Parameter Space for Collisionless RF Sheaths

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As plasma processing reactors approach higher ion density, the sheath models which neglect the rf response of the ions become invalid. This work shows that at arbitrary ion density, the nature of the collisionless rf ion sheath can be described in a number of different regimes of parameter space. These regimes can all be visualized on a single two dimensional plot where the horizontal axis is the ion plasma frequency divided by the frequency, and the vertical axis is the electron oscillating velocity divided by the ion sound speed.
PARAMETER SPACE FOR COLLISIONLESS RF SHEATHS

I. Introduction

Plasmas are now used extensively in material processing. In many instances, the plasma supplies a stream of energetic ions to the surface; and this stream of ions, along with the various neutral chemistry processes, gives rise to the appropriate surface modification. For a steady state plasma, the passive dc sheath (i.e., assuming the wall at the floating potential) gives an ion energy impinging on the surface of typically a few times the electron temperature. Since the electron temperature is typically a volt or two, the ion energy may be perhaps 5 to 10 volts. For many processing applications, higher ions energies are required. These are often generated by biasing the substrate with an rf current. (If the substrate is an insulator, an rf potential is the only way to bias it in a controlled manner.) The rf bias gives rise to a sheath which accelerates the ions to the surface. The ion current and energy distribution is controlled in some way by the rf biasing parameters, the power, current, frequency, etc. Here we discuss models of collisionless rf sheaths in various regimes. Particularly, we specify a parameter space which characterizes rf sheath in all density and current regimes, give the boundaries of the various regions, and give approximate analytic solutions for the sheath in each region.

The standard picture of an rf sheath has been developed by Godyak [1,2], Lieberman [3,4], and others. In this picture, the rf frequency is high compared to the ion plasma frequency, so the ions do not respond at the drive frequency. Thus the theory is particularly applicable to capacitively driven plasmas which typically have rather low density (typically less than \(10^{10}\) cm\(^{-3}\) and an rf frequency of 13 MHz). In this theory, as the electrons oscillate away from the bounding surface, a positive space charge is left behind, and this accelerates the ions to the surface. Thus the ions have a crucial dc response, but no rf response. The Godyak Lieberman (GL) theory is sketched out in Section II. There, in addition to the basic theory, the regime of validity is specified.

As modern processing steps put more demands on the plasma, usually one is forced to higher density plasmas, with a more controllable sheath. (In the capacitive discharge, the rf creates both the sheath and the plasma). Examples of such higher density plasma reactors would be the electron cyclotron resonance reactor [5], inductive discharge reactor [6], helicon reactor [7], and large area plasma processing systems (LAPPS) [8]. At higher density, the condition for neglect of ion rf effects becomes invalid. In fact at 13 MHz, the ion plasma frequency can be considerably larger than the drive frequency. One theory in this regime was developed by Metze, Ernie and Oscan [9] (MEO). This theory effectively assumes that the ions respond to the rf fields instantaneously. In the MEO reference, numerical results only were presented, so it is not clear just what the limits of validity for the theory are, and there were no simple analytic models. A more recent work [10] modeled the transition between the GL and MEO theories, and results of numerical simulations were presented. Again very little analytic insight into the different regimes was offered. In Section III we review the MEO theory, give simple analytic models and discuss the regimes of validity.

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As a simple way to envision the parameter space for rf sheaths, consider the single parameter \( \omega \tau \) where \( \omega \) is the rf frequency and \( \tau \) is the time it takes the ion to cross the sheath. An ion crosses the sheath for two reasons, first because it enters the sheath with some velocity, and second because the fields in the sheath accelerate the ion across the sheath. Consider first the convection with the initial ion velocity. The ion typically enters the sheath with the sound speed \( c_s = [T_e/M]^{1/2} \) where \( T_e \) is the electron temperature and \( M \) is the ion mass. In the bulk plasma, the electrons carry an rf current density \( J = n e v_{os} \) where \( n \) is the electron number density and \( v_{os} \) is the oscillating electron velocity in the bulk plasma. However close to the wall, the electrons cannot oscillate in this way because they bump into the wall. Thus, near the wall, there is a region devoid of electrons; the ions crossing this region in their self consistent fields form the rf sheath. A first rough estimate of the length of this region is \( s_0 = v_{os}/\omega \). Thus if the motion of the ions to the wall with their input velocity were the only consideration, an estimate for \( \omega \tau \) would simply be \( v_{os}/c_s \). If this parameter is small, the ion crosses the sheath quickly because the sheath is short and the ions enter at high speed. On the other hand, if this parameter is large, the ions cross the sheath slowly compared to \( \omega \tau \).

