AFRL-SR-BL-TR-99-
0062

REPORT DOCUMENTATION PAGE

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE
October, 1998

3. REPORT TYPE AND DATES COVERED
Final Report, Feb.1, 1998 - Oct. 31, 1998

4. TITLE AND SUBTITLE
Sensor Management and Nonlinear Filtering

5. FUNDING NUMBERS
F49620-98-1-0182

6. AUTHOR(S)
Avner Friedman
Keith Kastella
Wayne Schmaedeke

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Minnesota Center for Industrial Mathematics
University of Minnesota - 514 Vincent Hall, 206 Church St.
Minneapolis, MN
Lockheed Martin Tactical Defense Systems
P.O.Box 64525, MS.U1T23, St.Paul, MN 55164-0525

8. PERFORMING ORGANIZATION REPORT NUMBER
MCIM-Progress Report

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Office of Scientific Research
110 Duncan Avenue, Suite B115
Bolling AFB, DC 20332

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Distribution Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 Words)*

This grant is supporting development of mathematical foundations for sensor management and nonlinear filtering. The accomplishments so far are in two areas: (1) The use of Interactive Multiple Model Kalman Filters (IMMKF) with a metric called discrimination gain (DG); and (2) the use of nonlinear filtering, (NLF) in the tracking of target elevation for objects flying close to a reflecting surface. In the case of IMMKF, we demonstrate, using simulated data, that IMMKF can be used to compute the information gain when multiple sensors observe a collection of maneuvering airborne targets. In the case of NLF, we demonstrate the feasibility of using NLF methods for altitude tracking in multipath.

14. SUBJECT TERMS Data Fusion, Sensor Management, Bayesian statistics, classical probability, discrimination, Kullback-Liebler information, numerical methods

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18

DTIC QUALITY INSPECTED 1

1998 Final Report for AFOSR

Sensor Management and Nonlinear Filtering Research

Grant Number: AFOSRIF4962O-98-1-0182

Avner Friedman
Minnesota Center for Industrial Mathematics
School of Mathematics
University of Minnesota
127 Vincent Hall
206 Church Street, S.E.
Minneapolis, Minnesota 55455

Wayne Schmaedeke
Keith Kastella
Lockheed Martin Tactical Defense Systems
P.O. Box 64524 MS U1T23
St. Paul, Minnesota 55164-0525

Executive Summary:

The accomplishments are in two areas, sensor management and nonlinear filtering. Sensor management can be viewed as the problem of determining how to assign sensors to targets, and how to select sensor modes and sensor search patterns to maximize their effectiveness against a set of mission requirements. Nonlinear filtering refers to the problem of recursively and optimally determining the real-time evolution of the state of a target (kinematics, position, etc.) from discretely occurring data, even though both target motion may evolve in a nonlinear fashion, and even though measurements may be nonlinearly related to target state variables.

In the area of sensor management, it was found that the problem of assignment of sensors to targets could be posed as a linear programming problem with suitable constraints. One of the problems was how to select the objective function to be optimized. One approach is to use the ideas of information theory. The advantage of information theoretic metrics is that they measure the relative utility of a wide variety of sensor measurements that affect diverse features of the problem such as detection, kinematic localization and classification. This project extends the mathematical understanding of sensor management. This includes both numerical and analytical studies of various schemes. This year's accomplishments consist of using Interactive Multiple Model Kalman Filters (IMMKF) [Watson, 1992] with a metric called Expected Discrimination Gain (EDG) to determine effectiveness of various sensor choices. The IMMKF uses several target motion models and has been successfully applied to large multisensor applications [Yed, 1997; Bis,1996]. The optimization of the sensor assignments was calculated using Linear Programming. The results were presented at the Spring SPIE Conference in Orlando Florida in April 1998, and a preprint appeared in the University of Minnesota Institute for Mathematical Analysis (IMA) Preprint Series #1555 April 1998, [Sch,1998]. To summarize, we have demonstrated that the formalism of the IMMKF can be used as a framework for computing the expected information gain when multiple sensors observe a collection of airborne targets. When combined with the IMMKF, discrimination gain is able to recognize target maneuvers and respond by allocating additional sensor resource to the maneuvering target. The sensor target assignment that optimizes the total discrimination across all of the targets in a surveillance volume can be readily computed. Application of this optimal assignment results in improved tracking performance relative to random assignment of sensors to targets or fixed round-robin scheduling.

