RAY SOLUTIONS TO SOUND SCATTERING
BY COMPLEX BODIES: APPLICATION TO
ZOOPLANKTON

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ABSTRACT Acoustic surveys of zooplankton in the freshwater and marine
environments result in echograms displaying spatial patterns of volume scattering
strength and sometimes target strength data. A crucial element to the interpretation
of the data is understanding the acoustic scattering properties of the individual
animals. Because of the complexity of the boundary conditions of the animals,
approximate scattering models have been developed. In this paper, laboratory
backscattering data from various types of zooplankton are presented and compared
with some of the models. Much of the modeling has involved the use of ray
formulations-- a subject that was significantly advanced by Uberall.

1. INTRODUCTION

Acoustics provides an attractive means to rapidly and remotely survey
biological sound scatterers. In a relatively short period of time, the entire
water column can be mapped over distances of 10's to 100's of miles. The
result is echograms of spatial patterns of volume scattering strength. A major
challenge is interpreting the data in terms of meaningful biological parameters such as length and numbers of the organisms. In applications
involving commercial fishing, the acoustic data are related to the tonnage of
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the fish school.

A crucial element to interpretation of the acoustic backscattering data is the scattering amplitude of the animal which is a measure of the efficiency to which the animal scatters sound. The scattering amplitude is a function of the acoustic frequency as well as the size, shape, orientation, and material properties of the animal. There is a wide variety of types of marine organisms which span a wide range of sizes, shapes, and material properties (Fig. 1). Since there are so many species of animals, it is not practical to model the scattering of sound by each species. We have therefore systematically categorized the animals according to groups of gross anatomical features—fluid-like, elastic shelled, and gas-bearing. (The fluid-like group involves animals that are composed principally of tissue that does not support shear waves.) By categorizing animals into these groups, it is expected that an acoustic model developed for a given animal would apply to other species within the same group.

Because of the complexity of the boundary conditions of the animals, there are no exact analytical solutions to describe the scattering of sound by the animals. Further complicating the problem is the fact that since the animals are alive and moving, the shape of the animals is changing in time. Thus, even if an exact solution were available for a given shape, it is possible that the shape would not be known well enough to provide realistic input into the solution. Our approach to this issue is to make laboratory measurements of the scattering properties of the animals that contain enough information so that we can infer from the acoustic data what the dominant scattering mechanisms are. From that, approximate scattering models are developed and adjusted according to the data.

We have examined our acoustic data both in the spectral and time domain. The echoes have been studied one ping at a time, as well as in ensembles where the ping-to-ping fluctuations of the echoes (as the animal changes shape and orientation) and their averages have been investigated. The data generally indicate that the sound is scattering off of several major features from the animals. We have been successful in modeling the scattering, in part, by the use of ray formulations. Integral formulations and modal series solutions are also used in the modeling. There is a wide body of literature on ray formulations of sound scattering by various classes of objects. Much of the activity in this area has been due to significant advances made by Herbert Überall. While we are not going to attempt to present an exhaustive list of contributions made by Überall, Refs. 2-5 are examples of some of the work which we have drawn upon in our research.
FIGURE 1 Zooplankton and corresponding illustrations of certain important scattering components: (a) euphausiid, a shrimp-like animal (weakly scattering fluid-like body), (b) gastropod, a marine snail that swims (elastic shelled body), and (c) siphonophore, a gas-bearing animal (fluid-like tissue containing gas). From Ref. 1.
2. BASIC EQUATIONS

The scattering amplitude, $f$, of a target is defined in terms of the incident pressure, $P_{inc}$, and farfield scattered pressure, $P_{scat}$, as

$$P_{scat} = P_{inc} \frac{e^{ikr}}{r} f$$  \hspace{1cm} (2.1)

where $k$ is the acoustic wavenumber ($= 2\pi/\lambda$, where $\lambda$ is the acoustic wavelength) and $r$ is the distance between the target and point at which the scattered pressure is measured. The scattering amplitude in the backscatter direction is commonly expressed on a logarithmic scale:

$$TS = 10\log |f_{bs}|^2 = 10\log \sigma_{bs} = 10\log (\sigma/4\pi).$$  \hspace{1cm} (2.2)

