.55: & I_s &= 3.36 \frac{\Delta u}{g_0}, \text{ where } g_0 = 9.81 \text{ m/s}^2.
\end{align*}
\]

"TANKAGE": with 10% tankage penalty:

\[
\begin{align*}
\frac{m_{pl}}{m_o} &= 0.35: & I_s &= 2.27 \frac{\Delta u}{g_0}. \\
\frac{m_{pl}}{m_o} &= 0.45: & I_s &= 2.92 \frac{\Delta u}{g_0}. \\
\frac{m_{pl}}{m_o} &= 0.55: & I_s &= 3.94 \frac{\Delta u}{g_0}, \text{ where } g_0 = 9.81 \text{ m/s}^2.
\end{align*}
\]

For intermediate values of payload mass ratio, this same calculation can be accomplished using the ratios from Figures 8 and 9 along with the following equation,

\[
I_s = \left( \frac{v}{v_c} \right) \left( \frac{v_c}{\Delta u} \right) \left( \frac{\Delta u}{g_0} \right).
\]

4. **Designate a candidate electric propulsion engine.**

Referencing Figure 8 (on page 35), select the engine (or engines) whose available range of specific impulse, \( I_s \), most closely matches the calculated value necessary for the optimum profile.

Engine selection also locks in the characteristic engine performance values for specific power (\( \alpha \)), thruster efficiency (\( \eta_0 \)), and available thrust. These values can be found in the comprehensive summary of Table 5 on pages 36 and 37.
Again, if no engine proves an adequate match for the calculated values of $I_s$, it is necessary to use the graphs depicted in Figures 9 and 10 and attempt intermediate values of mass payload ratio within the range.

5. **Calculate burn time, $t_p$.**

Determine the thruster burn time for the profile from the following formula,

$$t_p = \frac{\left(\frac{v_c}{\Delta u}\right) \Delta u}{2 \alpha \eta_t}.$$  

At this point in the process, all of the above values are given: $\Delta u$ from mission profile, $v_c / \Delta u$ from step 2, $\alpha$ and $\eta_t$ from Table 5.

6. **Check that burn time, $t_p$, is less than both mission time, $t_m$, as well as the design lifetime of the designated engine.**

If burn time exceeds acceptable mission time or design life of the thruster, select another electric propulsion candidate (step 4) or vary the payload mass ratio (step 2).

7. **Calculate the anticipated masses of the propellant ($m_p$) and power-plant ($m_{pp}$) via the following formulas for the respective payload mass ratios:**

**IDEAL:** without “tankage” penalty (i.e., for Teflon PPT):

- $m_{pl} / m_o = 0.35$: $m_p = 1.152 m_{pl}$; $m_{pp} = 0.612 m_p$
- $m_{pl} / m_o = 0.45$: $m_p = 0.731 m_{pl}$; $m_{pp} = 0.672 m_p$
- $m_{pl} / m_o = 0.55$: $m_p = 0.469 m_{pl}$; $m_{pp} = 0.746 m_p$

**“TANKAGE”:** with 10% tankage penalty (i.e., for all other engines):

- $m_{pl} / m_o = 0.35$: $m_p = 1.078 m_{pl}$; $m_{pp} = 0.723 m_p$
- $m_{pl} / m_o = 0.45$: $m_p = 0.679 m_{pl}$; $m_{pp} = 0.801 m_p$
- $m_{pl} / m_o = 0.55$: $m_p = 0.432 m_{pl}$; $m_{pp} = 0.893 m_p$

For intermediate payload ratios, the following equations must be used...

(a) $m_{pl} / m_o = \text{desired ratio.}$
(b) $m_{pp} = \left(\frac{v}{v_c}\right)^2 m_p.$
(c) $m_o = m_p + m_{pp} + m_{pl}.$
(d) $m_{pl}$ is given.
8. Determine the total mass of the assembled vehicle (m₀) and verify the payload mass ratio.

\[ m_0 = m_{p} + m_{pp} + m_{pl}. \]

mass ratio = \( m_{pl} / m_{o} \).

9. Determine the required thrust based on the values for specific impulse, Iₛ, and propellant mass, mₚ. Determine the required power to be delivered to the thruster.

\[ F = \left( \frac{m_p}{t_p} \right) I_s g_0. \]
\[ P_e = \alpha \ m_{pp}. \]

Note: This calculated value of \( P_e \) should not greatly exceed the current “state of the art” for power generation (fuel cells, battery, or solar cells) of approximately 20 kW. If the power requirements cannot be met, return to step 4.

10. From the thrust calculation and the engine thrust characteristics (step 4), calculate the number of engines required to provide that thrust for the assigned mission.

11. Design check.

Considering component redundancy, is this design acceptable? That is, with current technology, is this a reasonable number of required engines? If not, return to design step 4 or attempt an intermediate payload ratio (step 2).

If yes, it may be advisable to iterate in order to account for redundancy thereby improving the \( \alpha \) parameter. In other words, are there any components that the thrusters may be able to share, thereby reducing \( m_{pp} \) and improving \( \alpha \). This will improve overall performance by reducing mass and possibly reducing the number of engines required for the mission.
Figure 9: Comprehensive Plot of $\Delta u / v_c$ and $v / v_c$ for Ideal Systems.

