Range Commanders Council

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IRIG OPTICAL TRACKING SYSTEMS CALIBRATION CATALOG

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PREFACE

The Range Commanders Council (RCC) was originated to preserve and enhance the efficiency, effectiveness, and economical operation of member ranges, individually and collectively, thereby increasing the national capability for research, development, and operational testing and evaluation. In the area of optical tracking, two sub-groups of the RCC have had a common interest. The two groups are the Optical Systems Group (OSG) and the Joint Range Instrumentation Accuracy Improvement Group (JRIAIG). The common interest in optical systems has been the overall accuracy of such instruments. Presently, optical systems are used for tracking a wide variety of targets ranging from sub-munitions and un-manned vehicles to aircraft, missiles, and satellites. In each of these tracking situations, the accuracy of the final results is of primary importance.

The raw range, azimuth, and elevation data from optical systems contain both systematic and random errors. Random errors are typically estimated, using statistical methods, and may be minimized by the use of optimal filter/smoother techniques. Systematic errors, on the other hand, require calibration (via satellite tracking or similar means), mathematical modeling, and mechanical alignment to remove or reduce their effects. The measurement and control of these errors can be very difficult and time consuming.

Since the error models (and error terms) for optical and radar tracking systems correspond strongly in most areas, this document was derived from RCC Document 256-93, *IRIG Radar Calibration Catalog*. Ranges which responded with calibration procedures for the error terms have been identified along with their procedures for assessment and measurement.

JRIAIG tasked the Air Force Flight Test Center (AFFTC) to create the first optical calibration catalog and to identify specific error models and procedures which will serve as generic starting points for future participants in optical calibrations. The current version of this document (Draft 1, August 1993) reflects the comments and suggestions of AFFTC local personnel, with inputs and comments from other ranges to be included in Draft 2. Some sub-sections for which discussions or derivations were not available in RCC Document 256-93, *IRIG Radar Calibration Catalog,* have been updated. A list of reference material is provided in the appendix primarily as a guide for further reading.

This document will serve as the IRIG reference for optical tracking systems calibrations; it may be reproduced as necessary for appropriate DOD agencies and their contractors. Please direct questions or comments to

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1. INTRODUCTION

Advancements in missile and space vehicle technology have generated stringent accuracy requirements for optical tracking systems. Because these systems make precise measurements of the angular direction of a test object in space, the measurements must be calibrated to a common or accepted reference to establish the accuracy of the measurements obtained. The Joint Range Instrumentation Accuracy Improvement Group (JRIAIG) of the Range Commanders Council (RCC) surveyed the range community to determine whether or not optical calibrations were being done, and if so, what procedures were being followed and what error models were being used. The response to this survey showed that no certified optical calibration procedures have been developed and that few established error models are in use. Only a small sample of the participating ranges were able to submit copies of the optical calibration procedures in use at their respective ranges. The documents that were submitted contributed to the completion of this document. Although the specifics of the procedures may deviate from one range to another, the theory and general techniques of the models were found to be the same.

No attempt was made to specifically address individual types of optical systems in the intial version of the *IRIG Optical Tracking Systems Calibration Catalog*. Instead, measurement procedures are described in general terms, focusing on the nature of the measurement rather than the details. The document therefore does not provide a discussion of all optical error sources identified by other IRIG or industry documents but rather addresses those *major* error sources for which mathematical models exist and calibration procedures have been developed. The document is also intended for application to optical systems that are pedestal mounted (i.e. on Askanias, Cinetheodolites, Kineto Tracking Mounts, etc.). Since the error models for optical tracking systems closely parallel those for radar systems, the initial draft of the document was derived from RCC Document 256-93, *IRIG Radar Calibration Catalog.*

The *IRIG Optical Tracking Systems Calibration Catalog* addresses a very large number of terms applied in known error models, accounting for most of the systematic errors contained in Time-Space-Position-Information (TSPI) derived from optical tracking systems. The focus is mainly upon identifying the error terms in common use at all ranges and structuring a standard error model to reflect these similarities. The remaining terms either cannot be estimated and removed, or result from random/undetermined error sources. The goal of the error model identified is to maintain the total contribution of uncorrected systematic errors to less than one least-significant-bit (LSB) for the instrument in question, although certain factors may make this impossible. The purpose of the document as a whole is to summarize information about existing calibration procedures in order that any range can use this document to tailor procedures to their local conditions. Although each range may develop different procedures, the end product should reflect methods which are traceable to a common reference standard (i.e., this document). It is assumed that the user of the document will be familiar with the calibration of ground-based optical tracking systems. The individual error model terms are presented as ***errors*** which are subtracted from the measured data to yield corrected data. These error sources represent the relationship between the true and measured data as follows:

Error = Measured - True

When using the error model presented in this document, particular attention should be given to the relationship between the physical meaning of the error source and the sign convention of its correction. Moreover, the term *normal* refers to azimuth and elevation values recorded when the measured elevation is less than 90° (0° to 90°). The term *plunge* (or *dump*) refers to those values recorded when elevation is greater than 90° (90° to 180°). The model ecounters difficulty at 90° elevation since the azimuth value is undetermined at this pointing angle.

In paragraph *2, SYSTEMATIC ERROR MODEL DEFINITION*, the optical error model is defined in terms of azimuth and elevation **errors**. In paragraph *3, SYSTEMATIC ERROR MODEL DESCRIPTION*, these terms are further explained with high-level descriptions of the common methods of measurement. Paragraph *4, SYSTEMATIC ERROR MODEL DERIVATION*, then provides derivations for the individual error model elements. In the concluding paragraph *5, APPLICATION OF ERROR MODEL*, a possible error coefficient collection methodology is discussed.

2. SYSTEMATIC ERROR MODEL DEFINITION

The phrase *error model* can be misleading. The implication can be that the *true* position or state of a target is known and that when the measured values are compared to the truth, errors are found which can be modeled.

In reality, the exact location of the target is unknowable. For practical applications, truth standards can be any source of information that is sufficiently more accurate than the system being calibrated. Often an *order of magnitude* is used as the criterion for a data source to function as the truth standard (i.e., the true-to-measured accuracy ratio is greater than ten). Under these conditions, comparison of the measured data with the truth standard yields error residuals that reflect the combined effect of both systematic and random errors in the measurement. The purpose of the error model is therefore to describe known systematic errors which can be mathematically subtracted from the data. The superposition of the systematic error terms make up the total correctable error. This combined term is divided among the two separate components of azimuth and elevation.

The error model definition is presented in paragraphs *2.1, Azimuth Component,* and *2.2, Elevation Component,* with component variables defined in paragraph *2.3, Component Definitions.* For cross-reference purposes, each equation in the document is numbered according to the paragraph and position within the paragraph (for example, 2.1-1 refers to the first equation in paragraph 2.1).

2.1 Azimuth Component

The following equation presents the error terms of the azimuth component of a pointing angle solution:

$A_c = A_o$ (2.1-1)

$- b_o$ ....Zeroset

$- b_1 \dot{A}_e$ ....Time Delay

$- b_1' \dot{A}_e$ ....Velocity Servo-lag ($b_1' = 1 / K \overset{A}{v}$)

$- b_2 \ddot{A}_e$ ....Acceleration Servo-lag ($b_2 = 1 / K \overset{A}{a}$)

$- b_3 \sin(A_1 + \alpha_1) \tan E_2$ ....Mislevel

$- b_4 \sin(2A_1 + \alpha_2) \tan E_2$ ....Mislevel Wobble

$- b_5 R_1 \dot{A}_e$ ....Transit Time (b_5 = 1 / speed of light)

$- b_6 \tan E_2$ ....Nonorthogonality (Standards)

$- b_7 \cos(m_1 A_o + \phi_1)$ .... m_1 Harmonic Encoder Nonlinearity

$- b_8 \cos(m_2 A_o + \phi_2)$ .... m_2 Harmonic Encoder Nonlinearity

$- b_9 \cos(m_3 A_o + \phi_3)$ .... m_3 Harmonic Encoder Nonlinearity

$- b_{10} \sec E_2$ .... Electrical Misalignment (Collimation)

$- \eta \tan \phi - \eta \tan E_2 \cos A_2 + \xi \tan E_2 \sin A_2$ .... Vertical Deflection

$- a_5 \dfrac{\cos A_2 \sec E_2}{R_{est}}$ .... X Survey

$- a_6 \dfrac{\sin A_2 \sec E_2}{R_{est}}$ .... Y Survey

2.2 Elevation Component

The following equation presents the terms of the elevation component of a pointing angle solution:

$$E_c = E_o \tag{2.2-1}$$

$- c_0$ .... Zeroset

$- c_1 \dot{E}_e$ .... Time Delay

$- c_1' \dot{E}_e$ .... Velocity Servo-lag ($c_1' = 1/K_v^E$)

$- c_2 \ddot{E}_e$ .... Acceleration Servo-lag ($\dot{c}_2 = 1/K_a^E$)

$- c_3 \cos(A_1 + \beta_1)$ .... Mislevel

$- c_4 \cos(2A_1 + \beta_2)$ .... Mislevel Wobble

$- c_5 R_{est} \dot{E}_e$ .... Transit Time ($c_5 = 1$ / speed of light)

$- c_6 \cos(n_1 E_o + \theta_1)$ .... n_1 Harmonic Encoder Nonlinearity

$- c_7 \cos(n_2 E_o + \theta_2)$ .... n_2 Harmonic Encoder Nonlinearity

$- c_8 \cos(n_3 E_o + \theta_3)$ .... n_3 Harmonic Encoder Nonlinearity

$- c_9 \cos E_1$ ....Antenna Droop

$- \eta \sin A_2 + \xi \cos A_2$ ....Vertical Deflection

$- a_5 \dfrac{\sin A_2 \sin E_2}{R_{est}}$ ....X Survey

$- a_6 \dfrac{\cos A_2 \sin E_2}{R_{est}}$ ....Y Survey

$- a_7 \dfrac{\cos E_2}{R_{est}}$ ....Z Survey

$- \rho_E$ ....Refraction

$- \rho_{E'}$ ....Residual Refraction

2.3 Component Definitions

The azimuth and elevation terms A_1, A_2, E_1, and E_2 are defined by the following equations:

$$A_1 = A_o - b_o - b_7 \cos(m_1 A_o + \phi_1) - b_8 \cos(m_2 A_o + \phi_2) - b_9 \cos(m_3 A_o + \phi_3) \quad (2.3\text{-}1)$$

$$A_2 = A_1 - b_{10} \sec E_2 - b_6 \tan E_2 - \{b_3 \sin(A_1 + \alpha_1) - b_4 \sin(2A_1 + \alpha_2)\} \tan E_2 \quad (2.3\text{-}2)$$

$$E_1 = E_o - c_o - c_6 \cos(n_1 E_o + \theta_1) - c_7 \cos(n_2 E_o + \theta_2) - c_8 \cos(n_3 E_o + \theta_3) \quad (2.3\text{-}3)$$

$$E_2 = E_1 - c_9 \cos E_1 - c_3 \cos(A_1 + \beta_1) - c_4 \cos(2A_1 + \beta_2) \quad (2.3\text{-}4)$$

The variables used in the above equations and in paragraphs *2.1, Azimuth Component,* and *2.2, Elevation Component,* are defined as follows:

A_c = Corrected Azimuth

E_c = Corrected Elevation

R_{est} = Estimated Range {derived from multi-point Best Estimate of Trajectory (BET)}

A_o = Measured Azimuth

$\dot{A}_e$ = Estimated Azimuth Velocity

$\ddot{A}_e$ = Estimated Azimuth Acceleration

E_o = Measured Elevation

$\dot{E}_e$ = Estimated Elevation Velocity

$\ddot{E}_e$ = Estimated Elevation Acceleration

α_1, β_1 = Mislevel Phase Angle

α_2, β_2 = Mislevel Wobble Phase Angle

ξ, η = Vertical Deflection Components

ϕ_A = Astronomic Latitude of optical system

ρ_E = Elevation Refraction Correction

$\rho_{E'}$ = Elevation Residual Refraction Correction

m_1, m_2, m_3 = Harmonics for Azimuth Nonlinearity

n_1, n_2, n_3 = Harmonics for Elevation Nonlinearity

ϕ_1, ϕ_2, ϕ_3 = Azimuth Nonlinearity Phase Angles

$\theta_1, \theta_2, \theta_3$ = Elevation Nonlinearity Phase Angles

a_i, b_i, c_i, d_i = Coefficients of Systematic Error Corrections

3. SYSTEMATIC ERROR MODEL DESCRIPTION

The following paragraphs discuss the previous error model equations in further detail. Where practical, a one-to-one correspondence has been maintained between the error terms in paragraph *2, SYSTEMATIC ERROR MODEL DEFINITION,* and those described in the following discussion.