where $TS$ is the target strength. The target strength is also expressed in terms of other commonly used terms, the differential backscattering cross section $\sigma_{bs}$ and backscattering cross section $\sigma$. It is also common to express the target strength in terms of the echo energy from a target averaged over an ensemble of realizations of orientation and shape. The "mean target strength" is given in this case as

$$\langle TS \rangle = 10\log \langle |f_{bs}|^2 \rangle.$$  \hspace{1cm} (2.3)

where the averaging is done before the logarithm is taken. In practical applications where repeated echoes from ensembles of targets are collected (such as from a school of fish or aggregation of zooplankton), the volume scattering strength is used:

$$S_v = 10\log \langle n |f_{bs}|^2 \rangle$$  \hspace{1cm} (2.4)

where $n$ is the number of targets per unit volume. It is desirable in aquatic applications to use multiple frequency echosounders so that the variability in volume scattering strength with respect to acoustic frequency can be inverted to produce biological parameters such as size distribution of the animals.\textsuperscript{6}
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All above logarithmic quantities are expressed on a decibel scale. The target strengths are in decibel units relative to $m^2$ and the volume scattering strength is in decibel units relative to $m^{-1}$.

3. EXPERIMENTS

We have conducted a wide variety of laboratory-style acoustic backscatter measurements on live individual zooplankton in a large tank filled with filtered seawater both on land and on the deck of a ship at sea.\textsuperscript{1,7-13} The acoustic system consisted of transducers spanning the frequency range 50 kHz to 1 MHz. There were two types of transducers—powerful narrowband transducers that emitted acoustic signals at essentially one frequency and less sensitive broadband transducers that emitted signals in an octave band. We typically transmitted a chirp (linear frequency modulated) signal with these broadband transducers. The frequencies of the narrowband transducers were distributed throughout the entire range of frequencies listed above while the broadband transducers covered the range 250 kHz to 1 MHz. The animals were tethered in the middle of the acoustic beam with fine monofilaments that were acoustically transparent. Hundreds of pings per frequency or band of frequencies were collected for each individual animal although, because of various constraints including time available, not all transducers were used for each animal. Since the broadband transducers yielded the most information, most data were collected with those transducers.

The data were processed and plotted for analysis in several different ways: 1) **Raw time series of echo for each ping**. Displaying this raw data helped us recreate the experiment later in order to assess system noise and other contaminants so that we could determine which data were of the best quality. 2) **Target strength versus frequency for each broadband ping**. These data produced the spectral characteristics for each realization of animal orientation and shape. 3) **Matched filter time series output for each broadband ping**. The replicate for this filter was a filtered and scaled version of the signal measured during the calibration of the system. The calibration involved separating the transmit and receive transducers, that were normally very close to each other for the backscatter measurements, and sending a low level signal from the transmitter to the receiver with the two facing each other. The replicate signal is related to the echo that would come from an ideal target with a uniform frequency response. Using this replicate in the matched filter with an ideal target would give a sinc-like output. Any deviations from this ideal output when applying the matched filter to actual data from animals is a source of information for the modeling. 4) **Average**
target strength (averaged first on a linear scale) versus frequency for sets of broadband pings. Although this averaging process tends to smooth out the structure in the patterns of target strength versus frequency, this average is most relevant to the volume scattering strength that one might measure in an actual aquatic environment. It is therefore important to investigate the degree to which the structure smooths out in the process as well as the degree to which the overall levels change.

