STUDY OF EXCEEDANCE CURVE SPREAD BY AIRCRAFT TAIL NUMBER BY MISSION TYPE & BASE FOR F-4 AND A-37B FLIGHT DATA

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FOREWARD

This report was prepared by Mr. David L. Banaszak of the Structural Integrity Branch, Structures Division, Air Force Flight Dynamics Laboratory. This effort was performed under Project 1367, "Structural Integrity for Military Aerospace Vehicles," Task 136701, "Structural Flight Loads."

This exercise was initiated largely in response to determinations reached in a series of meetings with personnel in ASD/ENF; namely Messrs. W. J. Crichlow, C. W. Luchsinger, and Troy King. These meetings were convened to discuss methods and techniques for analyzing and utilizing flight loads data in modifying and updating MIL-A-8866B specification.

Appreciation is extended to Messrs. W. J. Crichlow, C. W. Luchsinger, and Troy King for their guidance and suggestions in these data analyses.

This technical memorandum report has been reviewed and is approved.

DENNIS J. GOLDEN, Major, USAF
Chief, Structural Integrity Branch
Structures Division
AF Flight Dynamics Laboratory
SUMMARY

This report presents the results of the analysis of exceedance curves by aircraft tail number for the F-4 and A-37B aircraft.

The aim of this report is to provide a statistical bound for the different exceedance curves that are obtained from various aircraft tail numbers on a given aircraft type. The F-4 and A-37B aircraft were selected only because load factor (nz) exceedance data by tail number were available in table formats in References 1 and 2.

Results of this report indicate that reasonable bounds on exceedance curves may be obtained by tolerance limits which assume that for a fixed load factor (nz), the log of the exceedances per thousand flight hours has a normal distribution. Also, tolerance limits could be obtained using nonparametric techniques when the quantity of tail numbers is sufficiently large.

Finally, curves were fitted to the composite data. The designer may extrapolate these curves to determine design limit load factor and change the shape of the curves by varying values of coefficients.
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Nonparametric Bounds for F-4 Air-Ground Exceedances

Nonparametric Bounds for F-4 Air-Air Exceedances

Nonparametric Bounds for F-4 Inst. & Nav. Exceedances

Nonparametric Bounds for F-4 Reconnaissance Exceedances

90% Tolerance Limits with 90% Confidence for A-37B at England AFB (1969)

90% Tolerance Limits with 90% Confidence for A-37B at Bien Hoa AB (1970)

90% Tolerance Limits with 90% Confidence for A-37B at Binh Thuy AB (1971)

90% Tolerance Limits with 90% Confidence for F-4 Air-Ground

90% Tolerance Limits with 90% Confidence for F-4 Air-Air

90% Tolerance Limits with 90% Confidence for F-4 Inst. and Nav.

90% Tolerance Limits with 90% Confidence for F-4 Reconnaissance
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<tr>
<td>nz</td>
<td>vertical load factor in g's.</td>
<td></td>
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<tr>
<td>MED(nz)</td>
<td>the median value of exceedances per 1,000 hours for a fixed value of nz.</td>
<td></td>
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<tr>
<td>Ei(nz)</td>
<td>the number of exceedances per 1,000 hours for a fixed value of nz for the ith aircraft tail number.</td>
<td></td>
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<tr>
<td>LCI(nz)</td>
<td>the lower bound of the 90% confidence interval for MED(nz).</td>
<td></td>
</tr>
<tr>
<td>UCI(nz)</td>
<td>the upper bound of the 90% confidence interval for MED(nz).</td>
<td></td>
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<tr>
<td>C</td>
<td>the composite exceedance curve.</td>
<td></td>
</tr>
<tr>
<td>MAX(nz)</td>
<td>the maximum number of exceedances per 1,000 hours for a fixed value of nz.</td>
<td></td>
</tr>
<tr>
<td>MIN(nz)</td>
<td>the minimum number of exceedances per 1,000 hours for a fixed value of nz.</td>
<td></td>
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<tr>
<td>P</td>
<td>the percent of tail numbers that lie between 2 given limits.</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td>confidence, i.e. the probability that a given statement is true.</td>
<td></td>
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<tr>
<td>M(nz)</td>
<td>mean of exceedances per 1,000 hours for fixed nz.</td>
<td></td>
</tr>
<tr>
<td>s(nz)</td>
<td>standard deviation of exceedances per 1,000 hours for fixed nz.</td>
<td></td>
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<tr>
<td>NorUL(nz)</td>
<td>upper bound of 90% two-sided tolerance limit with 90% confidence assuming normal distribution.</td>
<td></td>
</tr>
<tr>
<td>NorLL(nz)</td>
<td>lower bound of 90% two-sided tolerance limit with 90% confidence assuming normal distribution.</td>
<td></td>
</tr>
<tr>
<td>M3=log₁₀(C(nz))</td>
<td>used as the mean for log normal assumptions.</td>
<td></td>
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DEFINITION OF SYMBOLS (cont'd)

\( s'(nz) \)  
standard deviation of the logs of the \( Ei(nz) \) for a fixed \( nz \).

LNUL  
Upper bound on log scale of 90% two-sided tolerance limits with 90% confidence of exceedances per 1,000 hours.

LNLL  
Lower bound on log scale of 90% two-sided tolerance limits with 90% confidence of exceedances per 1,000 hours.

LgNorUL  
10 (LNUL)

LgNorLL  
10 (LNLL)

\( F \)  
curve fitted to \( nz \) composite exceedance per 4,000 hours.

\( b(nz) \)  
best fit polynomial that \( \log F = b(nz) \).

\( M_w \)  
Weighted mean.
SECTION I
INTRODUCTION

As a result of a series of meetings with Mr. W. J. Crichlow, ASD/ENF, it appears that some changes are necessary in the presentation of flight loads data. This is expected to improve the development of more definitive structural design criteria. Mr. Crichlow, who has prime responsibility for MIL-A-8866B revision, explained the need for re-examining the statistical variations in vertical load factor \( n_z \) exceedance curves and this distribution's impact on more precise aircraft design, and fatigue, and fracture analyses. Accordingly, AFFDL/FBE initiated a program to analyze flight loads data, categorized by mission segment, type, and tail number as available, to determine the load's distribution mean exceedance curve and the statistical spread about the mean.