In the area of nonlinear filtering, we had proposed investigation of a Bayes-FPE-ADI approach: That is, nonlinear filtering based on direct numerical solution of the Fokker-Planck equation, and using the Alternating Direction Implicit (ADI) technique as the numerical solution approach. The baseline for this work was a two-dimensional Bayes-FPE-ADI algorithm previously developed under LMTDS IR&D funding. Our main progress involved verification of a critique of this baseline filter by LMTDS consultant Steve Shaffer (a professor at the New Mexico Institute of Mines and Technology and an expert in the development of practical PDE solvers). To summarize, we verified that the baseline algorithm incorrectly assumes that the filtering density is negligible on the boundaries. (In more concrete terms, this amounts to assuming that signal-to-noise-ratio is sufficiently high that the location of the target is already approximately known at all times.) We also verified that the central differencing scheme used in the baseline algorithm will cause it to be inherently numerically unstable regardless of the dimensionality of the problem.

The personnel supported were Avner Friedman, Keith Kastella, and Wayne Schmaedeke. The technical publications stemming from the work are:

Keith Kastella and Aleksandar Zatazelo,"Nonlinear Filtering for Real-time Joint Tracking and Recognition", submitted to *IEEE Transactions on Aerospace and Electronic Systems.*

Keith Kastella and Aleksandar Zatazelo, "Nonlinear Filtering for Low Elevation Targets in the Presence of Multi path Propagation", *Proceedings of SPIE Aerosense 98.*

K. Kastella. "Joint Multitarget Probabilities for Detection and Tracking", *Proceedings of SPIE AeroSense '97, 2 1-25* April, 1997.

K. Kastella, M. Biscuso, W. Kober, J. K. Thomas, and A. Wood, "Event-Averaged Maximum Likelihood Estimation Tracking for Fire-Control", *Proceedings of the 29th Southeastern Symposium on System Theory.* March 9-11, 1997, pp. 440-444.

Keith Kastella, "Discrimination Gain to Optimize Detection and Classification", *IEEE Transactions on Systems, Man and Cybernetics,* Vol. 27, No. 1, Jan. 1997, pp. 112-116.

Stan Musick , Keith Kastella and Ronald Mahler, "A practical implementation of joint multitarget probabilities", *Proceedings of SPIE Aerosense 1998 SPIE Vol. 3374, pp 26-37,*

Wayne Schmaedeke and Keith Kastella,"Discrimination Gain and Multiple Model Filters for Sensor Management in Multitarget / Multisensor Tracking", *Proceedings ofSPIEAerosense '98.*

1. Objectives

This project is developing mathematical foundations for sensor management systems and extending the mathematical understanding of sensor management and nonlinear filtering. Sensor management can be viewed as the problem of determining how to asign sensors to targets, how to select sensor modes, and sensor search patterns to maximize their effectiveness against a set of mission requirements. This requires a mathematical representation of the current and future probability distributions for targets. One of the central objectives in nonlinear filtering (NLF) is to develop efficient methods to compute these probability distributions.

Until recently, one of the problems facing designers of sensor management systems is that there was little consensus on what mathematical quantities should be optimized. Information theory was suggested as a basis for computing sensor effectiveness in [Hintz, 1991]. The advantage of information theoretic metrics is that they measure the relative utility of a wide variety of sensor measurements that affect diverse features of the problem such as detection, kinematic localization and classification.

2. Status of effort

Regarding the status of Sensor Management, we were able to successfully use the Integrated Multiple Model (IMM) filter concept [Watson, 1992] along with Kalman filters (IMMKF) to track simulated airborne targets with measurements obtained from two or more agile-beam radar systems. We selected a metric called Expected Discrimination Gain (EDG) to determine effectiveness of various sensor choices. Also, we allowed each radar to have capacity constraints on the number of targets it can observe on each scan. We demonstrated that the formalism of the IMMKF can be used as a framework for computing the expected information gain when multiple sensors observe a collection of airborne targets. The optimization of the sensor assignments was calculated using Linear Programming.

Regarding the status of nonlinear filtering, we had proposed investigation of a Bayes-FPE-ADI approach: That is, nonlinear filtering based on direct numerical solution of the Fokker-Planck equation, and using the Alternating Direction Implicit (ADI) technique as the numerical solution approach. The baseline for this work was a two-dimensional Bayes-FPE-ADI algorithm previously developed under LMTDS IR&D funding. Our main progress involved verification of a critique of this baseline filter by LMTDS consultant Steve Shaffer (a professor at the New Mexico Institute of Mines and Technology and an expert in the development of practical PDE solvers). To summarize, we verified that the baseline algorithm incorrectly assumes that the filtering density is negligible on the boundaries. (In more concrete terms, this amounts to assuming that signal-to-noise-ratio is sufficiently high that the location of the target is already approximately known at all times.) We also verified that the central differencing scheme used in the baseline algorithm will cause it to be inherently numerically unstable regardless of the dimensionality of the problem.

3. **Accomplishments:** This year's accomplishments are in two areas, sensor management and nonlinear filtering.

