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ABSTRACT

To maintain the continued airworthiness of military aircraft it is essential that the fatigue behaviour of components subjected to complex multi-axial stress conditions be both understood and predicted. This topic is extremely complex. Numerous fatigue failure criteria ranging from the purely empirical to the theoretical have been proposed. These criteria rely on the estimation of stress and strain at fatigue critical locations. This interim report focused on possible approaches which may be applicable to both low- and high-cycle fatigue regimes. It discusses the relative advantages of the Neuber and the Glinka methods for calculating localised notch strains as compared to the results from finite element analysis. These former techniques are at the core of several sequence accountable crack initiation prediction models, some of which are used in the life assessment of RAAF aircraft. Thus the accuracy of these techniques directly impact the estimated fatigue life of these aircraft.

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Executive Summary

To maintain the continued airworthiness of military aircraft it is essential that the fatigue behaviour of components subjected to complex multi-axial stress conditions be both understood and predicted. Numerous fatigue failure criteria ranging from the purely empirical to the theoretical have been proposed. These rely on the accurate estimation of the stress and strain state at fatigue critical locations, normally associated with stress concentrations in the form of notches. This interim report examines the relative advantages of the Neuber and the Glinka methods for calculating localised notch strains. These former techniques are at the core of several sequence accountable crack initiation prediction models, some of which are used in the life assessment of RAAF aircraft. Thus the accuracy of these techniques directly impact the estimated fatigue life of these aircraft. The accuracy of the Neuber and Glinka methods was assessed by comparison with the results of detailed finite element analysis. This analysis confirmed that the appropriateness of these techniques was dependant upon the stress state (plane stress or plane strain) at the notch root.

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Mr. Molent graduated in 1983 with a Bachelor of Engineering (Aeronautical). Since commencing employment at the then Aeronautical Research Laboratories in 1984, Mr Molent as worked in the fields of aircraft structural integrity, structural and fatigue testing, aircraft accident investigation and aircraft vulnerability. He has numerous publications in these technical areas. He has been attached to both the Civil Aviation Department (1985) and the US Navy (NAVAIR, 1990-1991) as an airworthiness engineer. Mr Molent is currently a senior research scientist and the task manager F/A-18 Life Assessment, at the Aeronautical and Maritime Research Laboratory.

DSTO Centres of Expertise

Dr Peter Preston, Chief, Airframes and Engines Division is fostering the development of Centres of Expertise (COE) in engineering science relevant to the area of responsibility of the Division. In addition to the COE in Structural Mechanics outlined further down this page, three other COEs have been established in Australian Universities. The COE in Damage Mechanics has the expertise in models and rules for crack growth and damage accumulation in metals and composites materials. The COE in Aerodynamic Loading contributes to the role of the Division in aerodynamic loading using real-time flight trial and wind tunnel data. The COE in Vibration Analysis specialises in application of advanced vibration analysis techniques for the detection, diagnosis and prognosis of faults in aircraft gas turbine engines and transmission systems.

DSTO Centre of Expertise in Structural Mechanics

The Centre of Expertise in Structural Mechanics has been established at Monash University, Melbourne and the Australian Defence Force Academy (ADFA) of the University of New South Wales in Canberra. Professor Rhys Jones, Chairman of the Department of Mechanical Engineering at Monash is Head of the Centre and Professor John Baird the ADFA Coordinator. Dr Alan Baker, the Research Leader in Aerospace Composite Structures in the Airframes and Engines Division, is the DSTO Liaison Officer for the Centre. As part of DSTO's role in lifeing studies of Defence Aircraft, the Centre has considerable expertise in stress and fracture mechanics analysis of metal and composites which contribute not only to the design of measures to extend the life of Defence aircraft but civilian aircraft also. This Centre brings together the largest engineering department in Australia at Monash University with the department responsible for training most of the future graduate engineers for the Australian Defence Force.

Contents

Notation

E	Young's modulus
e	remote strain
g	normal acceleration
K_t	theoretical elastic stress concentration factor
K_ε	elastic-plastic strain concentration factor
K_σ	elastic-plastic stress concentration factor
R	Ratio of minimum/maximum stress
S	remote stress
N_f	Number of cycles to failure
W	strain energy density
ε	strain
ε_e	elastic strain
ε_t	strain calculated using an elastic-plastic analysis (total strain)
ε_p	plastic strain contribution
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	local strain components in plane stress conditions; Cartesian co-ordinates
$\varepsilon_x', \varepsilon_y', \varepsilon_z'$	local strain components in plane strain conditions; Cartesian co-ordinates
ε_{ys}	strain at the yield strength
μ	generalised Poisson's coefficient
ν	Poisson's coefficient
σ	stress
σ_t	stress calculated using an elastic-plastic analysis (total stress)
$\sigma_x, \sigma_y, \sigma_z$	local stress components in plane stress conditions; Cartesian co-ordinates
$\sigma_x', \sigma_y', \sigma_z'$	local stress components in plane strain conditions; Cartesian co-ordinates
σ_{ys}	yield strength

1. Introduction

Aircraft structural components are generally subjected to complex loading spectra. The alternating loads tend to initiate fatigue cracks at notches and other regions of high stress. Historically the field of fatigue has been classified into a number of specific areas; viz: high-cycle and low-cycle fatigue; fatigue of notched and un-notched members; the initiation and propagation of cracks and fatigue life extension techniques. Initially the focus was on developing simple design guidelines. In [1] it was remarked that in the case of high cycle fatigue, the fatigue damage was associated with localised plasticity. This led to the use of a variety of approaches based on the concept of an equivalent von Mises strain.