Now let us consider the ion acceleration by the fields of the sheath. Let us assume that because the plasma is a good conductor, the electric field in the plasma is negligible. In the sheath region, the residual ions give rise to an electric field of \( 4\pi n e \varepsilon_0 \) where \( s \) is the distance from the instantaneous edge of the electrons (is a function of time because the sheath edge oscillates). Considering the acceleration of the ions in this field, we find that the acceleration of the ion to the wall gives a value of \( \omega \tau \) of about \( \omega^2/\omega_{pi}^2 \) where \( \omega_{pi} \) is the ion plasma frequency. For small values of this parameter, the ions accelerate across the sheath very quickly compared to \( \omega \tau \).

Thus we can envision a parameter space characterizing rf sheaths with \( \omega_{pi}/\omega \) being the horizontal axis, and \( v_{os}/c_s \) being the vertical axis. This values of these parameters compared to unity characterizes the various possible rf sheaths. It is only in the upper region along the vertical axis, \( \omega_{pi}/\omega < 1 \) and \( v_{os}/c_s > 1 \) that the ion remains in the sheath period for a time long compared to the oscillation period (here we use < and > rather than << and >> so as to more easily show the regions of validity on a two dimensional plot). This is the region of the GL sheath model.

Hence the GL theory is not only invalid at high density, it is also invalid at low rf current, a fact that does not seem to be generally realized. In Section IV we discuss the sort of rf sheath found in this regime. This sort of sheath (low density, low current) has parameters that are most likely not very interesting for plasma processing applications. Finally in Section V, we enumerate where all of the different models are valid on the two dimensional parameter space we have specified. Shown there is this chart with the regions of validity of the various sheath models denoted.
II The Godyak Lieberman [GL] rf Sheath Model.

Inside the bulk plasma there is a spatially uniform rf current carried by the electrons and given by

\[ J = J_0 \cos \omega t = ne \frac{ds}{dt} \]  \hspace{1cm} (1)

where \( n \) is the electron density in the bulk plasma at the sheath edge (for simplicity we assume here that the bulk plasma is uniform), and \( s \) is the position of the electron. However near the wall, the electron pulls away and an ion charge remain there. In the bulk the maximum excursion of an electron is

\[ 2s_o = \frac{2J_e}{ne\omega} \]  \hspace{1cm} (2)

However in the sheath, the ion density may not be constant, but has a density profile \( n(x) \) where \( s \) is the distance from the wall. If we assume that the plasma is a good conductor, and the electron sheath is at position \( s(t) \), we have from Poisson's equation that the electric field in the sheath is given by

\[ E = 4\pi e \int_{s(t)}^{s} dx n_i(x) \]  \hspace{1cm} (3)

The rf current, conduction plus displacement, must be continuous in the plasma and sheath, so we find

\[ n(s)e \frac{ds}{dt} = J_o \cos \omega t \]  \hspace{1cm} (4)