This paragraph is again subdivided into the two error components azimuth and elevation. The individual error terms within each component are indicated with **bold lowercase** and contain three further subdivisions indicated by *italic lowercase*. In order that each error term may stand alone as an individual reference item, repetition or redundancy among error term descriptions is necessary. References for specific optical systems or further reading are noted where appropriate. Derivations of the terms are presented in paragraph *4, SYSTEMATIC ERROR MODEL DERIVATION.*

3.1 Azimuth Component

3.1.1 Zeroset (or Static Error)

This term accounts for the constant offset of the measured value from the true value caused primarily by misalignment of the zero point of the azimuth encoder axis.

Error Definition and Effects

Azimuth zeroset is the difference between true north and the mechanical azimuth encoder zero position caused by the misalignment of the azimuth encoder axis. Static error is a broader-scoped term which attempts to encompass all constant offset errors in the azimuth data due to such factors as operator alignment error, misorientation, and alignment flaws. This is a bias value which alters all of the azimuth output data by a fixed amount.

Mathematical Form

All offset error sources are combined under the following equation. For purposes of systems analysis, some ranges may elect to subdivide this error term into its individual error elements.

$$\Delta A = b_o \tag{3.1-1}$$

Measurement

Using the optical system to track the North Star (Polaris), the azimuth position is recorded at known times. This value is then compared to a computed value for the position of Polaris, and the misalignment is deduced. By recording these measurements in the normal and plunge position, the need for precise boresight alignment of the optical instrument axes is eliminated.

3.1.2 Time Delay and Velocity Servo-lag

Velocity servo-lag describes the situation in which the optical system is not pointing directly at a dynamically moving target under track due to the pedestal's inability to sufficiently adjust for a tracking error before the angular velocity of the target creates a new tracking error. Time delay indicates an actual bias in an optical tracker's timing, but the effect is identical to that of velocity servo-lag.

Error Definition and Effects

As the optical system tracks a constant angular velocity target, the servo system responds to the displacement error and continually re-positions the pedestal to reduce the position error. A target moving with constant azimuth velocity will require the pedestal to rotate at a constant azimuth velocity to overcome the position error. The constant error, due to the constant angular velocity remaining between the actual target position and the pedestal position, is called a velocity servo-lag. Velocity servo-lag is only significant for Type 1 servo systems; this error is zero for Type 2 systems.

Mathematical Form

The error constants, b_1 and b'_2, have the units of time, therefore this error coefficient will yield the appropriate angular correction when multiplied by the azimuth angular velocity.

$$\Delta A = b_1 \dot{A}_e \qquad \text{(3.1-2a) (\textit{Time Delay})}$$

$$\Delta A = b_{1'} \dot{A}_e \qquad \text{(3.1-2b) (\textit{Velocity Servo-lag})}$$

In field procedures, the velocity servo-lag measurement is commonly referred to as K_v. The relationship between b'_1 and K_v is $b'_1=1/K_v$.

Measurement

Measurement of the velocity servo-lag value is optical system specific and will not be addressed in the current version of this document. It can be noted, however, that although the procedure performed in its entirety is lengthy and time consuming, many steps need not be repeated each time the velocity servo-lag constant is determined. The system's K_v should be checked daily; this check takes approximately 15 minutes. If a truth standard (i.e., visible satellite or star with known trajectory) is available, however, the time delay (if any) may be estimated from the slope of a plot of the azimuth residuals versus azimuth rate.

3.1.3 Acceleration Servo-lag

Acceleration servo-lag describes the situation in which the optical system is not pointing directly at a dynamically moving target being tracked due to the pedestal's inability to sufficiently adjust for a tracking error before the angular acceleration of the target creates a new tracking error.

Error Definition and Effects

As the optical system tracks a constantly accelerating target, the servo system must remain on track by continually overcoming the constantly changing velocity of the target. The acceleration constant is a measure of the optical system's ability to maintain track on the accelerating target and is related to the velocity constant. This value will change with each servo bandwidth setting.

Mathematical Form

The acceleration servo-lag error constant, b_2, has the units of time squared; therefore, this error coefficient will yield the appropriate angular correction when multiplied by the azimuth angular acceleration.

$$\Delta A = b_2 \ddot{A}_e \tag{3.1-3}$$

In field procedures, the acceleration servo-lag measurement is commonly referred to as K_a. The relationship between b_2 and K_a is $b_2=1/K_a$.

Measurement

Measurement of the acceleration servo-lag value is optical system specific and will not be addressed in the current version of this document. It can be noted, however, that although the procedure performed in its entirety is lengthy and time consuming, many steps need not be repeated each time the acceleration servo-lag constant is determined. The system's K_a can be determined and recorded on an operational basis for only the bandwidths which will be used during the operation. The K_a recordings can be made within a 20-30 minute period.

3.1.4 Pedestal Mislevel and Bearing Wobble (Azimuth Axis Rollerpath)

These terms account for the total amount of tilt of the azimuth axis in reference to that of the local vertical. This error is primarily caused by mounting irregularities and thermal gradients within the pedestal. In most cases, bearing wobble does not exist but rather is due to improper location of the levels.

Error Definition and Effects

Pedestal mislevel refers to the tilt of the azimuth axis from the local vertical. Azimuth axis rollerpath error (bearing wobble) is the result of imperfect azimuth axis bearings. Pedestal mislevel and azimuth axis rollerpath errors are discussed here together, because they have a similar effect on optical system azimuth and elevation angle error and both are measured in the same procedure. These errors are characterized by the first three harmonics of the cosine function. The first harmonic represents mislevel; the remaining harmonic terms describe the azimuth axis rollerpath error. Of these, the second harmonic only is described in the error model. The effect on azimuth is a function of the tangent of the elevation angle and the sine of the azimuth angle plus a phase bias. The azimuth mislevel error becomes pronounced at the higher elevation angles due to the convergence of the azimuth lines at the zenith, but it should also be noted that erroneous mislevel values will result from improperly installed equipment or faulty measurements.

Mathematical Form

In the mislevel and wobble equations below, b_3 and b_4 represent the desired coefficients of amplitude; α_1 and α_2 represent phase angle. A_1 is the measured azimuth corrected for zeroset and encoder nonlinearity. E_2 is the measured elevation corrected for zeroset, encoder nonlinearity, droop, and the elevation mislevel component.

$$\Delta A = b_3 \sin(A_1 + \alpha_1) \tan E_2 \qquad (3.1\text{-}4a)\ (\textit{Mislevel})$$

$$\Delta A = b_4 \sin(2A_1 + \alpha_2) \tan E_2 \qquad (3.1\text{-}4b)\ (\textit{Wobble Measurement})$$

Measurement

Mislevel is generally measured by mounting a level reading device (e.g. Talyvel, inclinometer, etc.) onto the pedestal and recording readings at uniform intervals throughout a 360° turn of the pedestal. These readings are then fit to a sinusoidal curve to determine amplitude and phase. Depending upon resources, the interval may range from a maximum of 90° to a minimum of 0° (continuous); however, generally speaking, the gross motion of the pedestal precludes continuous measurements due to vibration. Mislevel may also be determined by fitting a curve to the boresight measurements of several stars throughout a 360° turn.

3.1.5 Transit Time

This term accounts for the azimuth error induced by the motion of the target as the source image travels from the target at the finite speed of light. This error is typically only significant for great distances and velocities.

Error Definition and Effects

Transit time errors arise because optical wavefronts travel at a finite speed and, therefore, cannot report the instantaneous image (hence, position) of the target. During a mission, in the time that it takes for the image to travel back to the optical system, the target will have moved a distance equal to the velocity of the target times the transit time of the image. Therefore, at the time of completion of an optical system measurement, the target has moved to a new position and a measurement error is present.

Mathematical Form

The transit time equation derives from the assumption that azimuth deltas and time intervals are sufficiently small to accurately approximate azimuth rate. The estimated azimuth velocity, $\dot{A}_e$, derives from the measured azimuth value, and b_5 represents the reciprocal of the speed of light.

$$\Delta A = b_5 R_{est} \dot{A}_e \qquad (3.1\text{-}5)$$

The use of the vacuum speed of light for trans-atmospheric purposes is a valid assumption, since the atmospheric effects are considered in the refraction error term. Correction of transit time error can be achieved in one of two ways: by applying the foregoing equation to the measured data, or by changing the time tag of the data.

Measurement

Since transit time is functionally related to the range and azimuth rate values, the amount of error will vary throughout a mission; therefore, there is no fixed transit time error which can be measured and applied for an optical system. Accuracy is only affected by the selection of a speed of light standard which is accepted by the calibration community and the method by which $\dot{A}_e$ is determined.

3.1.6 Nonorthogonality (Standards)

This term accounts for the measured azimuth error induced by the tilt of the elevation rotation axis from orthogonality with the azimuth rotation axis.

Error Definition and Effects

The elevation axis is supported by the standards (i.e., trunions). In an ideal 2-axis gimbal azimuth/elevation tracking mount, the elevation axis of rotation is orthogonal (perpendicular in 3-space) to the azimuth axis of rotation. However, due to pedestal fabrication, assembly, and tooling jig misalignments inherent in the manufacturing process, the standards will not support each end of the elevation axis at the same height above the azimuth plane. As a result, an azimuth measurement error will be present and increase as a function of elevation angle. In a dynamic situation, the optical axis will require a rotation of the azimuth platform in order to maintain track of a target which increases in elevation but maintains constant azimuth.

Mathematical Form

Through the principles of spherical trigonometry, the nonorthogonality error equation is derived to show that the effect on azimuth is proportional to the tangent of the elevation angle. An upward tilt of the right side of the elevation axis, as viewed by an observer behind the optical system, causes a positive error in the azimuth data output. The coefficient, b_6, represents the angle of nonorthogonality; E_2 represents the measured elevation angle and assumes no zeroset, droop, or mislevel errors.

$$\Delta A = b_6 \tan E_2 \tag{3.1-6}$$

This error can significantly affect the azimuth data, but it has negligible effect on elevation angle data.

Measurement

No provision exists for the physical correction of this error after initial assembly of the pedestal; therefore, it is necessary to periodically measure this error in the field. The nonorthogonality coefficient is generally a fixed value which, once established, only requires further measurement to determine seasonal fluctuations (due to thermal expansion) and changes due to long-term bearing wear.

Nonorthogonality can also be stated as the amount of non-parallelism between the elevation axis and the azimuth plane of rotation; it is with respect to this equivalent definition that the following test actually measures nonorthogonality. A Talyvel Electronic Level, capable of measuring level indications to one arc-second or better, is used to establish the azimuth plane of rotation. A second Talyvel unit is mounted on an AA Gage ULTRADEX connected to the pedestal between the elevation axis bearings. A level curve is taken at 400 mil increments of azimuth, reading both units. One data pass is taken with the elevation of the pedestal at zero, and the other data pass is taken with the pedestal in the plunge position. In going from the normal to plunge position, the elevation axis mounted level unit becomes inverted by 180° plus twice the magnitude of the nonorthogonality error. A change to the upright position is accomplished by an exact 180° rotation of the ULTRADEX, leaving the two data passes biased by twice the magnitude of the nonorthogonality. Several data passes are taken to determine precision of the measurement.

3.1.7 Encoder Nonlinearity

A precision shaft angle encoder is a device which translates the mechanical rotation of a shaft into an incremental electrical digital representation. This term accounts for inaccuracies in the azimuth data output resulting from deviations in the straight line correlation of the input shaft rotation and the incremental output electrical digital representation due to various factors such as environmental conditions, inherent system errors, loading, and misalignment effects.

Error Definition and Effects

The error produced is the difference between the encoder output and the actual azimuth axis angular position resulting from misalignment in the mechanical linkage or manufacturing defects. The error is systematic and represents a nonlinear functional change which can be represented by an n-order harmonic series. Experience has indicated that, for a direct drive encoder coupling, measured nonlinearities for the first harmonic are very small and can usually be ignored. In most cases, the second harmonic is not related to the encoder but rather is induced by the operator during the test setup. The nonlinearity of the azimuth encoder causes a variable bias to be introduced into the azimuth output data. Although the effect would be relatively small at close range, the magnitude of the error could become quite significant at long range.

The error due to encoder coupling misalignment has a complex relationship to the input angle. The three components considered are:

-- Axial translation
-- Radial translation from concentricity

-- Angle between rotational axes.

Each of these contributions to the coupling error is, in general, a function of the shaft angle position. These functions usually possess a periodicity equal to some sub-multiple of 360° but may have different average values and arbitrary phase relationships with respect to the input angles. Other error sources such as velocity, acceleration, and temperature exist but are not specifically addressed in this discussion. Some pedestals may employ older systems where a coarse and fine encoder are used. Large errors in the 16th and 32nd harmonics are commonly found in these systems.