In this interim report we begin by briefly reviewing the existing strain based and energy based failure criteria theories. Attention is focused on approaches which may be applicable to both low- and high-cycle fatigue regimes. The relative advantages of the most common of these Neuber [2] and the Glinka approaches [3, 4] for calculating localised notch stresses and strains are discussed. These techniques are at the core of several sequence-accountable crack initiation prediction models, some of which are used in the life assessment of RAAF aircraft.

In a recent review [5] of elastic-plastic notch root stress-strain estimation methods, the following general conclusions as to the accuracy of Neuber and Glinka estimations were drawn:

- The relative error of the Neuber's rule is in general larger than that of Glinka. However, Glinka normally over-estimates where Neuber under-estimates the notch root strain. Neuber will therefore be generally conservative while Glinka is non-conservative.
- The closer the loading situation is to the elastic case, the smaller is the relative error between the two methods.
- Neuber's method generally gives the best prediction under predominantly plane stress state, whilst Glinka's method is best for nearly plane strain conditions.

These generalizations are investigated in this report by comparing the estimations based on the Neuber and Glinka methods with the results of a detailed finite element analysis. For this purpose a rectangular plate containing a circular hole was investigated. Both plane stress and plane strain conditions were investigated for two levels of monotonic loading.

A number of recommendations are then made to address some of the shortcomings inherent in the current crack initiation lifing methodology.

2. Common Fatigue Failure Criteria

2.1 HIGH CYCLE FATIGUE: Stress Based Criteria

Stress based methods are usually associated with high cycle fatigue. Whilst there are a large variety of failure criteria used many of these criteria can be written in the form; viz:

$$f(I_1, I_2) = q(N_f) \tag{1}$$

where $I_1 = \sigma_1 + \sigma_2 + \sigma_3$ and $I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3$, are the first and second invariants of the stress tensor. Here σ_1, σ_2 and σ_3 are the principal stresses, $\sigma_1 \geq \sigma_2 \geq \sigma_3$ and N_f is number of cycles to failure. One commonly used form for q is

$$q(N_f) = K\,N_f^{\alpha} + C \tag{2}$$

where K is a constant, $\alpha < 0$ and $C > 0$. If $N_f \rightarrow \infty$ then eqn. (1) becomes

$$f(I_1, I_2) = C \tag{3}$$

which resembles the yield criteria for isotropic metals. One of the most familiar, and widely used, yield criteria is:

$$1/2\; s_{ij}\, s_{ij} = C_0 \tag{4}$$

where s_{ij} is the deviatoric stress tensor, $C_0 = 3\sigma_Y$ and σ_Y is the uniaxial tensile yield stress. Many authors have adopted this analogy and some of the commonly used failure (yield) criteria are given in [6, 7, 8, 9]. Suitable adjustment of these criteria can reduce the majority to either the Tresca or the von Mises' yield condition. The fact that stress based criteria are more applicable to high cycle fatigue is discussed in [10].

2.2 LOW CYCLE FATIGUE: Strain-based criteria

In contrast strain based methods are usually associated with low cycle fatigue. In this post initial yield regime the Manson-Coffin relationship [11, 12]:

$$\Delta\varepsilon/2 = \Delta\varepsilon^e/2 + \Delta\varepsilon^p/2 = \sigma_f'/E\,(N_f)^b + \varepsilon_f'\,(N_f)^c \tag{5}$$

is widely used. Here Δ refers to its 'range'; and σ_f'/E and ε_f' are strain amplitudes corresponding to the elastic and the plastic intercept for one cycle respectively; E is Young's modulus, and b and c are experimental constants.

To extend this law to the multi-axial stress states requires the definition of an equivalent strain. This extension is often written in the form

$$\Delta\varepsilon_{eq}/2 = \sigma_f' / E\,(N_f)^b + \varepsilon_f'\,(N_f)^c \qquad (6)$$

A number of expressions for the equivalent strains have been used; viz: the octahedral shear strain [10, 13], the maximum shear strain [10, 14], the maximum normal strain [10, 15], the von Mises' equivalent strain [16], modified von Mises' [14, 17, 18] and the maximum total strain [19].

One expression commonly used as an equivalent strain measure is:

$$\varepsilon_{eq} = \alpha(\,(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_3 - \varepsilon_2)^2 + (\varepsilon_1 - \varepsilon_3)^2)^{1/2} \qquad (7)$$

When $\alpha = 1/3$ we find that $\varepsilon_{eq} = \gamma_{oct}$, the octahedral shearing strain. When $\alpha = \sqrt{2}/2(1+\upsilon)$ then ε_{eq} coincides with the expression for the von Mises equivalent strain.