3.1 Accomplishments in sensor management

The goal of sensor management systems can be regarded as selecting sensors and sensor dwells to increase the information that a data fusion system contains about a surveillance region. Discrimination is related to the notions of information and entropy in probability distributions [Hintz, 1991].The discrimination gain is a natural measure for use in sensor management systems. Expected discrimination gain is a measure of sensor effectiveness that has been used in a wide variety of model applications including multisensor/multitarget assignment problems [Sch95]. In the approach presented here this entails predicting the expected discrimination gain for each sensor dwell, determining the optimal global assignment of sensors to targets given sensor constraints and then applying this solution.

This proj ect extends the mathematical understanding of sensor management. This includes both numerical and analytical studies of various schemes. This year's accomplishments consist of using Interactive Multiple Model Kalman Filters (IMMKF) [Watson, 1992] with a metric called Expected Discrimination Gain (EDG) to determine effectiveness of various sensor choices. The IMMKF uses several target motion models and has been successfully applied to large multisensor applications [Yed, 1997; Bis,1996]. The optimization of the sensor assignments was calculated using Linear Programming. The results were presented at the Spring SPIE Conference in Orlando Florida in April 1998, and a preprint appeared in the University of Minnesota Institute for Mathematical Analysis (IMA) Preprint Series #1555 April 1998, [Sch,1998].

Some of our past work on sensor management for multitarget/multisensor problems developed discrimination gain for the case of a collection of targets described by conventional Kalman filters. In a generalization of this work, a method for computing the expected discrimination gain for multiple model Kalman filters was developed. To understand how expected discrimination gain can be computed in Kalman filter based tracking systems, recall that Kalman filters maintain estimates of both the target state and the state covariance. A very fortunate feature of the conventional Kalman filters is that the covariance estimate depends only on the uncertainty of the measurements, the state uncertainty, and the target motion model. but it does not depend directly on the measurements or the target state itself. On the other hand, in conventional Kalman filters, the uncertainty and the expected discrimination gain are insensitive to whether a target is maneuvering or not. This undesirable feature reduces the effectiveness of sensor management systems based on them. The IMMKF approach overcame this undesirable feature. (In an Extended Kalman Filter (EKF), the uncertainty can depend on the measurements indirectly through the linearization process, but we are not investigating EKF here.)

In [Sch, 1998], we demonstrated that the formalism of the IMMKF can be used as a framework for computing the expected information gain when multiple sensors observe a collection of airborne targets. When combined with IMMKF, this expected information gain, called discrimination gain (DG), is able to recognize target maneuvers and respond by allocating additional sensor resources to the maneuvering target(s). The sensor/target assignment that optimizes the total discrimination across all of the targets in a surveillance volume can be readily computed. Application of this optimal assignment results in improved tracking performance relative to random assignment of sensors to targets or fixed round-robin scheduling.

The natural coupling between the IMMKF and sensor management for single target applications was recognized in [Watson,1992] and exploited using measures of effectiveness based on maintaining a desired level of position error covariance. Coordination across multiple sensors is desirable in multitarget/multisensor applications. Typically, each sensor can observe a fixed number of targets on each sensing cycle. By coordinating sensing across platforms, sensing effort is not wasted by having several sensors expend all of their capacity on one target while other targets go unobserved.

The IMMKF used in obtaining our results was designed for an air traffic control application [Bis,1996]. For use in air-to-air tracking applications, different versions of the IMMKF must be developed to treat target motion on 3-dimensions, and to incorporate range-rate measurements. The IMMKF is based on a stochastic hybrid system model with a state evolution equation of the form

$$\mathbf{X}(k) = f\left[k-1, \mathbf{X}(k-1), m(k)\right] + g\left[m(k), k-1, \mathbf{X}(k-1), \mathbf{v}(k, m(k))\right],$$

where k is the time index; $\mathbf{X}(k)$ is a vector describing the state of the system at time k (e.g. position, velocity, turn-rate); f is the state transition function; g is the process noise function; $m(k)$ is the system mode during the interval prior to time k (e.g. uniform motion, or constant-rate turn modes); $\mathbf{v}(k, m(k))$ is a process noise vector. The process noise g generally depends on the mode m. For example, in straight and level flight, the process noise models the effect of small inaccuracies in guidance while for a constant rate turn, the process noise also models changes in the turn rate. The system mode m is modeled as a random event with Markov dynamics. The IMMKF used here has two modes: m_U for nearly uniform motion and m_T for a nearly constant rate turn. For example, $m_U(k-1)$ denotes the event where a target is undergoing uniform motion at time $k-1$ and $m_T(k)$ denotes the event where it is turning at time k.