where we have assumed that the electric field in the plasma vanished since the plasma is a good conductor. Thus, if \( n(s) \) decreases in the sheath, as one goes from the plasma edge to the wall, the size of the sheath increases. Now say the ions enter the sheath at \( s_{max} \) and there, satisfy the Bohm condition, so that the current density is \( nec_s \), and there is a potential drop of \( \phi \) across the sheath. Now at \( t=-\pi/2\omega \), the electron is at its maximum excursion on the left (the wall), and at \( t=\pi/2\omega \) it is at its maximum excursion on the right (the sheath edge). Multiply by \( dt \) and integrate the right hand side of the equation from \( -\pi/2\omega \) to \( \pi/2\omega \) and the left hand side from 0 to \( s_{max} \), the sheath width. To integrate the left side, express \( n(s) \) as a function of potential by using conservation of energy (assuming that \( e\phi/T_e >> 1 \)) and assume that the electric field in the sheath is has the dependence on potential as in a Langmuir Child diode (that is the \( \phi(x) = \phi(x/s_{max})^{4/3} \)). Then we find that

\[ s_{max} = \frac{2}{3} \sqrt{\frac{2e\phi}{T_e}} s_o \]  \hspace{1cm} (5)
To the ions entering, the sheath looks very much like an ion diode. However instead of vacuum, the ions are entering a sheath region where electron charge oscillates back and forth, canceling the ion charge where it exists. Thus the average ion charge is about half of the charge in a vacuum ion diode. Hence the simplest theory would predict

\[ n_0e\kappa = 2J_L^C \]  

(6a)

where \( J_L^C \) is the Langmuir Child value. For the current of an ion diode, we use the expression in cgs units

\[ J_L = \frac{2e}{M} \frac{\phi^{3/2}}{9\pi \kappa \kappa_{max}^2} \]  

(6b)

on more familiar units

\[ J_L(A/cm^2) = 10^{-8} \frac{M_A \phi^{3/2}(Volts)}{M \kappa_{max}^2(cm)} \]  

(6c)

where \( M_A \) is the mass of the argon atom. This then relates the potential to the current and diode width. A schematic of the GL sheath model is shown in Fig. (1).

The more precise theory of Lieberman [3,4] predicts a factor of 1.8 instead of 2 in Eq. (6a). At this point it might appear that there is a dc (ion) current to the sheath. However this is not necessarily so. Recall that the electron sheath oscillates in time between the maximum position and the substrate. When the electron sheath is in contact with the substrate, an electron current can flow to the substrate which is essentially unlimited. Thus when the electron sheath hits the surface, the dc current can be canceled or can take on whatever value is imposed by the external circuit.

In any case, solving the modified Langmuir Child diode law, Eq. (6), for the voltage, and using the fact that the sheath width now depends on voltage according to Eq. (5), we find

\[ \phi(Volts) = \frac{0.45J_e^4(mA/cm^2)}{T_e(eV)^2 \left( \frac{n(cm^{-3})}{10^{16}} \right) \left( \frac{f}{13MHz} \right)^4} \]  

(7)

where \( f \) is the rf frequency. This gives a simple, approximate law for the dc sheath voltage in terms of the plasma and rf parameters. Notice that the calculation of the sheath width \( \kappa_{max} \) depends on the functional form of \( \phi(x) \) in the sheath region. The assumption of a relation appropriate for a space charge limited diode gives the coefficients in Eqs.(5 and 7). Other functional forms for \( \phi(x) \) would give different coefficients in Eq.(5). For instance the assumption of a uniform electric field would give an \( \kappa_{max} \) larger by a factor of 1.5. However this factor, when raised to the fourth power would give a voltage larger by...
Fig. 1 - The ion density as a function of \( x \) between the wall (\( x=0 \)) and sheath edge \( x=S_{max} \) is shown as the solid curve. The electron density at a particular time \( t \) is also shown as the dashed curve. The electron sheath oscillates from \( x=0 \) to \( x=S_{max} \) and back again with period \( 2\pi/\omega \).
a factor of 5 in Eq.(7). Thus the voltage is difficult to predict accurately from simple considerations; however the sheath width should be predicted reasonably accurately. It might be best to regard Eq(7) as a scaling law, with a coefficient to be determined empirically.