3. Notch Root Stresses and Strains

The strain-life fatigue method requires that the notch root stresses and strains be known. These quantities can be determined in several ways, viz: via direct strain (mirco) measurements, using finite element analysis or by using approximate methods that relate local stresses and strains to their remote values.

The theoretical stress concentration factor, K_t, is often used to relate the nominal (or far field) stresses S, or strains, e, to the local values, σ and ε. Upon yielding, Hooke's law cannot be used to relate the local stress, σ, to the local strain, ε, and the local values are no longer related to the nominal values by K_t. Instead, the local stresses and strains are related to the remote values by their respective stress and strain concentration factors K_σ and K_ε, viz:

$$\sigma = K_\sigma S \text{ and } \varepsilon = K_\varepsilon e \qquad (8)$$

In an attempt to compute the local (notch) stress and strains the strain life approach frequently makes use of Neuber's rule [2]. In this approach the theoretical stress concentration is taken as the geometric mean of the stress and strain concentration factors. It is widely assumed that this relationship holds true for most notch geometries (see Potter in [20]). Problems associated with this and other related approximations are:

1. The notch root stress-strain response may not always be in phase with the global load [21];
2. The law does not account for time dependent processes such as creep and stress relaxation; and

3. The law does not account for cyclic stress relaxation.

A number of variants of Neuber's rule have been used to relate the remote stresses and strains to local values. The basic Neuber approach and a commonly used variant is outlined below:

The initial Neuber hypothesis [2] can be expressed in the form:

$$K_t^2 = K_\sigma K_\varepsilon \tag{9}$$

or alternatively

$$S\, e\, K_t^{\,2} = \sigma\ \varepsilon \tag{10}$$

When the remote stresses and strains are above yield Seeger and Heuler [22] proposed an extension of Neuber's rule, viz:

$$\underline{S}\ \underline{e}\, K_P^{\,2} = \sigma\ \varepsilon \tag{11}$$

where

K_P = Stress S at the onset of general yielding / Stress $\underline{S}$ at which the notch first yields

and $$\underline{S} = S\, K_t / K_P \tag{12}$$

Here $\underline{S}$ and $\underline{e}$ must lie on the cyclic stress-strain curve.

The formulation of equation (10) is widely accepted [20] and is used in many current sequence accountable crack initiation prediction models, see [23, 24, 25, 26, 27]. Currently the F/A-18 structure is monitored using a derivative of one of these programs.

In the authors opinion these formulations have a number of shortcomings; viz:

i) For cyclic loading it is common to modify Neuber's hypothesis by replacing K_t by a fatigue notch reduction factor K_f see Topper et al. [28].

The work of Glinka [3, 4] has shown that Neuber's hypothesis can incorrectly estimate the inelastic strain. The need to replace K_t by K_f etc, can be overcome by using the Glinka hypothesis to calculate $\Delta\varepsilon$. According to this hypothesis the strain energy density at the notch in a fully plastic analysis (small scale plasticity) equates to the strain energy obtained via a purely elastic analysis; viz:

$$1/2\, K_t^2\, Se = \int \sigma_{ij}\, d\varepsilon_{ij} = \int \sigma\, d\varepsilon \tag{13}$$

Once a valid stress strain relationship is known the notch stresses and strains can be readily evaluated.

ii) For plane stress or plane strain problems, one approach is to use simple power laws to relate σ to ε, or for saturated fatigue loops $\Delta\sigma$ to $\Delta\varepsilon$. These are normally formulated assuming the material exhibits Masing's hypothesis [31], which states that following initial yielding, the shape of the hysteresis curve can be approximated by assuming it is similar to the initial loading curve magnified by a factor of two. This assumption is not appropriate for all materials.

Alternatively, the stress-strain relationships can be "digitised" as an input into the lifing models [see 23, 24, 25, 26, 27].

iii) The relations presented above, and as used to life some military aircraft (eg the F/A-18), do not allow for the local level of constraint/triaxiality. One method for including constraint effects, which modifies the expression for the constant K in the fatigue law, is outlined in [29].

For complex 3D structural problems the use of a validated constitutive law may be necessary. In this case when calculating the $\Delta\sigma$ and $\Delta\varepsilon$ ranges the use of traditional yield surface plasticity should be avoided as this tends to produce "boxy" stress strain loops and gives poor estimates of the $\Delta\sigma$ and $\Delta\varepsilon$ ranges. Stiffener runout Number 2 in the F111 wing pivot fitting is a good example of this problem. Here when subjected to *g* loads going from 0*g* to 7.3*g* to 0*g* simple classical incremental plasticity gave a tensile stress range of approximately 2,000 MPa whilst more exact analysis, using a constitutive law, gave a stress range of only 1,200 MPa [30].

vi) Care should be taken to ensure these relations adequately allow for the load history effects, i.e. load interaction, stress relaxation, creep, and overload-underloads.

Here the stiffener runout Number 2 in the F111 wing pivot fitting is (again) a good example of the need to correctly follow load history. When subjected to *g* loads going from 0*g* to 7.3*g* to 0*g* simple classical approaches gave inspection interval of less than 500 hours. However, allowing for the load history enabled the inspection interval to be increased to almost 1,500 hours [30].