To compute the expected discrimination gain using an IMMKF filter, suppose that we have a set of measurements Z made up to time t_{k-1}. The basic idea is to compute the expected discrimination between the predicted density when no observation is made and the density obtained when a particular sensor is used. For multivariate Gaussian $q_0(\mathbf{X})$ and $q_1(\mathbf{X})$ with means $\mathbf{X}_0$ and $\mathbf{X}_1$ and variances Σ_0 and Σ_1, the discrimination q_1 with respect to q_0 is:

$$\begin{aligned} L(q_1; q_0) &= \int d\mathbf{X}\, q_1(\mathbf{X}) \log\left(q_1(\mathbf{X}) / q_0(\mathbf{X})\right) \\ &= \frac{1}{2} tr\left[\Sigma_0^{-1}\left(\Sigma_1 - \Sigma_0 + (\mathbf{X}_1 - \mathbf{X}_0)(\mathbf{X}_1 - \mathbf{X}_0)^T\right)\right] - \frac{1}{2}\log\frac{|\Sigma_1|}{|\Sigma_0|} \end{aligned}$$

To evaluate the relative utility of an observation made at time t_k, first note that if no observation is made then the target state density is obtained from the IMMKF prediction equations alone.

Now if a new measurement z obtained with a particular sensor is received at time t_k, the new observation set is $Z' = \{z\} \cup Z$. The IMMKF computes the joint conditional density for a target to be in state $\mathbf{X}$ and flight mode j, $p(\mathbf{X}, j|Z) = \mu_j p(\mathbf{X}|Z)$. The discrimination of $p(\mathbf{X}, j|Z)$ with respect to $p_0(\mathbf{X}, j|Z)$ is

$$L = \sum_j \int d\mathbf{X} p(j, \mathbf{X}|Z') \ln\left(p(j, \mathbf{X}|Z') / p_0(j, \mathbf{X}|Z)\right)$$

Note that this depends on the particular observation z. For sensor management, we require the expected value $\mathrm{E}L$ of this quantity with respect to the conditional density $p(z|Z)$. This is

$$\mathrm{E}L = \sum_j \int d\mathbf{X} dz\, p(z|Z) p(j, \mathbf{X}|Z') \ln\left(p(j, \mathbf{X}|Z') / p_0(j, \mathbf{X}|Z)\right)$$

The integral dz over the new observation density is complicated. However $p(z|Z)$ is sharply peaked about the expected value of the observation for scan k, $\hat{z}^{k|k-1} = H \sum_j \mu_j^- \mathbf{X}_j^{k|k-1}$. To lowest order we may

approximate this distribution by a delta function at $\hat{z}^{k|k-1}$. Then defining $\hat{Z}' = \{\hat{z}^{k|k-1}\} \cup Z$, we can approximate

$$\mathsf{E}L \approx \sum_j \int d\mathbf{X}\, p(j,\mathbf{X}|\hat{Z}') \ln\left(p(j,\mathbf{X}|\hat{Z}') / p_0(j,\mathbf{X}|Z)\right).$$

Given the gains $\mathsf{E}L$ computed above, we must now assign sensors to targets. The sensors are indexed $s = 1,\ldots,S$ and the targets are indexed $t = 1,\ldots,T$. Each sensor has a fixed capacity τ_s. This is the maximum number of targets that can be sensed on each sensor scan. The discrimination gain when sensor s is assigned to target t is denoted G_{st}. Our objective is to maximize the total gain across all of the targets.

The optimization problem considered in this paper is one of assigning sensors to targets in such a way that the one step ahead discrimination gain of the track set is maximized. To do this, the one step ahead predicted discrimination gain G_{ij} of a track j after it is updated with covariance data from sensor i is determined for all pairs i,j. This can be formulated as a Linear Programming problem with suitable constraints. The next step in the formulation is to consider the situation where more than one sensor may be assigned to the same target. As has been previously pointed out [Nash,1977], one method is to construct pseudo sensors comprised of combinations of the basic sensors. This allows any combination of sensors to be assigned to a single target simply by considering that a single pseudo sensor has been assigned to it. The number of "sensors" is thus increased from S to $2^S - 1$. The problem is now to make an assignment of these pseudo sensors to the targets in an optimal way.

There are however constraints on this assignment. One of the most important is the maximum tracking capacity of a sensor. That is, given a specified time interval, some sensors can scan a certain volume of space and also track a specified number of targets. They cannot exceed this maximum tracking capacity in the specified time period. If this maximum tracking capacity is known for each of the basic sensors, then this must be accounted for when the pseudo sensors are assigned. Surprisingly, the original maximum tracking capacities are all that is needed.

The constraints can be handled exactly as follows. Let the basic sensors be numbered from 1 to S. Let the pseudo sensors be numbered from $S + 1$ up to $2^S - 1$. For each basic sensor n, let J(n) be the set of integers consisting of n and the integer numbers of the pseudo sensors which contain sensor n in their combination. There will be 2^{S-1} integers in each set J(n). These sets J(n) will appear in the constraints that are given for the Linear Programming formulation to our optimal assignment problem.

Example 1.