There are two conditions under which the GL model is valid. First of all the ions are assumed to not react to the rf fields, but only the dc fields. Clearly this implies \( \omega_{pi} < \omega \), or the ions can respond to the rf as well as dc fields. (In the bulk plasma the ion dielectric constant \( \omega_{pi}^2/\omega^2 \) is simply added to the much larger electron dielectric constant \( \omega_{pe}^2/\omega^2 \), so the ions do not play a significant rf role in the bulk, at least for an unmagnetized plasma.) The second condition is that the ion transit time across the sheath is very long compared to \( 2\pi/\omega \), so that the ions only react to the average fields in crossing the sheath. The sheath width is about \( [2e\phi/T_e]^{1/2} s_o \), and the ion velocity across the sheath is about \( [2e\phi/M]^{1/2} \), where \( M \) is the ion mass. Hence the time for the ions to cross the sheath is about \( s_o/c_s \). In order for this to be large compared to the oscillation time, we must have \( v_{os}/c_s > \pi \), where \( v_{os} \) is the oscillating velocity,

\[
v_{os} = \frac{J_o}{ne}
\]  

(8)

If this condition is violated, the ions react to the time oscillation of the sheath and there will be a spread in ion energies impinging on the substrate. Notice that the GL model predicts monoenergetic ions impinging on the substrate. The monoenergetic nature of the ion flux can be disrupted by either ion rf effects, or ion transit time effects. Thus the two conditions for the validity of the GL model of the sheath are

\[
\frac{\omega_{pi}}{\omega} < 1
\]

(9a)

\[
\frac{v_{os}}{c_s} > 1
\]

(9b)

If we envision a parameter space where the horizontal axis is \( \omega_{pi}/\omega \) and the vertical axis is \( v_{os}/c_s \), then the GL model of the rf sheath is valid along the vertical axis above \( v_{os}/c_s > 1 \).
III The Metze, Ernie, Oskam rf Sheath Model

The MEO model assumes that the ions cross the sheath as if the sheath were constant in time. This means that in all cases, \( \omega \tau < 1 \). However the ion plasma frequency \( \omega_p \) must be large compared to \( \omega \) so that ion fluid responds to the instantaneous fields of the plasma. The sheath current is assumed to consist of both displacement current and conduction current. Thus there are two possible regimes for the MEO sheath one where conduction current dominates and one where displacement current dominates. We will devise simple analytic models for each. The Poisson’s equation for the sheath at any time is given by

\[
\frac{d^2 \phi}{dx^2} = -4\pi e(n_i - n_e) \tag{10}
\]

For the electron and ion densities we assume the normal formulae for Langmuir sheath theory,

\[
n_e = n \exp \left( \frac{e\phi}{T_e} \right) \tag{11a}
\]

\[
n_i = \frac{n}{\sqrt{1 - \frac{2e\phi}{Mc_i^2}}} \tag{11b}
\]

Here we have assumed that where each density is equal to \( n \), the ion velocity into the sheath is \( c_i \) and the potential at this point vanishes. Going further into the sheath, the potential gets more and more negative, accelerating ions into the wall and repelling electrons. Integrating Eq.(10) once by quadrature, we find the electric field in terms of the potential to be

\[
E = -\sqrt{8m_eT_e} \left[ 1 - \frac{2e\phi}{T_e} + \exp \left( \frac{e\phi}{T_e} \right) - 2 \right]^{1/2} \tag{12}
\]

where the sign of \( E \) is chosen such that ions are accelerated into the wall on the left, as in Fig. (1). Thinking of a potential drop across the sheath which is considerably larger than the electron temperature, we can adopt the following approximation for \( E \) in terms of \( \phi \)

\[
E = -\sqrt{8m_eT_e} \left[ - \frac{2e\phi}{T_e} \right]^{1/4} \quad \phi < 0 \tag{13a}
\]

\[
E = 0 \quad \phi > 0 \tag{13b}
\]
From Poisson’s equation, we can determine that the surface charge density \( \sigma \) on the substrate is simply equal to \( E/4\pi \), so one can derive a charge in terms of sheath potential. Defining the capacitance per unit area as \( C' = d\sigma / d\phi \), we find that

\[
C = 0.25 \sqrt{\frac{nT_e}{2\pi}} \left[ -\frac{2e}{\phi^3 T_e} \right]^{1/4} \quad \phi < 0 \tag{14a}
\]

\[
C' = 0 \quad \phi > 0 \tag{14b}
\]

Thus one equation relating the sheath voltage and current is

\[
C \frac{d\phi}{dt} + J_c = J_{rf} \tag{15}
\]

Here \( J_c \) is the conduction current, and \( J_{rf} \) is the rf current \( J_0 \cos \omega t \). The first term is of course the displacement current. There are two possible limits to Eq. (15), either the conduction current dominates or the displacement current dominates.

