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**Analytical Models for Battlespace  
Information Operations  
(Bat-IO)  
Part 2**

by

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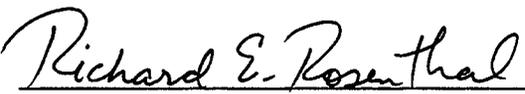
  
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# **Analytical Models for Battlespace Information Operations (BAT-IO)**

Donald P. Gaver  
Patricia A. Jacobs

## **Part 2**

### **Battlespace Information War: Attack and Defense of a Region with Adaptive Opponent Behavior, Including Deterrence**

#### **EXECUTIVE SUMMARY**

The coordination of information acquisition and interpretation to direct force application is increasingly recognized as a crucial military systems design and investment issue. This paper illustrates tradeoffs between Blue/own regional Attacker sensor and shooter capabilities: it studies a deep strike or SCUD-hunting scenario in a low-resolution, aggregated manner using an analytical state-space approach that recognizes gross aggregated regional Defender (Red), and regional Attacker (Blue), system capabilities and limitations. Emphasis is accordingly placed on explicitly modeling the availability and utilization of information to a striking Attacker, as it becomes available from a realistically finite sensor and C2 capability. The (imperfect) information on opposition units, the Defenders, that are candidates for prosecution by the Attackers is passed to

the finite, hence saturable (here missile-firing) Attacking force, the shooters, that then responds by prosecuting those units.

The models specifically recognize that regional Defenders will not be detected immediately, nor recognized perfectly, nor are Defender shots (e.g. SCUD launcher) fired perfectly, or immediately. Furthermore, attempts to effectively target are also realistically modeled as afflicted by *imperfect Attacker battle damage assessment* (BDA), an incapability that, if pronounced, will tend to non-linearly saturate shooters, increase their response times, and hence reduce targeting effectiveness and efficient ammunition expenditures. Such models can allow for adaptation by both attackers and defenders to recent fortunes: if Defender presence and activity is effectively countered by Attackers, then the former may tend to be deterred or withdraw; if not, the Defenders are motivated to press their apparent advantage. Sharp, threshold-like, responses can follow from the possibly multi-stable dynamics. This behavior will be explored in this report.

The present models are mainly deterministic or pseudo-stochastic in that they represent the non-linear *effect* of stochastic saturation approximately, but adequately. However, they can straightforwardly be "made stochastic", especially Markovian, and so realized using Monte Carlo simulations. Computer programs exist to provide numerical results; some are given. A simple one-dimensional stochastic (Markov birth-death) model is given as an appendix to this paper. This model can be shown analytically and numerically to exhibit "stochastic bi-stability" properties (two attractors) that under certain circumstances (parameter combinations) lead to *bimodal* steady-state distributions (if such are allowed to happen by the dynamics, and are of interest). Such a tendency will occur also in more detailed, but less analytically tractable models.

There are many problem elements that have been initially and purposefully ignored. They are addressed in later work; see Gaver and Jacobs (1999). For instance, the effects of different target types, false targets, and decoys must be added (some "decoys" are in effect present, in the form of killed Defenders, not

so recognized, that are mistakenly re-targeted). The effect of different principles for Attacker target prioritization under uncertainty, i.e. dynamic scheduling, requires systematic attention. In the present models Attackers are invulnerable to attack; this is not always realistic, and can be changed to a duel-like scenario involving suppression of enemy (Defender) air defense (SEAD); a first paper on this topic is Gaver and Jacobs (1998). In the current paper Attackers employ generic missiles only, but the use of (vulnerable, manned) Attack aircraft can similarly be modeled, as can combinations of Attack aircraft, Naval gunfire, and missiles, recognizing the coordination difficulties. Employment of cued reconnaissance aircraft, possibly UAVs, can likewise be represented quantitatively as state-space components. In addition, refinements that more faithfully represent spatial and perhaps other environmental constraints can be incorporated, as can details of communications assets and message-handling protocols in use by both Attackers and Defenders.

The present paper describes some of the possibilities for insights inherent in an enhanced state-space approach. As pointed out, many elaborations are possible. The objective is to recognize only that detail in the (preliminary) models that is sufficient to hint at payoff from adding suitable assets and strategies at appropriate points in the entire system. Finer detail and resolution is left to others to include, and possibly profit by. More elaborate and high-resolution models within such tools as NSS (METRON), and JWARS eventually can focus with greater intensity on some of the issues raised here.

In general we believe that this report is in accord with many of the views and suggestions of Ilachinski (1996), and also of Dockery and Woodcock (1993), and others. Those two publications contain many references, some to previous work on related topics. Our emphasis on explicit representation of protagonists' *information state* is of interest at the time of writing.