For example, with three basic sensors, $s = 3$, and $2^S - 1 = 7$. Let S1,S2,S3,S4,S5,S6, and S7 be the designations of the sensors with S4 = {S1,S2}, S5 = {S1,S3}, S6 = {S2,S3}, and S7 = {S1,S2,S3}. The integer sets J(n) then contain $2^{3-1} = 4$ integers and are then: J(1) = {1,4,5,7}, J(2) = {2,4,6,7}, J(3) = {3,5,6,7}.

We can now state the linear programming problem.

$$\text{maximize} \quad C = \sum_{i=1}^{2^S-1} \sum_{j=1}^{T} G_{ij} x_{ij}$$

subject to the constraints

$$\sum_{i=1}^{2^S-1} x_{ij} \qquad \text{for } j = 1,\ldots,T$$

$$\sum_{i \in J(n)} \sum_{j=1}^{T} x_{ij} \leq \tau_n \qquad \text{for } n = 1,\ldots,S$$

$$x_{ij} \geq 0 \quad \text{for all pairs } ij$$

and where τ_n is the maximum tracking capacity of the basic sensor n. In the LP solution, each x_{st} will be either 0 or 1. When $x_{st} = 1$, sensor s is assigned to target t.

Test Results

To quantify the efficacy of discrimination gain as a measure of sensor effectiveness we computed the RMS position error and total error covariance for three alternative sensor management schemes. These are given in Tables 1 and 2. The rows labeled "DG" use discrimination gain with the simplex optimization. The rows labeled "Fixed" use a fixed round-robin sensing schedule. There are many possible alternatives. We chose to use the schedule where sensor 1 observes target pair (1,5), then (2,6), (3,7) while sensor 2 observes targets (3,7), then (4,8) and so on. This is simple to implement and spreads the observations out among the targets. This coordinates sensing but does not adaptively compute the sensor effectiveness. For the rows labeled "Rand" each sensor selects targets at random for each scan cycle. This is the worst case approach in that it assumes that the sensing is completely uncoordinated.

In order to establish a baseline for comparison, we assumed that all of the scheduling schemes maintain central tracks updated via report fusion as soon as measurements are available. Although this may be unrealistic for some applications, it enables simple direct comparison of the three schemes.

Sensor and target parameters used to test sensor effectiveness measure.

	Tgt1	Tgt2	Tgt3	Tgt4	Tgt5	Tgt6	Tgt7	Tgt8	Avg
DG RMS (m)	85	76	83	93	76	74	73	75	80
DG Det	25	14	19	1470	14	14	454	10	252
Fixed RMS	90	76	81	110	78	76	79	84	85
Fixed Det	32	14	19	2824	19	14	525	25	434
Rand RMS	82	90	83	109	83	95	95	80	90
Rand Det	19	19	19	2293	19	32	902	19	415

Table 1 --

Test results obtained for 10 runs of 2-sensor closing target scenario. Use of discrimination gain (DG) to select the optimal sensor assignment reduces the average track error as measured by both the RMS position error and the determinant of the average covariance (highlighted column at right). DG is compared to a fixed sensor schedule and to a random sensor schedule (Rand) that simulates the situation where there is no coordination between the sensors. DG provides the most benefit against target 4 which maneuvers during the scenario.

	Tgt1	Tgt2	Tgt3	Tgt4	Tgt5	Tgt6	Tgt7	Tgt8	Avg
DG RMS (m)	77	81	80	102	87	73	80	88	84
DG Det	11	20	14	1850	25	7.6	454	24	300
Fixed RMS	91	86	85	111	78	77	85	91	89
Fixed Det	32	19	19	2824	14	11	19	32	371
Rand RMS	85	94	92	116	86	105	93	90	95
Rand Det	19	32	25	2824	19	52	792	19	472

Table 2 –

Test results obtained for 10 runs of transiting target scenario are qualitatively similar to results for closing target scenario (Table 1).

Discussion

To summarize, we have demonstrated that the formalism of the IMMKF can be used as a framework for computing the expected information gain when multiple sensors observe a collection of airborne targets. When combined with the IMMKF, discrimination gain is able to recognize target maneuvers and respond by allocating additional sensor resource to the maneuvering target. The sensor target assignment that optimizes the total discrimination across all of the targets in a surveillance volume can be readily computed. Application of this optimal assignment results in improved tracking performance relative to random assignment of sensors to targets or fixed round-robin scheduling.

The IMMKF used in this assessment was designed for an air traffic control application. For use in air-to-air tracking applications, different versions of the IMMKF must be developed to treat target motion in 3-dimensions and to incorporate range-rate measurements. Another topic that must be examined is the use of simpler filters that can still support discrimination gain evaluation. For example, the adaptive single model Kalman filter can also be used in this application but was not studied here. It may be that for some applications such as tracking low-priority targets, the computational simplicity of the single model filter outweighs its disadvantages relative to the IMMKF. This study has assumed that the target detection probability is 1 and that there is no clutter. Furthermore, it was assumed that the track is a confirmed target. However, if the track is a tentative target, then additional information is obtained when the target is observed with the sensor. These effects can be included in the discrimination gain calculations.