The data from the broadband transducers revealed significant structure in the pattern of target strength versus frequency for most individual pings from all animals (Fig. 2). The great majority of echoes from the fluid-like

![Graphs showing target strength vs frequency for different animals.](image)

**FIGURE 2** Target strength versus frequency for individual echoes from an individual euphausiid whose body is fluid-like, two similar sized gastropods (one animal per plot) whose bodies consist of an irregular elastic shell with an opening, and a siphonophore whose body is mostly a fluid-like tissue that contains a small gas inclusion. Two echoes per animal or animal type are shown (left/right plots) to indicate the variability in scattering characteristics for different realizations of an individual. Laboratory data are represented by thick curves. Models (thin curves) are plotted, when appropriate. From Ref. 1.
animals (euphausiids) produced both regular (left plot) and irregular (right plot) patterns of a series of peaks and deep nulls. The regular patterns tended to occur only when the animal was at broadside incidence. The irregular patterns would occur for all angles of incidence, although much more when the animal was well off of broadside incidence. The elastic-shelled animals (gastropods) also produced patterns with structure (left plot) although there were cases in which there was very little structure (right plot). Likewise, the gas-bearing animals (siphonophores) produced patterns that were sometimes mostly featureless (left plot) and sometimes contained much structure (right plot).

The matched filter output of the broadband echoes typically contained more than one major peak (not shown). As discussed above, for an ideal scatterer whose scattering response is uniform across frequencies, the matched filter output using our calibration signal replicate would be a sinc-like function that contains a main central lobe as well as sidelobes. The fact that there is more than one major lobe in the matched filter output for the animals indicates the fact that they are behaving in a manner different than an ideal scatterer (a major lobe in this case is defined as any lobe that is higher than a sidelobe calculated from an ideal scatterer).

The average target strengths of the animals showed much less, if any structure (Fig. 3). The amount of residual structure varied from animal to animal and set to set of data for a given animal type. For example, for euphausiids and decapod shrimp, there was quite often structure remaining after the average, although the structure was always much smoother. The data tended to be smooth after averaging of the data from the other animals (gastropods and siphonophores), although in certain cases some structure remained.

4. MODELING

Given the type of structure in the target strength versus frequency data, we hypothesize that the echo from a given animal is composed of multiple rays due to different parts of the body. The phase of each ray would depend upon the location of the part of the body that contributed to the scattering as well as the scattering mechanism (which may change the wave speed). The rays would add constructively or destructively, depending upon the frequency, resulting in peaks or nulls in the target strength versus frequency curve, respectively. The matched filter output is consistent with the hypothesis of
FIGURE 3  Echo data and model averaged over angle of orientation and distribution of animal sizes. Laboratory echo data from aggregations of decapod shrimp over the range 50 kHz to 1.2 MHz. An accurate two-ray model was analytically averaged for the predictions. From Ref. 9.

the echo consisting of multiple rays. There are multiple main lobes in the matched filter output and each lobe corresponds to at least one ray (or more than one unresolved ray). The multi-ray representation of the scattering by the individual animals can be written in symbolic form as

\[ f_{bs} = \sum_j f_j \]  

(4.1)

where the backscattering amplitude is shown to be the summation of rays from the animal body. Although written in simple form, determination of the individual ray contributions \( f_j \) is a challenge and depends upon the boundary conditions of each animal. The details of the analysis and derivations of the various ray contributions are given in Refs. 1, 8, 9, and 12 and will only be briefly summarized here.
4.1 Fluid-like Animals
Depending upon whether the patterns of target strength versus frequency were regular or irregular, we used either a two-ray or six-ray model, respectively. Since the regular pattern corresponded to broadside incidence, the two-ray model was derived in a straight-forward manner using a weakly scattering bent finite cylinder model that ignored end effects. In this case, it is hypothesized that one ray is from the front interface of the animal and the other ray is from the back interface. The resultant predictions agreed qualitatively with the single ping euphausiids (Fig. 2, left panel) and decapod shrimp data. It was not possible to make a quantitative comparison because the precise shape and orientation of the animals were not known. Thus the parameters associated with shape and orientation were adjusted within a reasonable range of values for the fit.

For the irregular patterns of target strength versus frequency, the six ray model was used. Six rays were required to generate the noise-like patterns. More than six rays produced essentially the same patterns, so the summation was truncated at six. Since the phases of the rays are random, there were too many variable parameters to make a meaningful comparison to irregular single ping patterns such as in the right plot of Fig. 2. Therefore, the statistics of the echoes were analyzed (not shown). In that study, the statistics of the magnitude of a particular frequency bin of the broadband echo was studied. Attempts to predict the magnitude of each ray were not made, so the horizontal scale was normalized. In that case, both the observed and simulated histograms were Rayleigh-like. The statistics of the echoes were also examined for the regular target strength pattern and two-ray model and those patterns, that were similar to each other, were clearly non-Rayleigh.