4. Comparison of Neuber's, Glinka's and Finite Element Results

As we have previously seen the accurate prediction of the stresses and the strains in a structure are important in fracture mechanics and fatigue life prediction. In this section of the report we will evaluate the relative ability of Neuber's rule and Glinka's method to calculate the local stresses and strains in a plate with a circular hole. To this end the

values predicted via Neuber's rule and Glinka's method were compared against finite element method (FEM) results obtained using classical incremental plasticity.

As an example let us consider a 22mm wide x 64mm long rectangular plate, 1mm thick with a 8mm diameter circular hole located at the centre of the plate, see Figure 1. The plate is subjected to a remote uniform stress. The load spectra applied to the plate involved monotonic loading from zero to a peak value of σ, the load was then fully reversed until a remote stress of $-\sigma$ was reached, at which stage the load was again reversed until a remote stress of σ was reached, ie. the cycle was $(0, \sigma, -\sigma, \sigma)$, R = −1.

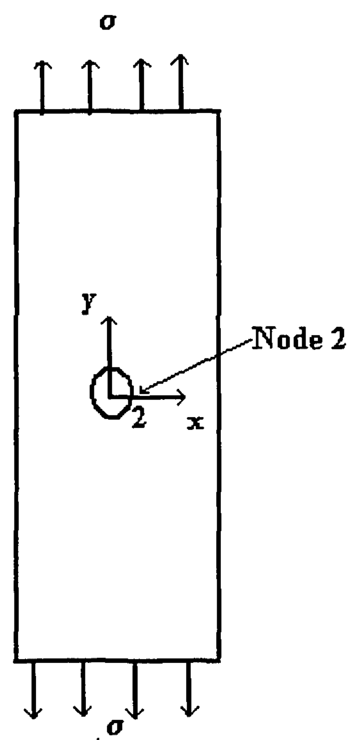

Figure 1: Schematic diagram of a rectangular plate with a circular hole

This analysis was repeated with two different levels of σ; viz: σ = 150 MPa and 225 MPa. These two values of the remote stress were selected so that the nominal section would be at different levels of stress which both approach the yield strength of the material. The load was applied in increments so the cyclic behaviour of the plate could be observed and the plate was considered to be in a state of either plane stress or plane strain. In this analysis the yield strength, σ_{ys}, of the idealised material was taken as 400 MPa, with a Young's Modulus, E, of 70,000 MPa and a post yield gradient of 2,003 MPa (elastic - linear strain hardening). The stresses and plastic strains at the notch were the calculated using the FEM, Neuber's rule and Glinka's method at node 2 as depicted in Figure 1.

The plate was modelled using 8-noded plate elements. A denser mesh was used around the hole to model the stresses and strains to a greater accuracy since steeper

stress gradients are present and the material yields at and in the vicinity of the notch tip. Because of symmetry only a quarter of the plate was modelled, a typical mesh is shown in Figure 2.

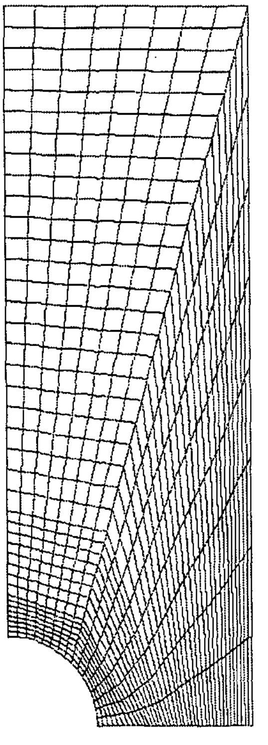

Figure 2: A typical mesh of the modelled plate

4.1 Neuber's Rule: - Plane Stress

For plane stress there is only a uniaxial stress, σ_y at node 2 since at this point $\sigma_x = \sigma_z = 0$. The monotonic stress-strain curve used for the material is given by equations (14) and (15).

$$= E\varepsilon \quad if \quad \sigma \leq 400MPa \tag{14}$$

$$= 2003\varepsilon + 388.55 \quad if \quad \sigma > 400 \tag{15}$$

The cyclic hysteresis stress-strain curve used is given by equations (16) and (17). This example assumed that the stress - strain hysteresis curve is approximately twice the value of the cyclic stress - strain curve (Masing's Hypothesis [31]). This assumption has a sound basis since the cyclic stress - strain curve is the locus of the tips of the hysteresis loops. When the material has been taken past the yield point, the new origin, in stress strain space, is located at the last value of the stress and strain at which a load reversal occurred.

$$= E\varepsilon \quad if \quad \sigma \leq 800MPa \tag{16}$$

$$= 2003\varepsilon + 777.11 \quad if \quad \sigma > 800MPa \tag{17}$$

Using Neuber's approach and the stress-strain curve for monotonic loading allows the values of stress, σ_t and the plastic strain, ε_t to be evaluated. Similarly using Neuber's approach and the cyclic hysteresis stress-strain curve for cyclic loading allows the values of stress, σ_t and the strain, ε_t to be evaluated at any point in the load cycle. Here the subscript "*t*" indicates that the values were calculated using an elastic-plastic stress analysis. For monotonic loading Neuber's approach gives:

$$2003\varepsilon_t^{\,2} + 388.55\varepsilon_t - \frac{(K_t S)^2}{E} = 0 \quad if \quad \sigma > 400MPa \tag{18}$$

Solving this quadratic equation allows ε_t to be evaluated.