# **Battlespace Information War: Attack and Defense of a Region with Adaptive Opponent Behavior, Including Deterrence**

**(BAT-IW)**

**Part 2**

**D.P. Gaver**

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## **1. Introduction: Generic Adaptive Staging**

Suppose a particular territorial region,  $\mathcal{R}$ , is selected by an attacking force, denoted as Red, as one possible stage from which to assemble, maneuver, and possibly launch attacks. An example is the designation of  $\mathcal{R}$  as a region in Iraq from which TELS/SCUDS missile systems launched attacks on Israel at the time of the Gulf War; another such region was the staging arena for attacks on Saudi Arabia. But the region could equally well be occupied by various armed units that would converge on a particular location within or on the boundary of  $\mathcal{R}$  in case it were attacked, e.g. by amphibious landing operation.

Let the force opposing Red utilization of region  $\mathcal{R}$  be denoted generically as Blue. In the present scenario Blue's assets consist of C4ISR systems ("sensors" plus a communication system) capable of detecting, tracking and targeting Red assets, plus weapons for attrition (or suppression) of those assets (the

“shooters”). Importantly and realistically, the Blue sensors and shooters are represented as of limited and fallible capability: Red forces in  $\mathcal{R}$  are detected and targeted by Blue C4ISR only after delay, and then with occasional error, and shooter assets are sometimes saturated and hit and kill Red assets with less than perfect success. Shooter assets considered here are a generic surface-surface missile system, e.g. ATACMS; with some additional effort a force of manned aircraft could alternatively, or as a complement, be envisioned and modeled. Included in the above model properties is the explicit recognition that follow-up of a Blue attack on a detected Red, i.e. *battle damage assessment* (BDA), is error-prone: not only may a dead target be mis-identified as alive and wastefully retargeted, acting effectively as a decoy, but an alive target, the subject of an unsuccessful attack, can be ignored and hence allowed to profitably reposition or re-attack.

### 1.1 Adaptive Staging

It is natural to expect that the conflict scenario portrayed occur dynamically. For example, Red forces may first infiltrate  $\mathcal{R}$  with intention of establishing a prescribed strength there, perhaps occasionally carrying out individual attacks. In response, Blue forces will focus increased sensor and shooter attention on the region. If such resistance is quick and effective Red activity will tend to adapt by tactics that reduce vulnerability, but also effectiveness: they will be *deterred* (at least in the region under consideration). Correspondingly, Blue force attention to the region may well then be limited, perhaps re-directed. Red occupancy of  $\mathcal{R}$  settles at an annoyance level. On the other hand, if Red forces once surpass the combined sensor-shooter forces of Blue the region will tend to settle at a considerably higher threat level – one considerably more difficult for Blue to dislodge.

The purpose of this paper is to connect various aspects of the above information-influenced scenario described by simple dynamic models that dramatically exhibit the sensitivity of conflict outcome to opponent's assets and operational behavior. The models proposed for first study are aggregated and deterministic, but still provocative. They can both be readily, and informatively, extended to far more detailed higher resolution tools that give more refined and hence perhaps useful (or at least easily defensible) guides to investment decisions (e.g. sensor and weapons mix, target prioritization, etc.). The approaches of the present paper provide a quick preliminary appraisal of a hypothetical situation; polish-up can be accomplished subsequently.

## **2. Model I: The "SCUDWORM" Analogy**

About twenty years ago, D. Ludwig identified and explored an intriguing dynamic situation pertaining to the interlinked behavior of a pest, the spruce budworm, and birds that feed upon populations of them. See Ludwig *et al.* (1978) and more recently Murray (1989). When spruce are hospitable, presumably when buds appear, populations of budworms appear to feed upon them and grow; in turn, the population of worms attracts birds that consume the worms, with attractiveness increasing with worm population size. If both budworm and bird population sizes are ultimately limited the dynamics are sometimes consistent with *two* (non-zero) local stable points for worm (and local bird) populations: one low, and one high. The simple deterministic model implies that once (initial) conditions bring worm and bird populations near either such point they remain there indefinitely, even though all descriptive rate parameters are otherwise unchanged. Only by adding more to the problem, e.g. by modifying the process to become stochastic or by incorporating time dependence, will escape from

quasi-steady state occur; this may occur as a result of smallish change of conditions, and hence appear instantaneous.

We exploit the analogy with the above predator-prey ecological situation in the first model below. Qualitative correspondence to the region-invasion scenario and simplicity and transparency are its major virtues. It is blatantly oversimplified, a fault that is partially rectified in Model II.

## 2.1 Mathematical Model I: the "SCUDWORM" Problem

What follows is a small modification of the work of Ludwig. Let

$A(t)$  = number of attackers in  $\mathcal{R}$  at  $t$ ;

$D(t)$  = number of defenders in  $\mathcal{R}$  at  $t$ .

Here  $t$  is any real number, although discrete-time versions of the problem are certainly possible, and possibly attractive.