Load factor \( n_z \) data from A-37B (Reference 1) and F-4 (Reference 2) aircraft were segregated by tail number, base, and mission type as available in the reports. The data from these subdivisions were analyzed and the results plotted in the form of exceedance curves.

These methods provide a precise procedure for criteria development by computing exceedance curves with a mean and tolerance limits for all possible exceedance curves in the given category. The results appear promising in providing a more descriptive USAF military specification, which in turn, necessarily results in more refined structural design for future aircraft.
SECTION II
DISCUSSION

To achieve the objectives stated above, several methods of analyzing the flight loads data as extracted from References 1 and 2 were employed. For each set of data by aircraft tail numbers, the following statistical functions were computed for the vertical load factor ($n_z$) exceedance curves:

a. Maximum, minimum and median exceedance curves with a 90% confidence interval for the median.

b. 90% two-sided tolerance limits with 90% confidence assuming the exceedances are normally distributed.

c. 95% two-sided tolerance limits with 95% confidence assuming the log of the exceedances are normally distributed.

d. Least squares curve fits to the log of the composite data.

Plots of the above results are shown in Figures 1 through 36 in Appendix I of this report. Detailed discussions of the flight data, computer programs, and assumptions used to obtain the above results are presented in the following paragraphs of this report.

1. Flight Data

The load factor ($n_z$) peak data used for this study were obtained from References 1 and 2. Reference 1 contained operational flight loads data from the A-37B aircraft while performing training flights from May 1969 to September 1971. A total of 4,001 hours of data was analyzed and is summarized in Table I. For a detailed description of the data, the reader is referred to Reference 1.
### TABLE I
SUMMARY OF A-37B DATA USED

<table>
<thead>
<tr>
<th>Base and Year</th>
<th>Hours</th>
<th>Tail Numbers</th>
<th>Hours/Tail Number</th>
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<tr>
<td>England AFB (1969)</td>
<td>541.47</td>
<td>11</td>
<td>49.22</td>
</tr>
<tr>
<td>Bien Hoa AB (1970)</td>
<td>2038.23</td>
<td>12</td>
<td>169.85</td>
</tr>
<tr>
<td>Binh Thuy AB (1971)</td>
<td>913.84</td>
<td>8</td>
<td>114.23</td>
</tr>
<tr>
<td>England AFB (1971)</td>
<td>507.87</td>
<td>4</td>
<td>126.97</td>
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### TABLE II
SUMMARY OF F-4 DATA USED

<table>
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<tr>
<th>Mission Type</th>
<th>Hours</th>
<th>Tail Numbers</th>
<th>Hours/Tail Number</th>
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<tr>
<td>Air-Ground</td>
<td>2308.6</td>
<td>46</td>
<td>50.19</td>
</tr>
<tr>
<td>Air-Air</td>
<td>149.0</td>
<td>23</td>
<td>6.48</td>
</tr>
<tr>
<td>In-Nav</td>
<td>481.2</td>
<td>40</td>
<td>12.03</td>
</tr>
<tr>
<td>Recon</td>
<td>515.2</td>
<td>19</td>
<td>27.12</td>
</tr>
<tr>
<td>Test</td>
<td>19.1</td>
<td>15</td>
<td>1.27</td>
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Reference 2 contained F-4 aircraft Southeast Asia (SEA) load factor data that were segregated by tail number, and mission types. These data were collected by Technology, Incorporated (TI), as part of Aircraft Structural Integrity Program (ASIP) during the period of 15 August 1969 through 31 December 1970. The 3,473 total hours analyzed are summarized in Table II.

The above referenced reports were selected since the data are in terms of aircraft tail number by mission type or base. However, these data are not in terms of mission segment as desired. It seems reasonable to assume that segregation by base, and mission type would be as applicable to the analysis methods as segregation by mission segment.

Before proceeding, some of the problems encountered in using the above data should be considered. One problem that appeared in Tables I and II was that both aircraft types have what appeared to be a large total number of hours, but may have too small a number of hours for any given tail number and category of mission or base. For example, there is 4,001 hours of F-4 data but only an average of 50 hours per tail number in the air-ground mission category, with other mission categories having even less hours. This raised the question about the validity of an exceedance curve for a given tail number. A small number of hours for a tail number would probably give the exceedance curves a wider spread than is actually the case; this would tend to make any estimates about the variability to be
unnecessarily large. If the criteria, that 1,000 hours of data are needed to plot a valid exceedance curve is used, it would be necessary to have at least $46(1,000) + 40(1,000) + 19(1,000) + 15(1,000) = 143,000$ total hours of F-4 data to obtain valid exceedance curves for all the aircraft tail numbers and categories listed in Table II. This is a large data requirement.

Another question concerns the type of distribution the exceedances have for a given value of $n_z$. It will be shown that normality should probably be ruled out, but the log of the exceedances being normally distributed is a plausible assumption. The assumption of a nonparametric (not normal) distribution would also provide some usable results, but more tail numbers would be required, and hence more data.

Lastly, the problem of zero exceedances at high $n_z$ values caused a number of interrelated problems, for example zero peaks in 50 hours implies zero peaks per 1,000 hours is not a valid assumption. That is, the real number of peaks per 1,000 hours at some high load factor may have really been 10 peaks implying .5 peaks in 50 hours. Since there can be no fractional count of a peak, 0.5 peaks in 50 hours of flight cannot occur. Even with over 1,000 hours of data, the result is questionable since occurrences of $n_z$ peaks at high $n_z$ values (above 7 g's) are rare. Handling of this zero occurrences problem is discussed throughout this report.

It was decided that these data would suffice for this study, since there did exist variability of the exceedance curves for each aircraft tail number. However, for better results, more data hours per aircraft and more tail numbers should be used.
2. **Computer Programs**

Several computer programs (See Figures 37-40) were used on the Hewlett-Packard (HP) 9830 calculator system to verify the validity of some new approaches to analyzing flight loads data. The computer programs were used to generate plots, tolerance limits, confidence intervals and curve fits for the $n_z$ exceedance data used herein.

