Statecharts via Process Algebra

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STATECHARTS VIA PROCESS ALGEBRA

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Abstract. Statecharts is a visual language for specifying the behavior of reactive systems. The language extends finite-state machines with concepts of hierarchy, concurrency, and priority. Despite its popularity as a design notation for embedded systems, precisely defining its semantics has proved extremely challenging. In this paper, a simple process algebra, called Statecharts Process Language (SPL), is presented, which is expressive enough for encoding Statecharts in a structure-preserving and semantics-preserving manner. It is established that the behavioral relation bisimulation, when applied to SPL, preserves Statecharts semantics.

Key words. bisimulation, compositionality, operational semantics, process algebra, Statecharts

Subject classification. Computer Science

1. Introduction. Statecharts is a visual language for specifying the behavior of reactive systems [7]. The language extends the traditional notation of finite-state machines with concepts of (i) hierarchy, so that one may speak of a state as having sub-states, (ii) concurrency, thereby allowing the definition of systems having simultaneously active subsystems, and (iii) priority, so that one may express that certain system activities have precedence over others. Statecharts has become popular among engineers as a design notation for embedded systems, and commercially available tools provide support for it [10]. Nevertheless, precisely defining the semantics of the language has proved extremely challenging, with a variety of proposals [8, 9, 18, 19, 21, 28, 29] being offered for several dialects [34] of the language. While the research results have yielded insight into different aspects of the notation, no definitive account has emerged. This has an obviously undesirable practical ramification; tool builders for Statecharts must resort to ad hoc decisions in their implementations of semantically-based tools, such as model checkers [16, 23], and this means that designs developed by engineers have a meaning that may vary from implementation to implementation.

The semantic subtlety of Statecharts arises from the language’s capability for defining transitions whose enablement disables other transitions. A Statechart may react to an event by engaging in an enabled transition, thereby performing a so-called micro step, which may generate new events that may in turn trigger new transitions while disabling others. When this chain reaction comes to a halt, one execution step, a so-called macro step, is complete. Technically, the difficulty for defining an operational semantics capturing the “macro-step” behavior of Statecharts arises from the fact that such a semantics should exhibit the following desirable properties: (i) the synchrony hypothesis [2], which guarantees that a reaction to an external event terminates before the next event enters the system, (ii) compositionality, which ensures that

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2. Statecharts. Statecharts is a specification language for reactive systems [27], i.e., concurrent systems which are characterized by their ongoing interaction with their environment. They subsume finite state machines whose transitions are labeled by pairs of events, where the first component is referred to as trigger and may include negated events, and the second component is referred to as action. Intuitively, if the environment offers the events in the trigger, but not the negated ones, then the transition is triggered and can be executed; it fires, thereby producing the events in the label's action. Concurrency is achieved by allowing complex Statecharts to be composed from more simple ones running in parallel, which may communicate via broadcasting events. Elementary, or basic states in Statecharts may also be hierarchically refined by injecting other Statecharts. Concurrency and hierarchy are especially important concepts, since they allow for bottom-up and top-down specifications of systems.

As an example, consider the Statechart depicted in Figure 2.1. It consists of a so-called and-state, labeled by $n_9$, which denotes the parallel composition of the two Statecharts labeled by $n_3$ and $n_8$. Actually, $n_3$ and $n_8$ are the names of or-states, describing sequential state machines. The first consists of two states $n_1$ and $n_2$ that are connected via transition $t_1$ with label $\neg a/b$. The label specifies that $t_1$ is triggered by $\neg a$, i.e., by the absence of event $a$, and produces event $b$. States $n_1$ and $n_2$ are not refined further and, therefore, are also referred to as basic states. Or-state $n_8$ is refined by or-state $n_6$ and basic state $n_7$, connected via a transition labeled by $b/a$. Or-state $n_3$ is further refined by basic states $n_4$ and $n_5$, and transition $t_2$ labeled by $b/c$.

It should be mentioned that the variant of Statecharts considered here does not include “features” present in some other variants. In particular, we prohibit interlevel transitions, i.e., transitions crossing borderlines of states, and triggers of the form $i_{n_i}$, where $n$ is the name of a state. Moreover, state hierarchy does not impose implicit priorities to transitions, where transitions on higher levels of the hierarchy have precedence over transitions on lower levels; e.g., transition $t_5$ does not have priority over transition $t_2$ in our example. The impact of altering our approach to accommodate these concepts is discussed in Section 6.