A similar substitution, using the cyclic hysteresis stress strain curve, can be made for cyclic loading to solve for ε_t. For the cyclic hysteresis stress strain curve it was assumed, for calculation purposes, that the origin was placed at the last stress and strain value before the load was reversed and the positive axes of stress and strain are in the direction of the reversed load.

4.2 Glinka's Method - Plane Stress

When using Glinka's method together with the stress strain curve for monotonic loading we obtain a similar quadratic equation allowing the stress, σ_t and the strain, ε_t to be calculated; viz:

$$0 = 2003\varepsilon_t^{\,2} + \varepsilon_t(388.55 + \sigma_{ys} - 2003\varepsilon_{ys}) +$$
$$\left[\frac{\sigma_{ys}^{\,2}}{E} - \frac{(K_t S)^2}{E} - \varepsilon_{ys}\left(388.55 + \sigma_{ys}\right)\right] \quad if \quad \sigma > 400MPa \quad (= \sigma_{ys}) \tag{19}$$

For cyclic loading use of the cyclic hysteresis stress strain curve, equation (17) again gives a quadratic equation allowing the stress, σ_t and the strain, ε_t to be evaluated.

4.3 Neuber's Rule and Glinka's Method in Plane Strain

For plane strain, an analogous analysis can be carried out as per that outlined above for the plane stress. However, in this case a biaxial stress state exists at the notch tip (because $\sigma_y' \neq 0$, $\sigma_z' \neq 0$). To allow for this Dowling et al. [32] derived the following relationships which allow for the translation of the uniaxial stress strain curve into a biaxial "plane strain" stress stain relation; viz:

$$\varepsilon_y' = \varepsilon_y\left(\frac{1-\mu^2}{\sqrt{(1-\mu+\mu^2)}}\right) \tag{20}$$

$$_y' = \sigma_y\left(\frac{1}{\sqrt{(1-\mu+\mu^2)}}\right) \tag{21}$$

where $$\mu = \frac{\left(\nu + \mathrm{E}\varepsilon^{\mathrm{p}}/2\sigma\right)}{\left(1+\mathrm{E}\varepsilon^{\mathrm{p}}/\sigma\right)}, \quad \varepsilon = \frac{\sigma}{\mathrm{E}} + \varepsilon^{\mathrm{p}} = \varepsilon_{\mathrm{e}} + \varepsilon^{\mathrm{p}} \tag{22}$$

Thus, under plane strain conditions, the same set of equations using Neuber's rule and Glinka's method can be used except that all the calculations are based on the "plane strain" stress-strain curve σ_y'-ε_y'.

4.4 Discussion

The accuracy of the FEM model was first checked against the elastic theoretical solution at the notch tip when the notch tip was still within the linear elastic range during monotonic loading of the plate. The theoretical solution at the notch tip when the material is linear elastic is given by,

$$\begin{cases}\sigma_y = K_t\sigma_{nom} \\ \sigma_x = \sigma_z = 0\end{cases} \quad \text{for plane stress} \tag{23}$$

and

$$\begin{cases}\sigma_y' = K_t\sigma_{nom} \\ \sigma_x' = 0 \\ \sigma_z' = \nu K_t\sigma_{nom}\end{cases} \quad \text{for plane strain} \tag{24}$$

In this case Peterson [33] gave a K_t of 3.658. The computed values for all the stress components are shown in Table 3. In each case the numerical results were within 2% of the theoretical value.

Table 3: Percentage error in the FEM model

(For node 2)

	FEM	THEORETICAL	FEM	THEORETICAL
	PLANE STRAIN (% error)	PLANE STRAIN	PLANE STRESS (% error)	PLANE STRESS
σ_x (MPa)	0.01	0	0.02	0
σ_y (MPa)	392.38 (-2.0%)	400.55	397.76 (-2.0%)	406.04
σ_z (MPa)	125.56 (-2.0%)	128.17	0	0

4.4.1 Comparison of Neuber's Rule and Glinka's method to the FEM model

Having confirmed the accuracy of the model the computed hysteresis loops, for both plane stress and plane strain, and for both load cases are shown in Figures 3 to 6. From these figures it can be observed that all three methods predicted a similar cyclic behaviour at the notch tip but with different magnitudes for the strains. All models displayed elastic unloading and symmetry in the cyclic stress-strain curve. Each method also predicted "boxy" stress-strain curves, as would be expected given that the theory for each method was based on incremental plasticity and an idealised material. In reality, "curved" stress-strain curves should be expected.

The percentage error of Neuber's rule and Glinka's method, as compared with the FEM results, in predicting the maximum stresses and strains in monotonic loading is shown in Table 4.