Stipulate that these differential equations describe the evolution of the populations:

$$\frac{dA(t)}{dt} = \underbrace{\lambda A(t)(1 - A(t)/\bar{A})}_{\text{(logistic) arrival rate of attackers into } \mathcal{R}} - \underbrace{\mu \bar{D}(D(t)/1 + D(t))p_K}_{\text{kill rate of attackers by defenders}} \quad (2.1)$$

$$\frac{dD(t)}{dt} = \underbrace{\beta A(t)}_{\text{rate of defender force allocation to region } \mathcal{R}} - \underbrace{\theta D(t)/A(t)}_{\text{rate of defender force removal from } \mathcal{R}} \quad (2.2)$$

The above setup cavalierly attributes certain simplistic behavior to both attacker (logistic increase in the region), and the defender (attacker queuing for deterministic service). More realistic and inclusive representations appear in the next model.

The model of (2.1) and (2.2) can be modified to allow attackers to enter the region when  $A(t) = 0$  and for numerical stability as follows.

$$\frac{dA(t)}{dt} = [\lambda_0 + \lambda A(t)][1 - A(t)/\bar{A}] - \mu \bar{D} [D(t)/(1 + D(t))] p_K \quad (2.2a)$$

$$\frac{dD(t)}{dt} = \beta A(t) - \theta D(t)/(A(t) + c) \quad (2.2b)$$

where  $c$  is a small constant.

The rightmost term of (2.1) deterministically represents the expected "service" rate of a  $\bar{D}$ -server/missile shooter; see Filipiak (1988) which relies on the ideas of Rider (1967) and Agnew (1976). The first right-side term of (2.2) states that defender force allocation to the region is enhanced at a rate proportional to the current attacker population size; presumably refinable to  $\beta = \beta^* \xi$  where  $\beta^*$  is a decision parameter and  $\xi$  is the rate of Blue sensor coverage of  $\mathcal{R}$ . The second, or rightmost, term of (2.2) postulates that defense force depletion is proportional to that force, but inversely proportional to attacker force size: if the latter becomes small, defenses become smaller. These rules are arbitrary but plausible (e.g.  $\beta A(t)$  could be replaced by almost any increasing function of  $A(t)$ , and  $\theta D(t)/A(t)$  by a decreasing function of  $A(t)$ , without altering qualitative effects). It is again acknowledged that certain limited-information or incomplete battle-damage assessment (BDA) is not represented in the present abbreviated model. For those features see Model II, detailed in Section 3.

Look for the possible stable points of (2.1), (2.2). These ( $A = A(\infty)$ ,  $D = D(\infty)$ ) satisfy

$$0 = \lambda A(1 - A/\bar{A}) - \mu p_K \bar{D} \frac{D}{1 + D} \quad (2.3)$$

$$0 = \beta A - \theta D/A. \quad (2.4)$$

Substituting the solution of (2.4), i.e.  $D = \left(\frac{\beta}{\theta}\right)A^2$ , into (2.3) we get

$$\lambda A(1 - A/\bar{A}) = \mu p_K \bar{D} \frac{\left(\frac{\beta}{\theta}\right)A^2}{1 + \left(\frac{\beta}{\theta}\right)A^2}, \quad (2.5)$$

a cubic with the possibility of one or three real roots. Notice that if we apply the *quasi-steady static approximation* (QSSA), cf. Segel and Slemrod (1989), letting  $\beta$  and  $\theta$  be relatively fast in (2.2) and putting the result into (2.1), the result is

$$\frac{dA(t)}{dt} = \lambda A(t)(1 - A(t)/\bar{A}) - \mu p_K \bar{D} \frac{\left(\frac{\beta}{\theta}\right)A^2(t)}{1 + \left(\frac{\beta}{\theta}\right)A^2(t)} \quad (2.6)$$

which is equivalent to the original model proposed by Ludwig and hence subject to nearly the same analyses, accessible in Murray (1989). These are based on identifying  $A(t) = A = 0$  as one stable point and factoring it out, then examining

$$1 - A/\bar{A} = \frac{\mu}{\lambda} p_K \bar{D} \left( \frac{\left(\frac{\beta}{\theta}\right)A}{1 + \left(\frac{\beta}{\theta}\right)A^2} \right) \quad (2.7)$$

graphically, i.e. plotting the left-hand and the right-hand sides:

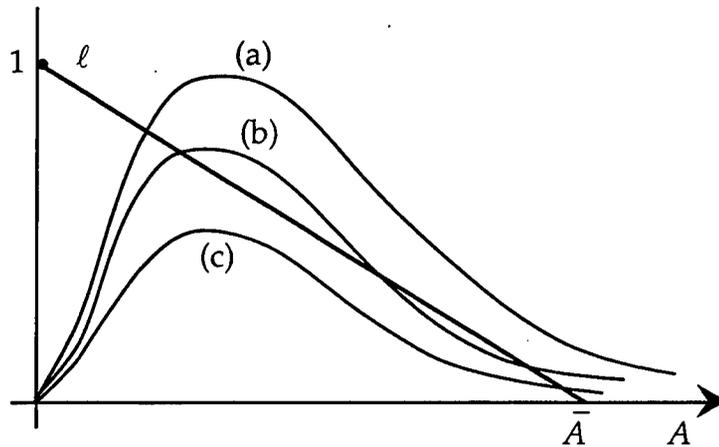


FIGURE 1

The crossings/intersections of left-hand and right-hand sides of (2.6) are stable or local equilibrium points. The curve (a) can represent relatively rapid and effective shooter service, and so provides for a small equilibrium attacker population (intersection of  $(\ell)$  and (a)). The curve (c) represents the opposite: slow shooter service or perhaps inadequate sensor coverage ( $\beta = \beta^* \xi$  with  $\xi$  small) hence a single equilibrium point very near the maximum regional occupancy chosen by Red (intersection of  $(\ell)$  and (c)). Finally, the curve (b) represents the interesting *bi-stable case*: the basic parameters are consistent *either* with a small attacker population (provided the initial population is small), (left-most intersection of  $(\ell)$  and (b)), or a much larger population if the initial population size is near the rightmost intersection of  $(\ell)$  with (b).

It is suggestive to “stochasticize” the above system, replacing the system (2.3) and (2.4), or more accurately (2.6) by a birth-death Markov process approximation that possesses a stationary distribution. See the appendix for details.

### 3. Model II: Model with Arrivals and Departures of Attackers and Defenders and Simplified Surveillance: Delayed BDA

Consider this more elaborate and realistic model: Red units (attackers) enter  $\mathcal{R}$  at a specified rate which is proportional to the number of Red units that are in the region and becomes 0 when the number of Red units reaches a maximum  $\bar{A}$ , i.e. following logistic growth as in (2.1); (modifications allowed without violating qualitative effects). The Red units are the subject of a prescribed level of surveillance/reconnaissance, are identified (possibly incorrectly) as alive, and are eventually targeted by members of a group of Defenders (Shooters). The Defenders also arrive at, or focus assets on, the region at a rate proportional to the number of attackers that have been detected. The arrival rate becomes 0 once a prescribed number of defenders is in the region.

Note that the present model explicitly accounts for realistic effects that Model I explicitly ignores, namely imperfect BDA, track loss, defender evasion (leaving the region to shake off pursuit – possibly inferred or imagined – and other such realisms).

### Parameters for Model II

- $\bar{A}$  = Maximum number of attackers in the region  $\mathcal{R}$
- $\bar{D}$  = Maximum number of defenders in the region  $\mathcal{R}$ . This is the number of individual shooters that can engage attacker targets at a time.
- $\beta$  = Arrival rate of defenders to the region  $\mathcal{R}$
- $\theta$  = Departure rate of defenders from the region  $\mathcal{R}$
- $c_s$  = constant
- $\lambda$  = Attacker arrival rate into region  $\mathcal{R}$ . Although this is first treated as a positive constant, it can be made a function of time, or even of the state of Red forces in the region. For example, Red may wish to amass a certain force size by a particular time to oppose an amphibious landing. In short,  $\lambda$  again stands for a simple control variable (“knob”) that in reality can be adjusted by Red.
- $\mu$  = Attacker attack/“service” rate ( $1/\mu$  = mean time to track, shoot, flight time of missile).
- $\nu$  = Track-loss rate ( $1/\nu$  = mean of a holding time of attacker in track).
- $\alpha$  = Attacker (e.g. TEL = SCUD launcher) shoot rate ( $1/\alpha$  = mean time between shots by a single attacker). This parameter is irrelevant under certain circumstances; it is relevant to SCUD-like attack.
- $\gamma$  = Rate attackers leave region ( $1/\gamma$  = mean holding time of attacker unit in region). An attacker decision variable.
- $\xi$  = Rate undetected/tracked attackers are acquired by sensors/ $C^2$  system.
- $p_K$  = Probability an attacked target is killed. This parameter is currently/ presently taken to be non-range-dependent.

$R_{aa}$  = Probability that an alive target that is correctly classified as being alive by the sensors/C<sup>2</sup> system.  $1 - R_{aa} = R_{ad} = P(\text{target classified or perceived as dead} | \text{alive})$ .

$R_{da}$  = Probability that a dead target is misclassified as being alive (hence potentially retargeted; effectively a *decoy*).

## State Variables for Model II

The following state variables are needed to describe the above dynamical system:

$A_u(t)$  = Number of *undetected* live (hence potentially active and threatening) Red attackers present in region  $\mathcal{R}$  at time  $t$ .