2.1. Statecharts Terms. For our purposes, it is convenient to represent Statecharts not visually but by terms. This is also done in related work [17, 18, 31], and our approach closely follows the one described in [18]. Formally, let $\mathcal{N}$ be a countable set of names for Statecharts states, $\mathcal{T}$ be a countable set of names for Statecharts transitions, and $\mathbb{II}$ be a countable set of Statecharts events. Moreover, we associate with every event $e \in \mathbb{II}$ its negated counterpart $\neg e$. We also lift negation to negated events by defining $\neg\neg e =_df e$. Finally, we write $\neg E$ for $\{\neg e \mid e \in E\}$, if $E \subseteq \mathbb{II} \cup \{\neg e \mid e \in \mathbb{II}\}$. Then, the set of Statecharts terms is defined to be the least set satisfying the following rules.


Table 2.2

<table>
<thead>
<tr>
<th>Step-construction function</th>
</tr>
</thead>
<tbody>
<tr>
<td>function step-construction( (s, E) ) ; var ( T := \emptyset );</td>
</tr>
<tr>
<td>while ( T \subseteq enabled(s, E, T) ) do choose ( t \in enabled(s, E, T) \setminus T ); ( T := T \cup {t} ) od;</td>
</tr>
<tr>
<td>return ( T )</td>
</tr>
</tbody>
</table>

Table 2.3

<table>
<thead>
<tr>
<th>Function update</th>
</tr>
</thead>
<tbody>
<tr>
<td>update(([n], T') =_{df} [n] )</td>
</tr>
<tr>
<td>update(([n : s], T') =_{df} [n : (update(s_1, T_1), \ldots, update(s_k, T_k))] )</td>
</tr>
</tbody>
</table>
| update\(([n : s]; l; T], T' \) =_{df} \begin{cases}  
\{ [n : s; l; T] \} & \text{if } T' = \emptyset \\
\{ [n : s_1, \ldots, update(s_i, T'), \ldots, s_k; l; T] \} & \text{if } \emptyset \neq T' \subseteq \text{trans}(s_i) \\
\{ [n : s_1, \ldots, default(s_m), \ldots, s_k; m; T] \} & \text{if } \emptyset \neq T' = \{ (t', l, E, A, m) \} \subseteq T \\
\{ [n] \} & \text{otherwise} 
\end{cases} |

micro steps, or transitions, that are triggered by events offered by the environment or generated by other micro steps, that are mutually consistent, compatible, and relevant, and that obey causality. The Statecharts principle of global consistency, which prohibits an event to be present and absent in the same macro step, is subsumed by the notions of triggered and compatible.

A transition \( t \in \text{trans}(s) \) is consistent with \( T \subseteq \text{trans}(s) \), in signs \( t \in \text{consistent}(s, T) \), if \( t \) is not in the same parallel component as any transition in \( T \). Formally,

\[
\text{consistent}(s, T) =_{df} \{ t \in \text{trans}(s) | \forall t' \in T. t \perp_s t' \}.
\]

Here, we write \( t \perp_s t' \), if \( t = t' \), or if there exists an and-state \([n : (s_1, \ldots, s_k)] \) in \( s \), i.e., \( n \in \text{states}(s) \), such that \( t \in \text{trans}(s_i) \) and \( t' \in \text{trans}(s_j) \) for some \( 1 \leq i, j \leq k \) satisfying \( i \neq j \).