This plot serves as a baseline plot of the values of $\Delta u / v_c$ and $v / v_c$ for all anticipated payload mass ratios between the optimum selection points of $m_{pl} / m_o = 0.35$ and $0.55$. The points roughly bracket the optimum point at $m_{pl} / m_o = 0.45$.

This plot is usable only for ideal estimates of the calculations discussed in the design calculation, and it is usable for systems that do not require fuel tanks – i.e. no “tankage penalty.” Such electric thrusters or engines include the pulsed MPD thrusters: Teflon PPT, LES 8/9 PPT, NASA Primex EO-1, and the Primex PRS-101.
This plot is acceptable for use with all electric propulsion thrusters that require fuel storage tanks. Therefore, it may be used for "tankage penalty" calculations for all of the engines listed except the pulsed MPD type.

Because of the fact that these plots take fuel tank considerations into account, this chart (or at least the determined values for the defined payload rations) should be consulted.
Figure 11: FLOWCHART -- Optimization Procedure for Electric Propulsion Engines
\[ m_0 = 1.152 \text{ m}, 1.078 \text{ m} \]
\[ m_w = 0.612 \text{ m}, 0.723 \text{ m} \]

\[ m_0 = m_0 + m_w + m_p \]
mass ratio = \( m_p / m_0 \)

\[ F = (m_v / t_v) l \cdot g \]
\[ P_v = \propto m_p \]

- yes

\[ P_v < 20 \text{ kW} \]

- no

- yes

\# engines = \( F / T \)

- no

- yes

Acceptable?

- no

redundancy?
reduce \( m_p \)
improve \( \propto \)
iterate

- yes

redundancy?
reduce \( m_p \)
improve \( \propto \)
iterate

\[ m_0 = 0.731 \text{ m}, 0.679 \text{ m} \]
\[ m_w = 0.572 \text{ m}, 0.801 \text{ m} \]

\[ m_0 = m_0 + m_w + m_p \]
mass ratio = \( m_p / m_0 \)

\[ F = (m_v / t_v) l \cdot g \]
\[ P_v = \propto m_p \]

- yes

\[ P_v < 20 \text{ kW} \]

- no

\[ m_0 = 0.469 \text{ m}, 0.432 \text{ m} \]
\[ m_w = 0.746 \text{ m}, 0.893 \text{ m} \]

\[ m_0 = m_0 + m_w + m_p \]
mass ratio = \( m_p / m_0 \)

\[ F = (m_v / t_v) l \cdot g \]
\[ P_v = \propto m_p \]

- yes

\[ P_v < 20 \text{ kW} \]

- no

\# engines = \( F / T \)

- no

- yes

Acceptable?

- no

redundancy?
reduce \( m_p \)
improve \( \propto \)
iterate

- yes

redundancy?
reduce \( m_p \)
improve \( \propto \)
iterate
V. APPLICATION OF METHOD

Note: In the following three examples, the defined parameters which had to be estimated by the author are preceded by the symbol "~".

A. "LIGHT" LEO TO GEO MISSION

For a particular LEO to GEO satellite, the following parameters are defined:

mission velocity requirement, $\Delta u = 4200$ m/s

payload mass, $m_{pl} = 250$ kg

mission time, $t_m = $ unrestricted / reasonable.

Using the defined optimization procedure, three potentially optimized engine runs are attempted at payload mass ratios of $m_{pl} / m_o = 0.35, 0.45, \text{ and } 0.55$.

In this example, the second ("tankage" considered) case for the payload mass ratio of 0.45, proves to be the optimized profile. Note that approximately 10 to 12 engines are required if the H$_2$ arcjet is selected as the thruster for this profile.

This example demonstrates that fuel tank mass considerations lead to the addition of more thrusters in order to compensate for the added mass. The mass penalty is easily scalable.
IDEAL:

\[
\frac{m_p}{m_o} = 0.35
\]

\[I_s = 829 \text{ sec}\]

engine selected:

NH₃ Arcjet

\[
\begin{align*}
I_s &= 650 \text{ sec} \\
\alpha &= 320 \text{ W/kg} \\
\eta_t &= 0.32 \\
T &= 0.2 \text{ N}
\end{align*}
\]

\[t_p = 5.28 \times 10^5 \text{ sec} \\
= 6.11 \text{ days}
\]

\[t_p < t_m
\]

\[
\begin{align*}
m_p &= 270 \text{ kg} \\
m_{pp} &= 195.2 \text{ kg} \\
m_o &= 715 \text{ kg}
\end{align*}
\]

payload ratio checks

\[F = 3.26 \text{ N}; \ P_e = 63 \text{ kW}
\]

\[\therefore 17 \text{ engines required; power req. excessive}
\]

with TANKAGE...

\[I_s = 972 \text{ sec}\]

engine selected:

H₂ Arcjet

\[
\begin{align*}
I_s &= 1000 \text{ sec} \\
\alpha &= 333 \\
\eta_t &= 0.4 \\
T &= 0.2 - 0.25 \text{ N}
\end{align*}
\]

\[t_p = 4.71 \times 10^5 \text{ sec} \\
= 5.45 \text{ days}
\]

\[t_p < t_m
\]

\[
\begin{align*}
m_p &= 270 \text{ kg} \\
m_{pp} &= 195.2 \text{ kg} \\
m_o &= 715 \text{ kg}
\end{align*}
\]

payload ratio checks

\[F = 5.47 \text{ N}; \ P_e = 65 \text{ kW}
\]

\[\therefore 22 \text{ to 28 engines required}
\]

\[m_p / m_o = 0.45
\]

\[I_s = 1075 \text{ sec}\]

engine selected:

NASA / Primex EO-1

\[
\begin{align*}
I_s &= 1150 \text{ sec} \\
\alpha &= 20 \text{ W/kg} \\
\eta_t &= 0.098 \\
T &= \text{pulsed}
\end{align*}
\]

\[t_p = 4.21 \times 10^7 \text{ sec} \\
= 487 \text{ days}
\]

\[t_p < t_m
\]

\[
\begin{align*}
m_p &= 182.8 \text{ kg} \\
m_{pp} &= 122.8 \text{ kg} \\
m_o &= 556 \text{ kg}
\end{align*}
\]

payload ratio checks

\[F = 0.0490 \text{ N}; \ P_e = 3 \text{ kW}
\]

\[\therefore 1,919 \text{ engines required!!!}
\]

\[m_p / m_o = 0.55
\]

\[I_s = 1439 \text{ sec}\]

engine selected:

ARC SPT-100

\[
\begin{align*}
I_s &= 1600 \text{ sec} \\
\alpha &= \sim 169.8 \text{ W/kg} \\
\eta_t &= 0.48 \\
T &= 0.083 \text{ N}
\end{align*}
\]

\[t_p = 1.638 \times 10^6 \text{ sec} \\
= 19.0 \text{ days}
\]

\[t_p < t_m
\]

\[
\begin{align*}
m_p &= 117.3 \text{ kg} \\
m_{pp} &= 87.5 \text{ kg} \\
m_o &= 455 \text{ kg}
\end{align*}
\]

payload ratio checks

\[F = 1.011 \text{ N}; \ P_e = 15 \text{ kW}
\]

\[\therefore 13 \text{ engines required}
\]

\[I_s = 1687 \text{ sec}\]

engine selected:

Hall (Xe)

\[
\begin{align*}
I_s &= 15-1600 \text{ sec} \\
\alpha &= 150 \\
\eta_t &= 0.5 \\
T &= 0.04 \text{ N}
\end{align*}
\]

\[t_p = 2.04 \times 10^6 \text{ sec} \\
= 23.6 \text{ days}
\]

\[t_p < t_m
\]

\[
\begin{align*}
m_p &= 108.0 \text{ kg} \\
m_{pp} &= 96.4 \text{ kg} \\
m_o &= 454 \text{ kg}
\end{align*}
\]

payload ratio checks

\[F = 0.831 \text{ N}; \ P_e = 15 \text{ kW}
\]

\[\therefore 21 \text{ engines required}
\]
B. "HEAVY" LEO TO GEO (COMM SAT) MISSION

Given a LEO to GEO mission for the orbit raising of a communication satellite, the optimization algorithm requires that we initially define the payload and required \( \Delta u \) for the transit.

The payload is defined to be relatively massive, weighing 1000 kg, and the defined \( \Delta u \) for the transfer is approximately 4640 m/s. For this mission, the transfer time cannot be unrealistically excessive due to the general mission urgency of communication satellite tasking. Using the planning algorithm, the results are illustrated as shown on the following page.

This example provides some interesting results. First, the payload is extremely heavy for most electrical propulsion engines. As stated in the earlier chapters, most high thrust electric propulsion engines are still in development. Yet, the Hall TAL D-55 engine is capable of accomplishing this mission in 88½ days with only 11 engines – excluding redundancy.

In both of the cases at the payload mass ratio of 0.45, it is evident that the increased burn time to maintain the profile allows for fewer engines as less thrust is required over a longer time. None-the-less, the number of engines required for this profile as \( m_{pl} / m_o = 0.45 \) is unacceptable.
IDEAL:

\[ m_{pl} / m_0 = 0.35 \]

\[ I_s = 916 \text{ sec} \]

engine selected:

NASA/Primex EO-1

\[ I_s = 1150 \text{ sec} \]
\[ \alpha = \sim 20 \text{ W/kg} \]
\[ \eta_t = 0.098 \]
\[ T = \text{pulsed} \]

\[ t_p = 3.37 \times 10^7 \text{ sec} \]
\[ = 389 \text{ days} \]
\[ = 1.067 \text{ years} \]

SHOWSTOPPER...

due \( t_p \gg t_m \)

with TANKAGE...

\[ I_s = 1074 \text{ sec} \]

engine selected:

H2 Arcjet

\[ I_s = 1000 \text{ sec} \]
\[ \alpha = 333 \text{ W/kg} \]
\[ \eta_t = 0.4 \]
\[ T = 0.2 - 0.25 \text{ N} \]

\[ t_p = 5.75 \times 10^5 \text{ sec} \]
\[ = 6.65 \text{ days} \]
\[ t_p < t_m \]

\[ m_p = 1078 \text{ kg} \]
\[ m_{pp} = 779 \text{ kg} \]
\[ m_0 = 2857 \text{ kg} \]

payload ratio checks

\[ F = 18.39 \text{ N}; \ P_e = 259 \text{ kW} \]
\[ : .74 \text{ to } 92 \text{ engines needed!!!} \]

\[ I_s = 1187 \text{ sec} \]

engine selected:

Hall (Xe)

\[ I_s = 15 -1600 \text{ sec} \]
\[ \alpha = 150 \text{ W/kg} \]
\[ \eta_t = 0.5 \]
\[ T = 0.04 \text{ N} \]

\[ t_p = 1.342 \times 10^6 \text{ sec} \]
\[ = 15.54 \text{ days} \]
\[ t_p < t_m \]

\[ m_p = 731 \text{ kg} \]
\[ m_{pp} = 491 \text{ kg} \]
\[ m_0 = 2222 \text{ kg} \]

payload ratio checks

\[ F = 8.02 \text{ N}; \ P_e = 74 \text{ kW} \]
\[ : .201 \text{ engines required!} \]

\[ I_s = 1589 \text{ sec} \]

engine selected:

Hall TAL D-55

\[ I_s = 1600 \text{ sec} \]
\[ \alpha = \sim 50.9 \text{ W/kg} \]
\[ \eta_t = 0.48 \]
\[ T = 0.082 \text{ N} \]

\[ t_p = 6.67 \times 10^6 \text{ sec} \]
\[ = 77.1 \text{ days} \]
\[ t_p < t_m \]

\[ m_p = 469 \text{ kg} \]
\[ m_{pp} = 350 \text{ kg} \]
\[ m_0 = 1819 \text{ kg} \]

payload ratio checks

\[ F = 1.104 \text{ N}; \ P_e = 18 \text{ kW} \]
\[ : .16 \text{ engines required} \]

\[ I_s = 1381 \text{ sec} \]

engine selected:

Hall (Xe)

\[ I_s = 15 -1600 \text{ sec} \]
\[ \alpha = 150 \text{ W/kg} \]
\[ \eta_t = 0.5 \]
\[ T = 0.04 \text{ N} \]

\[ t_p = 1.533 \times 10^6 \text{ sec} \]
\[ = 17.74 \text{ days} \]
\[ t_p < t_m \]

\[ m_p = 679 \text{ kg} \]
\[ m_{pp} = 544 \text{ kg} \]
\[ m_0 = 2223 \text{ kg} \]

payload ratio checks

\[ F = 6.52 \text{ N}; \ P_e = 82 \text{ kW} \]
\[ : .163 \text{ engines required!} \]

\[ I_s = 1864 \text{ sec} \]

engine selected:

Hall TAL D-55

\[ I_s = 1600 \text{ sec} \]
\[ \alpha = \sim 50.9 \text{ W/kg} \]
\[ \eta_t = 0.48 \]
\[ T = 0.082 \text{ N} \]

\[ t_p = 7.65 \times 10^6 \text{ sec} \]
\[ = 88.5 \text{ days} \]
\[ t_p < t_m \]

\[ m_p = 432 \text{ kg} \]
\[ m_{pp} = 386 \text{ kg} \]
\[ m_0 = 1818 \text{ kg} \]

payload ratio checks

\[ F = 0.886 \text{ N}; \ P_e = 20 \text{ kW} \]
\[ : .11 \text{ engines required} \]
C. LEO TO MARS MISSION

For a representative LEO to Mars mission, the optimization algorithm requires that we initially define the payload and required $\Delta u$.

The payload is defined to be a relatively small probe (mass equal to 250 kg), and the defined $\Delta u$ for the transfer is approximately 5,700 m/s. In this case, $\Delta u$ also defines a mission time ($t_m$) equal to 256 days. Thus, in order for the calculations to be valid, the burn time ($t_b$) must not exceed $t_m$.

The results of the calculations are demonstrated as shown.

Note that for the third case of $m_{pl} / m_o = 0.55$ either of the steady MPD thrusters would have been a better specific impulse fit. Those engines are scaleable in the range from $I_s = 2000$ to 5000 sec. However, since they are presently still under development, they have been excluded from this analysis even though it is strongly anticipated that such engines will be the desired propulsion systems for deep-space missions.
### Ideal:

<table>
<thead>
<tr>
<th>( m_p / m_o )</th>
<th>( I_s )</th>
<th>Engine Selected</th>
<th>( t_p )</th>
<th>( t_P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>1125 sec</td>
<td>Teflon PPT (pulsed)</td>
<td>1.422 x 10^9 sec</td>
<td>16,457 days</td>
</tr>
<tr>
<td>0.45</td>
<td>1458 sec</td>
<td>Hall SPT (Xe)</td>
<td>2.03 x 10^6 sec</td>
<td>23.4 days</td>
</tr>
<tr>
<td>0.55</td>
<td>1952 sec</td>
<td>SPT-100 (Fakel)</td>
<td>3.42 x 10^6 sec</td>
<td>39.5 days</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( I_s = 1125 ) sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine selected:</td>
</tr>
<tr>
<td>Teflon PPT (pulsed)</td>
</tr>
<tr>
<td>( \alpha = 1.0 ) W/kg</td>
</tr>
<tr>
<td>( \eta_t = 0.07 )</td>
</tr>
<tr>
<td>( T = \text{pulsed} )</td>
</tr>
<tr>
<td>( t_p = 1.422 \times 10^9 ) sec</td>
</tr>
<tr>
<td>( t_P = 16,457 ) days</td>
</tr>
<tr>
<td>( t_p &lt; t_m )</td>
</tr>
</tbody>
</table>

### Showstopper...

Due \( t_P \gg t_m \)

<table>
<thead>
<tr>
<th>( I_s = 1319 ) sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine selected:</td>
</tr>
<tr>
<td>Hall (Xe)</td>
</tr>
<tr>
<td>( I_s = 15-1600 ) sec</td>
</tr>
<tr>
<td>( \alpha = 150 ) W/kg</td>
</tr>
<tr>
<td>( \eta_t = 0.5 )</td>
</tr>
<tr>
<td>( T = 0.04 ) N</td>
</tr>
<tr>
<td>( t_p = 1.540 \times 10^6 ) sec</td>
</tr>
<tr>
<td>( t_P &lt; t_m )</td>
</tr>
</tbody>
</table>

### With tankage...

<table>
<thead>
<tr>
<th>( I_s = 1697 ) sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine selected:</td>
</tr>
<tr>
<td>ARC SPT-100</td>
</tr>
<tr>
<td>( I_s = 1600 ) sec</td>
</tr>
<tr>
<td>( \alpha = 131.4 ) W/kg</td>
</tr>
<tr>
<td>( \eta_t = 0.48 )</td>
</tr>
<tr>
<td>( T = 0.083 ) N</td>
</tr>
<tr>
<td>( t_p = 2.75 \times 10^6 ) sec</td>
</tr>
<tr>
<td>( t_P &lt; t_m )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( I_s = 2289 ) sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine selected:</td>
</tr>
<tr>
<td>XIPS</td>
</tr>
<tr>
<td>( I_s = 28-3500 ) sec</td>
</tr>
<tr>
<td>( \alpha = 100 ) W/kg</td>
</tr>
<tr>
<td>( \eta_t = 0.75 )</td>
</tr>
<tr>
<td>( T = 0.015-0.04 ) N</td>
</tr>
<tr>
<td>( t_p = 3.76 \times 10^6 ) sec</td>
</tr>
<tr>
<td>( t_P &lt; t_m )</td>
</tr>
</tbody>
</table>

### Payload ratio checks

- \( F = 2.58 \) N; \( P_e = 30 \) kW
  - .65 engines required
  - Power required is high
- \( F = 0.969 \) N; \( P_e = 18 \) kW
  - .12 engines required
- \( F = 0.538 \) N; \( P_e = 14 \) kW
  - .7 engines required
- \( F = 0.789 \) N; \( P_e = 10 \) kW
  - .20 to 53 engines required
VI. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

Routine implementation of electric propulsion for space thrusting applications is relatively new even though the governing concepts are almost 40 years old. The very latest engines, presented as examples in this thesis, demonstrate that some aspects of electric propulsion are still in their early stages of development. However, in light of time-tested theory and the limitations of chemical propulsion systems, it is evident that, right now, electric propulsion is the only potential stepping-stone to deep-space probes or other craft that could conceivably travel the great distances between the planets and beyond the solar system.

For any such mission profiles – interplanetary or interstellar in particular – the propulsion systems for these missions will have to be optimized. Such engines will have to be cost-effective, thrust efficient and therefore mass minimized. Someday there may even be considerations to make these systems potentially re-useable and refuelable. This thesis presents one technique with which mission profiles and engine selection can be matched and consequently optimized to theoretically achieve the greatest benefit of least overall mass with both maximum fuel and maximum payload in order to achieve the shortest mission time for a given payload.

Thus, a primary focus of this thesis has been to provide a new way of looking at electrical propulsion systems design in addition to attempting to provide an up-to-date and comprehensive summary of available electric propulsion thruster data. The selected approach (i.e., the Langmuir-Irving formulation) is based upon first principles as well as an analysis of other methods that are currently in use. In essence, an objective has been
to produce a design alternative to the satellite design engineers. This methodology or "procedure" is intended to generate some consideration into performance near a pre-determined yet not inflexible "dual-optimum" point. It is apparent in the literature that aspects of the discussed "dual-optimum" point have been plotted in various texts and somewhat addressed but never fully explored.

The most important concept to note is the idea that these results are "theoretical" in that systems are currently designed according to algorithms that are based more solidly on operational experience and power considerations than on any approach that considers pure optimization. In essence, no design algorithm is really in use that seems to consider optimization. Granted, electric propulsion is still in its early stages and most applications involving satellite re-positioning and orbit-lifting do not place heavy lift nor aggressive profile constraints upon engines. Consequently, such considerations or additional restraints may not really be required.