We consider first the former. The conduction current as a function of substrate voltage is as given for a Langmuir probe,

\[
J_c = ne[T_e / 2\pi m] \exp(e\phi / T_e) - n_{ec} \tag{16}
\]

where \( m \) is the electron mass, and \( J \) is plotted as a function of \( \phi \) in Figure (2). For positive potential, there is the large electron current, while for negative potential the current density asymptotes to a much smaller negative value, the ion saturation current \( n_{ec} \). The actual dependence of \( J \) on \( \phi \) for positive \( \phi \) is not important here, only the fact that it is large and rapidly increasing. Actually the current density asymptotes for negative values to \( n_{ec} \), and the voltage is determined by the input power. For the other half cycle the current is electron current, and the sheath voltage is small, as is the input power. The ions impinging on the substrate have a spread in energy from about zero up to \( \phi_{max} \), determined by the input power. The energy of a particular ion depends on what time it enters the sheath.

We now estimate the sheath potential in terms of the input power. To do so, we approximate the graph of Fig. (2) as that of a diode with back leakage current density equal to \( -n_{ec} \). For such a diode, connected to an external circuit, with negative voltage, the voltage is then specified as a function of time. In the negative half cycle, we assume that \( \phi(t) = -\phi \sin \omega t \) for \( 0 < t < \pi / \omega \). During this half cycle, the current density is \( -n_{ec} \). During the positive half cycle, both the current and voltage will be small, the main constraint being that the total dc current in the entire cycle is zero (or else is as specified by the external circuit). In terms of the power per unit area going into the sheath, \( P_{es} \), one can determined the sheath voltage. The result is
Fig. 2 - Conduction Current as a function of voltage.
\[ P_s = \frac{1}{\pi} \phi n e c_s \]  

(17)

where we have assumed that there is no power dissipated on the positive half cycle.

There are several conditions which must be satisfied in order to have a conduction current dominated rf sheath. First of all the maximum rf current, \( J_0 \), can only be equal to \( n e c_s \), or

\[ \frac{v_m}{c_s} < 1 \]  

(18a)

Secondly, the conduction current must dominate the displacement current. This gives rise to a maximum frequency. Denoting the maximum potential by \( \phi \), and comparing the conduction current to the displacement current, we find that conduction current dominates if

\[ 2.5 \left( \frac{\omega_{pi}}{\omega} \right) \left( \frac{T_e}{e\phi} \right)^{1/4} > 1 \]  

(18b)

Unless the potential is very large, this is nearly equivalent to the to the condition for the ions to cross the sheath quickly,

\[ \frac{\omega_{pi}}{\omega} > 1 \]  

(18c)

These define the region for an MEO sheath with conduction current dominant (we call this an MEOCC sheath). In the parameter space defined at the end of Section II, the MEOCC sheath exists along the horizontal axis for values of \( \omega_{pi}/\omega \) Greater than unity.

We now consider The MEO sheath where displacement current dominates We call this the MEO\( \omega \) sheath. The equation for this is

\[ 0.25 \frac{nT_e}{2\pi} \left[ \frac{2e}{\phi^3T_e} \right]^{1/4} \frac{d\phi}{dt} = -J_o \sin \omega t \]  

(19)

Of course the displacement current can only dominate for \( J<0 \), so we consider the solution to Eq.(19) for times \( 0<t<\pi/\omega \). Let us say that at \( t=0, \phi=0 \). Then we find that the potential as a function of time is

\[ -\phi(t) = \frac{32\pi^2 J_o}{n^2 T_e e^4} \left[ \cos \omega t - \frac{1}{2} \right]^2 \equiv \phi \left[ \cos \omega t - \frac{1}{2} \right]^2 \]  

for \( 0<t<\pi/\omega \)  