A transition \( t \in \text{trans}(s) \) is compatible to all transitions in \( T \subseteq \text{trans}(s) \), in signs \( t \in \text{compatible}(s, T) \), if no event produced by \( t \) appears negated in a trigger of a transition in \( T \). Formally,

\[
\text{compatible}(s, T) =_{df} \{ t \in \text{trans}(s) | \forall t' \in T. \text{act}(t) \cap \neg \text{ev}(t') = \emptyset \}
\]

A transition \( t \in \text{trans}(s) \) is relevant for \( s \), in signs \( t \in \text{relevant}(s) \), if the root of the source state of \( t \) is in the configuration of \( s \). Formally,

\[
\text{relevant}(s) =_{df} \{ t \in \text{trans}(s) | \text{root(out}(t)) \in \text{config}(s) \}
\]

A transition \( t \in \text{trans}(s) \) is triggered by a set \( E \) of events, in signs \( t \in \text{triggered}(s, E) \), if the positive, but not the negative, trigger events of \( t \) are in \( E \). Formally,

\[
\text{triggered}(s, E) =_{df} \{ t \in \text{trans}(s) | \text{ev}(t) \cap \Pi \subseteq E \text{ and } \neg (\text{ev}(t) \cap \neg \Pi) \cap E = \emptyset \}
\]

Finally, a transition \( t \) is enabled in configuration \( s \) regarding a set \( E \) of events and a set \( T \) of transitions, if \( t \in \text{enabled}(s, E, T) \), where

\[
\text{enabled}(s, E, T) =_{df} \text{relevant}(s) \cap \text{consistent}(s, T) \cap \text{triggered}(s, E) \cup \bigcup_{t \in T} \text{act}(t) \cap \text{compatible}(s, T)
\]
and synchronization in concurrent systems. The role of actions in process algebras corresponds to the one of events in Statecharts. A clock represents the progress of time, which manifests itself in a recurrent global synchronization event, the clock transition, in which all process components are forced to take part. However, action and clock transitions are not orthogonal concepts that can be specified independently from each other, but are connected via the maximal progress assumption [11, 35]. Maximal progress implies that progress of time is determined by the completion of internal computations and, thus, mimics the synchrony hypothesis of Statecharts. The key idea for embedding Statecharts terms in a timed process algebra is to represent a macro step as a sequence of micro steps that is enclosed by clock transitions, signaling the beginning and the end of the macro step, respectively. This sequence implicitly encodes causality and, thus, leads to a compositional semantics for Statecharts, whose practicality does not suffer from complicated transition labels including causal orders [17, 18, 31].

Unfortunately, existing timed process algebras are, in their original form, not suitable for embedding Statecharts. The reason is that Statecharts transitions may be labeled by multiple events and that some events may appear in their negated form. The former feature implies that – in contrast to standard process algebras [1, 12, 24] – processes may be forced to synchronize on more than one event simultaneously, and the latter feature is similar to mechanisms for handling priority [4]. Moreover, our framework must include an operator similar to the disabling operator of LOTOS [3] for resembling state hierarchy [32]. Our Statecharts Process Language combines these well-known concepts in a single process algebra, which is expressive and flexible enough for embedding several Statecharts variants, as we will show below.

3.1. Syntax. Formally, let $\Lambda$ be a countable set of events or ports, and let $\sigma \notin \Lambda$ be the distinguished clock event or clock tick. Based on $\Lambda$, we define input actions in SPL to be of the form $\langle E, N \rangle$, where $E, N \subseteq \Lambda$, and output actions $E$ to be subsets of $\Lambda$. In case of the input action $\langle \emptyset, \emptyset \rangle$, we speak of an unobservable or internal action, which is also denoted by $\bullet$. Moreover, we let $\mathcal{A}$ stand for the set of all input actions. In contrast to CCS [24], the syntax of SPL includes two different operators for dealing with input and output actions, respectively. The prefix operator $\langle E, N \rangle \sigma$ only permits prefixing with respect to input actions $\langle E, N \rangle$ which are instantly consumed in a single step. Output actions $E$ are signaled to the environment of a process by attaching them to the process via the signal operator $\langle E \rangle \sigma(\cdot)$. They remain visible until the next clock tick $\sigma$ occurs. The syntax of SPL is given by the following BNF

$$
P ::= 0 \mid X \mid \langle E, N \rangle P \mid [E]\sigma(P) \mid P + P \mid P \triangleright P \mid P \triangleright_\sigma P \mid P|P \mid P \setminus L
$$

where $L \subseteq \Lambda$ is a restriction set, and $X$ is a process variable taken from some countable domain $V$. We also allow the definition of equations $X \overset{\text{def}}{=} P$, where variable $X$ is assigned to term $P$. If $X$ occurs as a subterm of $P$, we say that $X$ is recursively defined. We adopt the usual definitions for open and closed terms and guarded recursion, and refer to the closed and guarded terms as processes [24]. The symbol $\mathcal{P}$ denotes the set of all processes and is ranged over by $P$ and $Q$. Finally, the operators $\triangleright$ and $\triangleright_\sigma$ – called disabling and enabling operator, respectively – allow us to model state hierarchy.