One computer program, which handles a maximum of 49 tail numbers and 16 different values of $n_z$ along the abscissa, used the $n_z$ peak data obtained from the references, and stored it on a file of a tape cassette. Hence, each file of the tape cassette contained $n_z$ data by tail number for a particular mission or base as shown in Tables I and II, and also the composite data which includes all tail numbers.

The tape cassette was used to input the $n_z$ data in an analysis program. The analysis program was used to analyze the data as follows:

a. Plot $n_z$ exceedance points per 1,000 hours for each tail number.

b. Plot $n_z$ exceedance curves for each aircraft tail number.

c. Compute and plot the minimum, maximum and median exceedance per 1,000 hours for each of the 16 $n_z$ values.

d. Compute and plot a 90% confidence interval for the median of the exceedances per 1,000 hours.

e. Compute and plot the mean and composite exceedances for each of the 16 $n_z$ values.

f. Compute and plot tolerance limits on the exceedances per 1,000
hours for each n stratum assuming that the exceedances or the log of the exceedances are normally distributed.

A third computer program, which was a revision of a HP Plot Pac Program, performs a least square fit of a polynomial to the log of the exceedance data. Composite data for all tail numbers for each category were input through the keyboard to fit curves of the form

\[ F = 10^{b(n_z)}, \]

where \( F \) is the number of exceedances per 4,000 hours and \( b(n_z) \) is the best polynomial that estimates log \( F \).

3. Data Presentation

Results of the analysis are presented in Figures 1 through 36 and Table III in Appendix I. A detailed discussion of the various methods that were studied is presented below.

3.1 Exceedance Curves by Aircraft Tail Numbers

For each of the nine categories listed in Tables I and II, the exceedance curve for each individual tail number has been plotted in Figures 1 through 9. These exceedance curves give a rough idea of the type of spread that might be expected for different aircraft tail numbers. The following sections primarily investigate the means by which statistical statements can be made in describing this spread in the exceedance curves.

3.2 Confidence Intervals for the Median

The median of the exceedances per 1,000 hours (MED\( (n_z) \)) for
each given value of $n_z$ have been plotted in Figures 10 through 18 for each of the nine cases listed in Tables I and II. Straight line segments were also drawn connecting the MED($n_z$) values. The median was calculated so that one can say that about 1/2 of the exceedances per 1,000 hours for each $n_z$ ($E_i(n_z)$ where $i$ is the tail number) lie above and below the MED($n_z$) value.

Next, a 90% two-sided confidence interval (CI) for the median was found using a distribution free procedure based on the sign test as outlined in Reference 3. The procedure was applied to each $E_i(n_z)$ for a given $n_z$ so that the 90% confidence interval for the median corresponds to the statement: the probability

$$LCI(n_z) \leq MED(n_z) \leq UCI(n_z) = 0.90$$

where $LCI(n_z)$ is the lower limit of the CI and $UCI(n_z)$ is the upper limit of the CI (i.e. the probability that the true median lies between $LCI(n_z)$ and $UCI(n_z)$ is 0.90).

All $E_i(n_z) = 0$ cases were included for the above computation. In addition, the composite was also plotted and found within the 90% CI for the median for the lower values of $n_z$. Figures 10 through 18 showed that a difference exists between the median and the composite (C) for the F-4 data whereas they are very similar for the A-37 data. In fact, for the F-4 data the median usually lies below the composite. This is probably due to the greater frequency of $E_i(n_z) = 0$, for the F-4 data which was caused by an inadequate number of hours per tail.
number as shown in Table II and discussed earlier in data presenta-
tion. It should be noted that 1 peak in 100 hours of data would
imply 10 peaks per thousand and may overestimate a theoretical value
of 1 peak per thousand hours by quite a margin. Intuitively, it then
seems that for the F-4, the composite would usually lie below any
exceedance per tail number due to the overestimation of the exceedance
for a limited number of hours.

3.3 Maximum and Minimum Values

The maximum and minimum values of $E_1(n_z)$ are also plotted in
Figures 10 through 18. The maximum ($\text{MAX}(n_z)$) and minimum values
($\text{MIN}(n_z)$) are plotted for each $n_z$ value and a line drawn connecting
the points. These curves may then be used to make distribution free
two-sided tolerance limit statements which means that with some confi-
dence gamma ($\gamma$), a percentage of the tail numbers ($P$) lies between the
two limit values. By using Table A-32 in Reference 4, values of $\gamma$
are obtained so that 90% of the tail numbers lie between the maximum
and minimum values of $n_z$ exceedances per 1,000 hours.

In Figures 10 through 18, the values of $\gamma$ have been included
on the plots. Note that gamma was higher as the number of tail numbers
increased. For example, $\gamma = 0.30$ for 11 tail numbers in Figure 10
and $\gamma = 0.95$ for 46 tail numbers in Figure 14. Hence, we can use the
maximum and minimum value of exceedances per 1,000 hours as statistical
bounds. This method would give a very conservative estimate of the
bounds on the exceedance curves since there are no assumptions made
about the underlying distribution. For Figures 10 through 14, the bounds determined in this manner appear to be relatively narrow. However, in Figures 15 through 18, the minimum is zero too often since these cases have a small average hours per tail number.

3.4 Tolerance Limits Assuming Normal Distribution

In Figures 19 through 27, a plot of a 90% two-sided tolerance limit with 90% confidence is shown with the assumption that for a given \( n \), the exceedances per 1,000 hours have a normal distribution.