Mathematical Form

The nonlinearity error effect causes a varying azimuth angle output bias which follows the cosine of the azimuth shaft angle change. The azimuth zeroset and collimation error measurements must be considered when encoder data is used for nonlinearity error determination purposes.

$$\left|\ \Delta A = b_7 \cos(m_1 A_o + \phi_1)\right. \qquad \text{(3.1-7a) } (m_1 \textit{ Harmonic})$$

$$\left|\ \Delta A = b_8 \cos(m_2 A_o + \phi_2)\right. \qquad \text{(3.1-7b) } (m_2 \textit{ Harmonic})$$

$$\left|\ \Delta A = b_9 \cos(m_3 A_o + \phi_3)\right. \qquad \text{(3.1-7c) } (m_3 \textit{ Harmonic})$$

In the foregoing equation, b_7, b_8, and b_9 are the coefficients representing the amplitudes of the harmonic error, while ϕ_1, ϕ_2, and ϕ_3 represent the phase angles. The variables m_1, m_2, and m_3 indicate the harmonic number; while they are indicated here as representing the first three harmonics, they may in practice represent any combination of harmonics.

Measurement

The testing of a precision angle encoder of any type should take into account all aspects of system performance as well as the interface between the encoder and the system with which it will be used. Measurement of the encoder nonlinearity is dependent upon the particular type and brand of encoding system. In general, however, the encoder output angle increment is compared against a precisely measured shaft angle increment through a turn of 360° in azimuth (via ULTRADEX, autocollimator, or similar). The recorded deviations of the encoder output from the true rotation are then modeled with the cosine series as discussed. The deviations will represent the summation of all contributing harmonics, therefore caution must be exercised when attempting to model the function.

Static accuracy or resolution is a measure of the encoder's ability to correlate an infinitesimal rotation of the shaft with the transition from one encoder quantum state to another. Encoder resolution is equal to the number of quantized positions per turn of the input shaft. It contributes an uncertainty to the system output which is a fraction of the smallest quantum, known as the Least Significant Bit (LSB), and is equal to one-half a quantum in the worst case. The quantum transition state is evidenced by the 'toggling' of the LSB from one number to the next and back again in a continuing rapid fluctuation.

3.1.8 Optical Misalignment (Collimation)

This term accounts for the measured azimuth error induced by the misalignment of the mechanical and optical axes.

Error Definition and Effects

The optical axis of the pedestal is intended to be coincident with the pedestal's mechanical axis as defined by the azimuth and elevation encoders (assuming all necessary corrections). In practice, however, there is some misalignment error due to mechanical, optical, or electrical effects. Mechanical misalignment results from non-orthogonal pedestal azimuth and elevation axes. Optical misalignment results from a non-parallel alignment of the optical and mechanical axes causing a constant bias if the optical axis is used to calibrate the electrical axis. Electrical misalignment results from an improper alignment of the positional encoders which causes an apparent shift of the optical axis from the mechanical axis. The misalignment can be decomposed into two perpendicular components: one along the elevation circle, and the other perpendicular to the plane of the elevation circle.

Mathematical Form

Through the use of spherical trigonometry, the effect of electrical misalignment on the azimuth measurement is shown to be functionally related to the secant of the elevation. The coefficient, b_{10}, represents the actual angular separation (azimuth component) of the electrical and mechanical axes.

$$\Delta A = b_{10} \sec E_2 \tag{3.1-8}$$

The elevation component of electrical misalignment is constant and therefore absorbed into the elevation zeroset coefficient.

Measurement

The azimuth error due to electrical misalignment should be determined on a pre-operational test-by-test basis. This error term is sensitive to mission polarization mode and received mission frequency. Satellite tracks are generally more desirable in determining this error, but the following example of collimation measurement will provide quick results using only a few data points.

Most static RF axis misalignment measurement procedures consist of pointing the radar electrically toward a fixed point in the normal position. Normal radar orientation is when the radar is directed toward a target with the elevation angle reading less than 90°. After recording the normal azimuth angle, the radar is plunged (elevation angle greater than 90°) and rotated in azimuth until it again electrically locks onto the same point in space. Because of the geometry of the rotations, the amount of necessary deviation from a 180° rotation is double the amount by which the RF axis is not perpendicular to the elevation axis (azimuth component). If the RF axis is perpendicular to the elevation axis, the azimuth rotation required to rotate the radar to lock on in the plunge position will be exactly 180°.

3.1.9 Vertical Deflection

This term accounts for the azimuth difference induced by the misalignment of the local gravity vector from the normal vector of the ellipsoid reference model.

Error Definition and Effects

Strictly speaking, the deflection of the vertical is not an error in the pedestal measurement. Optical system measurements must be made with respect to a coordinate system, and many systems use the astronomic vertical as an axis in their system. Systems that have their vertical axis aligned with the astronomic vertical make their measurements in an apparent or astronomic topocentric system referenced to the earth's geoid; while trajectory computations are most often performed on a mathematical ellipsoid, such as DOD WGS-84, which closely approximates the size and shape of the geoid. The ellipsoid is a mathematically defined regular surface with specific dimensions. The geoid coincides with the surface to which the oceans would conform over the entire earth if free to adjust to the combined effect of the earth's mass attraction and the centrifugal force of the earth's rotation. As a result of the uneven distribution of the earth's mass, the geoidal surface is irregular. Since the ellipsoid is a regular surface, the two will not coincide; the areas of separation between the geoid and ellipsoid are referred to as geoid undulations, geoid heights, or geoid separations.

The geoid is a surface along which the gravity potential is everywhere equal and to which the gravity vector is always perpendicular. The angle between the perpendicular to the geoid (plumb line) and the perpendicular to the ellipsoid is defined as the deflection of the vertical. The vertical deflection angle is usually resolved into a north-south component which is coincident with the local meridian and equal to the difference between astronomic and geodetic latitude; and an east-west component which is coincident with the prime vertical and proportional to the difference between astronomical and geodetic longitude. The north-south and east-west components of vertical deflection are referenced by the U.S. Geological Survey as ξ and η, respectively, with a north, south, east, or west identifier to indicate the direction in which the astronomic zenith is deflected relative to the geodetic zenith as viewed from a point in space. Thus the correction for vertical deflection is really a coordinate system transformation from the astronomic topocentric to the geodetic topocentric coordinate system.

The utility of performing this transformation is determined by processing requirements, and in some cases will lead to degradation in the data as a result of computer round-off. Typically, this transformation is made because users of the TSPI want it referenced to specific earth models such as WGS-84, or it will be combined with other instrumentation and the final trajectory estimate referenced to a specific earth model.

Mathematical Form

The equation describing vertical deflection uses the north-south and east-west components provided by the U.S. Geological Survey. The following equation provides the azimuth error as a function of azimuth and elevation; there are no coefficients to be determined. A_2 and E_2 are the adjusted azimuth and elevation angles of measurement, and ϕ_A is the astronomic latitude of the optical system.

$$\Delta A = \eta \tan \phi_A - \eta \tan E_2 \cos A_2 + \xi \tan E_2 \sin A_2 \qquad (3.1\text{-}9)$$

Confusion with the polarity of the variables of vertical deflection generally arises from the local sign convention. A review of local procedures is warranted to ensure proper use of this error term--particularly in regions of the world where vertical deflection is significantly large.

Measurement

Although measurement by each range is possible, it is generally better to use the values provided by the Defense Mapping Agency (DMA).

3.1.10 Survey

Not available at this time

3.2 Elevation Component

3.2.1 Zeroset (or Static Error)

This term accounts for the constant offset of the measured elevation angle from true caused by misalignment of the elevation encoder axis, optical collimation shift in the elevation plane, or both.

Error Definition and Effects

Elevation zeroset can represent two different types of constant offset. In the first representation, zeroset defines the mechanical offset of the encoder zero point from the true zero point. In the second representation, zeroset (or static error) defines the total offset error due to a combination of 1) the mechanical offset and 2) the optical collimation offset. The first definition is useful for encoder alignment while the second definition is useful (and necessary) for correction of measured data. The effect on the measured data of both definitions is a constant bias from the true position.

Mathematical Form

All offset error sources are combined under the following equation.

$$\Delta E = c_o \qquad (3.2\text{-}1)$$

For purposes of systems analysis, some ranges may elect to subdivide this error term into its individual error elements. In this case, the error equation may be represented as:

$$c_o = c_{\text{encoder bias}} + c_{\text{optical collimation bias}} \qquad (3.2\text{-}2)$$

Measurement

In its strictest definition, elevation zeroset is a measure of the difference between the encoder zero position and the perpendicular to the local gravity vector. The broader definition incorporates the difference produced by the optical axis collimation error along the elevation circle. The total static error is constant and is commonly determined by tracking a known truth standard (e.g. a satellite or star of known trajectory) and regressing the error from the trajectory solution.

3.2.2 Time Delay and Velocity Servo-lag

Velocity servo-lag describes the situation in which the optical system is not pointing directly at a dynamically moving target being tracked due to the pedestal's inability to sufficiently adjust for a tracking error before the angular velocity of the target creates a new tracking error. Time delay indicates an actual bias in an optical tracker's timing, but the effect is identical to that of velocity servo-lag.

Error Definition and Effects

As the optical system tracks a constant angular velocity target, the servo system responds to the displacement error and continually re-positions the pedestal to reduce the position error. A target moving with constant elevation velocity will require the pedestal to rotate at a constant elevation velocity to overcome the position error. The constant error, due to the constant angular velocity, remaining between the actual target position and the pedestal position is called a velocity servo-lag. Velocity servo-lag is only significant for Type 1 servo systems; this error is zero for Type 2 systems.

Mathematical Form

The error constants, c_1 and c'_1, have the units of time, therefore this error coefficient will yield the appropriate angular correction when multiplied by the elevation angular velocity.

$$\Delta E = c_1 \dot{E}_e \qquad \text{(3.2-3a) (\textit{Time Delay})}$$

$$\Delta E = c_{1'} \dot{E}_e \qquad \text{(3.2-3b) (\textit{Velocity Servo-lag})}$$

In field procedures, the velocity servo-lag measurement is commonly referred to as K_v. The relationship between c'_1 and K_v is $c'_1 = 1/K_v$.

Measurement

Measurement of the velocity servo-lag value is optical system specific and will not be addressed in the current version of this document. It can be noted, however, that although the procedure performed in its entirety is lengthy and time consuming, many steps need not be repeated each time the velocity servo-lag constant is determined. The system's K_v should be checked daily; this check takes approximately 15 minutes. If a truth standard (i.e., satellite or star of known trajectory) is available, however, the time delay (if any) may be estimated from the slope of a plot of the elevation residuals versus elevation rate.

3.2.3 Acceleration Servo-lag

Acceleration servo-lag describes the situation in which the optical system is not pointing directly at a dynamically moving target under track due to the pedestal's inability to sufficiently adjust for a tracking error before the angular acceleration of the target creates a new tracking error.

Error Definition and Effects

As the optical system tracks a constantly accelerating target, the servo system must remain on track by continually overcoming the constantly changing velocity of the target. The acceleration constant is a measure of the optical system's ability to maintain track on the accelerating target and is related to the velocity constant. This value will change with each servo bandwidth setting.

Mathematical Form

The acceleration servo-lag error constant, c_2, has the units of time squared; therefore this error coefficient will yield the appropriate angular correction when multiplied by the elevation angular acceleration.

$$\Delta E = c_2 \ddot{E}_e \tag{3.2-4}$$

In field procedures, the acceleration servo-lag measurement is commonly referred to as K_a. The relationship between c_2 and K_a is $c_2=1/K_a$.

Measurement

Measurement of the acceleration servo-lag value is optical system specific and will not be addressed in the current version of this document. It can be noted, however, that although the procedure performed in its entirety is lengthy and time consuming, many steps need not be repeated each time the acceleration servo-lag constant is determined. The system's K_a can be determined and recorded on an operational basis for only the bandwidths which will be used during the operation. The K_a recordings can be made within a 20-30 minute period.

3.2.4 Pedestal Mislevel and Bearing Wobble (Azimuth Axis Rollerpath)

These terms account for the total amount of tilt of the azimuth axis in reference to the local vertical. This error is primarily caused by mounting irregularities and thermal gradients within the pedestal. In most cases, bearing wobble does not exist but rather is due to improper location of the levels.

Error Definition and Effects

Pedestal mislevel refers to the tilt of the azimuth axis from the local vertical. Azimuth axis rollerpath error (bearing wobble) is the result of imperfect azimuth axis bearings. Pedestal mislevel and azimuth axis rollerpath errors are discussed here together, because they have a similar effect on optical system azimuth and elevation angle error and both are measured in the same procedure. These errors are characterized by the first three harmonics of the cosine function. The first harmonic represents mislevel; the remaining harmonic terms describe the azimuth axis rollerpath error. Of these, the second harmonic only is described in the error model. Mislevel error has a variable effect on the indicated-versus-true elevation parameter, depending upon the azimuth position of the pedestal. The error ranges between the peak-to-peak mislevel variation measured by the test; however, it should be noted that erroneous mislevel values will result from improperly installed equipment or faulty measurements.