One might expect the improved performance for the maneuvering target to come at the expense of some loss of performance for the non-maneuvering targets. However, this is not the case. By improving sensor allocation across all of the targets, average performance is improved for both the maneuvering and non-maneuvering targets. Finally, this study has addressed sensor effectiveness independent of the tactical or strategic utility of knowledge about the targets. In order to address this aspect of the problem, discrimination must be combined with a means of assessing preferences as well.

3.2 Accomplishments in nonlinear filtering (NLF)

The problem addressed in this task is as follows: A sensor with a nonlinear measurement model $f(z\,|\,x)$ collects a data-stream $Z^k : z_0, z_1, \ldots, z_k$ from a single moving target whose state x is assumed to evolve continuously in time in a nonlinear fashion governed by a stochastic PDE (partial differential equation). The filtering (i.e., *a posteriori*) density

$$f_{k\,|\,k}(x\,|\,Z^k)$$

contains all information necessary to compute the target's state after collection of the latest datum z_k. The goal of real-time nonlinear filtering is to provide a computationally tractable means of recursively computing the filtering density $f_{k+1\,|\,k+1}(x\,|\,Z^{k+1})$ at the time of the next measurement z_{k+1} from the filtering density $f_{k\,|\,k}(x\,|\,Z^k)$

at the time of the current measurement z_k. According to Theorem 6.1 of Jazwinksi's well-known textbook on filtering theory [Jazw,1970, p. 165], this recursion can be accomplished via Bayes' rule:

$$f_{k+1|k+1}(x|Z^{k+1}) = \frac{f(z_{k+1}|x)\, f_{k+1|k}(x|Z^k)}{\int f(z_{k+1}|y) f_{k+1|k}(y|Z^k)dy}$$

where $f_{k+1|k}(x|Z^k)$ is a solution of a partial differential equation called the Fokker-Planck equation (FPE). The obvious approach is to solve the FPE through direct numerical PDE solution techniques based on grid-based, finite-difference approximations of differential operators. When Jazwinski's textbook first appeared in 1970, good numerical FPE-solver techniquess were already well-known--the Alternating Direction Implicit (ADI) method, for example, dates from the mid-1960s [Wahs, 1966]. Nevertheless, the Bayes-FPE approach was dismissed at the time as computationally intractable for real-time application. For example, in introducing their "Gaussian sum" approximate-NLF technique in 1971, Sorenson and Alspach wrote:

> "Unfortunately, although the manner in which the [*a posteriori*] density evolves with time and additional measurement data can be described in terms of differential, or difference, equations...these relations are generally very difficult to solve either in closed form or numerically, so that it is usually impossible to determine the *a posteriori* density for specific applications." [Soren, 1971, p. 465]

In fact, until recently it was thought that true NLF was computationally intractable in real time applications except under very special conditions. In 1995, however, Prof. Boris Rozovskii of the University of Southern California Center for Applied Mathematics (CAMS) introduced LMTDSE to a radical new approach called the "spectral separation scheme" (S^3). S^3 makes real-time NLF possible by separating it into two parts: a computationally demanding part that can be completed off-line (on supercomputers, if necessary) and an online part that requires roughly the same computational load as a Kalman filter. S^3 has been implemented on conventional workstations for problems involving up to nine state variables. CAMS has also applied S^3 to very-low-observable Infra-Red Search and Track (IRST) problems, using it to detect and track targets in -9 dB SNR's.

Under 1998 LMTDSE IR&D funding, CAMS has extended this particular algorithm to detect and track *multiple* very-low-observable targets from IRST data. In addition, under a joint $150K/year grant by LMTDS and the Canadian national MITACS research consortium, Prof. Michael Kouritzin of the University of Alberta is developing the Infinite-Dimensional Exact (IDEX) filter {Kour, 1998], a new real-time nonlinear filtering approach he invented in 1996 under LMTDS funding [Kour, 1996]. LMTDS has applied an early version of Prof. Kouritzin's IDEX filter to an air traffic control problem [Kas-Kour, 1996]. LMTDS is also internally developing an innovative new approximate nonlinear filter called the Fast Approximate Bayesian (FAB) filter.