3.2. Operational Semantics. The operational semantics of an SPL process $P \in \mathcal{P}$ is given by a labeled transition system $\langle \mathcal{P}, \mathcal{A} \cup \{\sigma\}, \rightarrow, P \rangle$, where $\mathcal{P}$ is the set of states, $\mathcal{A} \cup \{\sigma\}$ the alphabet, $\rightarrow \subseteq \mathcal{P} \times (\mathcal{A} \cup \{\sigma\}) \times \mathcal{P}$ the transition relation, and $P$ the start state. We refer to transitions with labels in $\mathcal{A}$ as action transitions and to those with label $\sigma$ as clock transitions. For the sake of simplicity, we write $P \xrightarrow{E} P'$ instead of $\langle P, \langle E, N \rangle, P' \rangle \in \rightarrow$ and $P \xrightarrow{\sigma} P'$ instead of $\langle P, \sigma, P' \rangle \in \rightarrow$. We say that $P$ may engage in a transition labeled by $\langle E, N \rangle$ or $\sigma$, respectively, and thereafter behave like process $P'$. The transition relation
event in \( E \) and if all events in \( N \) are restricted. Finally, process variable \( X \), where \( X \overset{\text{def}}{=} P \), is identified with a process that behaves as a distinguished solution of the equation \( X = P \).

**Table 3.3**

**Operational semantics (clock transitions)**

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>tAct</code></td>
<td>( (E, N).P \overset{\sigma}{\rightarrow} (E, N).P ) ( (E, N) \neq \bullet )</td>
</tr>
<tr>
<td><code>tOut</code></td>
<td>( [E]_{\sigma}(P) \overset{\sigma}{\rightarrow} P )</td>
</tr>
<tr>
<td><code>tSum</code></td>
<td>( P \overset{\sigma}{\rightarrow} P' ) ( Q \overset{\sigma}{\rightarrow} Q' ) ( \frac{P + Q \overset{\sigma}{\rightarrow} P' + Q'}{P + Q \overset{\sigma}{\rightarrow} P' + Q'} )</td>
</tr>
<tr>
<td><code>tPar</code></td>
<td>( P \overset{\sigma}{\rightarrow} P' ) ( Q \overset{\sigma}{\rightarrow} Q' ) ( \cdot \notin \text{I}(P \cup Q) )</td>
</tr>
<tr>
<td><code>tNil</code></td>
<td>( 0 \overset{\sigma}{\rightarrow} 0 )</td>
</tr>
<tr>
<td><code>tDis</code></td>
<td>( P \overset{\sigma}{\rightarrow} P' ) ( Q \overset{\sigma}{\rightarrow} Q' ) ( P \triangleright Q \overset{\sigma}{\rightarrow} P' \triangleright Q' )</td>
</tr>
<tr>
<td><code>tRes</code></td>
<td>( P \overset{\sigma}{\rightarrow} P' ) ( P \setminus L \overset{\sigma}{\rightarrow} P' \setminus L ) ( \cdot \notin \text{I}(P \setminus L) )</td>
</tr>
<tr>
<td><code>tRec</code></td>
<td>( P \overset{\sigma}{\rightarrow} P' ) ( X \overset{\sigma}{\rightarrow} P' \overset{\text{def}}{=} P )</td>
</tr>
<tr>
<td><code>tEn</code></td>
<td>( P \overset{\sigma}{\rightarrow} P' ) ( P \triangleright Q \overset{\sigma}{\rightarrow} P' \triangleright Q )</td>
</tr>
</tbody>
</table>