Mathematical Form

In the mislevel and wobble equations below, c_3 and c_4 represent the desired coefficients of amplitude; β_1 and β_2 represent phase angle. A_1 is the measured azimuth corrected for zeroset and encoder nonlinearity.

$$\Delta E = c_3 \cos(A_1 + \beta_1) \quad \text{(3.2-5a) (\textit{Mislevel})}$$

$$\Delta E = c_4 \cos(2A_1 + \beta_2) \quad \text{(3.2-5b) (\textit{Wobble Measurement})}$$

Measurement

Mislevel is generally measured by mounting a level reading device (e.g. Talyvel, inclinometer, etc.) onto the pedestal and recording readings at uniform intervals throughout a 360° turn of the pedestal. These readings are then fit to a sinusoidal curve to determine amplitude and phase. Depending upon resources, the interval may range from a maximum of 90° to a minimum of 0° (continuous); however, generally speaking, the gross motion of the pedestal precludes continuous measurements due to vibration. Mislevel may also be determined by fitting a curve to the boresight measurements of several stars throughout a 360° turn.

3.2.5 Transit Time

This term accounts for the elevation error induced by the motion of the target as the source image travels from the target at the finite speed of light. This error is typically only significant for great distances and velocities.

Error Definition and Effects

Transit time errors arise because optical wavefronts travel at a finite speed and, therefore, cannot report the instantaneous position of the target. During a mission, in the time it takes for the image to travel back to the optical system, the target will have moved a distance equal to the velocity of the target times the transit time of the signal. Therefore, at the time of completion of an optical system measurement, the target has moved to a new position and a measurement error is present.

Mathematical Form

The transit time equation derives from the assumption that elevation deltas and time intervals are sufficiently small to accurately approximate elevation rate. The estimated elevation velocity derives from the measured elevation value, and c_5 represents the reciprocal of the speed of light.

$$\Delta E = c_5 R_{est} \dot{E}_e \qquad (3.2\text{-}6)$$

The use of the speed of light for trans-atmospheric purposes is a valid assumption, since the atmospheric effects are considered in the refraction error term. Correction of transit time error can be achieved in one of two ways: by applying the foregoing equation to the measured data, or by changing the time tag of the data.

Measurement

Since transit time is functionally related to the range and elevation rate values, the amount of error will vary throughout a mission; therefore, there is no fixed transit time error which can be measured and applied for an optical system. Accuracy is only affected by the selection of a speed of light standard which is accepted by the calibration community and the method by which $\dot{E}_e$ is determined.

3.2.6 Encoder Nonlinearity

A precision shaft angle encoder is a device which translates the mechanical rotation of a shaft into an incremental electrical digital representation. This term accounts for inaccuracies in the elevation data output resulting from deviations in the straight line correlation of the input shaft rotation and the incremental output electrical digital representation due to various factors such as environmental conditions, inherent system errors, loading, and misalignment effects.

Error Definition and Effects

The error produced is the difference between the encoder output and the actual elevation axis angular position resulting from misalignment in the mechanical linkage or manufacturing defects. The error is systematic and represents a nonlinear functional change which can be represented by an n-order harmonic series. Experience has indicated that, for a direct drive encoder coupling, measured nonlinearities for the first harmonic are very small and can usually be ignored. In most cases, the second harmonic is not related to the encoder but rather is induced by the operator during the test setup. The nonlinearity of the elevation encoder causes a variable bias to be introduced into the elevation output data. Although the effect would be relatively small at close range, the magnitude of the error could become quite significant at long range.

The error due to encoder coupling misalignment has a complex relationship to the input angle. The three components considered are:

-- Axial translation
-- Radial translation from concentricity

-- Angle between rotational axes.

Each of these contributions to the coupling error is, in general, a function of the shaft angle position. These functions usually possess a periodicity equal to some sub-multiple of 360° but may have different average values and arbitrary phase relationships with respect to the input angles. Other error sources such as velocity, acceleration, and temperature exist but are not specifically addressed in this discussion. Some pedestals may employ older systems where a coarse and fine encoder are used. Large errors in the 16th and 32nd harmonics are commonly found in these systems.

Mathematical Form

The nonlinearity error effect causes a varying elevation angle output bias which follows the cosine of the elevation shaft angle change. The elevation zeroset and collimation error measurements must be considered when encoder data is used for nonlinearity error determination purposes.

$$\Delta E = c_6 \cos(n_1 E_o + \theta_1) \qquad (3.2\text{-}7a)\ (n_1\ \textit{Harmonic})$$

$$\Delta E = c_7 \cos(n_2 E_o + \theta_2) \qquad (3.2\text{-}7b)\ (n_2\ \textit{Harmonic})$$

$$\Delta E = c_8 \cos(n_3 E_o + \theta_3) \qquad (3.2\text{-}7c)\ (n_3\ \textit{Harmonic})$$

In the foregoing equation, c_6, c_7, and c_8 are the coefficients representing the amplitudes of the harmonic error, while θ_1, θ_2, and θ_3 represent the phase angles. The variables n_1, n_2, and n_3 indicate the harmonic number; while they are indicated here as representing the first three harmonics, they may in practice represent any combination of harmonics.

Measurement

The testing of a precision angle encoder of any type should take into account all aspects of system performance as well as the interface between the encoder and the system with which it will be used. Measurement of the encoder nonlinearity is dependent upon the particular type and brand of encoding system. In general, however, the encoder output angle increment is compared against a precisely measured shaft angle increment through a turn of 180° in elevation (via ULTRADEX, autocollimator, or similar). The recorded deviations of the encoder output from the true rotation are then modeled with the cosine series as discussed. The deviations will represent the summation of all contributing harmonics, therefore caution must be exercised when attempting to model the function.

Static accuracy or resolution is a measure of the encoder's ability to correlate an infinitesimal rotation of the shaft with the transition from one encoder quantum state to another. Encoder resolution is equal to the number of quantized positions per turn of the input shaft. It contributes an uncertainty to the system output which is a fraction of the smallest quantum, known as the Least Significant Bit (LSB), and is equal to one-half a quantum in the worst case. The quantum transition state is evidenced by the 'toggling' of the LSB from one number to the next and back again in a continuing rapid fluctuation.

3.2.7 Optical Droop

This term accounts for the measured elevation error induced by gravitational loading on the various components of the instrument assembly (including pedestal trunions and arms, and optical system).

Error Definition and Effects

Due to the large mass of a tracking pedestal, gravitational forces will act upon it in sufficient measure to produce an elevation axis angle error that will depend upon the moment arm presented to the gravity vector. Intuitively, the moment arm is a maximum at 0° elevation and a minimum at 90° elevation. The functional relationship of the error follows the cosine of the elevation angle. The optical system components most affected by droop are the camera/lens subsystems and the pedestal arms (upon which the camera/lens subsystem is mounted).

Note that this discussion addresses center mounted optics only. The additional effects of side mounted optics are to be included in future revisions of this document.

Mathematical Form

From classical mechanics, it can be shown that the functional form of the droop effect is proportional to the cosine of the elevation angle. The coefficient, c_9, represents the maximum error value of droop (at 0° elevation), and E_1 assumes correction for zeroset and encoder nonlinearity.

$$\Delta E = c_9 \cos E_1 \qquad (3.2\text{-}8)$$

Measurement

The system droop coefficient is generally a constant term which can be applied to real-time data as well as post-flight data. Measurement of droop is best achieved by modeling the functional form in a known trajectory which spans a wide range of elevation angles. Droop measurements using a boresight tower are theoretically possible, but experience has shown this method to be unacceptable for instrumentation optics (use of satelites or visible stars is recommended, although this introduces additional error considerations).

3.2.8 Vertical Deflection

This term accounts for the azimuth difference induced by the misalignment of the local gravity vector from the normal vector of the ellipsoid reference model.

Error Definition and Effects

Strictly speaking, the deflection of the vertical is not an error in the pedestal measurement. Optical system measurements must be made with respect to a coordinate system, and many systems use the astronomic vertical as an axis in their system. Systems that have their vertical axis aligned with the astronomic vertical make their measurements in an apparent or astronomic topocentric system referenced to the earth's geoid; while trajectory computations are most often performed on a mathematical ellipsoid, such as DOD WGS-84, which closely approximates the size and shape of the geoid. The ellipsoid is a mathematically defined regular surface with specific dimensions. The geoid coincides with the surface to which the oceans would conform over the entire earth if free to adjust to the combined effect of the earth's mass attraction and the centrifugal force of the earth's rotation. As a result of the uneven distribution of the earth's mass, the geoidal surface is irregular. Since the ellipsoid is a regular surface, the two will not coincide; the areas of separation between the geoid and ellipsoid are referred to as geoid undulations, geoid heights, or geoid separations.

The geoid is a surface along which the gravity potential is everywhere equal and to which the gravity vector is always perpendicular. The angle between the perpendicular to the geoid (plumb line) and the perpendicular to the ellipsoid is defined as the deflection of the vertical. The vertical deflection angle is usually resolved into a north-south component which is coincident with the local meridian and equal to the difference between astronomic and geodetic latitude; and an east-west component which is coincident with the prime vertical and proportional to the difference between astronomical and geodetic longitude. The north-south and east-west components of vertical deflection are referenced by the U.S. Geological Survey as ξ and η, respectively, with a north, south, east, or west identifier to indicate the direction in which the astronomic zenith is deflected relative to the geodetic zenith as viewed from a point in space. Thus the correction for vertical deflection is really a coordinate system transformation from the astronomic topocentric to the geodetic topocentric coordinate system.

The utility of performing this transformation is determined by processing requirements, and in some cases will lead to degradation in the data as a result of computer round-off. Typically, this transformation is made because users of the TSPI want it referenced to specific earth models such as WGS-84, or it will be combined with other instrumentation and the final trajectory estimate referenced to a specific earth model.

Mathematical Form

The equation describing vertical deflection uses the north-south and east-west components provided by the U.S. Geological Survey. The following equation provides the elevation error as a function of azimuth; there are no coefficients to be determined. A_2 is the adjusted azimuth angle of measurement.

$$\Delta E = \eta \sin A_2 + \xi \cos A_2 \qquad (3.2\text{-}9)$$

Confusion with the polarity of the variables of vertical deflection generally arises from the local sign convention. A review of local procedures is warranted to ensure proper use of this error term--particularly in regions of the world where vertical deflection is significantly large.

Measurement

Although measurement by each range is possible, it is generally better to use the values provided by the Defense Mapping Agency (DMA).

3.2.9 Survey

Not available at this time

3.2.10 Refraction and Residual Refraction

This term accounts for the errors induced in the elevation measurement by an inappropriate or inaccurate refraction model correction.

Error Definition and Effects

Elevation errors due to refraction are the result of refractive index changes causing bending of the propagation path of electromagnetic energy. In a typical atmosphere, the refractive gradient will decrease smoothly with increasing height; however, anomalies will exist for various reasons and result in an inaccurate representation of the atmospheric characteristics.

Mathematical Form

Although presented as a constant, this variable could actually represent any number of functions designed to specifically address shortcomings in the refraction model applied to the data. The danger of such an open-ended approach is that, depending upon the functional form this variable takes, it may actually absorb errors attributable to other systematic error sources during a regression analysis routine.

$$\Delta E = \rho_E \qquad \text{(3.2-10a) } (\textit{Refraction})$$

$$\Delta E = \rho_{E'} \qquad \text{(3.2-10b) } (\textit{Residual Refraction})$$

Measurement

Refraction errors are determined through the use of sophisticated mathematical models based on inputs from the local environment. Residual refraction is intended to address errors associated with inaccurate weather condition inputs to the refraction model.

4. SYSTEMATIC ERROR MODEL DERIVATION

The following discussion provides a derivation for each of the error model terms described in the preceding paragraph. In this paragraph, however, the discussion does not follow a one-to-one correspondence with those terms in preceding paragraphs. Rather, the azimuth and elevation components of several error terms have been combined under one discussion in cases where the derivation proceeds from a common point or assumption (Paragraph 4.1). The derivation of the refraction term is complex and is left to a separate discussion (Paragraph 4.2).

Error is defined, for the purpose of this derivation, to be the difference between the measurement (or computed) value and the true value. The true value is, of course, unknowable; however, for practical applications it comes from some standard that is sufficiently more accurate than the system being calibrated. The total error is the resultant sum of all systematic error terms.

4.1 System Errors

4.1.1 Static Errors

The static error or bias is characterized by a constant offset from the true value. Sources of this error can be many -- encoder zeroset, image transit delay due to range, operator alignment, etc. As for the error model, the equations for azimuth and elevation are simply:

$$\boxed{\Delta A = \text{constant}_A} \tag{4.1-1}$$

$$\boxed{\Delta E = \text{constant}_E} \tag{4.1-2}$$

4.1.2 Servo-lag

There are two types of feedback control systems commonly found in tracking pedestals:

Type 1 System: Zero-displacement-error system. A constant reference input signal will produce a constant velocity of the controlled output.

Type 2 System: Zero-velocity-error system. A constant reference input signal will produce a constant acceleration of the controlled output.

The equation which relates the response or output function to the input function is

$$\psi_0(t) = \psi(t) - \varepsilon(t) \tag{4.1-3}$$

where

$\psi_0(t) =$ output function

$\psi(t)$ = input function
$\varepsilon(t)$ = error function

Servo-lag corrections deal with steady state error. A *Type 1* system can follow a constant velocity input with zero velocity error, but with a constant displacement error.