(20a)

or an expression for the maximum voltage in more familiar units is
\[
\phi (\text{volts}) = 2.4 \frac{J_o (mA/cm^2)}{\left( \frac{n (cm^{-3})}{10^{10}} \right)^2 T_e (eV) \left( \frac{f}{13 \text{MHz}} \right)^4}
\]  

(20b)

The scaling of the Voltage with parameters is the same as in the GL sheath, but the coefficient is larger by about a factor of 5. However the ions do not strike the wall with a mono-energetic energy distribution any longer, depending on the time they enter the sheath, their energy varies from zero to the maximum. The average value of the ion energy is three eighths the maximum value. Furthermore, this ion acceleration exists only for half of the rf cycle (we will discuss the other half cycle shortly). Hence a time average over the entire cycle reduces the effective potential by an additional factor of 2, so that when all the averaging is done, potential is reasonably comparable to that in the GL model.

Let us now consider the dynamics of the formation of the rf sheath in the regime where the displacement current dominates. At t=0, the plasma is in contact with the wall when a negative voltage is applied. Ions are accelerated into the wall and the electrons move away. At each instant, the sheath region appears to be a Langmuir Child ion diode as specified by Eq. (6). However now the current is the displacement current,

\[
J = ne \frac{ds}{dt}
\]  

(21)

For a voltage pulse specified as a function of time, we can multiply the Langmuir Child relation, Eq.(6) by \(s^2\) and integrate each side in time from zero to the voltage maximum. The left hand side simply gives \(s^3/3\). This is the basic idea used in calculating the ion dose to a substrate in plasma immersed ion implantation [11,12]. Assuming a voltage pulse given by Eq.(20), we find that

\[
s_{\text{max}} (cm) = 2.6 \times 10^{-3} \phi^{1/2} (\text{volts}) \left( \frac{M_{\Delta}}{M} \right)^{1/6} \left( \frac{n}{10^{10}} \right)^{1/3} \left( \frac{f}{13 \text{MHz}} \right)^{1/3}
\]  

(22)

Assuming that the ions accelerates through the sheath through the potential \(\phi\), we find that \(\omega \tau\) is given by

\[
\omega \tau \approx 0.4 \left( \frac{\omega}{\omega_{pi}} \right)^{2/3}
\]  

(23)

confirming our original condition for the validity of the rf sheath in this region. Thus the MEO sheath with conduction current dominant occurs for
\[
\frac{\omega_{pi}}{\omega} > 1, \quad \frac{v_{se}}{c_s} > 1
\] (24)

that is the upper region in the parameter space we have specified. Once the voltage reaches its maximum, the current changes sign, and the rf sheath quickly collapses. For positive current, both conduction and displacement current may be important, but in any case the voltage is low, and the total current in this half cycle simply cancels the total current in the first half cycle.

There is one additional problem regarding the MEOω sheath. Every cycle the sheath reaches into the plasma and pulls \( n s_{max} \) ions per unit area into the wall. Thus the average flux of ions is \( n s_{max} \omega / 2\pi \). However the maximum flux which can be supplied by the plasma external to the sheath is \( n c_s \). Here recall that \( n \) is the value at the edge of the sheath. Thus the MEOω sheath can only function as described if

\[
\frac{s_{max} \omega}{2\pi c_s} < 1
\] (25a)

Inserting the parameters for \( s_{max} \) which we have derived, we find that in the \( \omega_{pi}/\omega, v_{se}/c_s \) plane, this condition reduces to

\[
\left( \frac{v_{se}}{c_s} \right)^2 \left( \frac{\omega_{pi}}{\omega} \right)^{4/3} < 7
\] (25b)