$A_d(t)$  = Number of *detected* live Red attackers present at time  $t$ . These are on the Blue shooter's target list, and will be engaged unless lost by the sensor system (they may go into hiding, or even leave the region covered by the surveillance, e.g. JSTARS).

$D_a(t)$  = Number of *detected and perceived to be alive*, hence potentially Blue-targeted, but actually *dead* Red attackers at  $t$ . These are present because Blue battle damage assessment (BDA) is realistically imperfect.

$D_u(t)$  = Number of dead attackers in region that are not yet classified. Classification to be done by the surveillance/reconnaissance system.

$S(t)$  = Number of defenders (shooters) in the region  $\mathcal{R}$

For the deterministic modeling of saturable service by the defense shooters (presumably missile launchers in the present context) we make use of the following approximation, cf. Filipiak (1988), Agnew (1976), and also Rider (1967), for the saturable rate of processing by the shooters,

$$H_S(t) = \frac{[A_d(t) + D_a(t)]\mu S(t)}{1 + [A_d(t) + D_a(t)]}$$

## Transition Equations for Model II

$$\begin{aligned} \frac{dA_u(t)}{dt} = & \lambda(A_u(t) + A_d(t)) \left[ 1 - \frac{A_u(t) + A_d(t)}{A} \right] \\ & + \frac{A_d(t)}{A_d(t) + D_a(t)} H_S(t) [1 - p_K] \end{aligned} \quad (3.1a)$$

$$+ vA_d(t) - A_u(t)(\alpha + \gamma) - \xi R_{aa} A_u(t)$$

$$\begin{aligned} \frac{dA_d(t)}{dt} = & \xi R_{aa} A_u(t) + \alpha A_u(t) - (\gamma + v)A_d(t) \\ & - \frac{A_d(t)}{A_d(t) + D_a(t)} H_S(t) \end{aligned} \quad (3.1b)$$

$$\frac{dD_a(t)}{dt} = \xi R_{da} D_u(t) - \frac{D_a(t)}{A_d(t) + D_a(t)} H_S(t) \quad (3.1c)$$

$$\frac{dD_u(t)}{dt} = \frac{A_d(t)}{A_d(t) + D_a(t)} H_S(t) p_K - \xi D_u(t) + \frac{D_a(t)}{A_d(t) + D_a(t)} H_S(t) \quad (3.1d)$$

$$\frac{dS(t)}{dt} = \beta(A_d(t) + D_a(t)) \left[ 1 - \frac{S(t)}{D} \right] - \theta \frac{S(t)}{A_d(t) + D_a(t) + c_s} \quad (3.1e)$$

## Solutions

It is possible to obtain some initial information in the form of steady-state or long-run solutions. Set the derivatives of the equations (3.1a) – (3.1e) equal to 0 to find:

$$\begin{aligned} 0 = & \lambda[A_u + A_d] \left[ 1 - \frac{A_u + A_d}{A} \right] \\ & + \frac{A_d}{A_d + D_a} H_S [1 - p_K] + vA_d - A_u(\alpha + \gamma + \xi R_{aa}) \end{aligned} \quad (3.2a)$$

$$0 = \xi R_{aa} A_u + \alpha A_u - (\gamma + v)A_d - \frac{A_d}{A_d + D_a} H_S \quad (3.2b)$$

$$0 = \xi R_{da} D_u - \frac{D_a}{A_d + D_a} H_S \quad (3.2c)$$

$$0 = \frac{A_d}{A_d + D_a} H_S p_K - \xi D_u + \frac{D_a}{A_d + D_a} H_S \quad (3.2d)$$

$$0 = \beta(A_d + D_a) \left[ 1 - \frac{S}{\bar{D}} \right] - \theta \frac{S}{A_d + D_a + c_s} \quad (3.2e)$$

Solving (3.2d) for  $D_u$  and substituting the expression into (3.2c) results in

$$D_u = \frac{1}{\xi} \left\{ \frac{A_d}{A_d + D_a} H_S p_K + \frac{D_a}{A_d + D_a} H_S \right\}$$

and

$$0 = R_{da} \left\{ \frac{A_d}{A_d + D_a} H_S p_K + \frac{D_a}{A_d + D_a} H_S \right\} - \frac{D_a}{A_d + D_a} H_S$$

$$D_a(1 - R_{da}) = R_{da} A_d p_K$$

$$D_a = \frac{R_{da} p_K}{1 - R_{da}} A_d \equiv c A_d$$

Solving (3.2e) for  $S$  results in

$$S = \frac{\beta(A_d + D_a)}{\frac{\beta(A_d + D_a)}{\bar{D}} + \frac{\theta}{A_d + D_a + c_s}} = \frac{\beta(1+c)A_d \bar{D} ((1+c)A_d + c_s)}{\beta(1+c)A_d ((1+c)A_d + c_s) + \theta \bar{D}}$$

Equation (3.2b) results in

$$A_u = \frac{1}{\xi R_{aa} + \alpha} \left[ (\gamma + \nu) A_d + \frac{A_d}{1 + (1+c)A_d} \mu S \right]$$

Substitute  $A_u$  into equation (3.2a) and (numerically) solve for  $A_d$ .