Table 4: The percentage error in the maximum stresses and strains under monotonic loading using Glinka's method and Neuber's rule

Load	**Type of Loading**	**FEM model**		**Glinka's Method (% error)**		**Neuber's rule (% error)**	
MPa		ε	σ **(MPa)**	ε **(%)**	σ **(%)**	ε **(%)**	σ **(%)**
150	Plane Stress	0.00867	406.57	0.00821 (-5.22%)	405.01 (-0.38%)	0.01050 (21.1%)	409.59 (0.74%)
225	Plane Stress	0.01519	424.64	0.01474 (-2.94%)	418.09 (-1.54%)	0.02233 (46.9%)	433.29 (2.04%)
150	Plane Strain	0.00721	456.13	0.00761 (5.54%)	460.91 (1.05%)	0.00918 (27.3%)	466.71 (2.32%)
225	Plane Strain	0.01247	476.00	0.01330 (6.67%)	479.43 (0.72%)	0.01945 (55.9%)	496.55 (4.31%)

4.4.2 Remarks

For the particular problem studied this analysis has shown that, for the material stress-strain law, geometry and loading considered, Glinka's method estimates the stresses and strains to a greater accuracy than Neuber's rule. The following general observations can be made.

1. Both Neuber's rule and Glinka's method calculated the stresses, σ_t, to within 5% of the FEM results.

2. Neuber's rule significantly overestimated the strain, ε_t, for both load cases and both load types.

3. As the nominal stress was increased, the errors in the predictions made by Neuber's rule increased significantly.

4. For plane stress Glinka's method underestimates the value of the strain. However, it is within approximately 5% of the calculated values. For plane strain Glinka's method overestimates the value of strain and stress but is within approximately 5% of the calculated value.

5. As the nominal stress in the plate was increased (but was still below the yield strength of the material) Glinka's method predicted the stresses and strain to approximately the same degree of accuracy as the FEM.

These findings are generally in agreement with those of Shin [5].

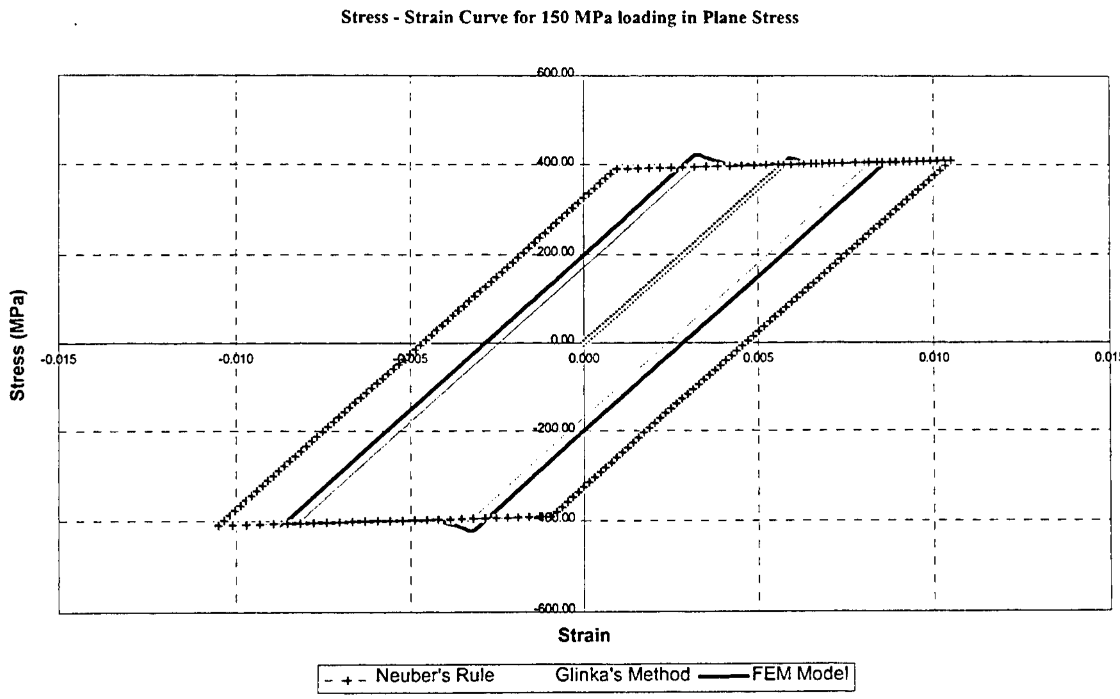

Figure 3: Stress Strain Curves for 155 MPa (Plane Stress)