The displacement error is due to a lag in the servo's ability to develop the required velocity and is given by

$$\varepsilon = \psi' / K_v \tag{4.1-4}$$

where ψ' is the constant velocity input and K_v is the servo's velocity error constant. Note that for a zero velocity input there is zero displacement error.

A *Type 2* system can do better; it can follow a constant acceleration input with zero velocity and acceleration errors. Under constant velocity (zero acceleration) input, it is able to zero-out the displacement error encountered with a *Type 1* system. Under a constant, non-zero acceleration input, however, a *Type 2* system also produces a displacement error due to servo-lag. This error is given by

$$\varepsilon = \psi'' / K_a \tag{4.1-5}$$

where ψ'' is the constant acceleration input and K_a is the servo's acceleration error constant. Note that for a constant velocity (zero acceleration) input there is zero displacement error.

Software can correct for these steady state errors by using calculated values for velocity and acceleration and input values of K_v or K_a (actually, values for $1/K_v$ and $1/K_a$ are generally used as input). Errors are calculated separately for the range, azimuth, and elevation channels.

$$\Delta A = (1/K_v^A)\dot{A} \text{ or } (1/K_a^A)\ddot{A} \tag{4.1-6}$$

$$\Delta E = (1/K_v^E)\dot{E} \text{ or } (1/K_a^E)\ddot{E} \tag{4.1-7}$$

where the K_v or K_a represent the appropriate servo error constants for range, azimuth, or elevation.

4.1.3 Pedestal Mislevel

Figure 4.1-1 shows the azimuth circle in the horizontal plane and a mislevel plane. The platform (or pedestal) has been misaligned with respect to the horizontal.

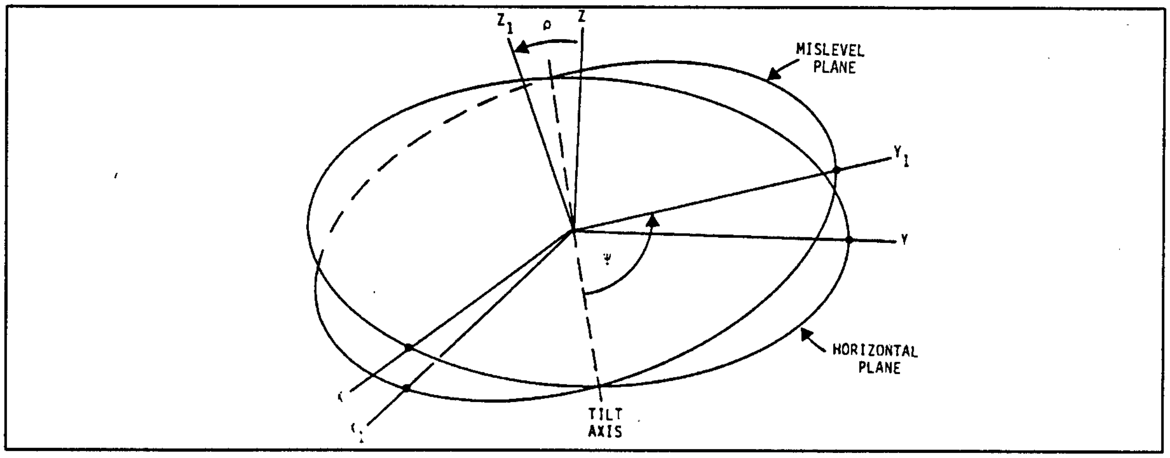

Figure 4.1-1 Mislevel Geometry

The following rotation will align the $X_1Y_1Z_1$ system with the XYZ system:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R_{Z_1}(-\psi)\,R_{Y_1}(\rho)\,R_{Z_1}(\psi)\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} \tag{4.1-8}$$

where

$$R_{Z_1}(\psi) = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4.1-9}$$

$$R_{Y_1}(\rho) = \begin{pmatrix} 1 & 0 & -\rho \\ 0 & 1 & 0 \\ \rho & 0 & 1 \end{pmatrix} \tag{4.1-10}$$

$$R_{Z_1}(-\psi) = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4.1-11}$$

Then equation 4.1-8 becomes

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X_1 - \rho Z_1 \cos\psi \\ Y_1 - \rho Z_1 \sin\psi \\ X_1 \rho\cos\psi + Y_1 \rho \sin\psi + Z_1 \end{pmatrix} \tag{4.1-12}$$

and

$$\Delta X = -\rho Z_I \cos\psi \tag{4.1-13}$$

$$\Delta Y = -\rho Z_I \sin\psi \tag{4.1-14}$$

$$\Delta Z = X_I \rho \cos\psi + Y_I \rho \sin\psi \tag{4.1-15}$$

These errors must now be transformed into ΔR, ΔA, *and* ΔE. This is done by noting that:

$$R = \sqrt{X^2 + Y^2 + Z^2} \tag{4.1-16}$$

$$A = \tan^{-1}(X/Y) \tag{4.1-17}$$

$$E = \tan^{-1}\left(\frac{Z}{\sqrt{X^2 + Y^2}}\right) \tag{4.1-18}$$

The first order differences are:

$$\Delta R = 0 \tag{4.1-19}$$

$$\Delta A = (\partial A / \partial X)\Delta X + (\partial A / \partial Y)\Delta Y \tag{4.1-20}$$

$$\Delta E = (\partial E / \partial X)\Delta X + (\partial E / \partial Y)\Delta Y + (\partial E / \partial Z)\Delta Z \tag{4.1-21}$$

Then, by substitution, we get:

$$\Delta A = \rho \sin(A + \phi) \tan E \tag{4.1-22}$$

$$\Delta E = \rho \cos(A + \phi) \tag{4.1-23}$$

where $\phi = \pi/2 - \psi$.

4.1.4 Optical Misalignment (Collimation)

The optical axis and the mechanical axis can be misaligned. There are two directions to this misalignment: one along the elevation circle, and the other perpendicular to the plane of the elevation circle. Figure 4.1-2 shows this relationship.

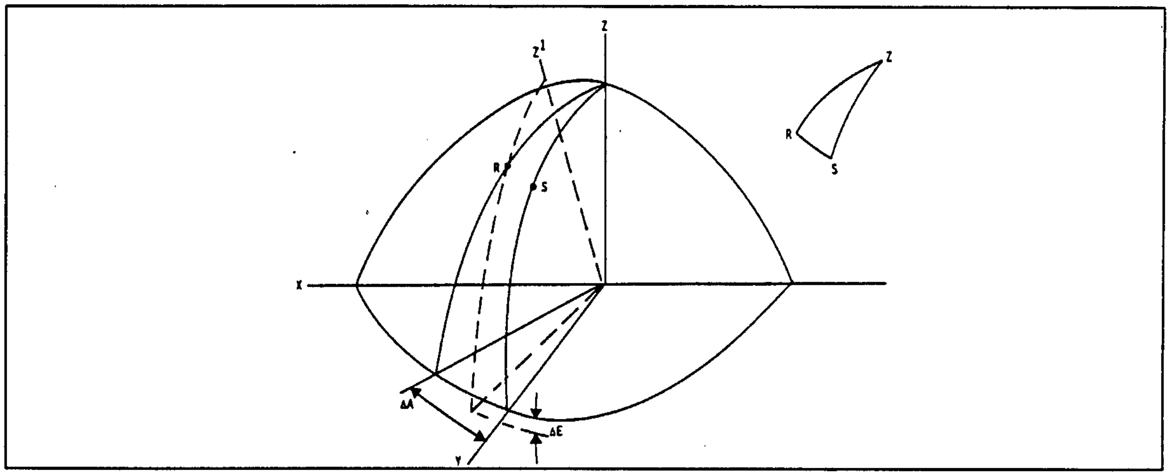

Figure 4.1-2 Optical Misalignment Geometry

From Figure 4.1-2 and the principles of spherical trigonometry, the errors are given as:

$$\tan(\Delta A) = \frac{\tan(A_M)}{\cos(E)} \approx \Delta A \qquad (4.1\text{-}24)$$

or in component form as:

$$\Delta A = A_M \sec E \qquad (4.1\text{-}25)$$

and

$$\Delta E = E_M \qquad (4.1\text{-}26)$$

which is constant and absorbed in the elevation zeroset, E_0.

4.1.5 Nonorthogonality (Standards)

The term standards is also given to this term because the elevation axis is supported by the standards (aka. trunions). If the standards are not the same height, the elevation axis will not be orthogonal to the azimuth axis. See Figure 4.1-3 for an illustration of this situation.

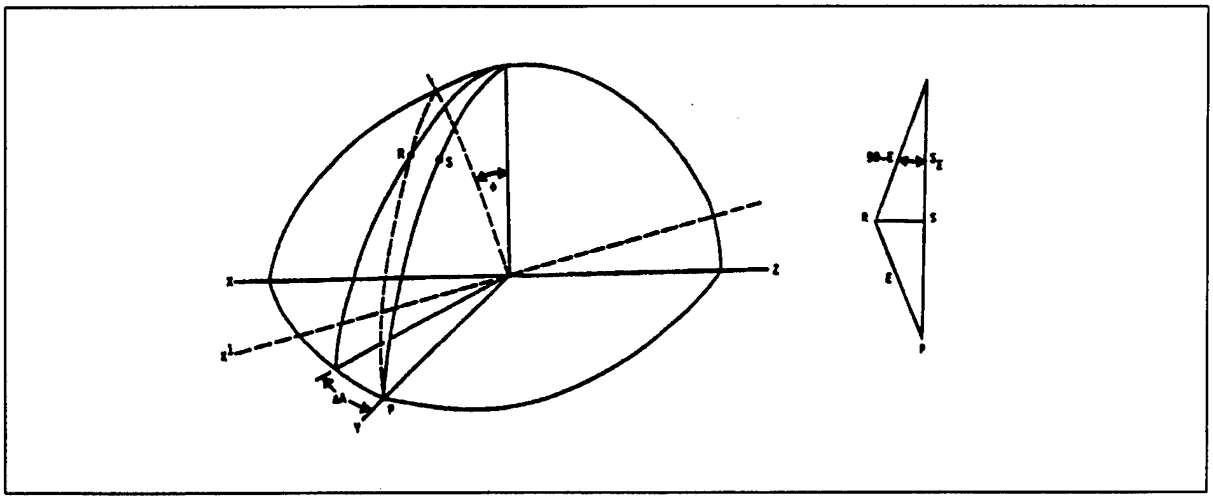

Figure 4.1-3. Nonorthogonality Geometry

From Figure 4.1-3 and the principles of spherical trigonometry, the following relationships are found to exist:

$$\tan(RS) = \tan(\phi)\sin(E) \tag{4.1-27}$$

$$\tan(RS) = \tan(S_E)\cos(E) \tag{4.1-28}$$

or by rearranging,

$$\tan(S_E) = \tan(\phi)\tan(E) \tag{4.1-29}$$

Now, let

$$K = \tan\phi \tag{4.1-30}$$

and for small angles,

$S_E = \tan S_E$. Substituting gives

$$S_E = K \tan E \tag{4.1-31}$$

$$\Delta A = K \tan E \tag{4.1-32}$$

$$\Delta E \approx 0 \tag{4.1-33}$$

4.1.6 Encoder Nonlinearity

Encoder nonlinearity is primarily due to the construction of the encoder itself. The encoder measures an angle based on the encoder's mechanical axis. When the encoder is coupled to an azimuth or elevation shaft, perfect mechanical alignment of the mechanical axis of the shaft and encoder is not possible. The problem is compounded when the angle measuring device is a multiple stage system such as a resolver. Figure 4.1-4 shows the situation.