There was great success in comparing analytical averages of the two-ray model and average echo data from both euphausiids and decapod shrimp (Fig. 3). First, the phase in the ray model was modified so the predictions would be accurate down to $ka = 0.1$, then it was analytically averaged over angle of orientation and animal size. The resultant predictions compared quite successfully with average echo data. Both the data and predictions show that some of the structure in the target strength pattern remains after the averaging.

In addition to the two-ray modeling which had many approximations as well as restrictions, the distorted wave Born approximation (DWBA) was used in making predictions of the average echoes. The comparisons of that volume integration approach with data were quite successful.
4.2 Elastic-shelled Animals

Both types of patterns of target strength versus frequency for the gastropods can be explained by the same model. The oscillatory behavior of the pattern that occurs for some orientations can be explained by the interference between the echo from the front interface of the shell and a subsonic Lamb wave. The period of the oscillations is consistent with a Lamb wave speed of about 1/8 of the surrounding water. The difference in arrival times between the two main peaks in the matched filter output is also consistent with that speed. The shell of the animal is not only irregular, but also has a large (opercular) opening which quite possibly prevents the propagation of Lamb waves for certain angles of orientation. If that is the case, then only the echo from the front interface will dominate the scattering resulting in flat patterns of target strength versus frequency as observed.

Modeling the scattering by the gastropod has been quite a challenge. The animal consists of an irregular shell of variable thickness and with a large opening on one side. Our first attempts to model it using the exact solution to a fluid-filled elastic spherical shell \(^{16}\) were unsuccessful, even after performing averaging over a range of sizes. The broad resonances that result from the solution were not seen in the data. A rigorous treatment would involve numerical approaches \(^{17}\) or sophisticated ray approaches for arbitrarily curved shells.\(^{18,19}\) Given the complexity of the shell and our lack of basic information on it such as precise profile of the shell cross section and material properties, we have begun our treatment by adapting a simple ray model for an elastic spherical shell to the problem. There are a number of formulations describing the scattering of sound by spherical shells and we have chosen an approximate one developed by Marston and colleagues.\(^{20,21}\)

We have adapted the ray model for the elastic spherical shell by randomizing the local radius of curvature which will result in Lamb wave paths that will vary in length according to what meridional line they follow. Furthermore, we have taken into account the encounter of the Lamb wave with the opercular opening in a heuristic manner by using a simple coefficient to the Lamb wave amplitude that one determines empirically for the various orientations. The resultant formulation shows the Lamb wave experiencing excess attenuation due to the random roughness and a reduction or complete elimination of the wave for certain orientations.

There was success in comparing the approximate formula with data (Fig. 2). In the data with an oscillatory pattern of target strength versus frequency, the roughness-induced attenuation of the Lamb wave had to be taken into account for a good fit with the data. For the zero roughness case, predictions produced a pattern with a similar period of oscillation, but with sharp peaks (not shown). Once the roughness was incorporated, the Lamb
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wave was reduced accordingly and the oscillations became rounded (Fig. 2, left plot). For other cases where the pattern of data was relatively flat, the heuristic factor which takes into account the opercular opening was used to completely eliminate the Lamb wave and the resultant flat pattern was predicted (Fig.-2, right plot).

In order to predict the average echo for an ensemble of different sized animals, our approximate model was numerically integrated over size. Some of the structure remained in the pattern of average target strength versus frequency in the averages and was consistent with the data.\textsuperscript{1,12}

4.3 Gas-bearing Animals
With the siphonophores, there are contributions from both the gas inclusion and the surrounding tissue. The flat target strength pattern (Fig. 2, left plot) and single peak matched filter output (not shown) for much of the echoes indicate that the gas sometimes dominates the echoes. However, for other echoes, the $TS$ pattern is irregular (Fig. 2, right plot) and the matched filter produces a secondary echo which indicates that under certain conditions, the tissue will also contribute significantly to the echo. Our scattering measurements on a certain siphonophore performed first with the whole animal and then with the gas removed showed the average target strength to drop by about 5 dB after removal which indicates that the tissue contributes, on average, about one third of the total echo energy.\textsuperscript{1,12} Echo statistics with both of those cases show a Rice-like PDF for the whole animal and a Rayleigh-like PDF for the tissue only.\textsuperscript{1,12} The degree of change in shape parameters of the PDFs for the two cases is consistent with the tissue contributing one third of the total energy.