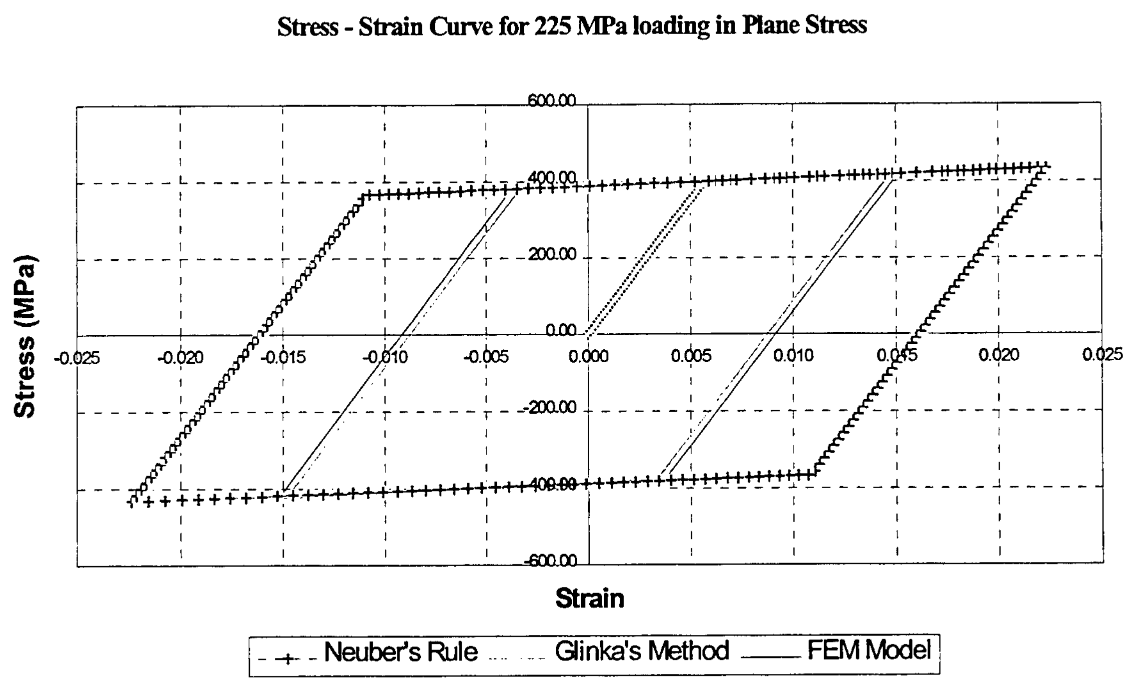

Figure 4: Stress Strain Curves for 225 MPa (Plane Stress)

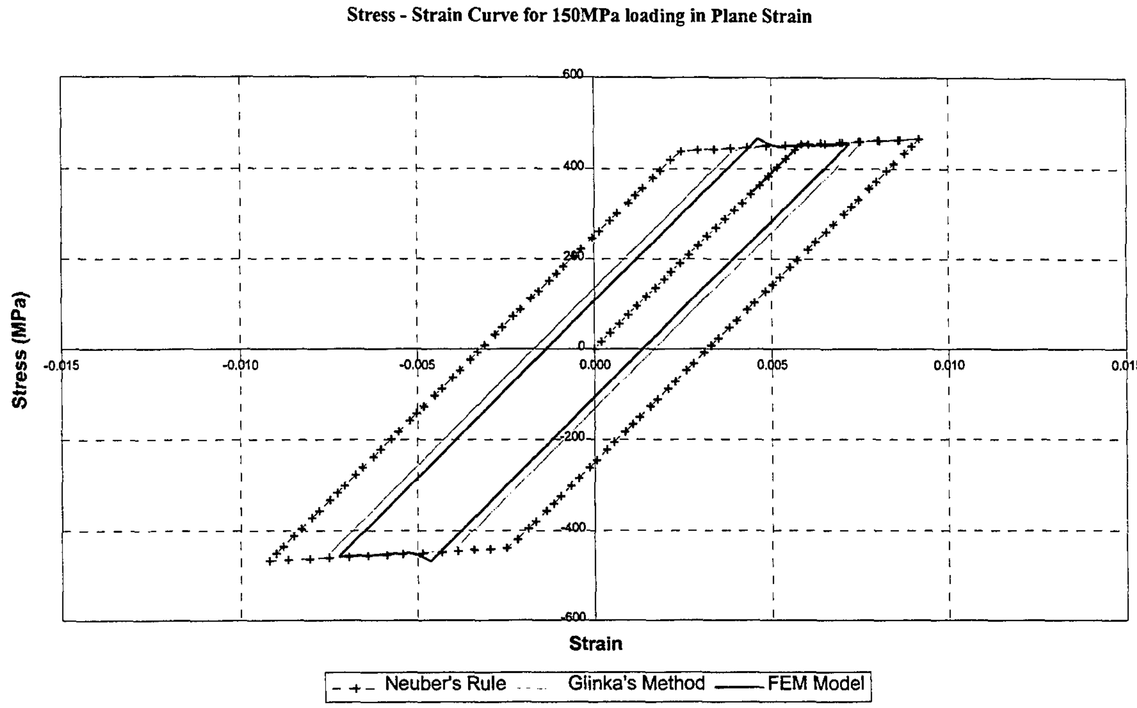

Figure 5: Stress Strain Curves for 155 MPa (Plane Strain)