The tolerance limit was computed by considering the \( E_i(n) \)'s for each given \( n \). The mean \( M(n) \) was calculated using

\[
M(n) = \frac{\sum_{i=1}^{N} E_i(n)}{N}
\]

where \( E_i(n) \) is the exceedances per 1,000 hours for a fixed \( n \) and \( N \) is the number of aircraft tail numbers. Then, the sample standard deviation(s) were computed by

\[
s(n) = \left( \frac{\sum_{i=1}^{n} E_i(n)^2 - \left( \sum_{i=1}^{n} E_i(n) \right)^2}{n(n-1)} \right)^{1/2}
\]

which was obtained from Reference 1 on page 1-10. The upper NorUL and lower limits NorLL were

\[
\text{NorUL}(n) = M(n) + K \cdot s(n)
\]

The upper NorUL and lower limits NorLL were
and

\[
\text{NorLL} = M(n_z) - K \cdot S(n_z)
\]

where \( K \) is obtained from Table A-6 of Reference 4 with \( \gamma = .90 \), \( P = 90\% \) and \( N \) = number of tail numbers. Before proceeding, it should be noted that all zero exceedances per 1,000 hours were included in the computations.

The lower limit did not appear in most of the figures, because the assumption, that the exceedances are normally distributed, allows the lower limit to take on negative values. This is not possible. Another factor to consider concerns plotting of limits on a log scale. Since

\[
| \log (M + KS) - \log M | \leq | \log (M-KS)-\log M |
\]

the upper limits would be closer to the mean than the lower limits on the log scale.

As was the case for the median, the mean and composite are approximately equal at low \( n_z \) values but differ at higher \( n_z \) values. This is again probably due to errors in exceedances per 1,000 hours that occur at the higher \( n_z \) values.

It was concluded from the above discussions and Figures 19 through 27 that the assumption of a normal distribution was not valid.

3.5 Tolerance Limits Assuming a Log Normal Distribution

Plots of 90\% two-sided tolerance limits with 90\% confidence are shown in Figures 19 through 27 with the assumptions that the log
of the exceedances per 1,000 hours for each \( n_z \) has a normal distribution. The mean \( M_3 \) was computed as the common logarithm of the composite \( (C(n_z)) \) at each \( n_z \). Note, the composite is the same as the weighted mean \( (M_w) \) defined by

\[
M_w = \sum_{i=1}^{N} \frac{E_i(n_z)}{\Sigma_t_i}
\]

which equals the total number of occurrences greater than \( n_z \) divided by total time. This result is the composite. Next, a standard deviation \( s'(n_z) \) is defined as

\[
s'(n_z) = \left( \frac{\sum_{i=1}^{N} (\log E_i(n_z) - \log C(n_z))^2}{N} \right)^{1/2}
\]

Since, the log of zero is undefined, zero values of \( E_i(n_z) \) were not included in the solution for \( s'(n_z) \). \( N \) was decreased by 1 each time a zero was excluded for a fixed value of \( n_z \).

Then, as in the previous section, upper (LNUL) and lower (LNLL) tolerance limits were computed by formulas:

\[
LNUL(n_z) = \log C(n_z) = K \cdot S'(n_z)
\]

and

\[
LNLL(n_z) = \log C(n_z) - K \cdot S'(n_z).
\]

These upper and lower tolerance limit (LNUL, LNLL) curves are then equidistant from the composite when plotted on semilog paper as is evident from Figures 19 through 27. For most cases, the log normal assumption gave some reasonable bounds but deviations do occur. For
instance, at the extreme $n_z$ values, the limits go to infinity and zero for the upper and lower bounds respectively. This is because at the extreme $n_z$ values there are many zeroes which make $N$ small, and $K$ which was obtained from Table A-6 becomes very large. Hence, LNUL and LNLL become very large and small respectively as can be seen clearly in Figure 26. The spike at $n_z = 6$ g's on Figure 22 resulted from one aircraft tail number exceedance curve being farther from the mean than the other three; thus, making a higher than probable standard deviation. It was decided, that overall, the log normal assumption did provide reasonable bounds for $n_z$ exceedance curves.

3.6 Composite Exceedance Curve Fitting

The final computer program provided for the fitting of polynomial equations to the log of the exceedances per 4,000 hours. Four thousand hours was chosen because it is the design life for most fighter-type aircraft. The fitted curves that were obtained are presented in Figures 28 through 36 and summarized in Table III.

In most instances, a third degree polynomial provided the best fit, but in instances where this was not a good fit, (e.g. the curve goes to plus infinity as $n_z$ goes to infinity) the best straight line was fitted through the data points. In most cases when the straight line fit was used it proved to be adequate.

These fitted curves to the $n_z$ exceedances per 4,000 hours can be used as a uniform method of extrapolation to one exceedance per 4,000 hours (i.e. the aircraft life to obtain an estimate of a realistic design limit load factor).
In addition, these equations may be used to change the magnitude and slope of the exceedance curves for test purposes. The equation has the form

\[ F(x) = P \cdot 10^{B(x) \cdot x} \]

where \( F \) is the curve function, \( x \) corresponds to \( n_z \) and \( B \) is a function of \( x \) of the form

\[ B(x) = b_2 x^2 + b_1 x + b_0 \]

For the case of the cubic equation for positive \( n_z \) in Figure 32, the equation for the exceedance curve for F-4 air-ground data was found to be

\[ F(x) = 10^{(6.9742 - 1.4829x + 0.2914x^2 - 0.0249x^3)} \]

(See Table II in Appendix I)

This equation can be reduced to

\[ F(x) = 10^{6.9742} 10^{x(-0.0249 + 0.2914x^2 - 0.0249x^3)} \]

This means we can place

\[ P = 10^{6.9742} = 9,423,234 \]

and

\[ B(x) = (-0.0249x^2 + 0.2914x - 1.4829) \]

\( P \) will define the point at which the curve intercepts the \( n_z = 0 \) axis and \( B(x) \) defines an instantaneous slope to the exceedance curve on the log scale. This takes into account the changes in slope which
is typical for fighter $n_z$ exceedance curves. As long as $b_2$ is a negative coefficient (-.0249 for the example cited above), we know that the fitted curve will approach zero as $n_z$ increases without bounds.

With these equations, a series of curves could be obtained by varying the magnitude $P$ and coefficients of the slope $B(x)$ such that they do not exceed the bounds determined by using one of the previous techniques. The effect of varying the $P$ and the $B(x)$ has not been studied in this report, but might be an approach that may prove effective.