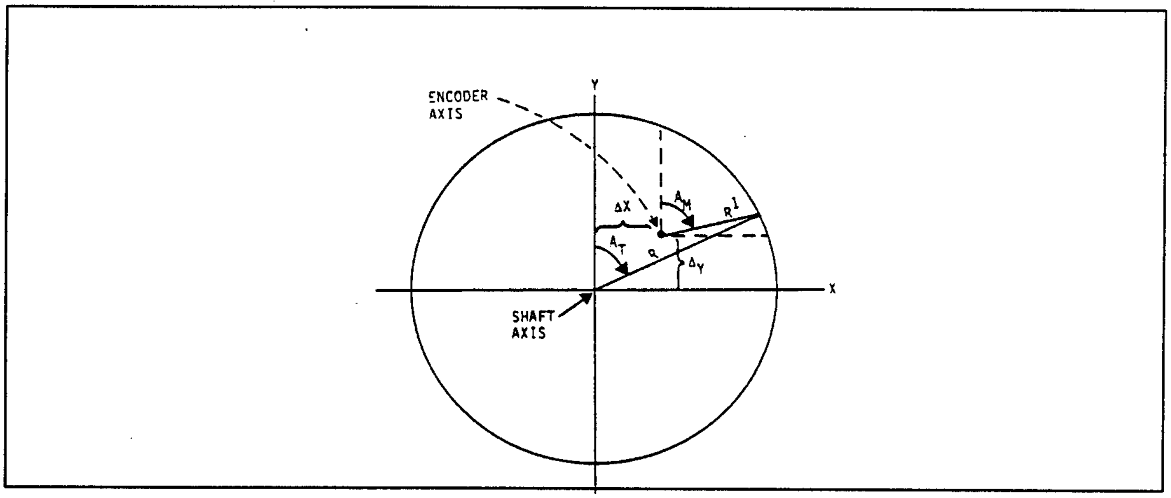

Figure 4.1-4 Encoder Nonlinearity Geometry

From figure 4.1-4 we have

$$X = R\sin A_T \ \ and \ Y = R\cos A_T \qquad (4.1\text{-}34)$$

$$R^2 = X^2 + Y^2 \ and \ A_T = \tan^{-1}(X/Y) \qquad (4.1\text{-}35)$$

Although a range term appears in the above equations, this term does not imply that range must be known to derive encoder linearity (as the range term drops out before the final solution is obtained). Linearizing equations 4.1-32 and 4.1-33 gives:

$$\Delta A_T = (\partial A_T/\partial X)\Delta X + (\partial A_T/\partial Y)\Delta Y \qquad (4.1\text{-}36)$$

$$= (Y/R^2)\Delta X - (X/R^2)\Delta Y \qquad (4.1\text{-}37)$$

$$= (Y/R)\cdot(\Delta X/R) - (X/R)\cdot(\Delta Y/R) \qquad (4.1\text{-}38)$$

Then, by substitution

$$\Delta A = (\Delta X/R)\cos A - (\Delta Y/R)\sin A \qquad (4.1\text{-}39)$$

or

$$\Delta A = A_N \sin A + B_N \cos A \qquad (4.1\text{-}40)$$

The X-Y coordinate system can be defined as $\Delta X = \Delta Y$ such that the error equations become:

$$\Delta A = A_N \cos(A + \phi) \qquad (4.1\text{-}41)$$

$$\Delta E = E_N \cos(E + \phi) \qquad (4.1\text{-}42)$$

4.1.7 Optical Droop

Due to the large mass of an optical system, gravity will act upon it to produce a deflection (droop) in elevation that will depend upon the moment arm presented to the gravity vector. Intuitively the moment arm is maximum at 0° elevation and zero at 90° elevation. Figure 4.1-5 shows this relationship.

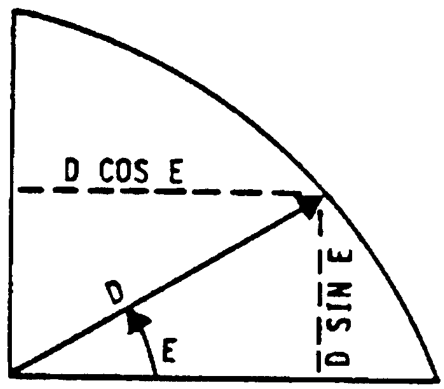

Figure 4.1-5 Droop Geometry

The derivation of droop can be simplified by thinking of the optical system assembly as a solid beam with the entire mass of the system assembly concentrated at a distance D from the center of rotation of the elevation system. It can be further assumed that no bending of the beam occurs. From classical mechanics, the deflection at the end of the beam at a distance $x=D\cos E$ is given by the following equation:

$$y = -x \frac{W}{2nA} \tag{4.1-43}$$

The mass of the system is acted upon by gravity in a downward direction only to produce a force which creates the deflection y. W constitutes this force, n is the shear modulus, and A is the cross-sectional area of the beam. The negative sign indicates the deflection to be downward.

Substituting a constant, K, which is equal to $-W/2nA$, the equation becomes,

$$\Delta E = y = K\,D\cos E \tag{4.1-44}$$

The product KD is the droop coefficient, c_9, in the error model description.

4.1.8 Vertical Deflection

Vertical deflection is a result of the fact that in geodesy the irregular shape of the earth is approximated by a mathematical surface. The irregular shape is known as the geoid, and represents the gravimetric equal potential surface. The geoid coincides with the surface to which the ocean would conform over the entire earth if free to adjust to the combined effects of the earth's mass attraction and the centrifugal forces of the earth's rotation. The mathematical surface is an ellipsoid of rotation that "best fits" the shape of the geoid. Vertical deflection results from the fact that the normals to these two surfaces are not coincident (Figure 4.1-6).

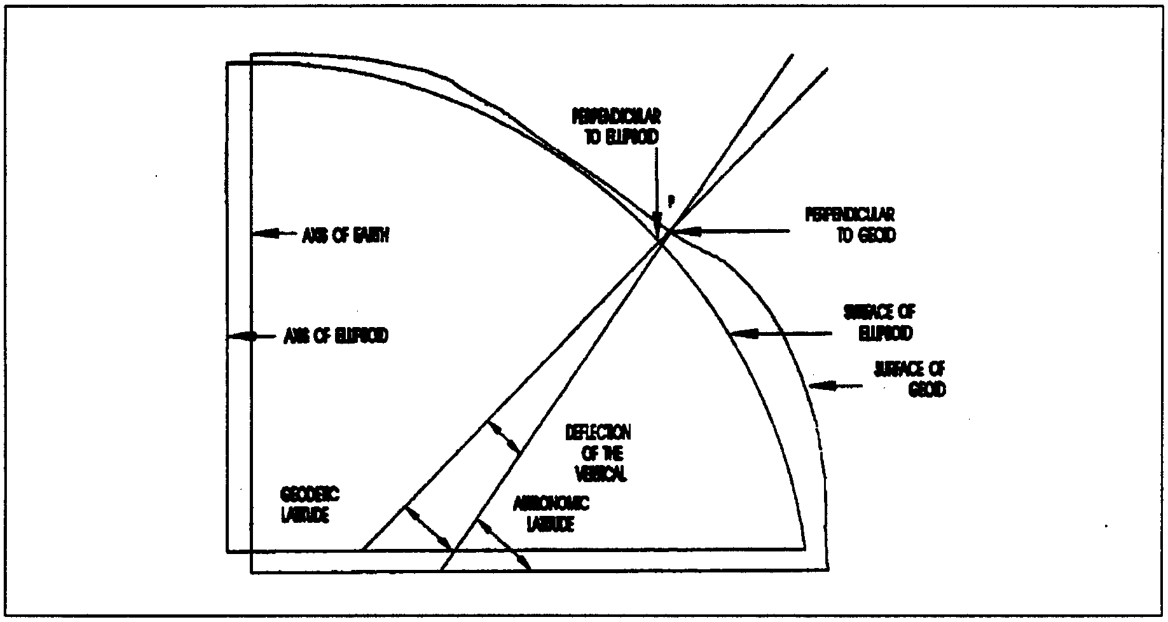

Figure 4.1-6. Vertical Deflection

Consider a point P near the surface of the earth (Figure 4.1-6). From point P there may be erected a geodetic vertical which is normal to the mathematical surface of the ellipsoid. There may also be erected a vertical which is normal to the irregular surface of the geoid at P. This vertical would be that of a *plumb line*. The angular separation of these two vertical lines is called the **vertical deflection**. As can bee seen in Figure 4.1-6 these two verticals have different latitudes and longitudes. The vertical referenced to the geoid is called the astronomic vertical and has the corresponding astronomic latitude and longitude. The vertical referenced to the ellipsoid is called the geodetic vertical and has corresponding geodetic latitude and longitude.

The deflection of the vertical at an instrument site, say point P, is defined by the deviation in the meridian,

$$\xi = (\phi_A - \phi) \tag{4.1-45}$$

by the deviation in longitude,

$$\beta = (\lambda_A - \lambda) \tag{4.1-46}$$

and by the deviation in the prime vertical,

$$\eta = \beta \cos\phi \tag{4.1-47}$$

Two rectangular coordinate systems with coincident origins both at P establish the astronomic and geodetic local level rectangular coordinate systems. For convention, let X be positive east, Y be positive north, and Z be the particular vertical in question. Then the transformation from one system to the other system is accomplished by the three axis rotation:

$$\begin{bmatrix} X_G \\ Y_G \\ Z_G \end{bmatrix} = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} \tag{4.1-48}$$

where the row subscripted with a 1 contains the direction cosines of the X_G axis in the astronomic system, the row subscripted with a 2 contains the direction cosines of the Y_G axis in the astronomic system, and the row subscripted with a 3 contains the direction cosines of the Z_G axis in the astronomic system. Using the exact definitions of these direction cosines, a rotation matrix can be defined as:

$$M = \begin{bmatrix} \cos\beta & -\sin\phi_A \sin\beta & \cos\phi_A \sin\beta \\ \sin\phi \sin\beta & \cos\xi - \sin\phi_A \sin\phi (1 - \cos\beta) & \sin\xi + \sin\phi \cos\phi_A (1 - \cos\beta) \\ -\cos\phi \sin\beta & -\sin\xi + \sin\phi_A \cos\phi (1 - \cos\beta) & \cos\xi - \cos\phi_A \cos\phi (1 - \cos\beta) \end{bmatrix} \tag{4.1-49}$$

The following relations are exact transformations between astronomic system and geodetic system for Equation 4.1.49:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_G = M \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_A \tag{4.1-50}$$

and

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_A = M^T \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_G \tag{4.1-51}$$

The typical TSPI coordinates, (R)ange, (A)zimuth, and (E)levation are related to the Cartesian coordinates X, Y, and Z as follows:

$$X = R\cos(E)\sin(A) \tag{4.1-52}$$
$$Y = R\cos(E)\cos(A) \tag{4.1-53}$$
$$Z = R\sin(E) \tag{4.1-54}$$

and

$$R = \sqrt{X^2 + Y^2 + Z^2} \tag{4.1-55}$$

$$A = \tan^{-1} \frac{X}{Y} \quad (4.1\text{-}56)$$

$$E = \tan^{-1} \frac{Z}{\sqrt{X^2 + Y^2}} \quad (4.1\text{-}57)$$

The steps in transforming TSPI in one system to another system involve first transforming range, azimuth, and elevation to X, Y, and Z; making the appropriate rotation; and then transforming the rotated X, Y, and Z back to range, azimuth, and elevation.

By making small angle assumptions, a first order approximation for the rotation matrix would be:

$$\begin{bmatrix} 1 & -\eta \tan\phi & \eta \\ \eta \tan\phi & 1 & \xi \\ -\eta & -\xi & 1 \end{bmatrix} \quad (4.1\text{-}58)$$

where ϕ is either the astronomic or geodetic latitude without further impairment of accuracy. Having made these assumptions, the vertical deflection is given by:

$$\Delta A = A_A - A_G = \eta \tan\phi - \eta \tan E \cos A + \xi \tan E \sin A \quad (4.1\text{-}59)$$

$$\Delta E = E_A - E_G = \eta \sin A + \xi \cos A \quad (4.1\text{-}60)$$

4.2 Atmospheric Errors

4.2.1 Refraction Technical Description

Due to the many refractive layers of atmosphere through which a wave front must travel, the refraction problem is complex. The refraction routines discussed below were developed for the Eastern Test Range by Gerald Trimble. Actually, two options exist for refraction corrections:

Option 1 -REEK, a completely rigorous ray trace method which solves the differential equation of a ray traveling through a spherically stratified atmosphere.

Option 2 -TRFR, a fast approximation to the refraction corrections provided by REEK (within 3%). TRFR uses REEK to build a table of refraction profiles prior to processing any data. Since TRFR is a subset of REEK, this option will not be discussed.

4.2.1.1 REEK Refraction

The subroutine REEK is designed to compute range and elevation refraction corrections in the troposphere and ionosphere for both pulse (group) or continuous wave (phase) radar systems.

There are no practical limitations on range or elevation values and the subroutine applies equally well for optical paths. The atmosphere characteristics may be supplied as an explicit profile of refractivity versus height or in terms of some reference profile plus a ground index to correct for local moisture conditions. Two different types of data input may be presented to REEK:

> Observed input of range (R) and elevation angle (ϕ_0) implies that the instrument is looking along that elevation angle (ϕ_0) and the return pulse is sensed 2R/C seconds after it is emitted. The curved path is $(R - \varepsilon_R)$ long where ε_R is the retardation refraction correction (see Figure 4.2-1).