### Numerical Example

For the parameters  $R_{ad} = R_{da} = 0.3$ ,  $\beta = 10$ ,  $\lambda = 10$ ,  $\bar{D} = 10$ ,  $\xi = 500$ ,  $\alpha = 5$ ,  $p_K = 0.7$ ,  $\nu = 0.5$ ,  $\gamma = 0.1$ ,  $\theta = 10$ ,  $c_s = 0.1$ ,  $\mu = 20$ , the steady state equations (3.2a) – (3.2e) have 4 solutions

$$(A_u, A_d, D_a, D_u, S) = (0, 0, 0, 0, 0)$$

$$(A_u, A_d, D_a, D_u, S) = (0.04, 1.07, 0.32, 0.03, 1.72)$$

$$(A_u, A_d, D_a, D_u, S) = (0.4, 13.4, 4.0, 0.28, 9.7)$$

$$(A_u, A_d, D_a, D_u, S) = (0.48, 33.7, 10.1, 0.30, 9.95).$$

The solutions  $(0, 0, 0, 0, 0)$  and  $(0.4, 13.4, 4.0, 0.28, 9.7)$  are unstable equilibrium points. The solutions  $(0.04, 1.07, 0.32, 0.03, 1.72)$  and  $(0.48, 33.7, 10.1, 0.30, 9.95)$  are stable equilibrium points. That is, for the present example there are two distinct steady states: inhabitation of one or the other depends on where the system started from, i.e. the initial conditions.

Thus, the differential equations can have different limiting solutions depending on the initial conditions, a classical multistable situation. In particular, for the present numbers:

$$\text{if } A_u(0) = 5, A_d(0) = 0, D_a(0) = 0, S(0) = 0, D_u(0) = 0,$$

$$\text{then } A_u(\infty) = 0.05, A_d(\infty) = 1.07, D_a(\infty) = 0.32, S(\infty) = 1.73; D_u(\infty) = 0.031.$$

In this case the defenders were able to oppose the attackers before buildup occurred, and to force the number present to nearly zero. This might well be the fruit of superior intelligence *and* the available assets to capitalize on it. However,

$$\text{if } A_u(0) = 30, A_d(0) = 0, D_a(0) = 0, S(0) = 0, D_u(0) = 0,$$

$$\text{then } A_u(\infty) = 0.48, A_d(\infty) = 33.7, D_a(\infty) = 10.1, S(\infty) = 9.95, D_u(\infty) = 0.30.$$

Then in this case the Blue force was presumably oblivious to attackers until their number became substantial, (possibly a failure of intelligence and/or overall surveillance) after which defenders swung into action and detected and targeted those defenders in the region. The number of shooters appears inadequate because the number of attackers queued for shooter service eventually is higher than the initial number in the region. If assets are available this deficiency might well be corrected in real time. But the present model allows anticipation of such a possible state of affairs, and might allow forestalling it. This is "real dynamics" that is unmodeled, but for which models are useful.

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## APPENDIX

### A Birth-Death Markov "SCUDWORM" Model

Suppose that  $A(t)$  now represents a Markov chain in continuous time with state space  $\{0, 1, 2, \dots, \bar{A}\}$ . Here  $\bar{A}$  is an integer. Furthermore, let defender adaptation dynamics, as represented by  $\beta$  and  $\theta$  be fast as compared to parameters  $\lambda$  and  $\mu$ , and invoke QSSA. Then we replace the deterministic differential equation (2.6) by a birth-death process with generator

$$P\{A(t+dt) = A(t) + 1 | A(t)\} = \lambda A(t) \left(1 - \frac{A(t)}{\bar{A}}\right) dt + o(dt) \quad (\text{A.1})$$

and

$$P\{A(t+dt) = A(t) - 1 | A(t)\} = \mu p_K \frac{\bar{D} \left(\frac{\beta}{\theta}\right) A^2(t)}{1 + \left(\frac{\beta}{\theta}\right) A^2(t)} dt + o(dt); \quad (\text{A.2})$$

there is a natural boundary at  $A(t) = 0$  so no transition to negative values is possible. Additionally,

$$P\{A(t+dt) = A(t) | A(t)\} = 1 - \left[ \lambda A(t) \left(1 - \frac{A(t)}{\bar{A}}\right) + \mu p_K \frac{\bar{D} \left(\frac{\beta}{\theta}\right) A^2(t)}{1 + \left(\frac{\beta}{\theta}\right) A^2(t)} \right] dt + o(dt). \quad (\text{A.3})$$

All other transitions have negligible probability  $o(dt)$  as  $dt \rightarrow 0$ . Because of the non-linearity of the transition functions it is *not* true that  $E[A(t)]$  satisfies the differential equation (2.6), although there will be qualitative resemblance between  $E[A(t)]$  and the differential equation solution  $A(t)$  as functions of  $t$ . It should also be possible to show that  $A(t)/\bar{A}$  tends weakly to  $a(t) = A(t)/\bar{A}$  as  $\bar{A}$  becomes large, provided  $\bar{D} = \bar{d}\bar{A}$ ; furthermore it should be the case that, for large  $\bar{A}$ ,  $A(t)$  can be well approximated as the sum of a deterministic trajectory and diffusion process. This option is not investigated here; something simpler is.