We model the siphonophore by adding the exact solution for a gas sphere \textsuperscript{22} to a model for the tissue. The choice of model for the tissue depends upon the application. If only the statistics of the echoes are to be examined, then a six-ray model is used where the phases are random ($0 - 2\pi$) for these high frequencies and the amplitudes are determined empirically by comparisons with data. However, if predictions of average levels are required, then a more precise approach is used. For fluid-like bodies, we have successfully used both the DWBA approach numerically averaged over angle of orientation as well as an accurate two-ray model analytically averaged over orientation.\textsuperscript{9}

Comparisons between the single ping data and the gas-only model have been successful under the conditions when a flat $TS$ pattern occurs (Fig. 2, left plot). For the conditions when an irregular pattern occurs, then it is not meaningful to examine the echoes on a ping-by-ping basis, but rather on a
statistical basis. In this latter case, the echo fluctuations for both the whole animal and the gas-less tissue are predicted well with the ray models. Using a constant ray representing the gas added to six randomized rays representing the tissue and adjusted so that the total energy of the six rays is equal to one half of that of the gas echo energy (i.e., one third of the total energy) the resultant simulations of echo fluctuations are quite consistent with the Ricean behavior of the observed data. Removing the gas ray from the simulations results in a Rayleigh-like PDF which is consistent with the PDF for the gas-less tissue.

Adding the echo energy contributions as calculated with the modal series solution for the gas sphere and the accurate two-ray fluid bent cylinder model for the tissue produces predictions that are consistent with the average echo data from a siphonophore over a wide range of frequencies. Both data and theory show a flat $T$ pattern at the highest frequencies and an increase in level with decreasing frequency as the scattering approaches the resonance of the gas.

5. COMPARISON OF MODELS WITH ACOUSTIC SURVEY DATA IN THE OCEAN

Various forms of the above models have been used in our efforts to interpret acoustic scattering levels measured in the ocean. In a recent study, a 420 kHz acoustic echosounder was towed over parts of Georges Bank, a shallow water region near Cape Cod, Massachusetts. Zooplankton were collected in nets at the same time. The animals were sorted according to anatomical group, size, and location at which they were collected so that direct comparison could be made between the acoustic records and net samples. Once sorted, acoustic models were used to make predictions of the volume scattering strength one might expect for the animals present at each location. There was excellent agreement between the trend of the predictions and observed volume scattering strengths. The slope of the plot of predictions versus observations was unity, which is the same as what one would expect for a perfect set of predictions. However, the data were offset uniformly from the ideal line by 3.5 dB. This systematic bias has not been explained, but certainly system calibration could be one of the contributing factors of a uniform bias. Also, slight changes in parameters defining the material properties of the animals (density and sound speed contrast) could be a contributing factor.
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6. CONCLUSIONS

Modeling the scattering of sound by zooplankton has required approximate methods. Because of the complex nature of the boundary conditions, ray methods have been an integral part of the modeling. The methods have been based upon simpler shapes and adapted to the zooplankton problem through both physical arguments and empirical parameters. Use of broadband laboratory scattering data has been crucial in the development of the models.

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13. ABSTRACT (Maximum 200 words)

Acoustic surveys of zooplankton in the freshwater and marine environments result in echograms displaying spatial patterns of volume scattering strength and sometimes target strength data. A crucial element to the interpretation of the data is understanding the acoustic scattering properties of the individual animals. Because of the complexity of the boundary conditions of the animals, approximate scattering models have been developed. In this paper, laboratory backscattering data from various types of zooplankton are presented and compared with some of the models. Much of the modeling has involved the use of ray formulations—a subject that was significantly advanced by Überall.

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