The operational rules for clock transitions deal with the maximal progress assumption, i.e., if \( \bullet \in \text{I}(P) =_{\text{def}} \{ (E, N) \mid \exists P', \ P \overset{E}{\rightarrow} P' \} \) then a clock tick \( \sigma \) is inhibited. The reason that transitions other than those labeled by \( \bullet \) do not have pre-emptive power is that these only indicate the potential of progress, whereas \( \bullet \) denotes real progress in our framework. Rule `tNil` states that inaction process \( 0 \) can idle forever. Similarly, process \( (E, N).P \) may idle for clock \( \sigma \), whenever \( (E, N) \neq \bullet \). The signal operator in process \( [E]_{\sigma}(P) \), which offers communications on the ports in \( E \) to its environment, disappears as soon as the next clock tick arrives and, thereby, enables process \( P \). Time has to proceed equally on both sides of summation, parallel composition, and disabling, i.e., \( P + Q, P \cup Q \), and \( P \triangleright Q \) can engage in a clock transition if and only if both \( P \) and \( Q \) can. The side condition of Rule `tPar` implements maximal progress and states that there is no pending communication between \( P \) and \( Q \). The reason for the side condition in Rule `tRes` is that the restriction operator may turn observable input actions into the internal, unobservable input action \( \bullet \) (see Rule `tRes`) and, thereby, may pre-empt the considered clock transition. Finally, Rule `tEn` states that a clock tick switches the enabling to the disabling operator. Rule `tRec` does not require extra explanation.

The operational semantics for SPL possesses several pleasant algebraic properties which are known from timed process algebras [11, 35], such as (i) the idling property, i.e., \( \bullet \notin \text{I}(P) \) implies \( \exists P' \in \mathcal{P}. P \overset{E}{\rightarrow} P' \), for all \( P \in \mathcal{P} \), (ii) the maximal progress property, i.e., \( \exists P' \in \mathcal{P}. P \overset{E}{\rightarrow} P' \) implies \( \bullet \notin \text{I}(P) \), for all \( P \in \mathcal{P} \), and (iii) the time determinacy property, i.e., \( P \overset{E}{\rightarrow} P' \) and \( P \overset{E}{\rightarrow} P'' \) implies \( P' = P'' \), for all \( P, P', P'' \in \mathcal{P} \).

Moreover, the summation and parallel operators are associative and commutative.

### 3.3. A Behavioral Equivalence

As shown above, the SPL operational semantics interprets processes as labeled transition systems. However, from a semantic point of view, several transition systems might describe the same observable system behavior. For coping with this situation, standard process algebras introduce behavioral equivalences which relate processes, or transition systems, that describe the same intuitive behavior. One popular behavioral equivalence is bisimulation [24] which may be adapted to cater for SPL as follows.

**Definition 3.1** (Bisimulation). Bisimulation equivalence, \( \sim \subseteq \mathcal{P} \times \mathcal{P} \), is the largest symmetric relation such that whenever \( P \sim Q \), the following conditions hold.

1. \( \mathcal{I}(P) \subseteq \mathcal{I}(Q) \)
2. If \( P \overset{E}{\rightarrow} P' \) then \( \exists Q' \in \mathcal{P}. Q \overset{E}{\rightarrow} Q' \) and \( P' \sim Q' \).
Table 4.1
Embedding of the Example Statechart

\[
\begin{align*}
[s_9] &= n_9 \overset{\text{def}}{=} n_3 \mid n_8 \\
[s_3] &= n_3 \overset{\text{def}}{=} \bar{n}_1 \\
\bar{n}_1 &= n_1 \overset{\text{def}}{=} (\emptyset, \{a, \neg a\}').t_1 \\
\bar{t}_1 &= \{(b, \neg a)\} \sigma(\bar{n}_2) \\
[s_1] &= n_1 \overset{\text{def}}{=} \bar{n}_1 \overset{\text{def}}{=} 0 \\
[s_2] &= n_2 \overset{\text{def}}{=} \bar{n}_2 \overset{\text{def}}{=} 0 \\
[s_8] &= n_8 \overset{\text{def}}{=} \bar{n}_6 \\
\bar{n}_6 &= n_6 \overset{\text{def}}{=} \{\{b\}, \{\neg a\}\}' .t_3 \\
\bar{t}_3 &= \{(a)\} \sigma(\bar{n}_7) \\
[s_6] &= n_6 \overset{\text{def}}{=} \bar{n}_4 \\
\bar{n}_4 &= n_4 \overset{\text{def}}{=} \{\{b\}, \{\neg c\}\}' .t_2 \\
\bar{t}_2 &= \{(a)\} \sigma(\bar{n}_5) \\
[s_4] &= n_4 \overset{\text{def}}{=} \bar{n}_4 \overset{\text{def}}{=} 0 \\
[s_5] &= n_5 \overset{\text{def}}{=} \bar{n}_5 \overset{\text{def}}{=} 0 \\
[s_7] &= n_7 \overset{\text{def}}{=} \bar{n}_7 \overset{\text{def}}{=} 0
\end{align*}
\]