Computational processing power has increased greatly in the thirty years since the Bayes-FPE and ADI techniques were introduced. For this reason, in 1996 LMTDSE decided that it would be worthwhile to re-investigate numerical FPE-solver approaches for NLF. A multigrid-based FPE solver and the S^3 filter were the main elements of a white paper submitted to Wright Labs that helped initiate the Nonlinear Filtering for Target Identification (NOFTID) solicitation. Because the multigrid FPE solver subsequently proved to be computationally unfeasible, LMTDS solicited the advice of Steve Shaffer (a professor at the New Mexico Institute of Mines and Technology and an expert in the development of practical PDE solvers). Prof. Shaffer directed our attention to the ADI approach and, in particular, to the specific pages in Strikwerda's textbook [Stri, 1989, pp. 142-153] describing its implementation. He also informed us that the development of real-time FPE-solver techniques is a basic research problem and sketched the outlines of an appropriate research program. A two-dimensional Bayes-FPE-ADI nonlinear filtering algorithm was coded under LMTDS IR&D funding. In the 97/98 continuation proposal for this year's project [Fried, 1998], this algorithm was cited as the baseline for the NLF task that was initiated this year; and Prof. Shaffer's recommendations formed the heart of the NLF research program proposed there:

"ADI is a so-called *O(N)* algorithm, which means that its computational complexity grows linearly with the size of the finite-element grid used to discretize the problem. One of the primary objectives of this proposed new research will be to extend and quantify these preliminary results...[T]he issue of boundary conditions for the class of Fokker-Planck equations that arise in NLF is not well-understood at this point and requires further study. A promising approach to the boundary value problem is to use multi-resolution methods to embed the grid for the region of interest into a larger but lower resolution grid. This approach lends itself to Fast Adaptive Composite (FAC) methods that have been developed in the context of problems such as oil field modeling. Extension of these rules to NLF appears very promising, exploiting the fact that the boundary condition on the coarse grid is simple because it occurs at a large distance from the region of interest (usually the density goes to 0 exponentially at ∞). Once the coarse grid solution is computed, it can be used to approximate an improved boundary condition for the fine-grid region of interest." [Fried, 1998]

Our main progress this year involved verification of Prof. Shaffer's critique of the baseline Bayes-FPE-ADI filter. On the one hand, Prof. Shaffer pointed out that the baseline algorithm does not take proper account of boundary conditions because it incorrectly assumes that the filtering density is negligible on the boundaries. This assumption is valid only if the filtering density can be assumed to be contained in the computational grid. (In more concrete terms, assuming that the tails of the filtering density are negligible at the boundaries amounts to assuming that signal-to-noise-ratio is sufficiently high that the location of the target is already approximately known at all times.) Otherwise, significant portions of the "tails" of the filtering density will lie outside the grid and probability mass will be, in effect, "clipped off." This results in loss of probability mass at the boundaries and subsequent corruption of the solution in the interior of the region of interest. Though this problem can be avoided by making the grid large enough to contain the entire region of interest, this would result in computational intractability. In order to avoid intractability, one must develop new adaptive composite grid methods.

On the other hand, Prof. Shaffer pointed out that the central differencing scheme used in the baseline Bayes-FPE-ADI algorithm will lead to serious numerical instability. We have verified that this also is the case. In filtering problems, the convection term of the FPE dominates the diffusion term. Under such circumstances, central differencing results in loss of probability mass not only at the boundaries but throughout the region of interest, and thus can result in negative-valued solutions to the FPE. Indeed, it is well known in the computational fluid dynamics community that central differencing of the convection term can result in numerical instabilities [Fletch, 1998, p. 296]. Central differencing of a dominant convection term is so well known an error, in fact, that it is cited as such in *Numerical Recipes in C* [Press, 1992, p. 840].

Lockheed Martin/IMA postdoctoral fellow Dr. Aleksander Zatezalo verified the predicted misbehavior of central differencing of the FPE in a simple one-dimensional model problem. He showed that the predicted difficulties (numerical instability, negative probability mass) become apparent as the target is localized and the probability density function becomes sharply peaked. He also verified that these problems can be eliminated by replacing central-differencing of the convection term with upwind-differencing. Based on Dr. Zatezalo's experiments, LMTDS believes that the "blooming" effects observed in the performance of the baseline Bayes-FPE-ADI filter result from improper (e.g. nonconservative and unstable) finite-difference approximations to the FPE. As a result, the baseline algorithm will be inherently numerically unstable regardless of the dimensionality of the problem.

Though in principle these problems could be addressed by using upwind differencing in place of central differencing, this would produce difficulties of a different nature (e.g., reduction of accuracy from second-order to first-order). More advanced differencing techniques exist which avoid the problems associated with both central differencing and upwind differencing. These concepts represent the logical direction that further work should take.

4. Personnel Supported:

Avner Friedman

Keith Kastella
Wayne Schmaedeke

5. Technical References:

[Bis, 1996] M. Biscuso and K. Kastella *Tracking Algorithms for Air Traffic Control Applications,* Air Traffic Control Quarterly, pp. 19-43, Vol. 3(1), Jan. 1996.