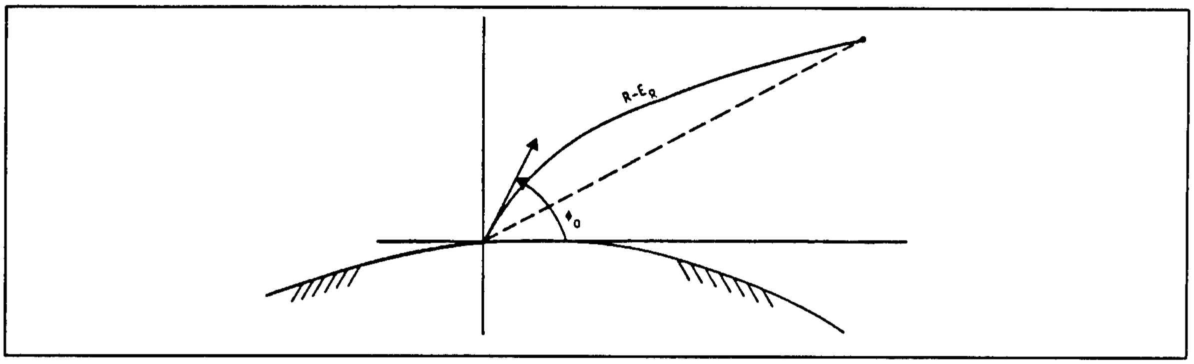

Figure 4.2-1 Geometry of Observed Input

> True input of range (R) and elevation (ϕ_0) implies that the instrument sees an object located on a straight line of elevation angle ϕ_0 and range R (see figure 4.2-2).

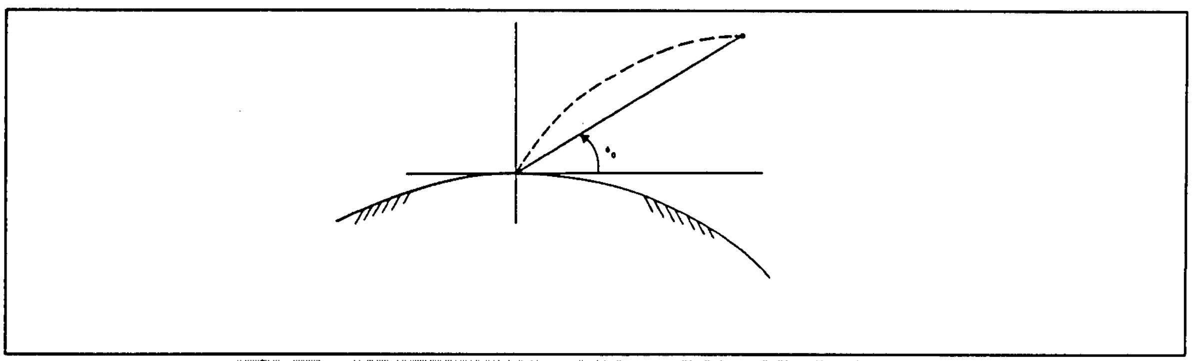

Figure 4.2-2 Geometry of True Input

The retardation correction is due to the speed difference between a ray travelling in a refractive medium and a ray travelling in a vacuum. In the troposphere, the phase refractivity values (n) are greater than 1 and profile values, which are in terms of n-1, are positive. Also, no difference exists between group and phase corrections. In the ionosphere, the phase refractivity is less than 1 and the profile terms are negative. Also, considerable differences exist between group and phase retardation corrections.

4.2.1.2 Differential Equation of a Ray (Spherically Stratified Case)

The velocity of an electromagnetic wave in a refractive medium is:

$$V = \frac{ds}{dt} = \frac{C}{n} \qquad (4.2\text{-}1)$$

where

s = distance along the path
C = wave propagation speed in a vacuum
N = phase refractivity

The components of Equation 4.2.1 expressed in a two-dimensional polar coordinate system are (see Figure 4.2-3):

$$\frac{d\theta}{dt} = \frac{C}{n} \cdot \frac{\cos\phi}{(R_e + h)} \qquad (4.2\text{-}2)$$

$$\frac{dh}{dt} = \frac{C}{n} \cdot \sin\phi \qquad (4.2\text{-}3)$$

where

θ = geocentric angle from site
h = height above the earth
ϕ = angular direction of the ray relative to local horizontal
R_e = radius of earth (assumed spherical)

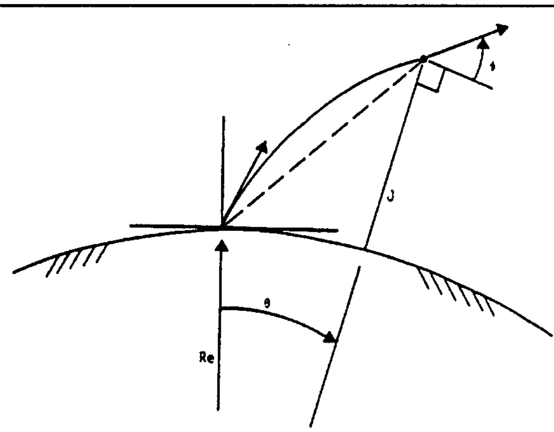

Figure 4.2-3 Definition of Terms

Rearranging Equation 4.2.1 to

$$dt = \frac{n}{C}\, ds\,,$$

Equations 4.2.2 and 4.2.3 become

$$\frac{d\theta}{ds} = \frac{\cos\phi}{(R_e + h)} \tag{4.2-4}$$

$$\frac{dh}{ds} = \sin\phi \tag{4.2-5}$$

A vector K which describes the three dimensional ray path curvature in a refractive medium is:

$$\mathrm{K} = \frac{1}{\mathrm{n}}\,(\hat{\mathrm{I}}_{\mathrm{T}} \times \nabla \mathrm{n}) \tag{4.2-6}$$

where

$\hat{I}_T$ = ray tangent unit vector
∇n = refractivity gradient
n = local refractivity scalar

The initial three dimensional ray tangent vector, assuming for convenience that it lies in the (θ, R) plane, is:

$$\mathrm{I} = \hat{\mathrm{i}}_\theta \left(\mathrm{R}\,\frac{\mathrm{d}\theta}{\mathrm{ds}}\right) + \hat{\mathrm{i}}_{\mathrm{R}} \left(\frac{\mathrm{dR}}{\mathrm{ds}}\right) + \hat{\mathrm{i}}_{\mathrm{a}}\,(0) \tag{4.2-7}$$

Substituting Equations 4.2.4 and 4.2.5 into 4.2.7, and noting that

$$\frac{dR}{ds} = \frac{dh}{ds}$$

gives:

$$\hat{I}_T = \hat{I}_\theta\,(\cos\phi) + \hat{I}_R\,(\sin\phi) + \hat{I}_a\,(0) \tag{4.2-8}$$

where

$\hat{I}_a$ = unit vector normal to R and $\hat{I}_T$
$\hat{I}_R$ = unit vector along R
$\hat{I}_\theta$ = unit vector normal to $\hat{I}_a$ and $\hat{I}_R$
R = position vector from earth center

Based on the assumption of a spherically stratified atmosphere, the refractivity gradient is:

$$\nabla n = \hat{I}_\theta\,(0) + \hat{I}_R \left(\frac{\partial n}{\partial R}\right) + \hat{I}_a\,(0) \tag{4.2-9}$$

or since

$$\frac{\partial n}{\partial R} = \frac{\partial n}{\partial h},$$

Equation 4.2.9 becomes

$$\nabla n = \hat{I}_\theta\,(0) + \hat{I}_R \left(\frac{\partial n}{\partial h}\right) + \hat{I}_a\,(0) \tag{4.2-10}$$

Combining these results, Equation 4.2.6 becomes:

$$K = \frac{1}{n}\left[\hat{I}_\theta\,(0) + \hat{I}_R\,(0) + \hat{I}_a \left(\cos\phi \frac{\partial n}{\partial h}\right)\right] \tag{4.2-11}$$

which implies that the ray path remains in the (θ, R) plane, and hence the *a* component remains zero length.

If δ is the path angle in the (θ, R) plane, referenced to the initial horizontal, then $d\delta/ds$ defines the signed curvature magnitude,

$$\frac{d\delta}{ds} = \pm|K| = \frac{1}{n}\cos\phi\,\frac{dn}{dh} \tag{4.2-12}$$

It follows that for Figure 4.2-4, the following relationships are true:

$$\frac{d\phi}{ds} = \frac{d\delta}{ds} + \frac{d\theta}{ds}$$

and

$$\phi = \delta + \theta$$

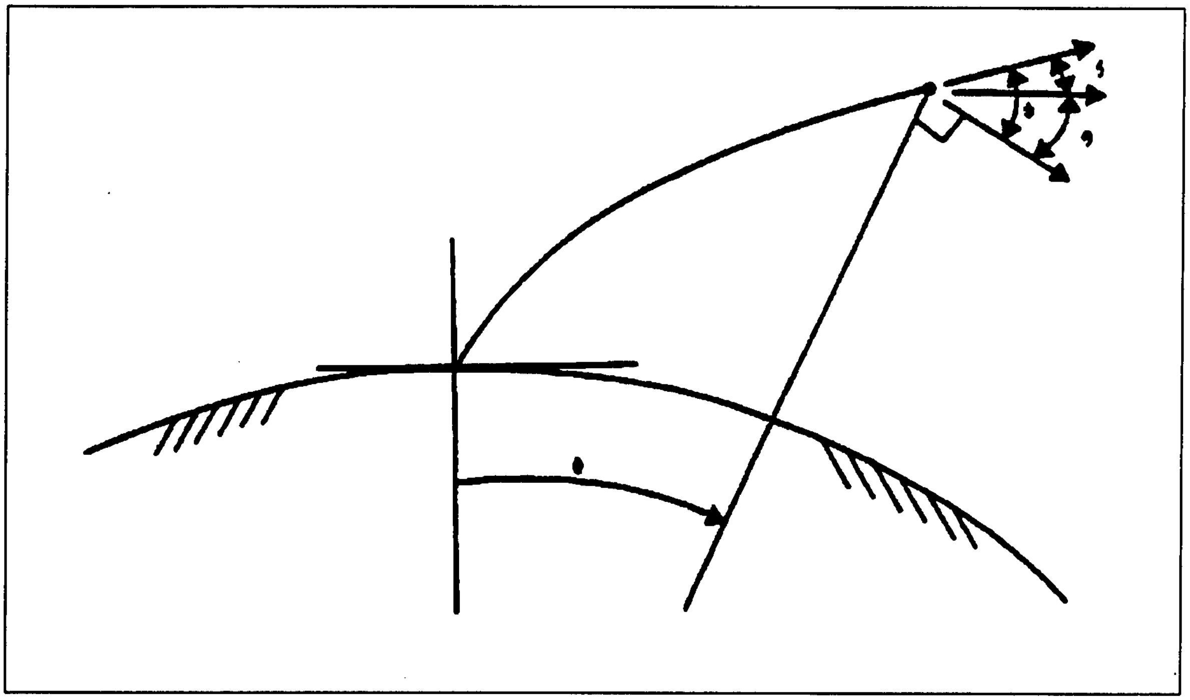

Figure 4.2-4. Relationship for Bending.

The basic differential equations of the ray are 4.2.4, 4.2.5 and 4.2.12, assuming spherical stratification of the atmosphere.

4.2.1.3 Range and Elevation Refraction Correction

An additional differential equation is derived which will accumulate only the range refraction corrections as a ray trace solution of the previous differential equations proceeds. This is done in the interest of accuracy due to the large numbers associated with range. The derivation individually includes both the bending portion and the retardation portion of the range correction.

4.2.1.3.1 Range Bending Correction

The range bending correction (ε_B) is defined as the difference in the length of the actual ray path traversed (S) and the straight line (R) connecting the end points of the ray. ε_B is expressed as (see figure 4.2-5):

$$\varepsilon_B = S - R \qquad (4.2\text{-}13)$$

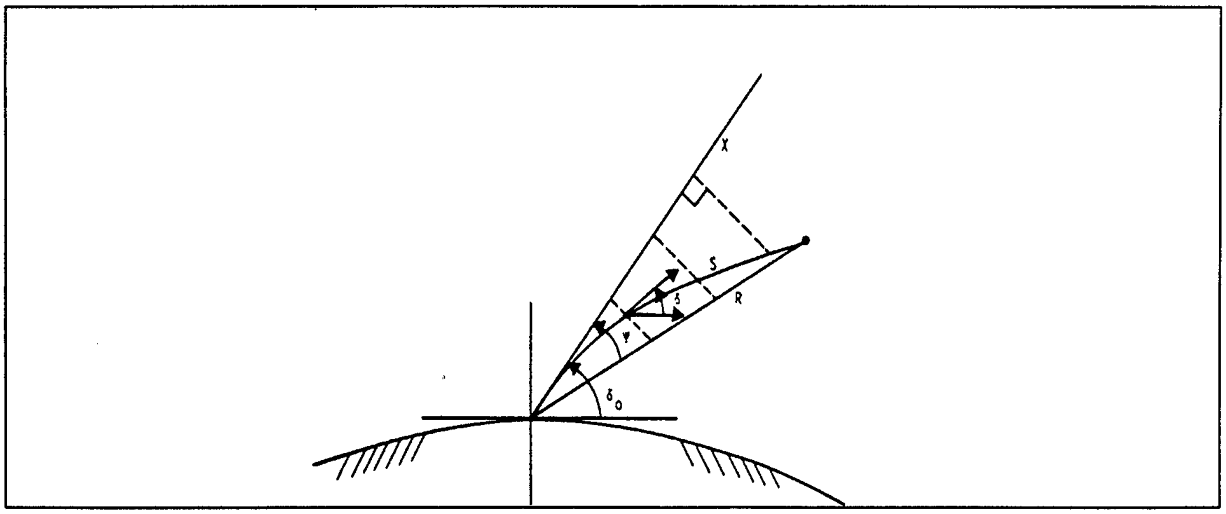

Figure 4.2-5 Derivation of Bending Error

Neither the length nor the direction of R are known a-priori to the ray trace. An arbitrary straight line (χ) is defined and the differences in length between the ray and (χ) are accumulated. Subsequent to the ray trace, this is corrected in order to account for the difference in direction (ψ) between (χ) and line R. With this in mind, the following equation will be used in place of 4.2.13:

$$\varepsilon_B = S - \chi - (R - \chi) \qquad (4.2\text{-}14)$$

where (χ) is assumed the component distance along the straight line defined by the initial direction of the ray. This choice in direction is arbitrary and picked only for convenience.