Due to the relatively limited number of electrical propulsion engines that are currently operational, the examples of this thesis show that it is sometimes difficult to find an identical match to an optimum profile's specific impulse, $I_s$. In fact, some of the most powerful and most promising electric propulsion engines – those most likely to be desirable for high thrust and high payload mass missions – are still in some stages of development.

Specific impulse, $I_s$, is comparable to the required "gas mileage" for a particular trip through space. After the profile is determined through orbital mechanics, the required incremental change in vehicular velocity ($\Delta u$) is known. Then, the required tank of fuel can be established for an expected level of performance or fuel efficiency – "gas
mileage.” Hence, the objective is to match the profile to specific impulse (or engine). This is accomplished using the “dual-optimum” procedure. Nonetheless, the “dual-optimum” establishes not only $I_s$ but also suggests desired values of specific power ($\alpha$) and thruster efficiency ($\eta_t$) as well as the likely burn time ($t_p$) for a desired mission profile, $\Delta u$.

As demonstrated in the three provided examples in the previous chapter, it is not obvious how a particular engine will perform on a specific mission profile. In essence, the design optimum is dependent as much on the payload mass ($m_{pl}$) and mass payload ratio ($m_{pl} / m_o$) as upon the specific characteristic of the engine itself.

The most important of these engine characteristics are specific power ($\alpha$), thruster efficiency ($\eta_t$), and available thrust ($T$). Specific power, $\alpha$, is the most critical to the designer because it drives the amount of available power to the thruster versus the required mass of the overall system that provides that power. As power conditioning systems improve and the “state-of-the-art” in the field advances, this parameter will improve and enhance electrical propulsion system performance. In this thesis, it is evident that specific power has been the most difficult parameter to establish for the majority of the listed engines. A worked example demonstrates that this is primarily due to the fact that the value given to $\alpha$ varies depending somewhat upon arbitrary considerations such as thruster configuration, redundancy, etc. In addition, it shows that mass reduction is equally as important as where that mass is “billed” within the calculations.

Thruster efficiency, $\eta_t$, generates a feel for the engine’s efficiency in terms of nozzle losses, off-axis losses, as well as other thruster inefficiencies. Generally, these
values vary widely depending upon the type of thruster under consideration. For example, the LES 8/9 PPT thruster has an efficiency of only 0.0068 to 0.009 while the N₂H₄ resistojet turns in numbers in the 0.8 to 0.9 range. The reasons for this are discussed in depth.

Available thrust, \( T \), although extremely small on most electrical propulsion systems, gives a measure of the number of engines required for a necessary profile. This value can be a major player as demonstrated in the examples because it drives largely the additional issues of component redundancy and additional mass – which again affects alpha, \( \alpha \).

Another consideration which became apparent in the course of this work is the capability to allow for the required fuel tanks within the design and thus within the optimum profile. Fuel considerations are certainly mission or profile dependent. Consequently, this is a substantial capability for the designer to predict optimum performance with variable tank mass as per mission requirements.

These results, as presented, are easily scalable depending upon the particular profile and its fuel requirements. Exploration of the topic of allowing for “tankage” created estimation possibilities that allowed for even more realistic comparisons between different electric propulsion systems. As evident in the overall engine parameter summary in Table 5, some engines (of the pulsed MPD variety) require no fuel storage tanks at all.

If the engine summary shows one thing, it certainly lets one know that there are a host of possibilities out there being developed by aggressive companies. These systems use a variety of physical principles to create thrust. Some at greater expense of the other
relevant parameters — $\alpha$, $\eta$, and $T$. Consequently, the examples provided for the design algorithm (on the three different mission profiles) demonstrate that all engines are not created equal for various mission profiles — certainly not if one endeavors to fly an optimum profile.

Of course, industry may argue that there are important additional considerations on any profile. For example, a communication satellite that takes six months to be in position is probably not cost-effective. That could be a mission failure. Nonetheless, success depends on what parameter is being optimized. In the case of this research, the objective was to optimize both the mass and time constraints. At the present time, in the development of the "state-of-the-art" in the evolution of electrical propulsion, this was not always possible. This fact is certainly demonstrated in the examples provided, which show that there are no obvious trends with the systems currently available. Electric propulsion is not totally there yet; however, scientists and engineers are pushing the frontier forward and expanding to new horizons and possibilities.

B. RECOMMENDATIONS

In any branch of research involving a rapidly growing technology (particularly one in which corporate vendors on the "cutting-edge" of that technology are especially unwilling to give up data that they consider proprietary), it is extremely difficult to track the leading edge of developments. Consequently, acquiring the latest available data can be next to impossible. Hence, best-estimate scenarios can be flawed or only approximate.

In the course of this research, the author frequently ran into barriers when attempting to answer questions regarding particular thruster systems. A more open forum could certainly benefit not only science but also the vendors themselves, particularly if
the data contained within this thesis proves useful to designers. On the other hand, the
developed procedure might demonstrate that certain engines are less effective for
necessary mission profiles. As a result, one design might prove less effective than
another.