[Fletch, 1988] C.A.J. Fletcher (1988) *Computational Techniques for Fluid Dynamics: Fundamental and General Techniques, Vol. 1*, Springer-Verlag

[Fried, 1998] A. Friedman, K. Kastella, and W. Schmaedeke, "Sensor Management and Nonlinear Filtering," 97/98 proposal to AFOSR, July 1997

[Hintz 1991] K.J. Hintz and E.S.McVey. Multi-Process Constrained Estimation. *IEEE Transactions on Systems. Man, and Cybernetics,* Vol. 21, No. l,pp237-244, January/February 1991

[Jazw,1970] A.H. Jazwinski, 1970, *Stochastic Processes and Filtering Theory,* Academic Press, New York.

[Kas-Kour, 1996] K. Kastella, M.A. Kouritzin, and A. Zatezalo (1996) "A nonlinear filter for altitude tracking," *Proc. of Air Traffic Control Assoc.*, Oct. 1996, pp. 1-5

[Kour, 1996] M.A. Kouritzin, "On exact filters for continuous signals with discrete observations", Technical Report No. 1409, University of Minnesota Institute for Mathematics and Its Applications, May 1996.

[Kour, 1998] M.A. Kouritzin (1998) "On exact filters for continuous signals with discrete observations," *IEEE Trans. Auto. Control*, vol. 43, pp. 709-715

[Nash,1977] J. M. Nash,"Optimal allocation of tracking resources",*Proceedings IEEE Conference on Decision and Control,* pp. 1177-1180, 1977.

[Peeb,1971] P.Z. Peebles and L. Goldman, "Radar Performance with Multipath Using the Complex Angle", *IEEE Trans. OnAero. And Elec.* Sys.,Vol. AES-7, No. 1, pp. 171-178, Jan. 1971.

[Press, 1992] W.H. Press and S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery (1992) *Numerical Recipes inC: The Art of Scientific Computing, Second Edition*, Cambridge University Press

[Sch, 1998] W.S. Schmaedeke and Keith Kastella, Sensor Management Using Discrimination Gain and Interacting Multiple Model Kalman Filters, *JMA Preprint Series ~U555 April 1998.*

[Sher,1971] S.M. Sherman, "Complex Indicated Angles Applied to Universal Radar Targets and Multipath",JEEE *Trans. OnAero. And Elec. Sys.* ,Vol. AES-7, No. 1, pp. 160-170, Jan. 1971.

[Soren, 1971] H.W. Sorenson and D.L. Alspach (1971) "Recursive Bayesian Estimation Using Gaussian Sums," *Automatica*, vol. 7, pp. 465-479

[Stri, 1989] J.C. Strikwerda, *Finite Difference Schemes and Partial Differential Equations,* Chapman and Hall, New York, 1989.

[Wahs, 1966] E.L. Wahspress (1966) *Iterative Solution of Elliptic Systems and Application to the Neutron Diffusion Equations of Reactor Physics*, Prentice-Hall

[Watson, 1992] G. A. Watson and W. D. Blair, "IMM Algorithm for Tracking Targets that Maneuver Through Coordinated Turns", *Proc. SPIE Signal and Data Processing of Small Targets,* 1992, Orlando, FL, Apr. 1992, pp 236-247

[Yed,1997] M. Yeddanapudi, Y. Bar-Shalom, and K. R. Pattipati, "1MM Estimation for Multitarget-Multisensor Air Traffic Surveillance", *Proc. IEEE,* Vol *85,* No. 1 Jan, 1997, pp. 80-94.

6. Technical Publications:

Keith Kastella and Aleksandar Zatazelo,"Nonlinear Filtering for Real-time Joint Tracking and Recognition", submitted to *IEEE Transactions on Aerospace and Electronic Systems.*

Keith Kastella and Aleksandar Zatazelo, "Nonlinear Filtering for Low Elevation Targets in the Presence of Multi path Propagation", *Proceedings of SPIE Aerosense 98.*

K. Kastella. "Joint Multitarget Probabilities for Detection and Tracking", *Proceedings of SPIE AeroSense '97, 2 1-25* April, 1997.

K. Kastella, M. Biscuso, W. Kober, J. K. Thomas, and A. Wood. "Event-Averaged Maximum Likelihood Estimation Tracking for Fire-Control", *Proceedings of the 29th Southeastern Symposium on System Theory,* March 9-11, 1997, pp. 440-444.

Keith Kastella. "Discrimination Gain to Optimize Detection and Classification", *IEEE Transactions on Systems, Man and Cybernetics,* Vol. 27, No. 1, Jan. 1997, pp. 112-116.

Stan Musick , Keith Kastella and Ronald Mahler, "A practical implementation of joint multitarget probabilities", to appear, *Proceedings of SPIE Aerosense 1998 SPIE Vol. 3374, pp 26-37,*

Wayne Schmaedeke and Keith Kastella,"Discrimination Gain and Multiple Model Filters for Sensor Management in Multitarget / Multisensor Tracking". *Proceedings of SPIE Aerosense '98.*