Differentiating and integrating Equation 4.2.14 with respect to distance gives:

$$\varepsilon_B = \int_s \left(1 - \frac{d\chi}{ds} \right) ds - (R - \chi) \qquad (4.2\text{-}15)$$

and since $\frac{d\chi}{ds} = \cos(\delta - \delta_o)$,

$$\varepsilon_B = \int_s [1 - \cos(\delta - \delta_o)]\, ds - (R - \chi) \qquad (4.2\text{-}16)$$

At the completion of the ray trace, the final direction angle ψ can be determined to correct for the bias due to the arbitrary choice of the χ direction.

Expressing $(R - \chi)$ as $R(1 - \cos\psi)$ and using a trigonometric identity,

$$1 - \cos\psi = 2\sin^2\left(\frac{\psi}{2}\right)$$

greater precision is maintained and Equation 4.2.16 becomes:

$$\varepsilon_B = \int_s 2\sin^2\left(\frac{\delta - \delta_o}{2}\right) ds - R \cdot 2\sin^2\left(\frac{\psi}{2}\right) \qquad (4.2\text{-}17)$$

The first term in the above equation is solved during the ray trace solution of the differential equations and the last term is a correction applied after the ray trace is complete.

4.2.1.3.2 Range Retardation Correction

The range refraction correction due to retardation (ε_R) is defined as the difference between the distance traveled by the ray in a vacuum minus the distance traveled in the atmosphere. The equation is:

$$\varepsilon_R = \int_t \left(C - \frac{C}{n} \right) dt \qquad (4.2\text{-}18)$$

We can change variables by noting $\frac{ds}{dt} = \frac{C}{n}$ to yield:

$$\varepsilon_R = \int_s (n - 1) ds \qquad (4.2\text{-}19)$$

4.2.1.3.3 Total Range Correction

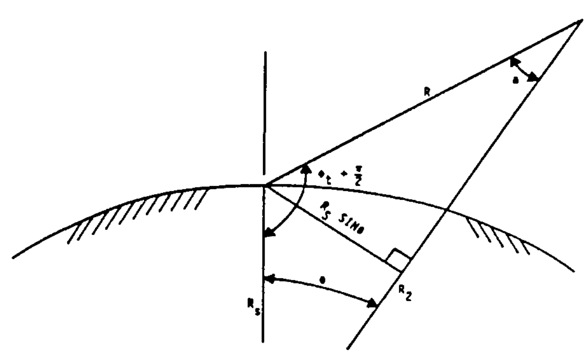

Figure 4.2-6 Geometry for Elevation Correction

The total range refraction correction is the sum of Equations 4.2.18 and 4.2.19:

$$\varepsilon_T = \varepsilon_B + \varepsilon_R \tag{4.2-20}$$

and the true range is:

$$R_T = R_O - \varepsilon_T \tag{4.2-21}$$

4.2.1.3.4 Elevation Angle Correction

Figure 4.2-6 shows the relationship which will be used to define the elevation correction.

From Figure 4.2-6, the law of cosines can be expressed as:

$$R_S^2 + 2R_2\cos(a) = R_2^2 + R^2$$

Solving for cos(a), we get:

$$\cos(a) = \frac{R_2^2 + R^2 - R_S^2}{2R_2R} \tag{4.2-22}$$

Again, using Figure 4.2-6, we see that

$$\sin(a) = \frac{R_S \sin\theta}{R} \tag{4.2-23}$$

so that a, with the proper sign, can be computed as follows:

$$a = \tan^{-1}\left(\frac{\sin(a)}{\cos(a)}\right) \tag{4.2-24}$$

and the true elevation angle ϕ_t is:

$$\phi_t = \frac{\pi}{2} - \theta - a$$

or

$$\phi_t = \frac{\pi}{2} - \theta - \tan^{-1}\left[\frac{\left(\frac{R_S\sin\theta}{R}\right)}{\left(\frac{R_2^2 + R^2 - R_S^2}{2R_2R}\right)}\right] \tag{4.2-25}$$

Then, the elevation angle correction is:

$$\Delta E = \phi_o - \phi_t \qquad (4.2\text{-}26)$$

4.2.1.4 Solution to the Differential Equations (Ray Trace)

The equations to be solved are:

$$\frac{d\theta}{ds} = \frac{\cos\phi}{(R_e + h)} \qquad (4.2\text{-}27)$$

$$\frac{dh}{ds} = \sin\phi \qquad (4.2\text{-}28)$$

$$\frac{d\delta}{ds} = \frac{1}{n}\cos(\phi)\frac{dn}{dh} \qquad (4.2\text{-}29)$$

$$\frac{d\varepsilon_B}{ds} = 2\sin^2\left(\frac{\delta - \delta_o}{2}\right) \qquad (4.2\text{-}30)$$

$$\frac{d\varepsilon_R}{ds} = n - 1 \qquad (4.2\text{-}31)$$

where

$$\phi = \delta + \theta$$

The differential equations are solved using the Runge-Kutta-Gill numerical method until the input range (S) is satisfied or until vacuum conditions are encountered ($n-1 \leq 10^{-30}$) and no additional profile exists.

4.2.2 Transit Time

Derivation Not Available At This Time

5. APPLICATION OF ERROR MODEL

The stability and reliability of the coefficients derived using the error model described in this document depend on several factors outside the scope of this material; however, for completeness, some discussion is warranted at this point. Error coefficients are classified into two separate categories: long-term and short-term. Long-term coefficients change very slowly; therefore, the frequency of measurement for these is of the order of several months. Short-term coefficients, however, can change very rapidly due to mechanical or climatic influences. Measurement of these should occur more frequently depending on the type of error. The type of error also determines the method of measurement. Some error terms are best determined using electro-optical equipment, while others are best determined with satellite or star tracks, or by some other means. The following table summarizes the methodology used for the optical systems at the Air Force Flight Test Center and is provided as an example for practical optical tracking systems calibration.

EXAMPLE OF ERROR MODEL COEFFICIENT COLLECTION METHODOLOGY
Used At AFFTC For Fixed-Site Cinetheodolites

ERROR TERM	PRIMARY METHOD	SECONDARY METHOD	FREQUENCY OF CALIBRATION
Azimuth Terms			
ZEROSET	STARS	TARGET BOARDS	BI-WEEKLY
TIME DELAY	REGRESSED IN BET	N/A	EVERY MISSION
MISLEVEL	TARGET BOARDS	STARS	EVERY MISSION
WOBBLE	TARGET BOARDS	STARS	EVERY MISSION
TRANSIT TIME	PURE MATHEMATICAL	N/A	EVERY MISSION
NON-ORTHOGONALITY	E/O CALIBRATIONS	N/A	ANNUAL
NON-LINEARITY	E/O CALIBRATIONS	N/A	SEMI-ANNUAL
OPTICAL MISALIGNMENT	STARS	TARGET BOARDS	EVERY MISSION
SURVEY	RE-SURVEY	N/A	AS NEEDED
VERTICAL DEFLECTION	SURVEY	N/A	AS NEEDED
ELEVATION TERMS			
ZEROSET	STARS	TARGET BOARDS	EVRERY MISSION
TIME DELAY	REGRESSED IN BET	N/A	EVERY MISSION
MISLEVEL	TARGET BOARDS	STARS	EVERY MISSION
WOBBLE	TARGET BOARDS	STARS	EVERY MISSION
TRANSIT TIME	PURE MATHEMATICAL	N/A	EVERY MISSION
NON-LINEARITY	E/O CALIBRATIONS	N/A	SEMI-ANNUAL
OPTICAL DROOP	STARS	TARGET BOARDS	EVERY MISSION
SURVEY	RE-SURVEY	N/A	AS NEEDED
VERTICAL DEFLECTION	SURVEY	N/A	AS NEEDED
REFRACTION	RTREF(PMTC)	REEK(ETR)	EVERY MISSION

APPENDIX A: RELATED DOCUMENTS

RANGE REFERENCE TABLE

OLD NAME	NEW NAME
EASTERN TEST RANGE	45TH SPACE WING
PACIFIC MISSILE RANGE	NAVAL AIR WARFARE CENTER-WEAPONS DIVISION
POINT MUGU	NAVAL AIR WARFARE CENTER-WEAPONS DIVISION
SAMTEC	30TH SPACE WING
WESTERN TEST RANGE	30TH SPACE WING
USAKA	KWAJALEIN MISSILE RANGE

STATIC ERROR

None Available

AZIMUTH/ELEVATION ENCODER NONLINEARITY

None Available

SERVO LAG

Procedure for Field Determination of K_V, K_A
May 1972
Pacific Missile Range

VERTICAL DEFLECTION

DMA Technical Report Geodesy for the Layman, DMA TR 80-003
December 1983
Defense Mapping Agency

J. J. O'Connor, Methods of Trajectory Mechanics, ESMC-TR-80-45
May 1981
Eastern Space and Missile Center

PEDESTAL MISLEVEL AND AZIMUTH ROLLERPATH ERROR

Pedestal Mislevel and Azimuth Rollerpath Error Measurement Procedure (using Brunson Electronic Level)
February 1970
Pacific Missile Range

Pedestal Mislevel and Azimuth Bearing Wobble Error Measurement Procedure Brunson Electronic Level Method (RADEM No. 2.2.2.1, 2.2.2.2., 2.2.2.3)
February 1970
Pacific Missile Range

TRANSIT TIME

None Available

AZIMUTH/ELEVATION AXIS NORTHOGONALITY MEASUREMENTS

None Available

DROOP

None Available

ATMOSPHERIC REFRACTION

Atmospheric Ray Tracing and Refraction Correction, Technical Publication, TP-82-01
October 1981
Pacific Missile Range

Determination of Elevation and Slant Range Errors Due to Atmospheric Refraction, Technical Note No. 3280-6, December 1962 (Revised 1964)
Pacific Missile Range

Atmospheric Refraction Correction Program, Tech. Note. No. 3430-35-68
December 1968
Pacific Missile Range

Altitude Error at 50 NMI Due to Refraction, For a Range of Atmospheric Profiles Observed at Point Mugu, Project RIMCOM, Geophysics Division, Date Unknown
Pacific Missile Range

G. D. Trimble, REEK-REEK: Spherically Stratified & Two Dimensional Profile Refraction Corrections for Range and Elevation (Technical Memorandum 5350-70-4), ETV-70-90, April 1970
Eastern Test Range

OPTICAL SYSTEM ERROR MEASUREMENT

Boresight Telescope Optical Axis Nonsymmetry Error Measurement Procedure (RADEM No. 2.3.5.1 and 2.3.5.2), December 1970
Pacific Missile Range

Boresight Telescope Optical Calibration Target Nonlevel Error Measurement Procedure (RADEM No. 2.3.5.3), December 1970
Pacific Missile Range

MISCELLANEOUS

Range Commanders Council Organization Policy Document November 1990
Range Commanders Council

REFERENCES

1. *Range Commanders Council Organization Policy Document, November 1990, Range Commanders Council*

2. *IRIG Radar Calibration Catalog, Version 2, IRIG Document #TBD, November 1992, Range Commanders Council*

3. *Derivation of Systematic Errors for WTR Radar Systems, FDEA-66-4, May 1966, Western Test Range*

4. *Radar Data Correction Program, ER-021-85, May 1985, Air Force Flight Test Center*

5. *Error Sources in Precision Trajectory Radar Calibration, IRIG Document 117-69, February 1969, Range Commanders Council*

6. *K. R. Symon, Mechanics 3rd Edition, Addison-Wesley Publishing Company, 1971*

7. *J. J. O'Connor, Methods of Trajectory Mechanics, ESMC-TR-80-45, May 1981, Eastern Space and Missile Center*

8. *DMA Technical Report, Geodesy for the Layman, DMA TR 80-003, December 1983, Defense Mapping Agency*

9. *G. D. Trimble, REEK-REEK: Spherically Stratified & Two Dimensional Profile Refraction Corrections, for Range and Elevation (Technical Memorandum 5350-70-4), ETV-70-90, April 1970, Eastern Test Range*