been triggered. Accordingly, it offers the events in A until the current macro step is completed, i.e., until a clock transition is executed. In order to ensure global consistency, process t also offers the events in \(E \cap \neg \Pi\). It is worth noting that SPL’s two-level semantics of action and clock transitions allows for broadcasting events using SPL’s synchronization mechanism together with its maximal progress assumption.

We now return to our introductory example by presenting its formal translation to SPL in Table 4.1, left-hand side. The embedding’s operational semantics is depicted on the right-hand side of Table 4.1, where \(\bar{t}_2 \overset{\text{def}}{=} \bar{t}_2 \sigma \{\{b\}, \{\neg a\}\}' .t_3\), and \(\bar{h} \overset{\text{def}}{=} \emptyset \overset{\text{def}}{=} \{\{b\}, \{\neg a\}\}' .t_3\). Moreover, the initial output action set \(\Pi(P)\), for some \(P \in \mathcal{P}\), is denoted next to the ellipse symbolizing state \(P\), and the sets \(\mathcal{N}'\) appearing in the label of transitions are underlined in order to distinguish them from the sets \(\mathcal{E}'\). Let us have a closer look at the leftmost path of the transition system, which passes the states \((n_3 \mid n_8)\), \((t_1 \mid n_8)\), \((t_1 \mid \bar{t}_2)\), \((\emptyset \mid \bar{h})\), \((\emptyset \mid t_3)\), and \((\emptyset \mid \emptyset)\). The first three states are separated from the last three states by a clock transition. Hence, the considered sequence corresponds to two “potential” macro steps. We say “potential,” since macro steps only emerge when composing our Statecharts embedding with an environment which triggers macro steps. The events needed to trigger the transitions and the actions produced by them can be extracted from a macro-step sequence as follows. For obtaining the trigger, consider all transition labels \(\langle E, \mathcal{N} \rangle\) occurring in the sequence, add up all events in components \(E\), and include the negations of all positive events in components \(\mathcal{N}\). Regarding the generated actions, consider the set of positive events in the initial output action sets of the states preceding the clock transition which signals the end of the macro step. Thus, the first potential macro step of the example sequence is triggered by \(\neg a\) and produces events \(b\) and \(c\), whereas the second is triggered by \(b\) and produces \(a\). The state names along a sequence also indicate the transitions which have fired. More precisely, whenever a state includes a variable \(t \in \mathcal{T}\) at its top-level, transition \(t\) participates in the current macro step. Thus, for the first potential macro step, transitions \(t_1\) and \(t_2\) are chosen, whereas
corresponds to the firing of \( t_i \) in \( s \). Vice versa, if \( (\text{Env}_E \mid [s]) \setminus \Lambda \) is the origin of an SPL path to a process which can only engage in a clock transition to \( (0 \mid P') \setminus \Lambda \) and which mimics the triggering of a transition sequence \( T = (t_1, \ldots, t_k) \), then \( T \) can be generated by the step-construction function relative to \( s \) and \( E \). Moreover, \([\text{update}(s, T)] \equiv P'\).

The formalization of the above intuition requires the following auxiliary properties, where \( s \in \text{SC} \) and \( E, A \subseteq \Pi \). Here, \( T \) stands for an arbitrary prefix of the above transition sequence \( (t_1, \ldots, t_k) \) interpreted as set, i.e., \( T = \{t_1, \ldots, t_i\} \) for some \( 0 \leq l \leq k \), and \( \text{act}(T) \) stands for \( \bigcup_{t \in T} \text{act}(t) \).