The stochastic model (A.1), (A.2), (A.3) is clearly richer than its deterministic counterpart or approximation, (2.1) – (2.6). Furthermore, the present simplified stochastic version can be studied reasonably explicitly and *analytically*, i.e. without Monte Carlo simulation or intricate numerical computation. Various questions will now be addressed. For simplicity put

$$P\{A(t+dt) = a+1 | A(t) = a\} = \lambda_a dt + o(dt) \quad (\text{A.4})$$

and

$$P\{A(t+dt) = a-1 | A(t) = a\} = \mu_a dt + o(dt) \quad (\text{A.5})$$

where  $\lambda_a$  and  $\mu_a$  are gotten from (A.1) and (A.2). Look at some options.

### (1) Stationary solution

As (A.1) is presented, it describes an absorbing chain with certain absorption at  $A(t) = 0$ . Modify this by introducing an *attacker import rate*  $\lambda_0$  ( $\lambda_0 > 0$ ): with probability  $\lambda_0 dt$  a new attacker appears if the region becomes empty. Now using well-known formulas one can find

$$\lim_{t \rightarrow \infty} P\{A(t) = a | A(0)\} = \pi_0 \frac{\lambda_0 \lambda_1 \dots \lambda_{a-1}}{\mu_1 \mu_2 \dots \mu_a} \equiv \pi_a, \quad 0 \leq a \leq \bar{A}, \quad (\text{A.6})$$

where  $\pi_0$  normalizes  $\pi_a$  to sum to unity. This probability mass function  $\pi_a$  can be studied analytically, or, more easily, evaluated and studied numerically. Exploration shows that it *can* be bimodal, with modes corresponding roughly to the two stable points that may occur for the deterministic model. An example is provided in Figure A1. The operational interpretation is that, over a long period of time attacker numbers may typically reside near either a low value, or a higher – obviously more dangerous – level. The chance of being at that level can be controlled by enhancing defense shooter assets. In a rough sense the  $A(t)$  process tends to be near one or the other of the two stable points, remaining at

one place for a nearly exponential time period, then jumping to (near) the other, and so on back and forth until the parameters change. The defender's job is to cost-effectively lengthen the time attacker state resides at a low level, while the attacker presumably wants her state to remain high, but also to carry out aggressive acts, such as launching SCUDs.

## (2) First-Passage or Exit Times

Suppose that initially, i.e. at  $t = t'$  (where  $t'$  can be taken to be 0 because of stationary transitions)  $A(t') = i$ , any particular state value. Then it is of interest to study

$$\tau_{ij} = \inf\{t : A(t+t') = j | A(t') = i\} \quad (\text{A.7})$$

the *first-passage time* (fpt) from  $i$  to  $j \neq i$ ; this is the random time required to pass, for the first time, from state  $A(t') = i$  to state  $j$ . For models of the present type it is well-understood how to calculate all interesting probabilistic features of  $\tau_{ij}$ . For example,

**(2.1) Expected fpt.** Let  $m_{ij} = E[\tau_{ij}]$ . Then it is easily shown that  $m_{ij}$  satisfies a backward equation:

$$m_{ij}(\lambda_i + \mu_i) = m_{i+1,j}\lambda_i + m_{i-1,j}\mu_i + 1 \quad (\text{A.8})$$

with boundary condition  $m_{ii} = 0$ ,  $m_{\ell j} = 0$  if  $\ell < 0$ .

**Example:** Suppose  $A(t') = 0$ . Find  $E[\tau_{02}] = m_{02}$ .

From (A.8),

$$\begin{aligned} m_{02} &= \frac{1}{\lambda_0 + \mu_0} [m_{12}\lambda_1 + m_{-1,2}\mu_{-1} + 1] \\ &= \frac{1}{\lambda_0} [m_{12}\lambda_1 + 1] \end{aligned} \quad (\text{A.9.1})$$

by virtue of boundary conditions. Also

$$\begin{aligned}
m_{12} &= \frac{1}{\lambda_1 + \mu_1} [m_{22}\lambda_2 + m_{02}\mu_1 + 1] \\
&= \frac{1}{\lambda_1 + \mu_1} [m_{02}\mu_1 + 1].
\end{aligned}
\tag{A.9.2}$$

The two equations (A.9.1) and (A.9.2) can now be solved simultaneously. This pattern can be followed more generally.