Extrapolation of these curves to design limit load factor is now a simple procedure. Since $F$ is the exceedances per 4,000 hours, find the value of $n_z$ such that the $\log F = \log 1 = 0$. For example in Figure 28 for A-37B, England AFB, 1969, we would find a design limit load factor of 6.8 g's.
SECTION III
CONCLUSIONS

The plotting of $n_z$ exceedance curves by aircraft tail number to obtain statistical bounds did appear to be feasible. Finding two-sided tolerance limits for exceedance curves with the assumption that the log of the exceedances per 1,000 hours are normally distributed appeared to give reasonable bounds. Nonparametric techniques gave reasonable, but more conservative bounds. The assumption that the exceedances per 1,000 hours were normally distributed appeared to give the widest bound and hence were not as useful. Curve fitting polynomials to the log of the exceedances per 4,000 hours for composite $n_z$ data provided usable equations for representing $n_z$ exceedance curves in most cases for the two aircraft types that were considered. In addition, the polynomials provide a standard mean of extrapolating $n_z$ exceedance curves to design limit load factor.
SECTION IV
RECOMMENDATIONS

As noted earlier, the data used for the F-4 represented a small amount of flight time per tail number. A further study should be made to determine what minimum time on a tail number is required in each category to ensure that a representative exceedance curve can be obtained for that particular aircraft. The A-37B data indicated that 100 hours per aircraft tail number would probably be sufficient, but more information would have to be studied before this statement could be made with certainty.

Separating the data into mission segments would increase the need for a larger data base. For example, if nonparametric techniques are used, then at least 50 tail numbers should be used for each mission segment. If 100 hours per tail number are required and five mission segments for a typical aircraft type are assumed, there would need to be at least 25,000 hours of data on the particular aircraft type. If the data were not categorized by mission segment, 5,000 hours would suffice, and hence, it is seen that the total number of hours needed is directly proportional to the number of categories within the data base. Thus, it should be remembered that segregating data into mission segment would greatly increase the amount of data that needs to be collected.

Also, the uncertainty of the number of exceedances per 1,000 hours at higher $n_z$ values is of concern. A minimum of 100 hours per tail number would be a minimum requirement; however, there is still a problem
when zero exceedances per 1,000 hours occur, because exceedance per 1,000 hours approaches zero as $n_z$ increases, but does not really reach zero. These problems at high $n_z$ values could easily form the major topic of further study.

Lastly, the aircraft for this report were both fighter-type aircraft. For other aircraft, like transport, other techniques may be more applicable than those cited in this study.
APPENDIX I

DATA RESULTS
FIGURE 1. $N_z$ Exceedance Curves by Tail Number for A-37B at England AFB (1969)
FIGURE 2. $N_z$ Exceedance Curves by Tail Number for A-37B at Bien Hoa AB (1970)
FIGURE 3. $N_z$ Exceedance Curves by Tail Number for A-37B at Binh Thuy AB (1971)
FIGURE 4. $N_z$ Exceedance Curves by Tail Number for A-37B at England AFB (1971)
FIGURE 5. $n_z$ Exceedance Curves by Tail Number for F-4 Air-Ground
FIGURE 6. $N_z$ Exceedance Curves by Tail Number for F-4 Air-Air
FIGURE 7. $N_z$ Exceedance Curves by Tail Number for F-4 Inst. & Nav.
FIGURE 8. $N_z$ Exceedance Curves by Tail Number for F-4 Reconnaissance
FIGURE 9. $N_z$ Exceedance Curves by Tail Number for F-4 Test
Confidence that 90% of tail numbers lie between the maximum and minimum is 0.30.

12 Tail Numbers
2038.23 Total Hours
ASD-TR-72-1 Volume I

Confidence that 90% of tail numbers lie between the maximum and minimum is 0.34


1 - Maximum
2 - Upper Bound of 90% CI for the Median
3 - Median
4 - Lower Bound of 90% CI for the Median
5 - Minimum
6 - Composite
8 Tail Numbers
913.84 Total Hours
ASD-TR-72-1 Volume I

Confidence that 90% of tail numbers lie between the maximum and minimum is 0.19

Confidence that 90% of tail numbers lie between the maximum and minimum is 0.05

1 - Maximum
2 - Upper Bound of 90% CI for the Median
3 - Median
4 - Lower Bound of 90% CI for the Median
5 - Minimum
6 - Composite

Confidence that 90% of tail numbers lie between the maximum and minimum is 0.95

FIGURE 14. Nonparametric Bounds for F-4 Air-Ground Exceedances
23 Tail Numbers
149.0 Total Hours
ASD-TR-71-37

Confidence that 90% of tail numbers lie between the maximum and minimum is 0.64

FIGURE 15. Nonparametric Bounds for F-4 Air-Air Exceedances

40 Tail Numbers
481.2 Total Hours
ASD-TR-71-37

Confidence that 90% of tail numbers lie between the maximum and minimum is 0.92
FIGURE 17. Nonparametric Bounds for F-4 Reconnaissance Exceedances

19 Tail Numbers
515.2 Total Hours
ASD-TR-71-37

Confidence that 90% of tail numbers lie between the maximum and minimum is 0.58
15 Tail Numbers
19.1 Total Hours
ASD-TR-71-37

Confidence that 90% of tail numbers lie between the maximum and minimum is 0.45.

FIGURE 18. Nonparametric Bounds for F-4 Test Exceedances
FIGURE 20. 90% Tolerance Limits with 90% Confidence for A-37B at Bien Hoa AB (1970)
FIGURE 21. 90% Tolerance Limits with 90% Confidence for A-37B at Binh Thuy AB (1971).
FIGURE 22. 90% Tolerance Limits with 90% Confidence for A-37B at England AFB (1971).
FIGURE 23. 90% Tolerance Limits with 90% Confidence for F-4 Air-Ground.
23 Tail Numbers  
149.0 Total Hours  
ASD-TR-71-37

**FIGURE 24.** 90% Tolerance Limits with 90% Confidence for F-4 Air-Air.
FIGURE 25. 90% Tolerance Limits with 90% Confidence for F-4 Inst. and Nav.