Additional work could be extremely advantageous if a more in-depth study could
be made into attempting to combine the various schools of thought in design criteria.
Like missile design, there is no clear-cut or established method that is followed due to the
fact that a design must consider the whole picture – i.e., mass, time, and power
considerations – all at once. Considerations of power, time, and cost seem to be the
biggest factors in design at the present time, and obviously contractors wanting to sell
their products are not going to be quick to admit that a system could be better optimized
with another company’s thruster. Certainly, perfection is infinitely expensive, but that is
not what this process advocates. Systems could simply be better if any attempt were
made to “ride the dual-optimum profile.” The author would certainly be curious if
industry were to consider the presented ideas and improve upon them.
APPENDIX A. DEVELOPMENT - PAYLOAD MASS FRACTION EQUATION:

By definition, specific power, $\alpha = P_e / m_{pp}$. Thus, $m_{pp} = P_e / \alpha$. (Sutton, 1992, pg. 597)

By definition, electrical power available (as previously shown), $P_e = m_p v^2 / (2 t_p \eta t)$. Hence, $m_{pp} = (1/\alpha) (m_p v^2 / (2 t_p \eta t))$. Solving for payload mass, $m_p$, yields...

$m_p = m_{pp} (2 \alpha t_p \eta t / v^2)$ or $m_{pp} / m_p = v^2 / 2 \alpha t_p \eta t$. (Sutton, 1992, pg. 596)

By definition of allocated spacecraft masses, $m_o = m_p + m_{pl} + m_{pp}$. Define final mass, $m_f = m_o - m_p$ (i.e. initial mass less propellant mass). Thus, redefine initial mass as follows: $m_o = m_p + m_f$. (Sutton, 1992, pg. 596)

Derived from the equation for maximum velocity at propellant burnout,

$\Delta u = v \ln (m_o / m_i)$, know that $e^{\Delta u/v} = m_o / m_i = m_p / (m_o - m_p) = 1 / (1 - m_p / m_o)$. This equation, when solved for $m_p / m_o$, gives $m_p / m_o = 1 - e^{\Delta u/v}$. (Sutton, 1992, pg. 123)

Rewriting the equation for initial mass yields $m_o = m_p (1 + m_{pp} / m_p) + m_{pl}$. Now, when substituting the above equation for $m_{pp} / m_p$, the initial mass is rewritten as follows: $m_o = m_p (1 + v^2 / 2 \alpha t_p \eta t) + m_{pl}$.

Again, rewriting the equation... $1 = (m_p / m_o) (1 + v^2 / 2 \alpha t_p \eta t) + m_{pl} / m_o$. Substituting the equation for $m_p / m_o$ and solving for $m_{pl} / m_o$, the equation transforms to $m_{pl} / m_o = 1 - (1 - e^{\Delta u/v}) (1 + v^2 / 2 \alpha t_p \eta t)$.

When this equation is multiplied through and solved for $m_{pl}/m_o$, the payload mass fraction appears as illustrated: $m_{pl} / m_o = (1 - (v / v_o)^2 (e^{\Delta u/v} - 1)) / e^{\Delta u/v}$. 59
APPENDIX B. FIRST PRINCIPLES – CODES

The following three pages contain the MATLAB codes that were utilized in the
generation of Figures 2, 3, and 4 respectively.
% THESIS
% Plot2(2): mpl / mo vs. v / vc
% "payload fraction vs. v / vc"

% LT John Jay De Bellis, USN
% 30 JUN 99

% variables:
% x() = v / vc
% a() = deltau / vc
% y() = mpl / mo

x1=linspace(0,3,300)
x2=linspace(0,3,300)
x3=linspace(0,3,300)
a1=0.2
a2=0.3
a3=0.5
y1=(1-x1.^2.* (exp(a1./x1)-1))./exp(a1./x1)
y2=(1-x2.^2.* (exp(a2./x2)-1))./exp(a2./x2)
y3=(1-x3.^2.* (exp(a3./x3)-1))./exp(a3./x3)

figure(1), plot(x1,y1,x2,y2,x3,y3), grid
axis([0 3 0.0 1.0])
ylabel('mpl / mo'),xlabel('v / vc')
gtext('deltau/vc = 0.2'), gtext('deltau/vc = 0.3'), gtext('deltau/vc = 0.5')
% THESIS
% Plot1:  deltau / vc vs. v / vc (or Is / vc)
%      "incremental change of vehicle velocity vs. impulse"