1. \( \exists t \in \text{enabled}(s, E, A, T) \setminus T \) implies \( [s, T] \overset{t'}{\rightarrow} P' \) for some \( E', N' \subseteq \Lambda \) and \( P' \in \mathcal{P} \), such that \( P' \equiv [s, T \cup \{t\}] \), \( E' = (\text{ev}(t) \cap \Pi) \setminus \text{act}(T) \), and \( N' = \neg(\text{ev}(t) \cap \neg \Pi) \cup \neg \text{act}(T) \).

2. \( [s, T] \overset{t'}{\rightarrow} P' \) for some \( E' \subseteq E \), \( N' \cap (E \cup \neg A) = \emptyset \), and \( P' \in \mathcal{P} \) implies \( \exists t \in T \). \( P' \equiv [s, T \cup \{t\}] \), \( t \in \text{enabled}(s, E, A, T) \setminus T \), \( E' = (\text{ev}(t) \cap \Pi) \setminus \text{act}(T) \), and \( N' = \neg(\text{ev}(t) \cap \neg \Pi) \cup \neg \text{act}(T) \).

3. \( \text{enabled}(s, E, A, T) \setminus T = \emptyset \) implies \( [s, T] \overset{\omega}{\rightarrow} P' \) for some \( P' \in \mathcal{P} \), where \( P' \equiv [\text{update}(s, T), \emptyset] \), and \( \forall (E', N') \in I([s, T]) \). \( E' \setminus E \neq \emptyset \) or \( N' \cap (E \cup \neg A) \neq \emptyset \).

4. \( [s, T] \overset{t'}{\rightarrow} P' \) for some \( P' \in \mathcal{P} \) and \( E' \setminus E \neq \emptyset \) or \( N' \cap (E \cup \neg A) \neq \emptyset \) for all \( (E', N') \in I([s, T]) \) implies \( \text{enabled}(s, E, A, T) \setminus T = \emptyset \) and \( P' \equiv [\text{update}(s, T), \emptyset] \).

The above properties establish a micro-step level relationship between Statecharts terms and the processes occurring in their embedding. The proof of each property can be done by induction on the structure of \( s \) and uses our extensions of the enabled function (cf. Section 2.3) and the embedding mapping (cf. Section 4.2).

5.2. Preservation Results. We close the technical part by returning to the behavioral relation \( \sim \) of bisimulation equivalence. First, we state a preservation result involving \( \sim \) and SPL’s macro-step semantics.

Theorem 5.3. Let \( P, P', Q \in \mathcal{P} \) such that \( P \sim Q \) and \( P \overset{E}{\Rightarrow} P' \). Then \( \exists Q' \in \mathcal{P} \). \( Q \overset{E}{\Rightarrow} Q' \) and \( P' \sim Q' \).

The validity of this theorem relies on the congruence property of \( \sim \) for SPL. When combining the insights obtained by establishing Theorems 5.2 and 5.3, one may derive the following corollary which relates bisimulation equivalence and Statecharts macro-step semantics.

Corollary 5.4. Let \( E, A \subseteq \Pi, s \in \text{SC} \), and \( P \in \mathcal{P} \) such that \( [s] \sim P \). Then

1. \( \forall s' \in \text{SC} \). \( \overset{E}{\Rightarrow} s' \) implies \( \exists P' \in \mathcal{P} \). \( P \overset{E}{\Rightarrow} P' \) and \( [s'] \sim P' \).

2. \( \forall P' \in \mathcal{P} \). \( P \overset{E}{\Rightarrow} P' \) implies \( \exists s' \in \text{SC} \). \( \overset{E}{\Rightarrow} s' \) and \( [s'] \sim P' \).

6. Adaptability to Other Statecharts Variants. For Statecharts, a variety of different semantics has been introduced in the literature. The comparison paper [34] surveys over twenty Statecharts variants. In this section, we show how our approach can be adapted to these variants and, thereby, testify to its flexibility. We focus on the most relevant issues of Statecharts semantics, which are identified in [34].