Thus in our parameter space, in the upper most region, the plasma is not able to supply the necessary flux demanded by the rf sheath. In this upper region, for the rf sheath to exist, it appears the rf must modify the ambient plasma. It may do this in at least three ways. First it can supply additional electron heating so as to appropriately increase \( c_s \); secondly, it may drain the plasma until the density is low enough that Eq.(25b) is satisfied; and thirdly, it may produce additional ionization in the sheath so that the ion drain is equal to the ion input (flowing in at the sound speed) plus the ions produced in the sheath region by the rf fields. In other words, the rf sheath not only accelerates the existing plasma to the wall, it also must modify the plasma in the sheath region and/or in the bulk. The region of our parameter space where the sheath does not modify the plasma is rather limited, and considering the approximations made, the MEOω sheath without plasma modification may exist only as an idealization.

IV The Modified DC (MDC) Sheath

Let us now consider the rf sheath in the case of low current and low density. By low density, we mean that the ions are unable to respond to the rf fields, and only respond to the
dc fields. This means of course that $\omega_{pl}/\omega < 1$. Also we assume that $v_{\infty}/c_s < 1$, so as to distinguish it from the low density Godyak/Lieberman model of an rf sheath. At low density and low current, a matrix sheath model [13] is a reasonable one; that is a model in which the electrons oscillate away from the wall, but the ion charge density is unperturbed. The width of the sheath is now $2s_e = 2J_e/ne\omega$, the maximum potential drop between the sheath edge and wall is

$$\phi = 2\pi ne_{\infty}^2 = 8\pi ne\frac{v_{\infty}^2}{\omega^2}$$

(26)

and the maximum ion energy gain compared to the entrance energy, $0.5Mc_s^2$ is given by

$$\frac{\Delta E_{max}}{2Mc_s^2} = \frac{4\omega_{pl}^2v_{\infty}^2}{\omega^2c_s^2}$$

(27)

Thus, in the regime in which we have defined the sheath, the ion energy change is relatively small. This justifies the assumption that ion density is unperturbed. The time for an ion to cross the sheath is roughly $s_e/c_s$. Comparing this to $\omega^{-1}$, we see that the crossing time is smaller than $\omega^{-1}$ by a factor of $v_{\infty}/c_s$.

However there is normally a dc sheath if the wall is at the floating potential ($J=0$). Thus the dc sheath has an additional negative potential drop of $0.5T_e\ln[M/2\pi n]$. Hence from the dc sheath alone the ions strike the wall with energy $0.5T_e\{1 + \ln[M/2\pi n]\}$. The effect of the rf is then only to slightly modify the dc sheath, by spreading the energy of ions striking the wall over an energy range

$$0.5T_e\left(1 + \ln\frac{M}{2\pi n}\right) < E < 0.5T_e\left(1 + \ln\frac{M}{2\pi n} + \frac{4\omega_{pl}^2v_{\infty}^2}{\omega^2c_s^2}\right)$$

(28)

In the parameter space we have defined, this sheath is characterized by the region near the origin. Since it only modifies the dc sheath slightly, and in doing so, only spreads out the nearly uniform energy distribution, one would not normally use an rf sheath in this regime for plasma processing.

V Parameter Space for Collisional rf Sheaths

We have seen that there are a number of different possible regimes for the behavior of rf sheaths in collisionless plasmas. It is possible to plot these regions on a simple two dimensional plot where the horizontal axis is $\omega_{pl}/\omega$, and the vertical axis is $v_{\infty}/c_s$. Figure 3 shows such a plot. We have found simple analytic models of the sheath in each of the regions. Also shown in the $\text{MEO}\omega$ region is the curve, above which the sheath cannot exist unless the rf which forms the sheath also increases the electron density and/or electron temperature at the sheath edge, so that the exterior plasma can supply the
Fig. 3 - The Parameter Space
necessary particle flux. The basis of these models are that the ions density is either so low, that the ions cannot respond to the high frequency fields; or is so high that they respond to the rf fields as if they were dc. The only model which predicts a monoenergetic flux of ions impinges on the substrate is the low density, high current limit, the GL sheath. The ions pick up a distribution in energy as one goes to either high density or low current.

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References


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