Such stochastic models are usually not available in a form even as simple as the above. Monte Carlo simulation will ordinarily be required in order to elicit lessons and discover sensitivities. But a quite abbreviated model such as the present "SCUDWORM" example is useful in that it may quickly indicate particular sensitivities and tradeoffs, in this case to increased surveillance defense surveillance *or* increased attacker activity (perhaps putting  $\beta = (\xi + \alpha)\beta^*$ , where  $\xi$  is surveillance rate and  $\alpha$  is attacker rate of fire).

LIMITING DIST OF BIRTH DEATH SCUDWORM MODEL

ABAR=20; MUXPKxDBAR=4; BETA/THETA=1/16; LAM=LAMO=1

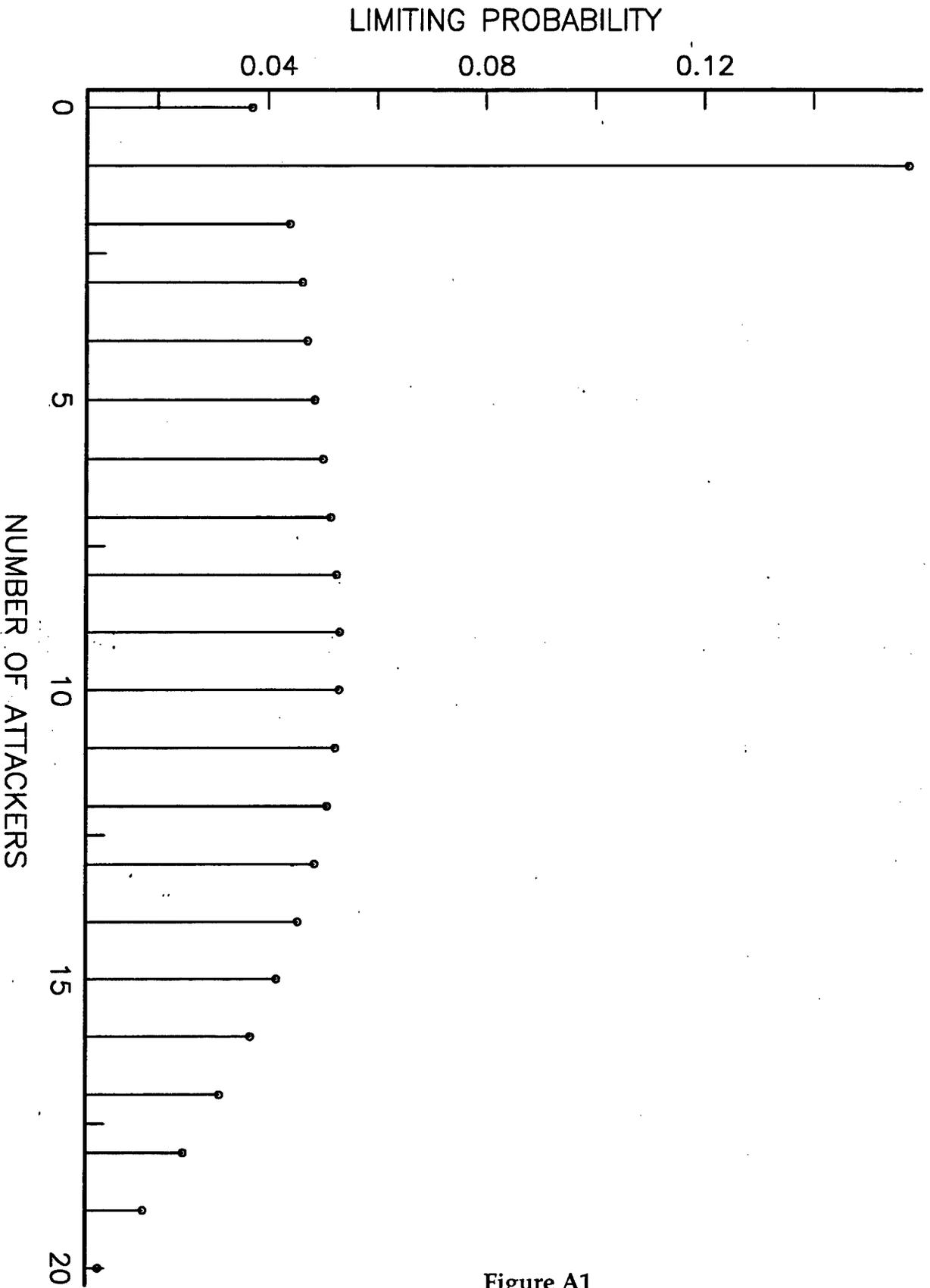


Figure A1

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