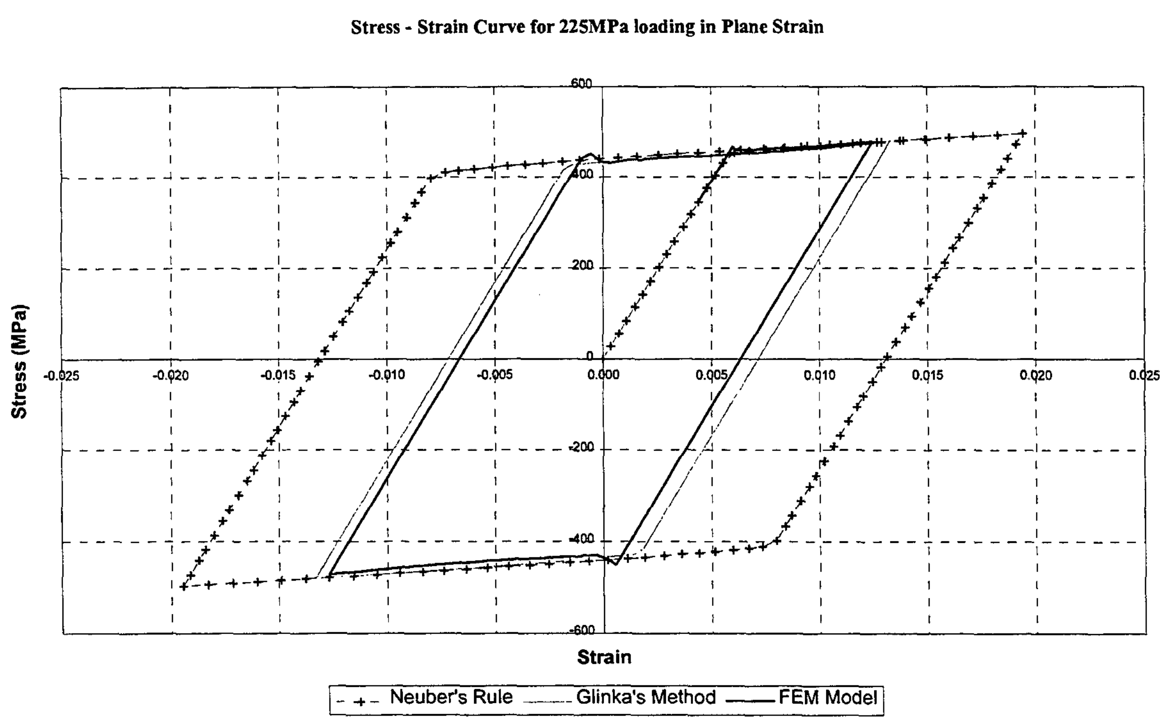

Figure 6: Stress Strain Curve for 250 MPa (Plane Strain)

5. Conclusion

This report has presented a brief summary of some common relationships used to estimate notch stress and strain for both low- and high-cycle fatigue. Particular attention has been paid to the relative advantages of the Neuber and the Glinka approaches.

Current lifing methodology is frequently based on the an early version of the strain life approach and has a number of shortcomings which should be addressed; viz:

1. Neuber's hypothesis is frequently used to compute the strain range. When using this approach K_t should be replaced by the fatigue notch reduction factor K_f, see Topper et al [28]. However, it is desirable that both Neuber's hypothesis and Glinka's hypothesis should be available for use in any lifing analysis package. This would require the user to have a prior knowledge of the relative accuracy of each method for the particular material, geometry and loading spectrum under consideration.
2. Some analyses use power laws to compute $\Delta\sigma$ and $\Delta\varepsilon$. Some relationships used are only accurate for Masing type materials. The use of generalisations of this approximation for non Masing materials should be investigated.
3. The strain life notch relations, as currently used, do not allow for the local level of constraint/triaxiality. This can have a very significant effect. Methods for including constraint effects should be adopted.
4. Care should be taken to ensure these relations adequately allow for the load history effects, i.e. load interaction, stress relaxation, creep, and overload-underloads. General ways of accounting for these effects should be included in the lifing formulae.

In comparison with the results of a finite element analysis of a plate with a central hole:

5. Neuber's rule significantly overestimated the notch strain for both plane stress and plane strain for the two load cases considered.
6. As the nominal stress was increased, the errors in the predictions made by Neuber's rule increase significantly.
7. For plane stress Glinka's method underestimated the value of the strain. However, it is within approximately 5% of the calculated values. For plane strain Glinka's method overestimated the value of strain and stress but is within approximately 5% of the calculated value.

8. As the nominal stress in the plate was increased (but was still below the yield strength of the material) Glinka's method predicted the stresses and strain to approximately the same degree of accuracy as the finite element method.

5.1 Summary

The accuracy of the current analysis methodology generally accepted to compute the fatigue (initiation) life may have the potential to be dramatically improved. Before conducting a lifing analysis, the relative accuracy of the Neuber or Glinka hypotheses for the particular geometry under consideration should be assessed, and the appropriate hypothesis chosen. Further attention should be given to accounting for the level of local constraint, and on the use of methodologies capable of accounting for the effect of overloads and stress relaxation.

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19. ABSTRACT

To maintain the continued airworthiness of military aircraft it is essential that the fatigue behaviour of components subjected to complex multi-axial stress conditions be both understood and predicted. This topic is extremely complex. Numerous fatigue failure criteria ranging from the purely empirical to the theoretical have been proposed. These criteria rely on the estimation of stress and strain at fatigue critical locations. This interim report focused on possible approaches which may be applicable to both low- and high-cycle fatigue regimes. It discusses the relative advantages of the Neuber and the Glinka methods for calculating localised notch strains as compared to the results from finite element analysis. These former techniques are at the core of several sequence accountable crack initiation prediction models, some of which are used in the life assessment of RAAF aircraft. Thus the accuracy of these techniques directly impact the estimated fatigue life of these aircraft.

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