As is imminent in this paper, we favor an operational semantics over a denotational one, since we feel that operational models are more intuitive and, therefore, easier to understand. Moreover, operational models provide an immediate interface to verification tools which implement state-exploration techniques. An important observation of this paper is that the concept of a single, global clock together with maximal progress is the key to providing a compositional, causal state-machine semantics for Statecharts. Although the semantics is defined on the micro-step level, it allows for an easy identification of macro steps. The clock enforces global synchronizations which mark the beginning and end of macro steps. Thus, macro steps are represented as sequences of micro steps, which encode the underlying causality of Statecharts semantics.
compositionality holds on the micro-step level, i.e., the level of SPL action transitions, whereas responsiveness and causality is guaranteed on the macro-step level, i.e., the level on which sequences of SPL action transitions between global synchronizations, caused by clock ticks \( \sigma \), are bundled together.

Uselton and Smolka [31] and Levi [17] also focused on achieving a clean, compositional semantics for Statecharts by referring to process algebras. In contrast to our approach, Uselton and Smolka’s notion of transition system involves complex labels of the form \( (E, \prec) \), where \( E \) is a set of events and \( \prec \) a transitive, irreflexive order on \( E \), for encoding causality. Unfortunately, their semantics suffers from some serious problems, as pointed out in [17, 18]. Essentially, the semantics does not correspond – as intended – to the Statecharts semantics of Pnueli and Shalev [28]. Levi repaired this shortcoming by modifying the domains of the arguments of \( \prec \) to sets of events and by allowing empty steps to be represented explicitly. However, we believe that our semantics, where labels do not contain any order at all, profits from improved readability.

Maggiolo-Schettini et al. considered a hierarchy of equivalences for Statecharts, including isomorphism and bisimulation, and studied congruence properties with respect to Statecharts operators [18]. For this purpose, they defined a compositional, operational macro-step semantics of Statecharts, which slightly differs from the one of Pnueli and Shalev since it does not allow the step-construction function to fail. In their semantics, labels of transitions consist of four-tuples which include information about causal orderings, global consistency, and negated events. This complexity prohibits an intuitive understanding of Statecharts semantics and an easy integration with existing analysis and verification tools. However, it should be noted that the semantic framework presented in [18] serves well for the purpose of studying certain algebraic properties of equivalences on Statecharts, such as fully-abstractness results and axiomatizations [14, 15].

Another popular design language with a visual appeal like Statecharts and, moreover, a solid algebraic foundation is Argos [20]. However, the semantics of Argos, defined via SOS rules as labeled transition systems, significantly differs from classical Statecharts semantics. For example, Argos is deterministic, abstracts from “non-causal” Statecharts by semantically identifying them with a failure state, and allows a single parallel component to fire more than once within a macro step.

Interfacing Statemate [10] to model-checking tools is a main objective in [16] and most recently also in a series of papers by Milik et al. [21, 22, 23]. The first paper of this series includes a formalization of the semantics of Statemate, as defined in [8], within the specification formalism Z [30]. The second paper describes a translation from a subset of Statemate to hierarchical state automata which may be mapped to the specification language of the verification tool Spin [13], as shown in Milik’s third paper.

8. Conclusions and Future Work. This paper presented a process-algebraic approach to defining a compositional semantics for Statecharts. Our technique translates Statecharts terms to terms in the process algebra SPL which is expressive enough to model the semantic principles underlying Statecharts. SPL allows one to encode a “micro-step” semantics of Statecharts in the traditional SOS-style; it is at this level that our semantics is compositional, as bisimulation may be shown to be a congruence for the language. The macro-step semantics may then be given in terms of a derived transition relation. This semantics cannot be compositional, as results of Huizing and Gerth have shown [15]. However, the algebraic basis of our semantics permits the investigation of, e.g., the largest congruence consonant within this semantics. Also, since these sequences essentially encode total closures of causal orders, partial order methods might be useful for avoiding unnecessary state explosion in practice [6]. Note that, although SPL is a newly developed process algebra, all of its semantic ingredients have already been studied in the process-algebra community.


Statecharts via process algebra

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Statecharts is a visual language for specifying the behavior of reactive systems. The language extends finite-state machines with concepts of hierarchy, concurrency, and priority. Despite its popularity as a design notation for embedded systems, precisely defining its semantics has proved extremely challenging. In this paper, a simple process algebra, called Statecharts Process Language (SPL), is presented, which is expressive enough for encoding Statecharts in a structure-preserving and semantics-preserving manner. It is established that the behavioral relation bisimulation, when applied to SPL, preserves Statecharts semantics.