1 - Upper bound of tolerance limit assuming normal
2 - Mean
3 - Lower bound of tolerance limit assuming normal
4 - Upper bound of tolerance limit assuming lognormal
5 - Composite
6 - Lower bound of tolerance limit assuming lognormal

40 Tail Numbers
481.2 Total Hours
ASD-TR-71-37
FIGURE 26. 90% Tolerance Limits with 90% Confidence for F-4 Reconnaissance.
FIGURE 27. 90% Tolerance Limits with 90% Confidence for F-4 Test.
### TABLE III. CURVE FIT COEFFICIENTS

F(X) = 10\(^{(a_3X^3 + a_2X^2 + a_1X + a_o)}\) where \(x=n_z\) and \(F(X) = \) Exceedances/4,000 Hours

#### A. Coefficients for \(n_z > lg\)

<table>
<thead>
<tr>
<th>Figure</th>
<th>(a_3)</th>
<th>(a_2)</th>
<th>(a_1)</th>
<th>(a_o)</th>
<th>R-Square</th>
<th>Hours</th>
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<td>7.6166</td>
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<td>-1.1635</td>
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<td>.99913</td>
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<td>.99681</td>
<td>2308.6</td>
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<tr>
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<tr>
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<td>-.8723</td>
<td>6.3584</td>
<td>.99442</td>
<td>19.1</td>
</tr>
</tbody>
</table>

#### B. Coefficients for \(n_z < lg\)

<table>
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<th>(a_2)</th>
<th>(a_1)</th>
<th>(a_o)</th>
<th>R-Square</th>
<th>Hours</th>
</tr>
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<td></td>
<td></td>
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<td>.99835</td>
<td>19.1</td>
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APPENDIX II

COMPUTER PROGRAMS USED
FIGURE 37. Program to Store Data on Tape Cassette

10 COM A[I:50], X[I:50], T[I:50], Y[I:16]
12 FOR J=1 TO 50
14 K[I,J]=0
15 T[I,J]=0
16 Y[I,J]=0
17 NEXT K
18 NEXT J
20 DISP "ROWL=,COLUMN=":
22 INPUT L,M
24 FOR I=1 TO L
26 DISP "A("I")=":
28 INPUT A[I]
30 NEXT I
32 FOR J=1 TO M
34 DISP "X("I","J")=":
36 INPUT X[I,J]
38 NEXT J
40 DISP "Y("I","J")=":
42 INPUT Y[I,J]
44 NEXT J
46 FOR J=1 TO L
48 PRINT A[I,J]
50 FOR K=1 TO 8
52 PRINT X[I,K];
54 NEXT K
56 PRINT T[I,J];
58 NEXT J
60 PRINT
62 FOR I=1 TO 8
64 PRINT Y[I,J];
66 NEXT I
68 PRINT
70 PRINT
72 FOR J=1 TO L
74 FOR K=9 TO 16
76 PRINT X[I,K];
78 NEXT K
80 PRINT T[I,J];
82 NEXT J
84 PRINT
86 NEXT I
88 PRINT
90 NEXT J
92 FOR K=9 TO 16
94 NEXT K
FIGURE 37. Continued

250 DIM NL(J),D1
260 PRINT R
270 NEXT J
280 PRINT
290 FOR I=9 TO 16
300 PRINT YEL
310 NEXT I
320 PRINT
330 DISP "ENTER 1 IF DATA IS GOOD"
340 INPUT T1
350 IF T1=1 THEN 390
360 DISP "DATA BAD IN COLUMNS 3
370 INPUT L1,M1
380 DISP "AD:LI,YC(L1),"M1")",L1,YC(M1"
390 INPUT AEL1;NEL1,H1;YEL1,YM1
400 PRINT AEL1;NEL1,H1;YEL1,YM1
410 DATA 350
420 DISP "FILE NO.= "
430 INPUT 2
440 STORE DATA 2
450 CLS
460 END

FIGURE 38. Program to Read Data on Tape Cassette

20 DISP "FILE NO.,ROWS,COLUMNS"
25 INPUT TSM
30 LOAD DATA T
35 DISP "COLUMN TO PRINT"
40 INPUT M1
50 TO PRINT YM1
60 FOR J=1 TO L
70 PRINT AEL1;NEL1,YM1
75 NEXT J
80 READ 50
90 END
FIGURE 39. Program to Analyze the Data by Tail Number

```plaintext
10 READ X, K, M, L, T, N
20 DATA 100, 200, 300, 400, 500, 600
30 FOR H=1 TO 6
40 READ X, K, M, L, T, N
50 IF X=H THEN 60
60 NEXT H
70 IF H=1 THEN 30
80 IF H=2 THEN 30
90 IF H=3 THEN 30
100 IF H=4 THEN 30
110 IF H=5 THEN 30
120 NEXT H
130 IF H=6 THEN 30
140 NEXT H
150 DISP "ENTER 1 TO PLOT ALL EXCEEDANCES."
160 IF T=1 THEN 450
170 FOR J=1 TO N
180 NEXT J
190 DISP "ENTER 1 TO PLOT ALL EXCEEDANCE POINTS."
200 IF T=1 THEN 450
210 FOR K=1 TO M
220 NEXT K
230 IF T=1 THEN 450
240 IF T=1 THEN 450
250 IF T=1 THEN 450
260 IF T=1 THEN 450
270 NEXT T
280 NEXT J
290 NEXT K
```