% LT John Jay De Bellis, USN
% 25 AUG 99

% Variables:
% x() = v / vc
% a() = mpl / mo
% y() = deltau / vc

xl=linspace(0,3,300)
x2=linspace(0,3,300)
x3=linspace(0,3,300)

a1=0.35
a2=0.45
a3=0.55

y1=xl.*log((1 + xl.^2)./(a1 + xl.^2))
y2=x2.*log((1 + x2.^2)./(a2 + x2.^2))
y3=x3.*log((1 + x3.^2)./(a3 + x3.^2))

figure(1), plot(xl,y1,x2,y2,x3,y3), grid
axis([0.0 2.0 0.0 0.5])
ylabel('deltau / vc'),xlabel('v / vc')
gtext('mpl/mo = 0.35'), gtext('mpl/mo = 0.45'), gtext('mpl/mo = 0.55')
"THESIS"
% Plot3: optimization curves -- joint optimum
% "product of incremental change of vehicle velocity and payload ratio vs.
% specific impulse" (mpl/mo * deltau vs. Is)
% LT John Jay De Bellis, USN
% 25 AUG 99

Variables:
% x() = v / vc
% a() = mpl / mo
% y() = deltau / vc

x1=linspace(0,3,300)
x2=linspace(0,3,300)
x3=linspace(0,3,300)
a1=0.35
a2=0.45
a3=0.55

y1=x1.*log((1 + x1.^2)./(a1 + x1.^2))
y2=x2.*log((1 + x2.^2)./(a2 + x2.^2))
y3=x3.*log((1 + x3.^2)./(a3 + x3.^2))

%zl=y1.*a1
%z2=y2.*a2
%z3=y3.*a3

figure(1), plot(x1,y1,x2,y2,x3,y3), grid
axis([0.1 0.15 0.5 1.5])
ylabel('deltau/vc * mpl/mo'), xlabel('v/vc')
gtext('mpl/mo = 0.35'), gtext('mpl/mo = 0.45'), gtext('mpl/mo = 0.55')
APPENDIX C. DUAL OPTIMUM CODE

This appendix lists the necessary Maple program code that is utilized in the production of Figure 4, which illustrates the "dual optimum."
THESIS: Plot #5: OPT. CURVES: 3-D plot: optimization of $v/v_c$ and $mpl/mo$ curves

LT John Jay De Bellis, USN
30 JUN 1999

defined variables:
- $\Delta u =$ defined incremental change of vehicle velocity (m/s) -- defined by mission
- $v_c =$ characteristic velocity (m/s)
- $v =$ effective exhaust velocity (m/s)
- $m_{ratio} =$ payload ratio = $mpl/mo$

$x = v/v_c$
$y = mpl/mo$

> restart; with(plots); with(plottools);
plot3d($x*y*ln((1+x^2)/(y+x^2))$, $x=0.5..1.3$, $y=0.3..0.6$, axes=BOXED, title="optimization: product of $v/v_c$ and $mpl/mo$");
APPENDIX D. TANKAGE PENALTY CODES

The following pages list the MAPLE program codes that were used in the creation of Figures 6 and 7 respectively.
% THESIS
% PlotLT: deltau / vc vs. v / vc (or Is / vc) with 10% TANKAGE PENALTY
% "incremental change of vehicle velocity vs. impulse"

% LT John Jay De Bellis, USN
% 25 AUG 99

% Variables:
% x() = v / vc
% a() = mpl / mo
% y() = deltau / vc

x1=linspace(0,3,300)
x2=linspace(0,3,300)
x3=linspace(0,3,300)
a1=0.35
a2=0.45
a3=0.55

y1=x1.*log((1.1 + x1.^2)./(a1 + x1.^2 + 0.1))
y2=x2.*log((1.1 + x2.^2)./(a2 + x2.^2 + 0.1))
y3=x3.*log((1.1 + x3.^2)./(a3 + x3.^2 + 0.1))

figure(1), plot(x1,y1,x2,y2,x3,y3), grid
axis([0.0 2.0 0.0 0.5])
ylabel('deltau / vc'),xlabel('v / vc')
gtext('mpl/mo = 0.35'), gtext('mpl/mo = 0.45'), gtext('mpl/mo = 0.55')
% THESIS
% Plot1T: deltau / vc vs. v / vc (or Is / vc) with/without 10% TANKAGE PENALTY
%    "incremental change of vehicle velocity vs. impulse"

% LT John Jay De Bellis, USN
% 25 AUG 99

% Variables:
%    x() = v / vc
%    a() = mpl / mo
%    y() = deltau / vc

x1=linspace(0,3,300)
x2=linspace(0,3,300)
x3=linspace(0,3,300)
a=0.45

y1=x1.*log((1.1 + x1.^2)./(a + x1.^2 + 0.1))
y2=x2.*log((1 + x2.^2)./(a + x2.^2))

figure(1), plot(x1,y1,x2,y2), grid
axis([0.0 2.0 0.0 0.5])
ylabel('deltau / vc'),xlabel('v / vc')
gtext('mpl/mo = 0.45'), gtext('mpl/mo = 0.45 with 10% tankage penalty')
LIST OF REFERENCES


Electronic correspondence between Mr. Craig Clauss (clauss@arceng.com), Atlantic Research Corporation, and the author, 19 July 1999.


Osborn, Mike, Naval Research Labs, 14 June 1999.
Electronic correspondence between Mr. Mike Osborn (mosborn@space.nrl.navy.mil), Naval Research Labs and the author, 14 June 1999.


BIBLIOGRAPHY


THIS PAGE INTENTIONALLY LEFT BLANK.