60
FIGURE 39. Continued

600 PRINT "ENTER 1 TO PROCEED"
610 INPUT T1
620 IF T1=1 THEN 730
630 REM "COLUMN TO PRINT"
640 L=90
650 PRINT T1
660 FOR J=1 TO L
670 PRINT HL REPEAT M
680 NEXT J
690 GOTO 540
700 IF T1=1 THEN 800
710 REM PLOT COMPOSITE
720 R1=L
730 GOSUB 730
740 PLOT
750 GOTO 800
760 STOP
770 FOR J=1 TO M
780 IF XLR1,J=0 THEN 790
790 PLOT Y=J LET XLR1,J=1
800 IF K#1 THEN 780
810 PLOT
820 NEXT K
830 RETURN
840 DISP "ENTER 1 TO FIND COXNAT."
850 INPUT T1
860 IF T1=1 THEN 1350
870 REM THE ELEMENTS OF EACH COLUMN ARE RANKED
880 IF N>25 THEN 800
890 DISP "INPUT 5(A+/2,1/2)M FOR N=151;
900 INPUT B
910 GOTO 900
920 DISP "12(A/2)=")
930 INPUT 2
940 FOR K=1 TO M
950 FOR J=1 TO N
960 REJ1=REJ1 K
970 NEXT J
980 FOR J=1 TO N
990 FOR J1=J+1 TO N
100 IF REJ1#REJ1 J THEN 500
101 GOTO 1910
102 TS=REJ1
103 REJ1=REJ11
104 REJ11=TS
105 NEXT J1
106 NEXT J
107 IF XLR1,K#1 THEN 500
108 GOTO 1910
109 IF K#1 THEN 100
110 GOTO 100
111 IF T1=1 THEN 110
112 IF C=0 THEN 110
113 C=C+1
114 C3=C33
115 GOTO 115
116 END
FIGURE 39. Continued

1. IF $i>25$ THEN 1100
2. $i = i+1$
3. IF $i > 75$ THEN 1100
4. $i = 3(i/2)-
5. FOR $K=1$ TO $i/2$
6. FOR $R1=1$ TO $5$
7. CALL 25500
8. NEXT $R1$
9. NEXT $K$
10. PRINT "ENTER 1 TO PRINT C/S";
11. INPUT TI
12. IF $TI=1$ THEN 1300
13. FOR $K=1$ TO $i$
14. CALL 25500
15. FOR $R1=1$ TO $5$
16. NEXT $R1$
17. NEXT $K$
18. IF $H1<0$ THEN 25500
19. IF $H1>0$ THEN 25500
20. FOR $J=1$ TO $i$
21. IF $J=2$ THEN 25500
22. FOR $K=1$ TO $H$
23. PRINT 3+H
24. NEXT $K$
25. NEXT $J$
26. IF $N=0$ THEN 25500
27. IF $H1+1<K+2$ THEN 25500
28. FOR $J=2$ TO $i$
29. IF $H1+1<K+2$ THEN 25500
30. NEXT $J$
31. NEXT $H$
32. NEXT $K$
33. PRINT "ENTER 1 TO FIND 1.1, 2.1, 3.1, NORMAL DISTRIBUTION";
34. INPUT TI
35. IF $TI=1$ THEN 1100
36. IF $N=0$ THEN 1100
37. PRINT "ENTER 1 IF $P=0.05$ OR $P=0.1$ THEN 1100";
38. INPUT TI
39. IF $TI=1$ THEN 1100

FIGURE 39. Continued

100 INPUT H
110 M=0
120 IF M=H THEN 200
130 L=H M
140 GOTO 100
150 NEXT K
160 STOP "ENTER 1 TO FIND M2, M3; 82; ASSUMING LOG NORMAL DISTRIBUTION"
170 INPUT T1
180 IF T1=1 THEN 2000
190 G="32"
200 FOR I=1 TO N
210 IF X(I,J)=0 THEN 2100
220 M2=LGTM(1,J)
230 FOR J=1 TO N
240 IF X(I,J)=0 THEN 1900
250 Ni=J+1
260 G=LGTM(1,J,K)-M2*K2+2
270 NEXT J
280 GOTO 1900
290 Ni=K-1
300 NEXT J
310 IF Ni=1 THEN 2100
320 IF Ni=2 THEN 2100
330 V=0
340 S2=0
350 NEXT K
360 STOP "ENTER 1 TO FIND 1, G2; LOG NORMAL DISTRIBUTION"
370 INPUT T2
380 IF T2=1 THEN 2100
390 G=Kx2 FOR K=190, 35, Ni=H
400 INPUT K
410 L1=H2-K5*82
420 L2=M2*K5*82
430 C1(K)=V2
440 C2(K)=L1
450 C3(K)=M2
460 C4(K)=L2
470 C5(K)=S2
480 NEXT K
490 STOP "ENTER 1 TO PLOT L1, M2, L2"
500 INPUT T1
510 IF T1=1 THEN 2200
520 G="32"
530 FOR I=1 TO N
540 GOTO 2200
550 NEXT M
560 STOP "ENTER 1 TO PLOT MP=4MC; STORED L2"
570 INPUT T1
580 WRITE ""
FIGURE 39. Continued

100 FOR K=1 TO M
110 LET K1 = K
120 DISP "ENTER I TO DRAW VS. L1 (0.10-3.02)"
130 INPUT T1
140 IF T1<1 THEN 2580
150 FOR K=1 TO M
160 COSUB 2530
170 FOR R1=1 TO 5
180 COSUB 2560
190 NEXT R1
200 NEXT K
210 STOP
220 FOR K=1 TO M
230 IF T2=1 THEN 2450
240 IF CIR1.K1 = 0 THEN 2490
250 PLOT Y21+LST(CIR1.K1)
260 GOTO 2470
270 IF CIR1.K1 = 106 THEN 2490
280 PLOT Y21+CIR1.K1
290 IF K#1 THEN 2490
300 PEN
310 NEXT K
320 PEN
330 RETURN
340 END 2500
350 PRINT
360 PRINT Y21;
370 PRINT "RETURN"
380 PRINT CIR1.K1;
390 RETURN
400 END 2560
FIGURE 40. Revised HP Plot Pac Program

10: L4 = (E0+O3)+E1011
20: FOR I = 1 TO 11
30: CL1: =111-0
40: IF I = 1 THEN
50: EHP T=12 TO 6
60: E(1) = 0
70: NEXT I
80: IF I#1
90: M=SI=S2=S3=S0=S5=S0
100: DISP "RAD. DEGREE="I
110: INPUT D2
120: IF D2>9 THEN 130
130: DISP "XMIN,XMAX,INCRN,="I
140: INPUT X1,X2,X3
150: DISP "YMIN,YMAX,INCRN,="I
160: INPUT Y1,Y2,Y3
170: I=(X2-X1)/27
180: J=(Y2-Y1)/17
190: X5=Y1+2#J
200: Y6=Y3+J
210: SCALE X1-2#I,X2+I,Y3+Y6
220: PLOT X3,Y1
230: PLOT X1,Y1
240: PLOT X1,Y1
250: PLOT I,Y2,-1
260: U=Y1
270: U=Y2
280: U=Y3
290: Z=PUL1
300: XV=U
310: Z=PUL1
320: Z=PUL0
330: LABEL (-0.3,1.0,2/3)
340: DISP "ENTER 1 TO PRINT"PUNT"
350: INPUT P9
360: IF P91 THEN 420
400: PRINT
410: PRINT "PLOT "TAB14"X,Y"TAB14"
420: PRINT
430: DISP "PRESS "ENTER" KEY"
440: END
450: FORMAT 2F7.2
460: SET PUL(C2)
470: Y=ABSU
480: Y=ABSU
490: =E0-CH(X+E(Y-Z)+Z(E-Z))
500: P=0 OR P2)
520: LABEL (-1.5,2;Z=ATL(+P2)-B/2)
530: FOR E=O TO Y STEP 1
540: PLOT I=HOT 7K=Z+K; NOT 2Y=2+Z=1
550: GLOT I=-7.3,9.3
560: LABEL (+560.07; NOT 55sP3+16P2"
570: RETURN I
580: T=0 WITH 580
590: LABEL (+"X"TAB18P3
600: TURN 0
FIGURE 40. Continued

10 IF W THEN 60
15 DISP "T(HOURS)";
20 INPUT T
30 PRINT B[2]; "CUMULATIVE PEAKS ="Y
34 Y=Y/T
35 Y=Y*4000
36 PRINT B[2]; "CUMULATIVE PEAKS PER 4000 HR.="Y
37 Y=LGTY
40 IF FNX1 THEN 20
50 END
60 DISP "NOT ALLOWED"
70 END

10 IF W THEN 60
20 DISP "WRONG X; Y=";
30 INPUT B[2]; Y
40 IF FNY(-1) THEN 20
50 END
60 DISP "NOT ALLOWED"
70 END

10 S8=SQR((S2-S1^2/N)/(N-1))
20 S5=SQR((S4-S1^2/N)/(N-1))
30 R9=(S5-S1*S3/N)/(N-1)/S8/S9
40 PRINT "NO. POINTS ="N
50 PRINT "X: MEAN="S1/N; "ST. DEV.="S8
60 PRINT "Y: MEAN="S3/N; "ST. DEV.="S9
70 PRINT "CORR. COEFF.="R9
80 PRINT "END"

10 IF N <= D2-W THEN 260
20 DISP "DEG.REF.="!
30 INPUT D1
40 IF D1 <= D2-W THEN 70
50 DISP "MAX DEG="D2-W
60 END
70 IF W=0 THEN 250
80 T=0
90 FOR I=1 TO D1+1
100 BI[I]=6
110 FOR J=1 TO D1-I+2
120 R=(I+J-1)*(D2+2-0.5*(I+J))/
130 BI[I]=BI[I]+C[T+J]*C[R]
140 NEXT J
150 T=T*(R2+(3-T)/2)
FIGURE 40. Continued

160 NEXT I
170 R1=0
180 FOR I=2 TO D1+1
190 R1=R1+CEI*(D2+(3-I)/2)+2
200 NEXT I
210 T0=CI(D2+1)*(D2+2)/2
220 TO=TO-CT(D2+1)+2
230 DISP "DONE"
240 END
250 IF N>D2 THEN 280
260 DISP "NOT ENOUGH POINTS"
270 END
280 P=W=1
290 D2=D2+1
300 FOR J=1 TO D2
310 C[P]=SQR[C[P]
320 FOR I=1 TO D2-J+1
330 C[P+I]=C[P+I]/C[P]
340 NEXT I
350 R=P+I
360 S=R
370 FOR L=1 TO D2-J
380 P=P+1
390 FOR M=1 TO D2+2-J-L
400 C[R+M-I]=C[R+M-I]-C[P]*C[P+M-I]
410 NEXT M
420 R=R+M-1
430 NEXT L
440 T=(D2+1)*(D2+2)/2
450 FOR I=1 TO D2-1
460 T=T-I-I
470 FOR J=1 TO D2-I
480 P=D2+1-I-J
490 P=P*(D2+1-(P-1)/2-I)
500 R=P-J
510 S=0
520 U=I+J+1
530 V=P
540 FOR K=1 TO J
550 V=V+U-K
560 S=S-C[R+K]*C[V]
570 NEXT K
580 C[P]=S/C[R]
590 NEXT J
600 NEXT I
610 C[1]=1/C[1]
620 GOTO 80
FIGURE 40. Continued

10 IF N=0 THEN 120
20 PRINT
30 PRINT "COEFFICIENTS"
40 PRINT
50 FORMAT F3.0,F12.4
60 FOR I=1 TO D1+1
70 WRITE (15,50)"B","I-1"="B(I-1)
80 NEXT I
90 PRINT
100 PRINT "R SQUARE = R1/T0"
110 PRINT
120 END

14 FOR X=X1 TO X2 STEP (X2-X1)/100
15 Y=FNZX
16 IF Y<Y5 OR Y>Y6 THEN 60
17 PLOT X,Y
18 GOTO 70
19 PEN
20 NEXT X
21 Z=FMMK0
22 END

16 DISP "CHARACTER HEIGHT(X)";
20 INPUT H
30 LABEL (*,H,2,0,2/3)
40 LETTER
50 Z=FMMK0
60 END

18 DISP "X=";
20 INPUT X
30 DISP "Y(CALC)="FMMX
40 END
REFERENCES


